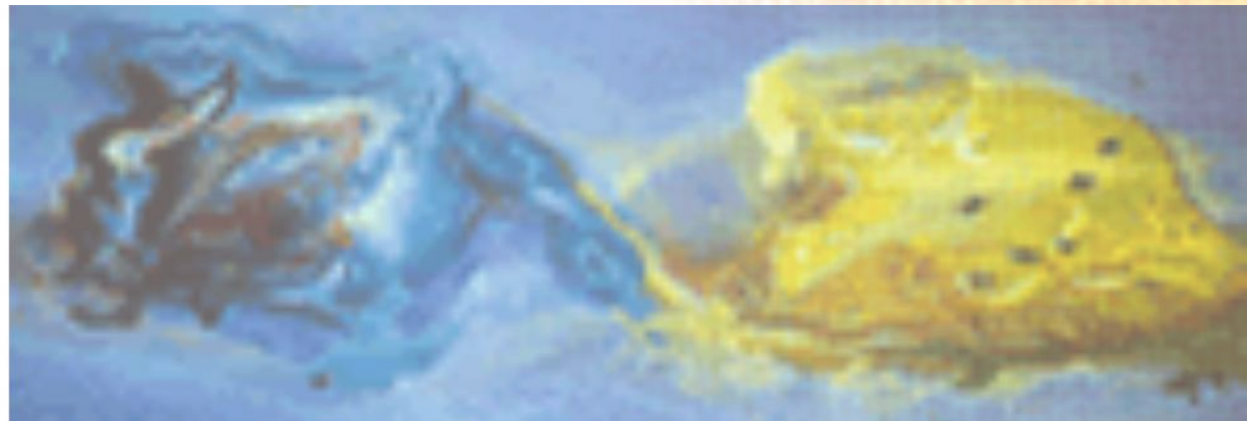


# Anomalous chiral matter: from QCD to condensed matter

**Igor Shovkovy**  
**Arizona State University**

**XIIIth Quark Confinement  
and the Hadron Spectrum,  
Maynooth University,  
Ireland, August 1-6, 2018**



- **Early Universe, e.g.,**

[Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]

- **Heavy-ion collisions, e.g.,**

[Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

- **Super-dense matter in compact stars, e.g.,**

[Yamamoto, Phys.Rev. D93, 065017 (2016)]

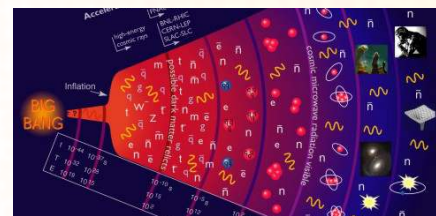
- **Ultra-relativistic jets from black holes**

- **Superfluid  $^3\text{He-A}$ , e.g.,**

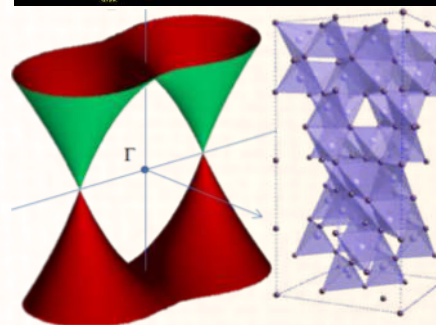
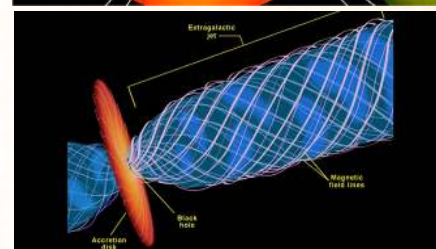
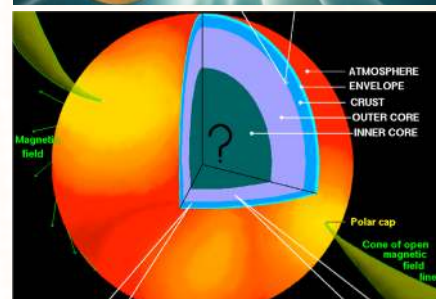
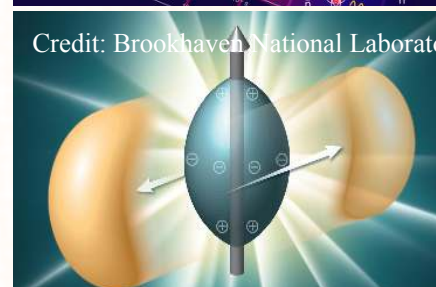
[Volovik, JETP Lett. 105, 34 (2017)]

- **Dirac/Weyl (semi-)metals, e.g.,**

[Li et. al. Nature Phys. 12, 550 (2016)]



Credit: Brookhaven National Laboratory



- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma} \cdot \vec{p}\right) \Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \Psi = \text{sign}(p_0) \gamma^5 \Psi$$

For particles ( $p_0 > 0$ ):                      chirality = helicity

For antiparticles ( $p_0 < 0$ ):                      chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime

– High temperature:                       $T \gg m$

– High density:                               $\mu \gg m$

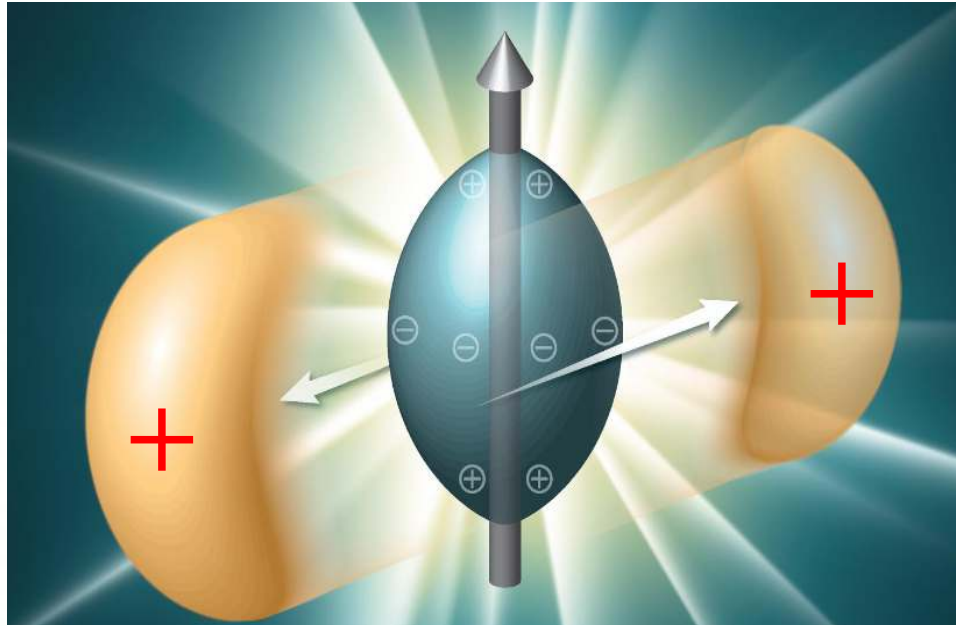
- Matter made of chiral fermions may allow  $n_L \neq n_R$
- The chiral charge ( $n_R - n_L$ ), unlike the electric charge ( $n_R + n_L$ ), is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c}$$

- The chiral anomaly can have *macroscopic* implications in chiral matter

$$B \approx 10^{18} \text{ G}$$



# ANOMALOUS EFFECTS

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]

[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]

[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

# Chiral Magnetic Effect ( $\mu_5 \neq 0$ )

- Dirac equation @  $B \neq 0$

$$\left[ i\gamma^0 \partial_0 - i\vec{\gamma} \cdot (\vec{\nabla} + ie\vec{A}) \right] \Psi = 0$$

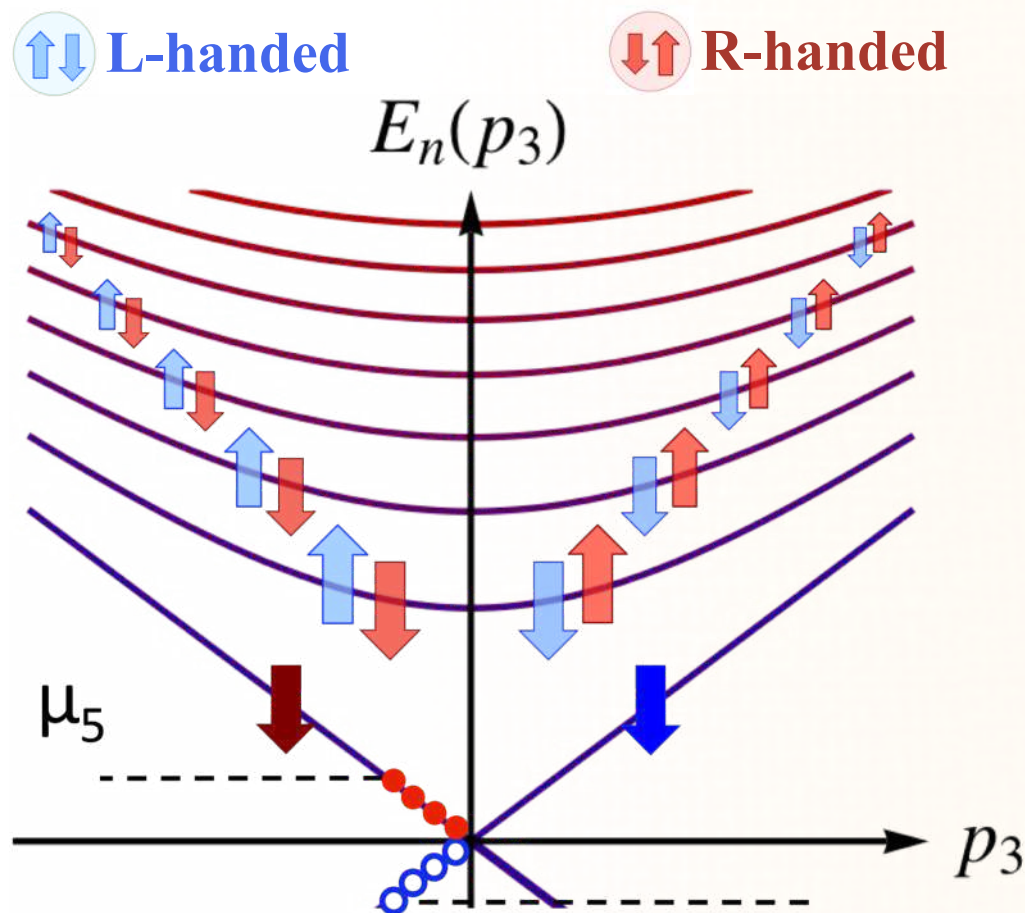
Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm \sqrt{2n|eB| + p_3^2}$$

Spin polarized LLL ( $s=\downarrow$ ):

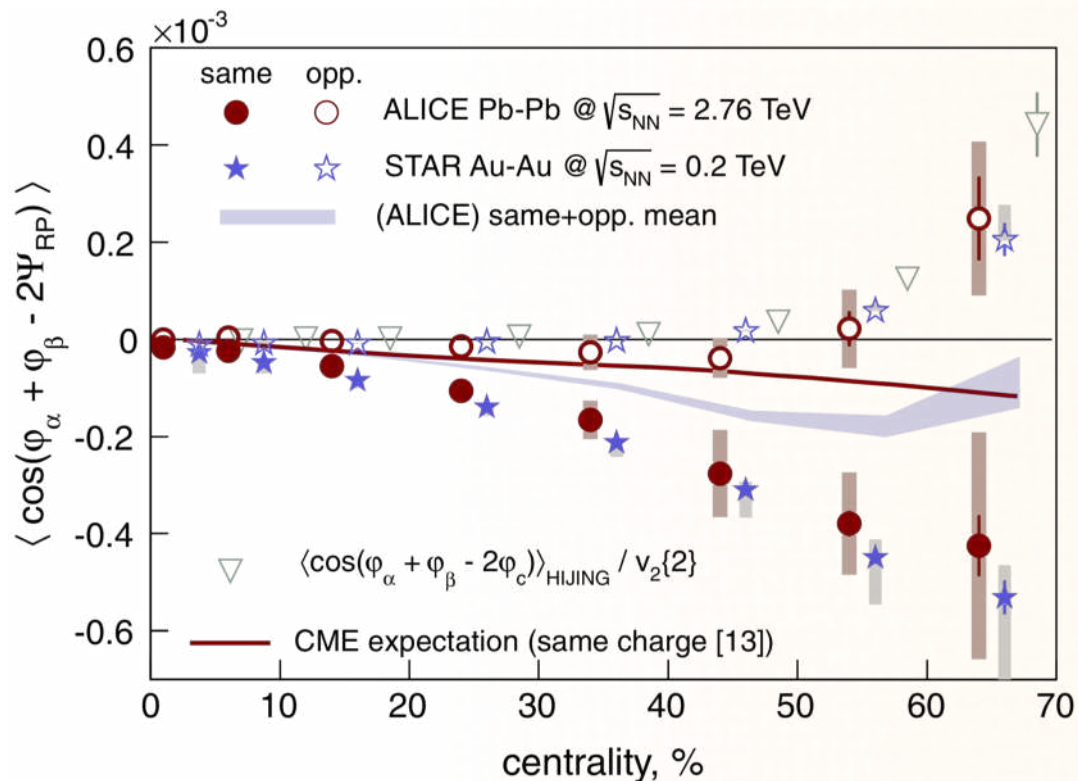
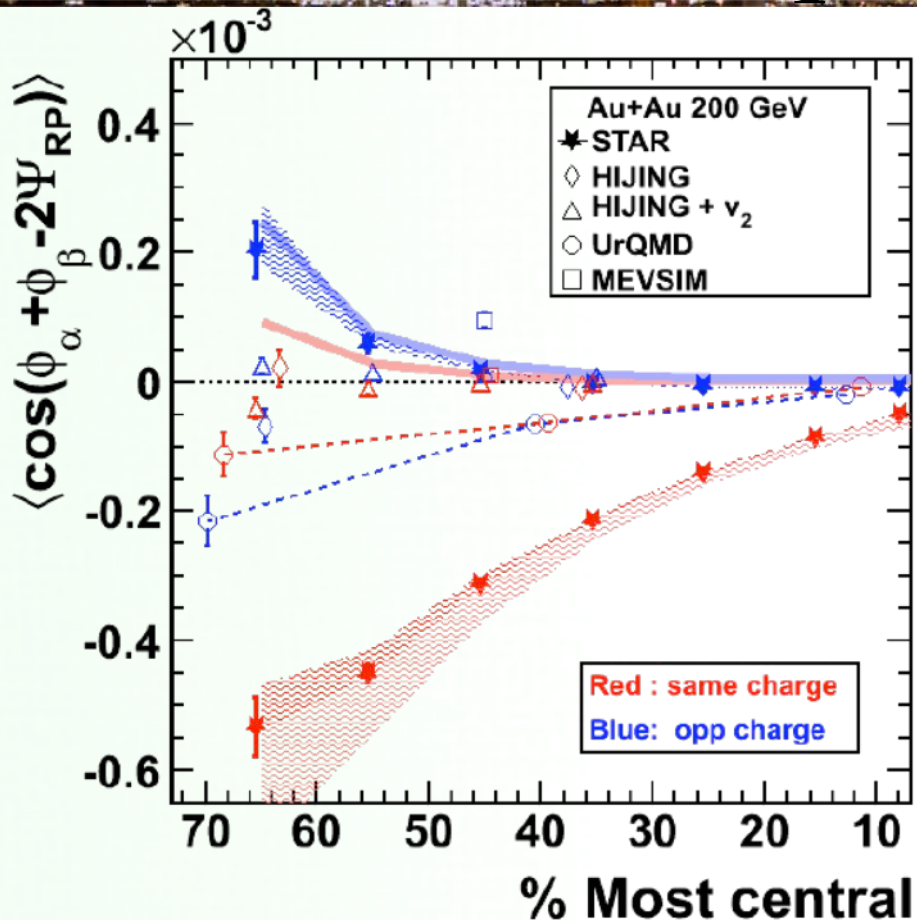
- R-handed electrons  $p_3 < 0$
- L-handed positrons  $p_3 > 0$

Topological fluctuations could induce nonzero chiral charge ( $\mu_5 \neq 0$ )



CME current: 
$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]



Correlations of same & opposite charge particles:

- [Abelev et al. (STAR), PRL **103**, 251601 (2009)]
- [Abelev et al. (STAR), PRC **81**, 054908 (2010)]
- [Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
- [Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

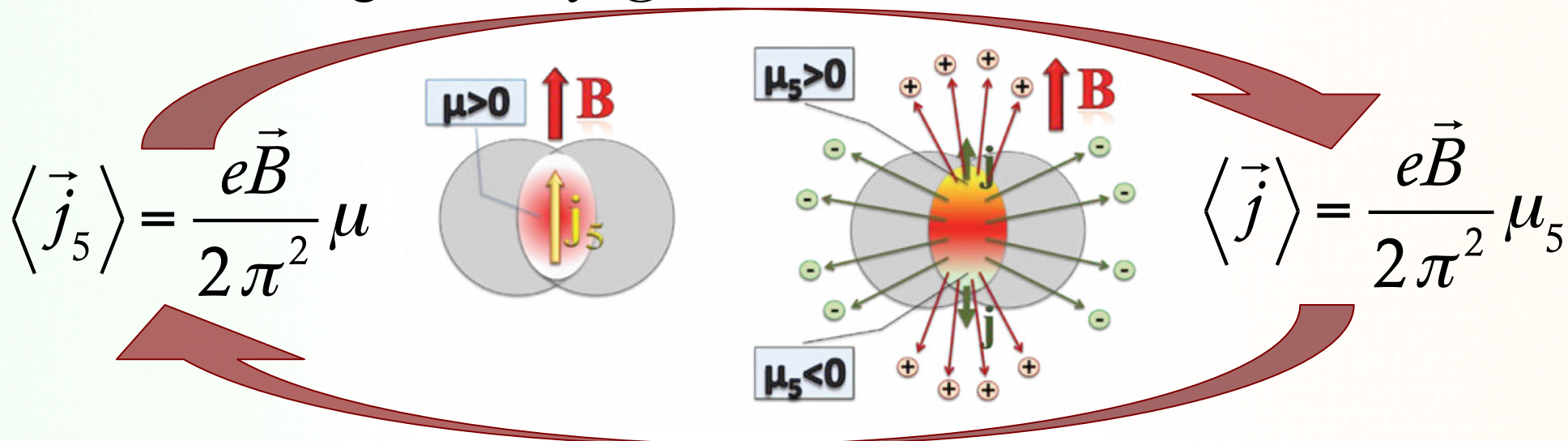
$$\left\{ \begin{array}{l} \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\mp - 2\Psi_{RP}) \rangle \\ \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}) \rangle \end{array} \right.$$

**LARGE BACKGROUND EFFECTS!**

[Belmont & Nagle, PRC **96**, 024901 (2017)], [ALICE Collaboration, Phys. Lett. B **777**, 151 (2018)]

# Chiral Magnetic Wave

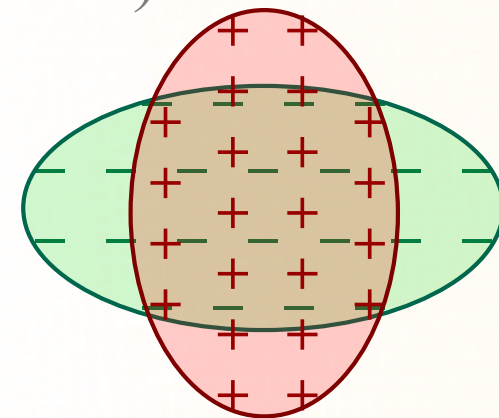
- Nonzero charge density @  $B \neq 0 \rightarrow$  CMW



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in  $\pi^+$  and  $\pi^-$ )

$$\frac{dN_{\pm}}{d\phi} \approx \bar{N}_{\pm} [1 + 2v_2 \cos(2\phi) \mp A_{\pm} r \cos(2\phi)]$$

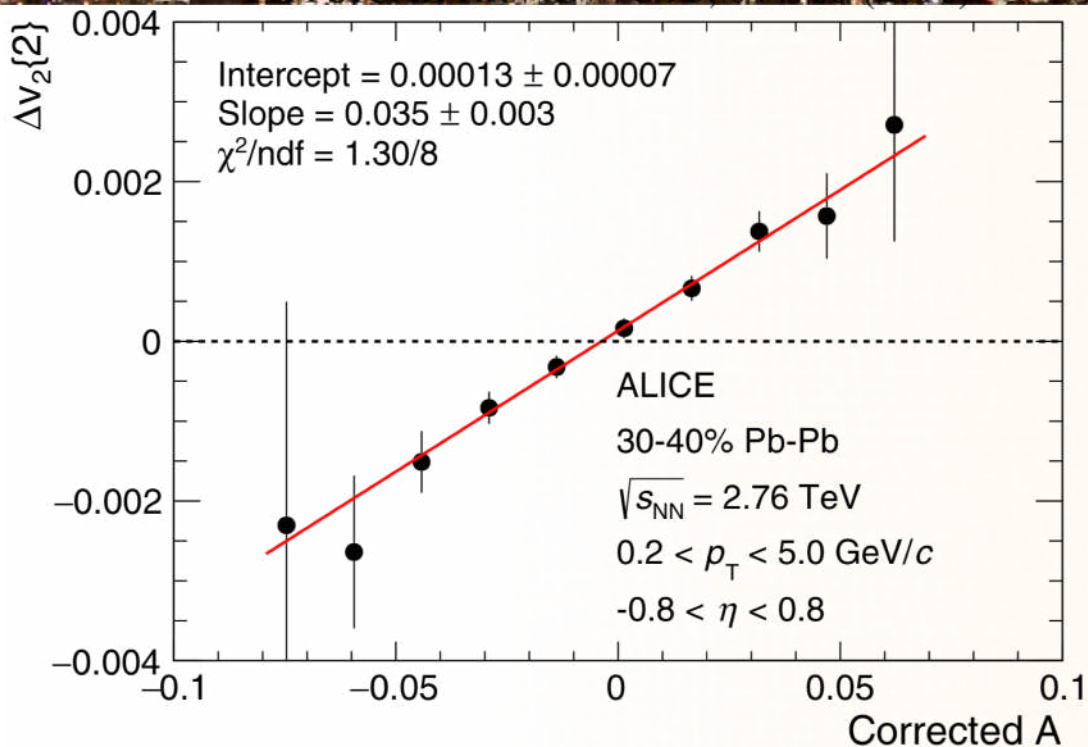
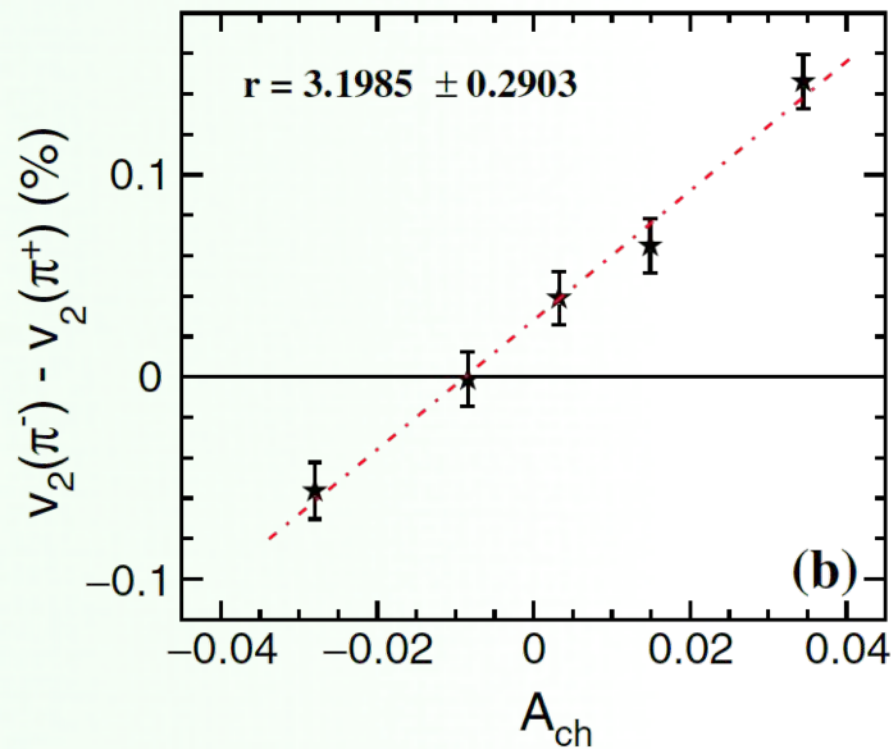


where  $A_{\pm}$  is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]



# CMW: Experimental evidence



[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

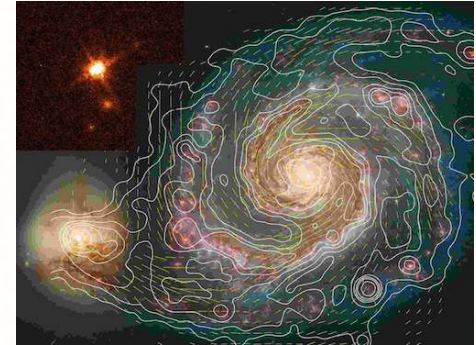
Higher harmonics of particle correlations indicate a possible strong background

**BACKGROUND EFFECTS MAY DOMINATE OVER THE SIGNAL!**

[CMS Collaboration, arXiv:1708.08901]

# Anomalous plasmas elsewhere?

- Inverse magnetic cascade may produce seeds of helical magnetic fields in the early Universe



[Vilenkin, Phys. Rev. D22, 3080 (1980)],  
 [Joyce & Shaposhnikov, astro-ph/9703005],  
 [Giovannini & Shaposhnikov, hep-ph/9710234]

- Eigenmodes of long wavelength and fixed helicity grow:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma} (4\pi C_5 \mu_5 - ck) B_k$$

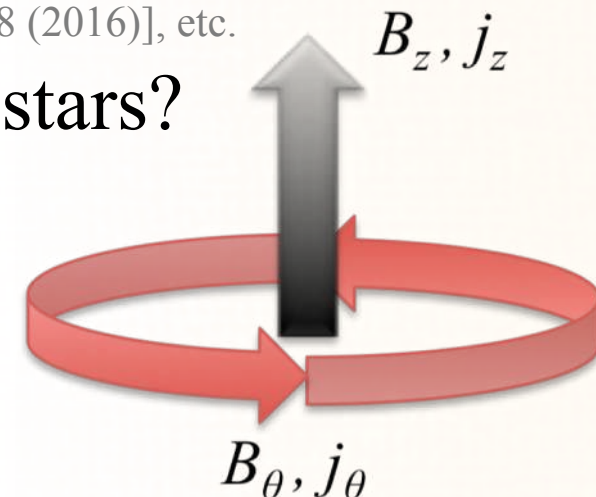
[Boyarsky et al., PRL **108**, 031301 (2012)], [Tashiro et al., PRD **86**, 105033 (2012)],  
 [Manuel et al., PRD **92**, 074018 (2015)], [Hirono et al., PRD **92**, 125031 (2015)],  
 [Buividovich et al., PRD **94**, 025009 (2016)], [Gorbar et al., PRD **94**, 103528 (2016)], etc.

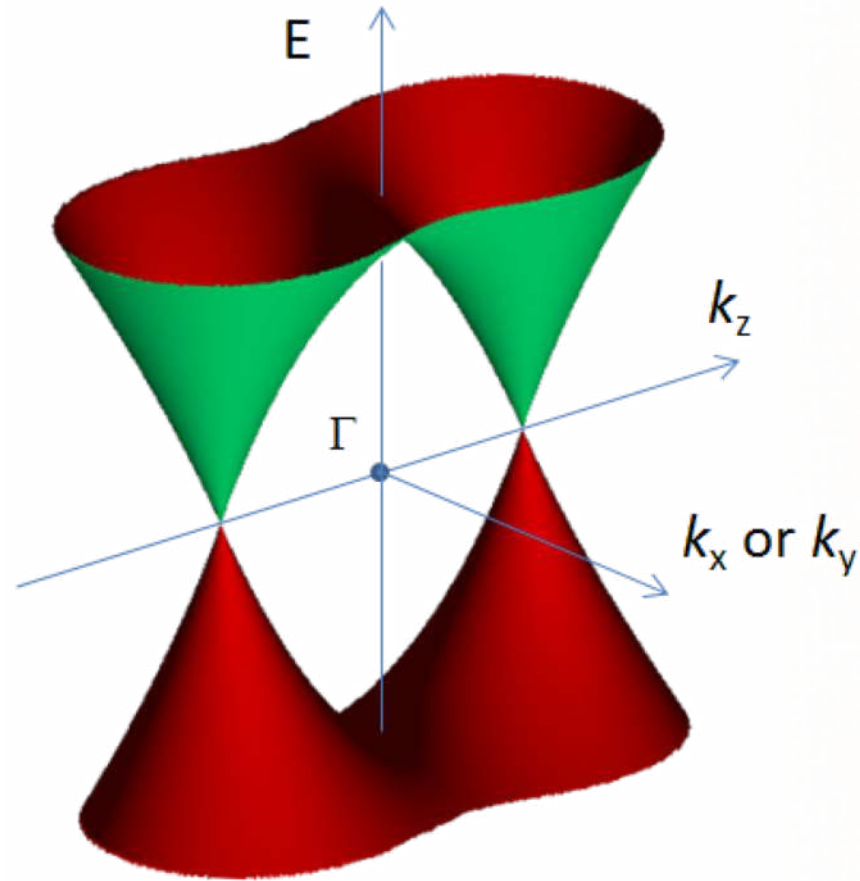
- Strong helical magnetic field in compact stars?

[Ohnishi, Yamamoto, arXiv:1402.4760]  
 [Yamamoto, Phys. Rev. D **93**, 065017 (2016)]  
 [Dvornikov, J. Exp. Theor. Phys. **123** 967 (2016)]

- Perhaps, not...

[Grabowska, Kaplan, Reddy, Phys. Rev. D **91**, 085035 (2015)]





Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

# DIRAC & WEYL MATERIALS

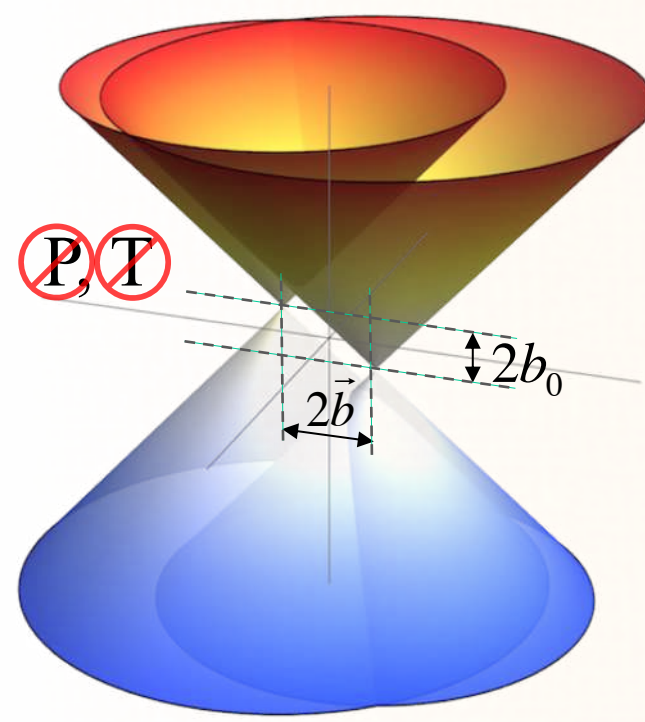
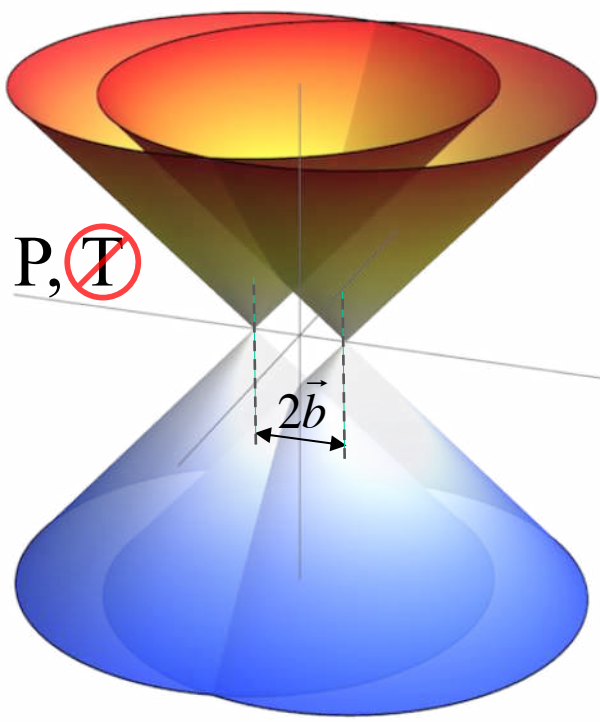
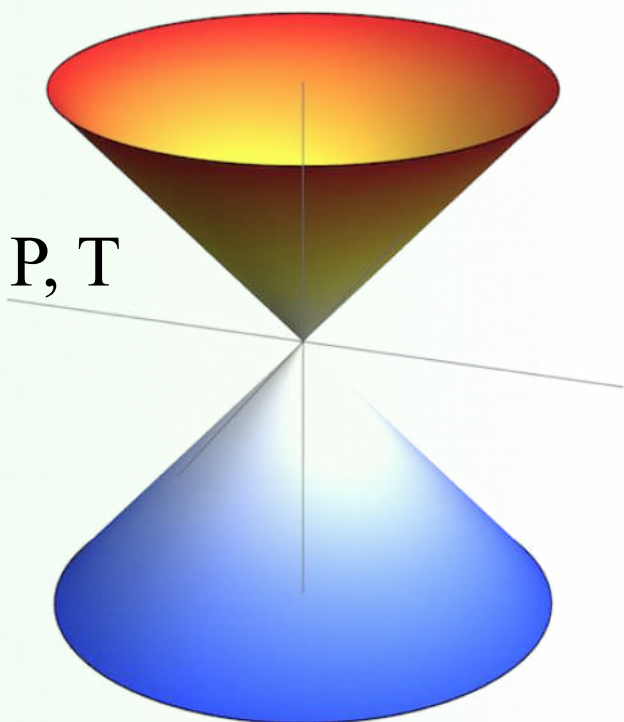
# Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H = \int d^3\mathbf{r} \bar{\psi} \left[ -iv_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - \underbrace{(\vec{b} \cdot \vec{\gamma})}_{\cancel{T}} \gamma^5 + \underbrace{b_0 \gamma^0 \gamma^5}_{\cancel{P}} \right] \psi$$

**Dirac** (e.g., Na<sub>3</sub>Bi, Cd<sub>3</sub>As<sub>2</sub>, ZrTe<sub>5</sub>)

**Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe<sub>2</sub>)



# Weyl fermions on a lattice

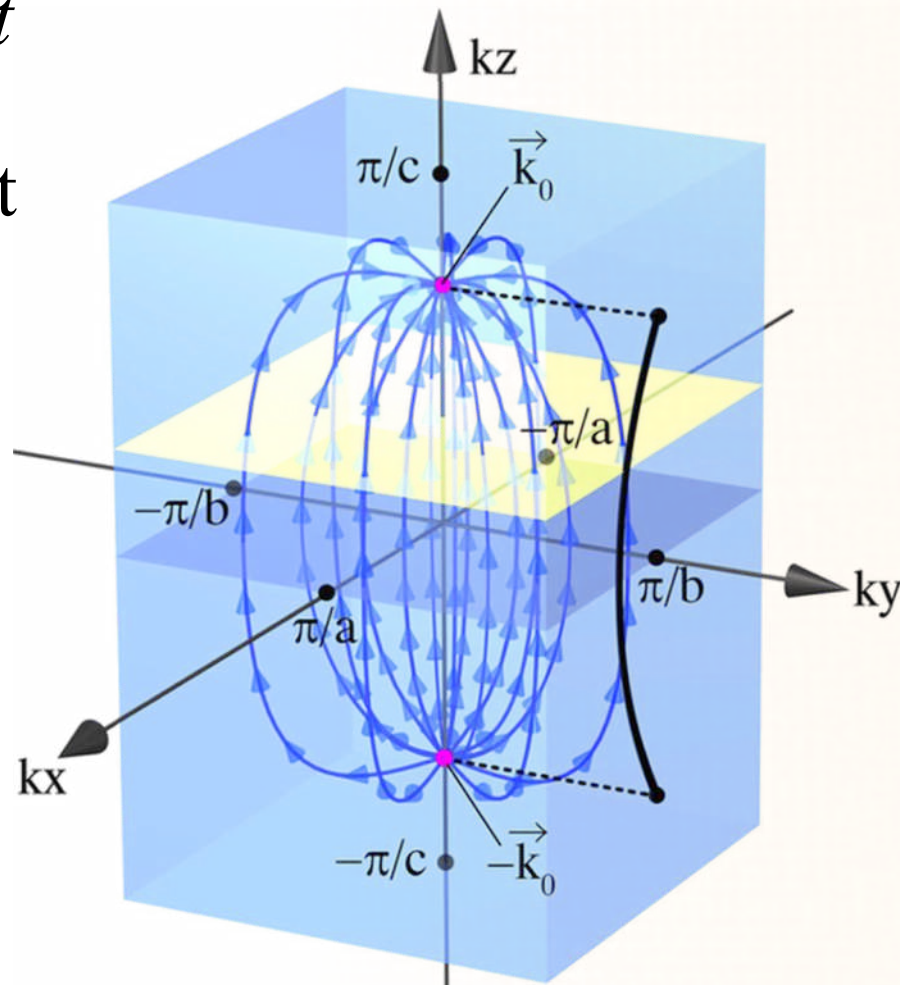
- In solid state physics, the momentum space (Brillouin zone) is *compact*
- *Berry curvature* is nonzero at Weyl nodes

$$\vec{\Omega}_k = \lambda \frac{\vec{k}}{2k^3}$$

- Weyl nodes carry nonzero *topological charges*

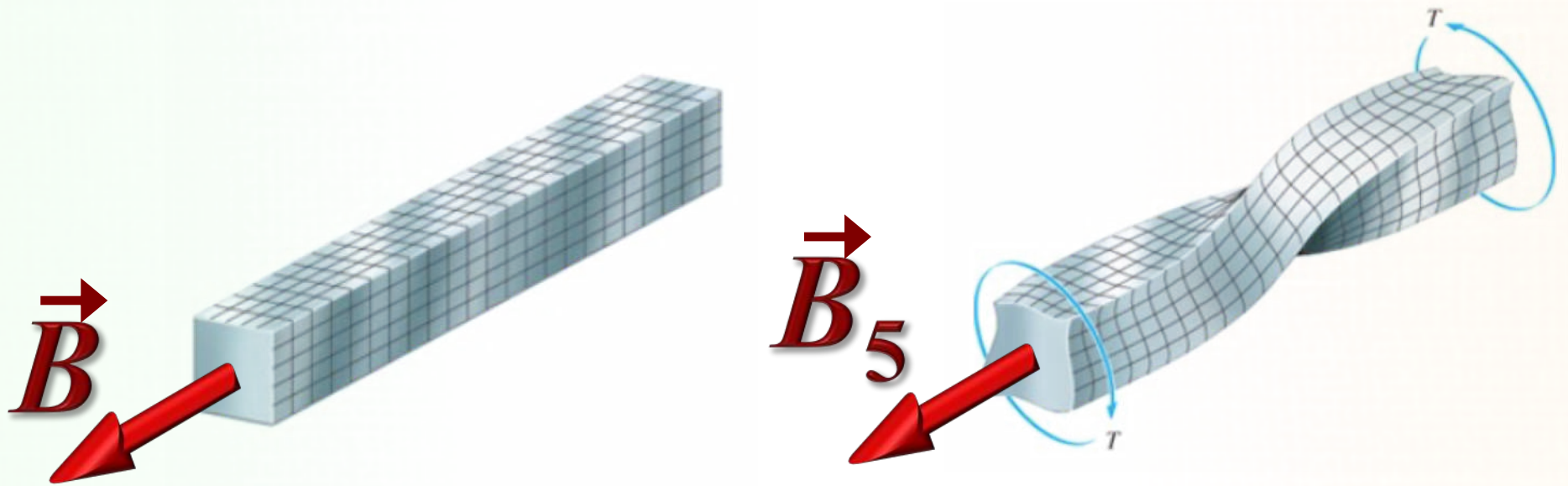
$$C = \frac{1}{2\pi} \oint \vec{\Omega}_k \cdot d\vec{S} = \lambda = \pm 1$$

- Weyl fermions come in pairs of opposite chirality



[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]

[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]



# PSEUDO-ELECTROMAGNETIC FIELDS

[Zubkov, Annals Phys. **360**, 655 (2015)]

[Cortijo, Ferreira, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]

[Grushin, Venderbos, Vishwanath, Ilan, Phys. Rev. X **6**, 041046 (2016)]

[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B **94**, 241405 (2016)]

[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

# Strain in Weyl materials

- Strains in the low-energy effective Weyl Hamiltonian

$$H = \int d^3 \mathbf{r} \bar{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - (\vec{b} + \vec{A}_5) \cdot \vec{\gamma} \gamma^5 + (b_0 + A_{5,0}) \gamma^0 \gamma^5 \right] \psi$$

where the chiral gauge fields are

$$A_{5,0} \propto b_0 \left| \vec{b} \right| \partial_{||} u_{||}$$

[Zubkov, Annals Phys. **360**, 655 (2015)]

[Cortijo, Ferreira, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]

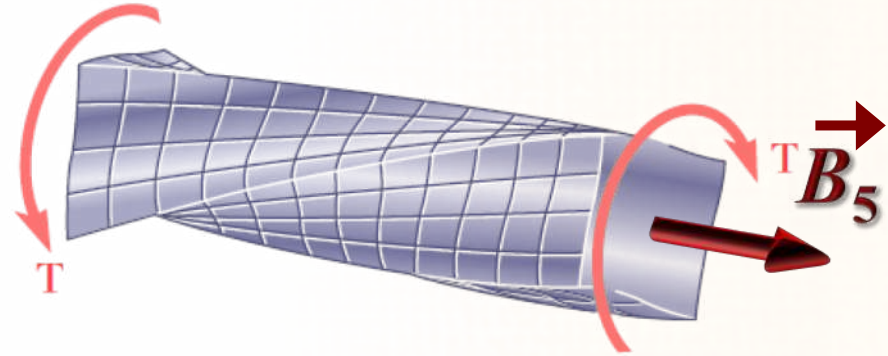
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]

$$A_{5,\perp} \propto \left| \vec{b} \right| \partial_{||} u_{\perp}$$

[Grushin, Venderbos, Vishwanath, Ilan, PRX **6**, 041046 (2016)]

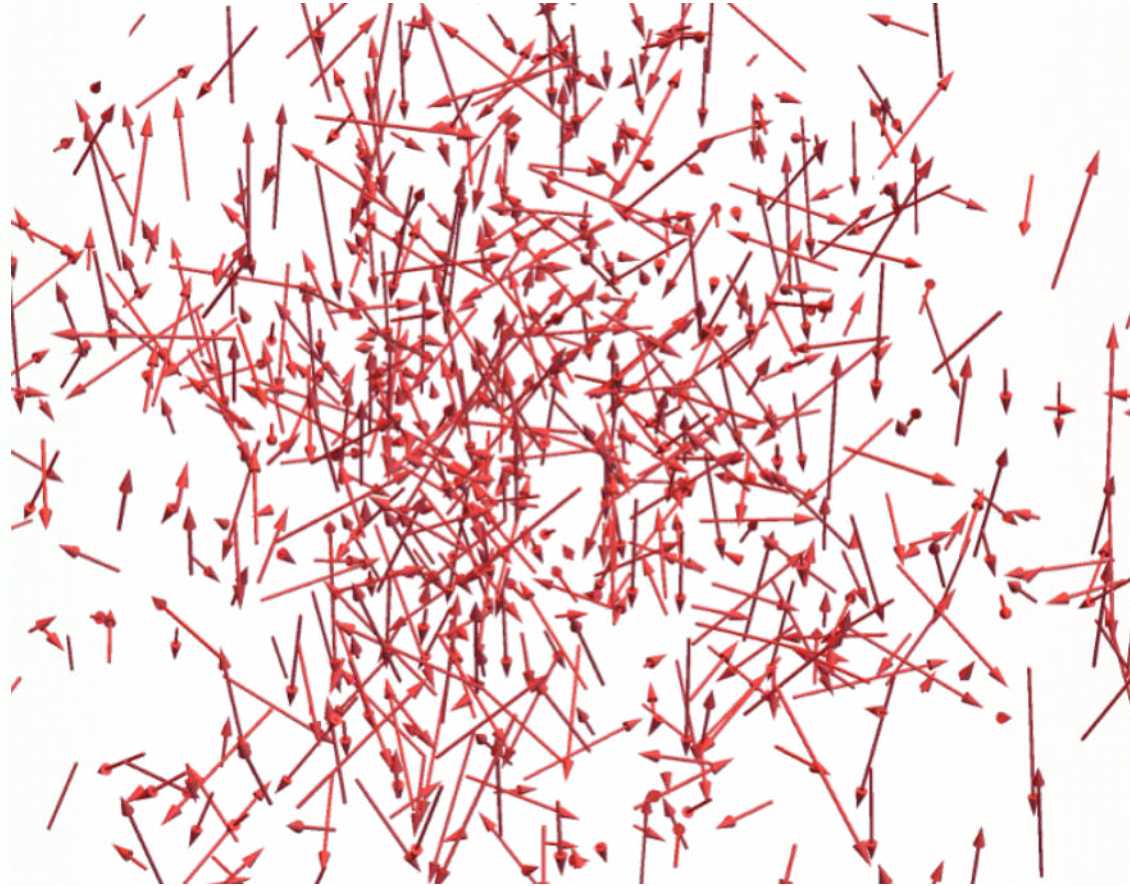
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

$$A_{5,||} \propto \alpha \left| \vec{b} \right|^2 \partial_{||} u_{||} + \beta \sum_i \partial_i u_i$$



leading to the pseudo-EM fields

$$\vec{B}_5 = \vec{\nabla} \times \vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla} A_0 - \partial_t \vec{A}_5$$



# CONSISTENT CHIRAL KINETIC THEORY

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]  
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]



- Kinetic equation: [Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]  
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[ e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v} \times \mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda \cdot \mathbf{B}_\lambda)\mathbf{\Omega}_\lambda \right] \cdot \nabla_{\mathbf{p}} f_\lambda}{1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)} + \frac{\left[ \mathbf{v} + e(\tilde{\mathbf{E}}_\lambda \times \mathbf{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v} \cdot \mathbf{\Omega}_\lambda)\mathbf{B}_\lambda \right] \cdot \nabla_{\mathbf{r}} f_\lambda}{1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)} = 0$$

where  $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\nabla_{\mathbf{r}}\epsilon_{\mathbf{p}}$ ,  $\mathbf{v} = \nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}$ , and

$$\epsilon_{\mathbf{p}} = v_{Fp} \left[ 1 - \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda) \right]$$

and  $\mathbf{\Omega}_\lambda = \lambda\hbar \frac{\hat{\mathbf{p}}}{2p^2}$  is the Berry curvature

- The definitions of density and current are

$$\rho_\lambda = e \int \frac{d^3 p}{(2\pi\hbar)^3} \left[ 1 + \frac{e}{c} (\mathbf{B}_\lambda \cdot \boldsymbol{\Omega}_\lambda) \right] f_\lambda,$$

$$\mathbf{j}_\lambda = e \int \frac{d^3 p}{(2\pi\hbar)^3} \left[ \mathbf{v} + \frac{e}{c} (\mathbf{v} \cdot \boldsymbol{\Omega}_\lambda) \mathbf{B}_\lambda + e (\tilde{\mathbf{E}}_\lambda \times \boldsymbol{\Omega}_\lambda) \right] f_\lambda$$

$$+ e \nabla \times \int \frac{d^3 p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}} \boldsymbol{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2 \hbar^2 c} \left[ (\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5) \right] \quad \checkmark$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2 \hbar^2 c} \left[ (\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B}) \right] \quad \times$$

# Consistent definition of current

- Bardeen-Zumino (Chern-Simons) term is needed,

$$\delta j^\mu = \frac{e^3}{4\pi^2 \hbar^2 c} \epsilon^{\mu\nu\rho\lambda} A_\nu^5 F_{\rho\lambda}$$

or

[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]

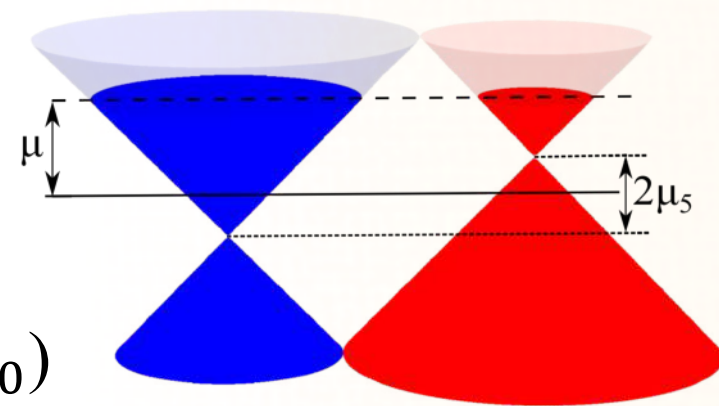
$$\delta \rho = -\frac{e^3}{2\pi^2 \hbar^2 c^2} (\mathbf{b} \cdot \mathbf{B})$$

$$\delta \mathbf{j} = -\frac{e^3}{2\pi^2 \hbar^2 c} b_0 \mathbf{B} + \frac{e^3}{2\pi^2 \hbar^2 c} [\mathbf{b} \times \mathbf{E}]$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **96**, 085130 (2017)]

- Its role and implications:

- Electric charge is conserved ( $\partial_\mu J^\mu = 0$ )
- Anomalous Hall effect is reproduced
- CME vanishes in equilibrium ( $\mu_5 = -eb_0$ )



- Any observable properties of Weyl materials directly sensitive to  $b_0$  and  $\vec{b}$ ? Chiral anomaly? Topology?
- Potentially, there are many observable effects:
  - Anomalous Hall effect
  - Negative magnetoresistance
  - Strain/torsion induced CME
  - Quantum oscillations of the density of states
  - Strain/torsion dependent resistance
  - Unusual features of collective modes
  - Anomalous electric/chiral/thermal transport
  - Unusual features of nonlocal transport



# CHIRAL MAGNETIC PLASMONS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

# Chiral magnetic plasmons

Non-degenerate plasmon frequencies @  $k=0$ :

$$\omega_l = \Omega_e, \quad \omega_{\text{tr}}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2} \left( \mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right)}$$

and

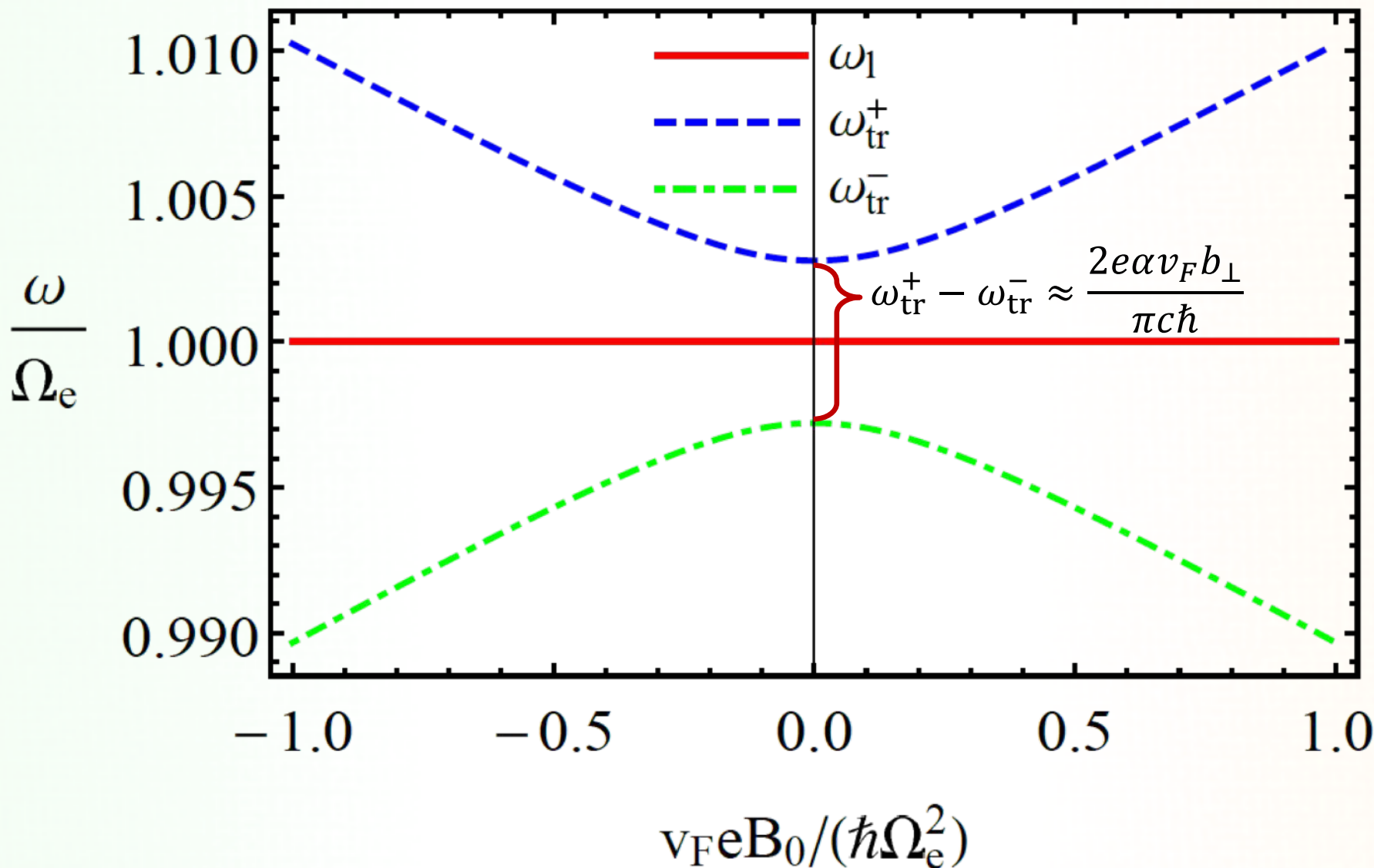
$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2} \left\{ 9\hbar^2 b_{\perp}^2 + \left[ \frac{2v_F}{\Omega_e^2} (B_0\mu + B_{0,5}\mu_5) - 3\hbar b_{\parallel} - \frac{v_F\hbar^2}{4T} \sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_{\lambda}}{T}\right) \right]^2 \right\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

# Plasmon frequencies, $\vec{B} \perp \vec{b}$

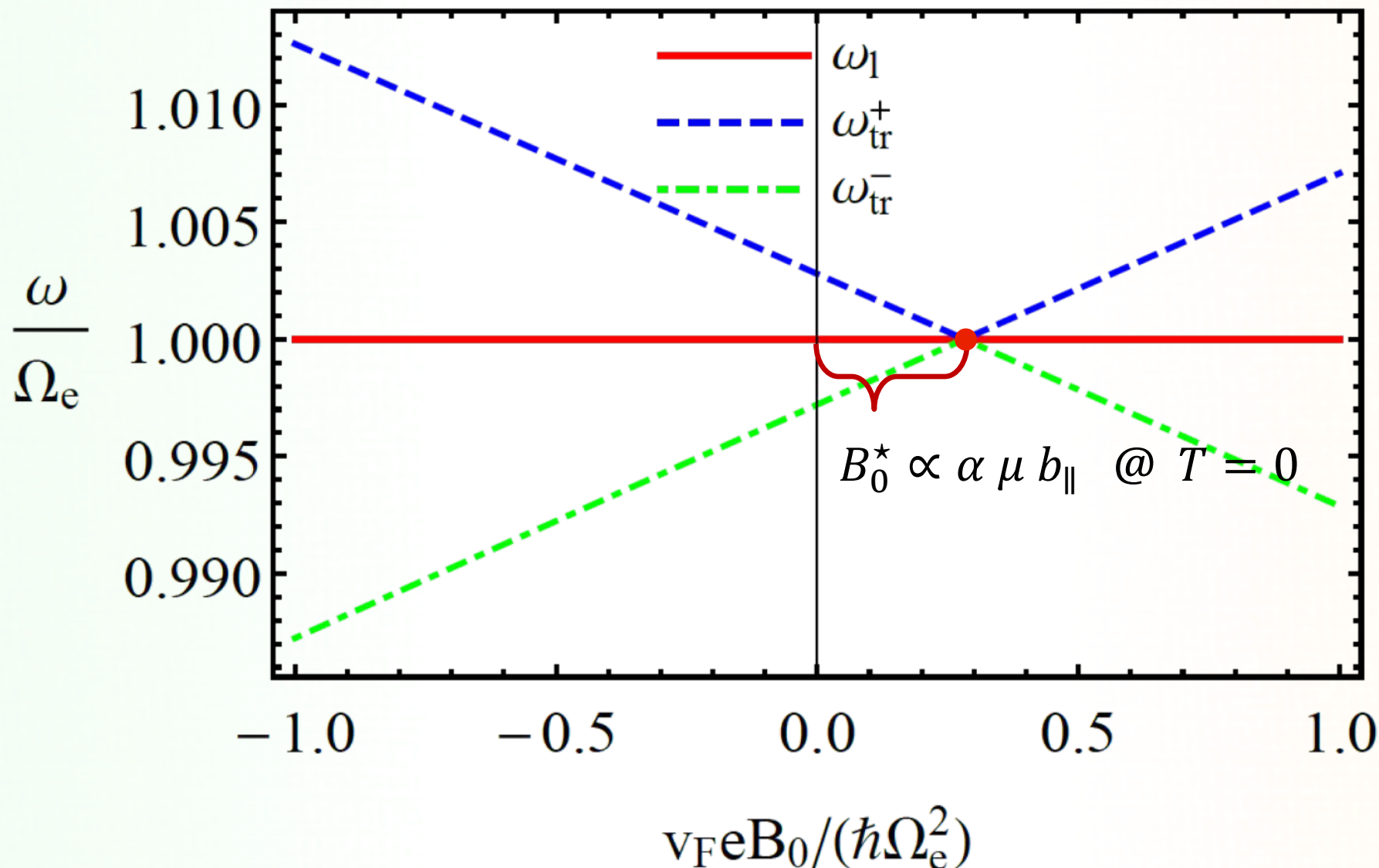
$$b_{\perp} = 0.2\hbar\Omega_e/e$$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

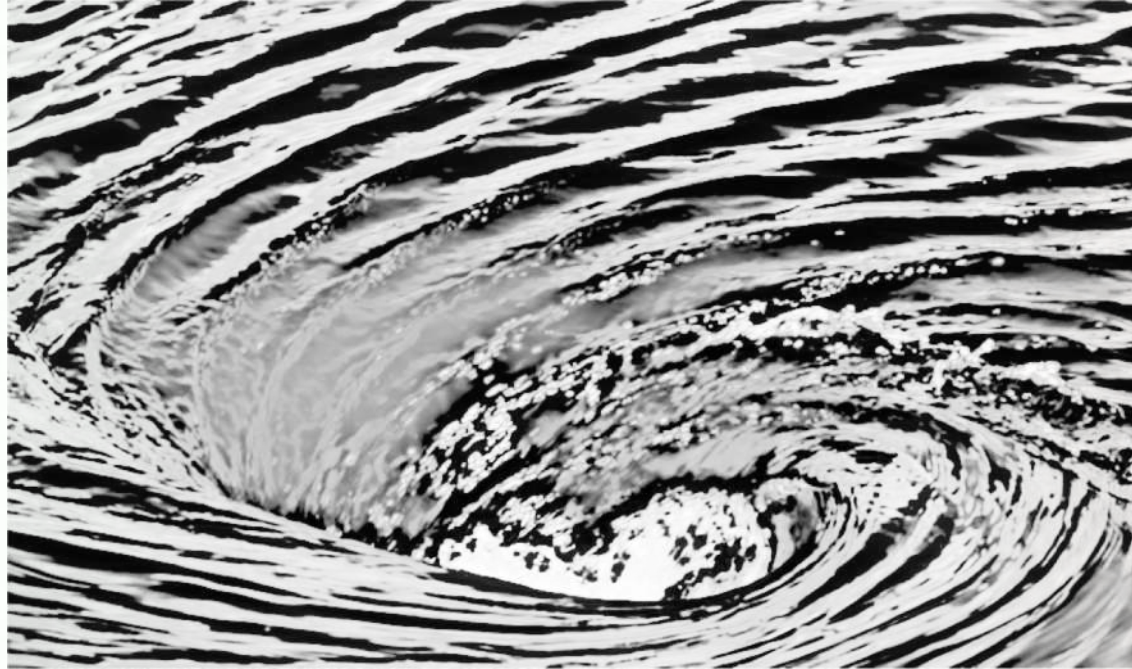
# Plasmon frequencies, $\vec{B} \parallel \vec{b}$

$$b_{\parallel} = 0.2 \hbar \Omega_e / e$$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]





# **ELECTRON TRANSPORT IN HYDRODYNAMIC REGIME**

The Euler equation for electron fluid:

[Gurzhi, JETP 17, 521 (1963)]

$$\frac{1}{v_F} \partial_t \left( \frac{\epsilon + P}{v_F} \mathbf{u} + \sigma^{(\epsilon, B)} \mathbf{B} \right) = -en \left( \mathbf{E} + \frac{1}{c} [\mathbf{u} \times \mathbf{B}] \right) + \frac{\sigma^{(B)} (\mathbf{E} \cdot \mathbf{B})}{3v_F^2} \mathbf{u} - \frac{\epsilon + P}{\tau v_F^2} \mathbf{u} + O(\nabla_r)$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB 97, 121105(R) (2018)]

The energy conservation

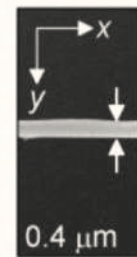
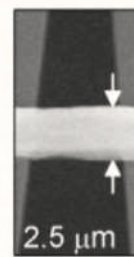
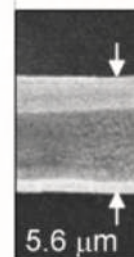
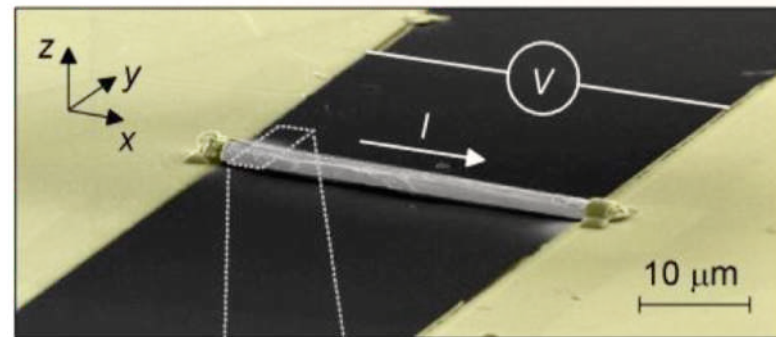
$$\partial_t \epsilon = -\mathbf{E} \cdot (en\mathbf{u} - \sigma^{(B)} \mathbf{B}) + O(\nabla_r)$$

+ Maxwell equations with the Chern-Simons currents

$$\rho_{CS} = -\frac{e^3 (\mathbf{b} \cdot \mathbf{B})}{2\pi^2 \hbar^2 c^2}$$

$$\mathbf{J}_{CS} = -\frac{e^3 b_0 \mathbf{B}}{2\pi^2 \hbar^2 c} + \frac{e^3 [\mathbf{b} \times \mathbf{E}]}{2\pi^2 \hbar^2 c}$$

Experimental evidence in tungsten diphosphide ( $WP_2$ ) [Gooth et al., arXiv:1706.05925]



- Magneto-acoustic wave ( $\rho = 0$ ):

$$\omega_{s,\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau} \sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)} [2|\mathbf{k}|^2 B_0^2 - (\mathbf{k} \cdot \mathbf{B}_0)]}{3w_0}}$$

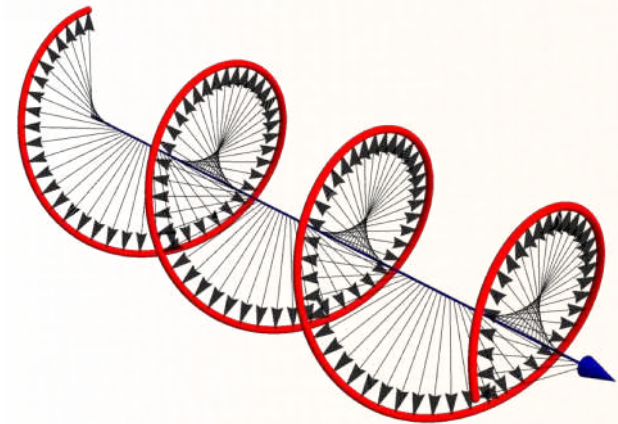
- *Gapped* chiral magnetic wave ( $\rho = 0$ ):

$$\omega_{\text{gCMW},\pm} = \pm \frac{eB_0 \sqrt{3v_F^3 \left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_{\parallel}^2\right)}}{2\pi^2 T_0^2 c \sqrt{\varepsilon_e \hbar}}$$

- Helicons ( $\rho \neq 0$ ):

$$\omega_{h,\pm} \approx \mp \frac{ck_{\parallel}^2 B_0}{4\pi\mu_m \rho_0} - \frac{i}{\tau} \frac{c^2 k_{\parallel}^2 w_0}{4\pi\mu_m v_F^2 \rho_0^2} + O(k_{\parallel}^3)$$

- New anomalous Hall waves at  $\vec{b} \neq 0$ , etc.



[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

- **Neutral** chiral plasma has chiral vortical waves
  - Speeds depend on the direction @  $\mu_5 \neq 0$   
[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]  
(might be important for neutrino flow in supernovas/protoneutron stars)
- Propagating (not overdamped) hydrodynamic modes in **charged** chiral plasma are [Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]
  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density
- **Dynamical electromagnetism** plays a crucial role
  - no chiral magnetic waves, special chiral Alfven waves, or other exotic modes obtained in background-field approximation
  - chiral magnetic wave predictions for HIC should be revised

- Anomalous *macroscopic* effects are expected in many forms of chiral plasmas
- Experimental search for anomalous signatures in high-energy physics is extremely *difficult*
- Low-energy *chiral fermions* can be realized in Dirac/Weyl materials
- Fundamental anomalous physics could be tested in *table-top experiments*
- Chirality and anomaly could be valuable in *applied* research