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QUANTUM ANOMALIES ETC.
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NORDITA

Hydrodynamic modes in magnetized chiral plasma with vorticity*

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NORDITA

*Based mostly on [Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Quantum Anomalies and Chiral Magnetic Phenomena

from 17 September 2018 to 12 October 2018 Nordita, Stockholm

CHIRAL MATER

Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{p}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

For particles $(p_0 > 0)$: chirality = helicity

For antiparticles $(p_0 < 0)$: chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

Anomalous chiral matter

- Matter made of chiral fermions may allow $n_\mathrm{L} \neq n_\mathrm{R}$
- Unlike the electric charge $n_\mathrm{R} + n_\mathrm{L}$, the chiral charge $n_\mathrm{R} - n_\mathrm{L}$, is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

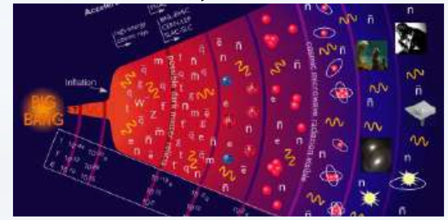
$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral anomaly can have *macroscopic* effects in chiral matter

Chiral forms of matter $(\vec{B}, \vec{\omega})$

- **Early Universe**, e.g.,

  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]



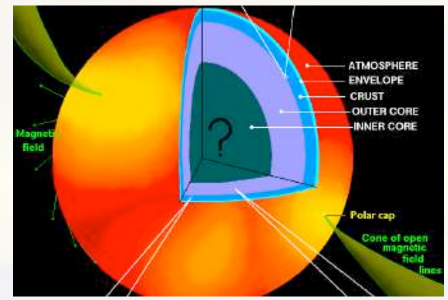
- **Heavy-ion collisions**, e.g.,

  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

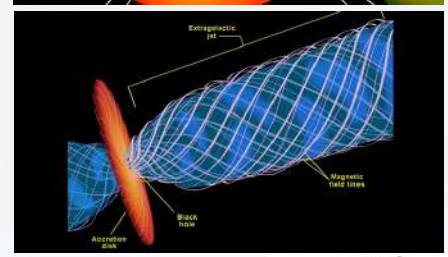

Credit: Brookhaven National Laboratory

- **Super-dense matter in compact stars**, e.g.,

  [Yamamoto, Phys.Rev. D93, 065017 (2016)]

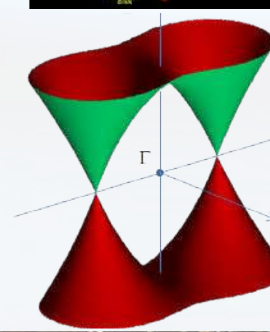


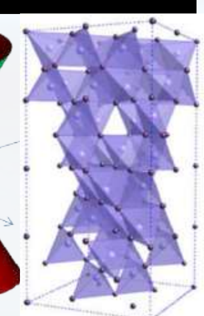
- **Ultra-relativistic jets from black holes**



- **Dirac/Weyl (semi-)metals**, e.g.,

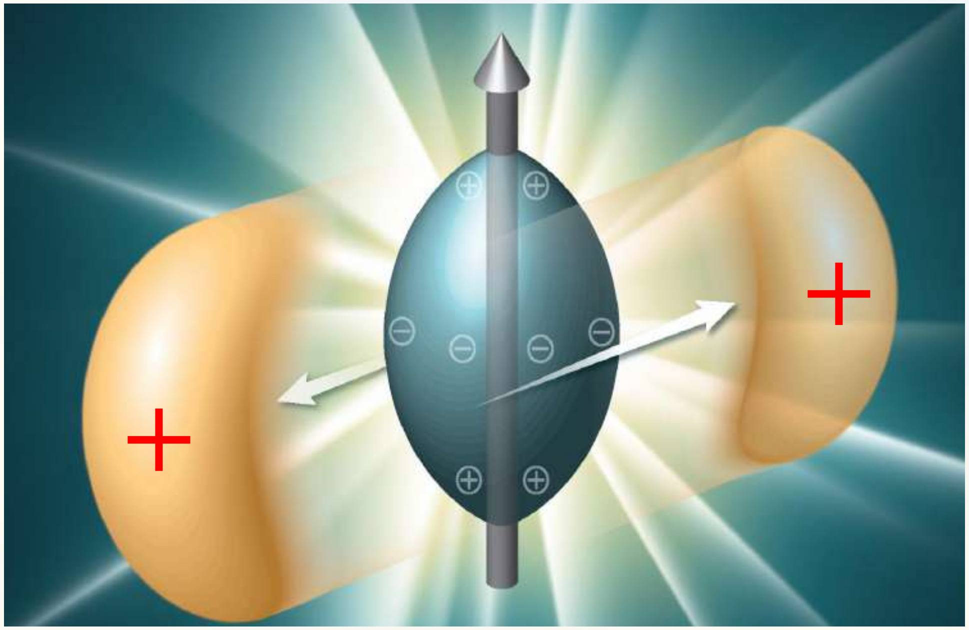
  [see talk by Pavlo Sukhachov]





- **Superfluid ^{3}He-A**, e.g.,

  [Volovik, JETP Lett. 105, 34 (2017)]



ANOMALOUS EFFECTS IN HEAVY-ION COLLISIONS

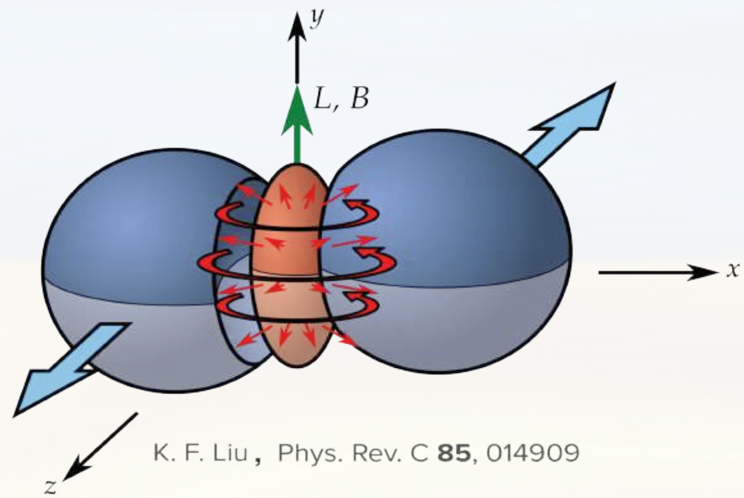
[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



K. F. Liu , Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak &. Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108]

- Magnetic field estimate:

  B ~ 10^{18} to 10^{19} G (~ 100 MeV)

- Vorticity estimate:

  $\omega \sim 10^{21} s^{-1}$ (~ 10 MeV)

Effect of magnetic field

- Dirac equation with massless fermions

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum

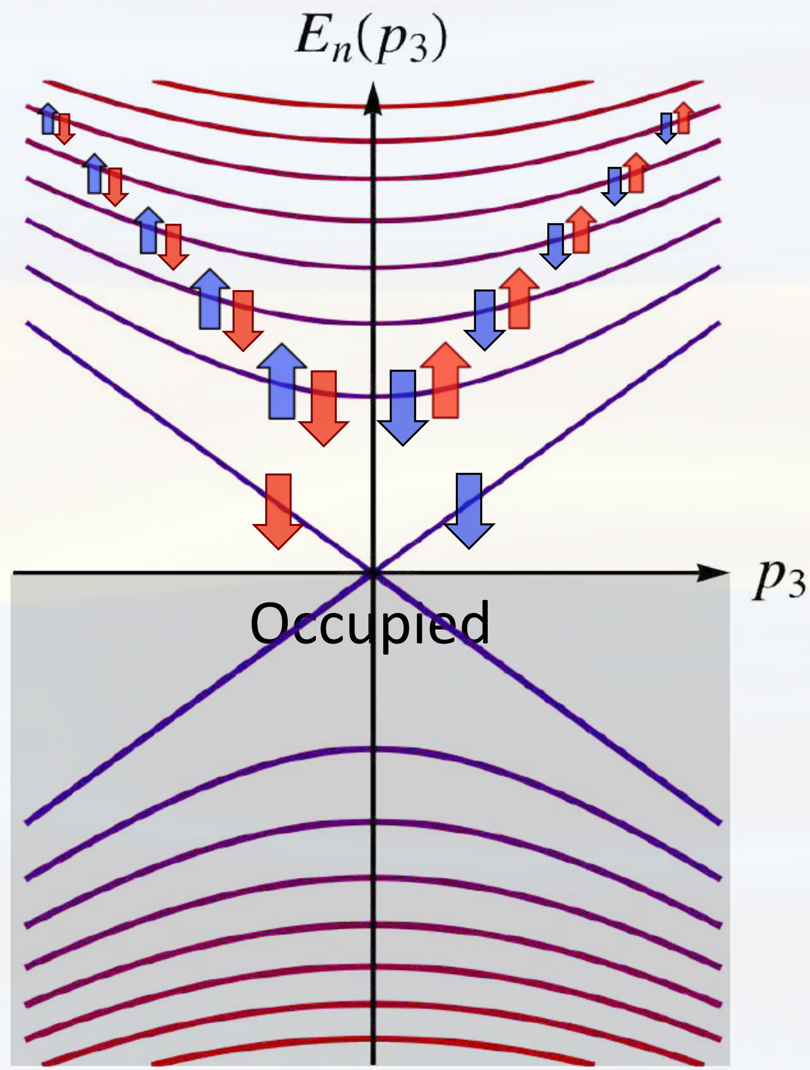
$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

$$n = \underbrace{s}_{s=\pm 1/2 \text{ (spin)}} + \underbrace{k + \frac{1}{2}}_{k = 0,\,1,\,2,\ldots \text{ (orbital)}}$$

$s = \pm 1/2$ (spin)

$k = 0, 1, 2, \ldots$ (orbital)

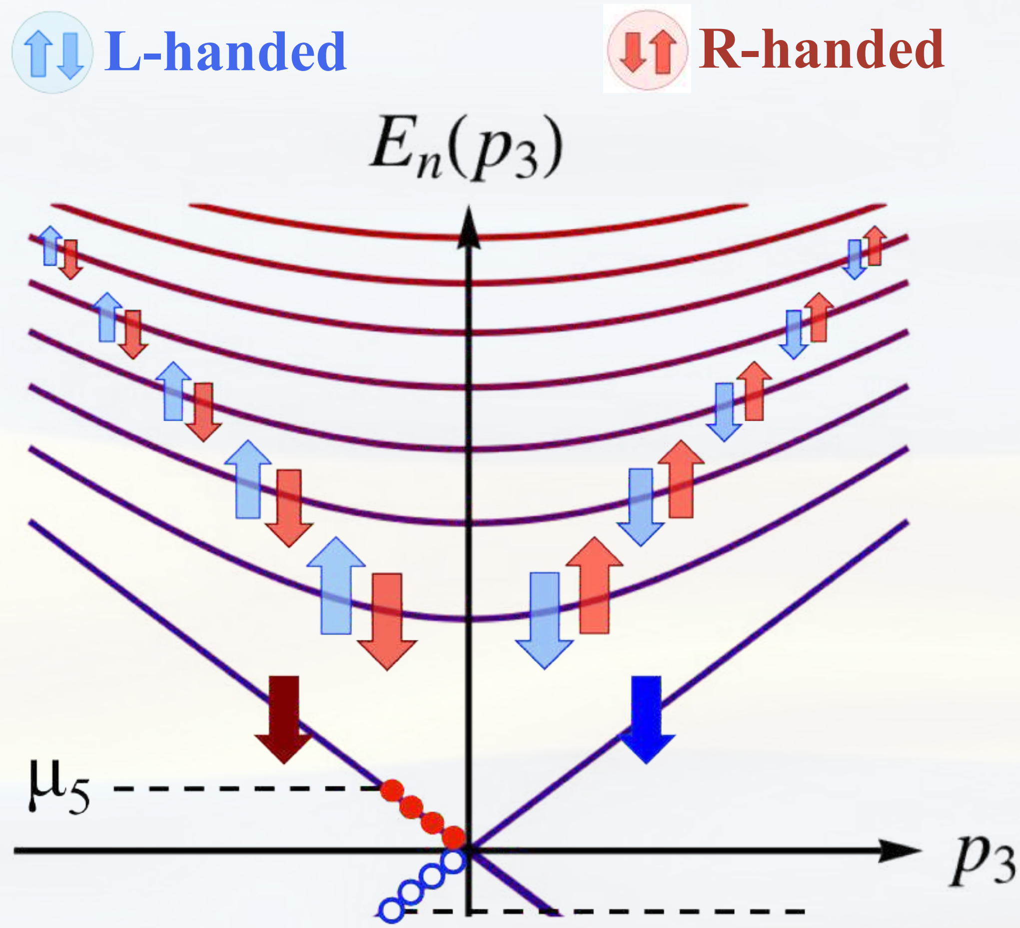


Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

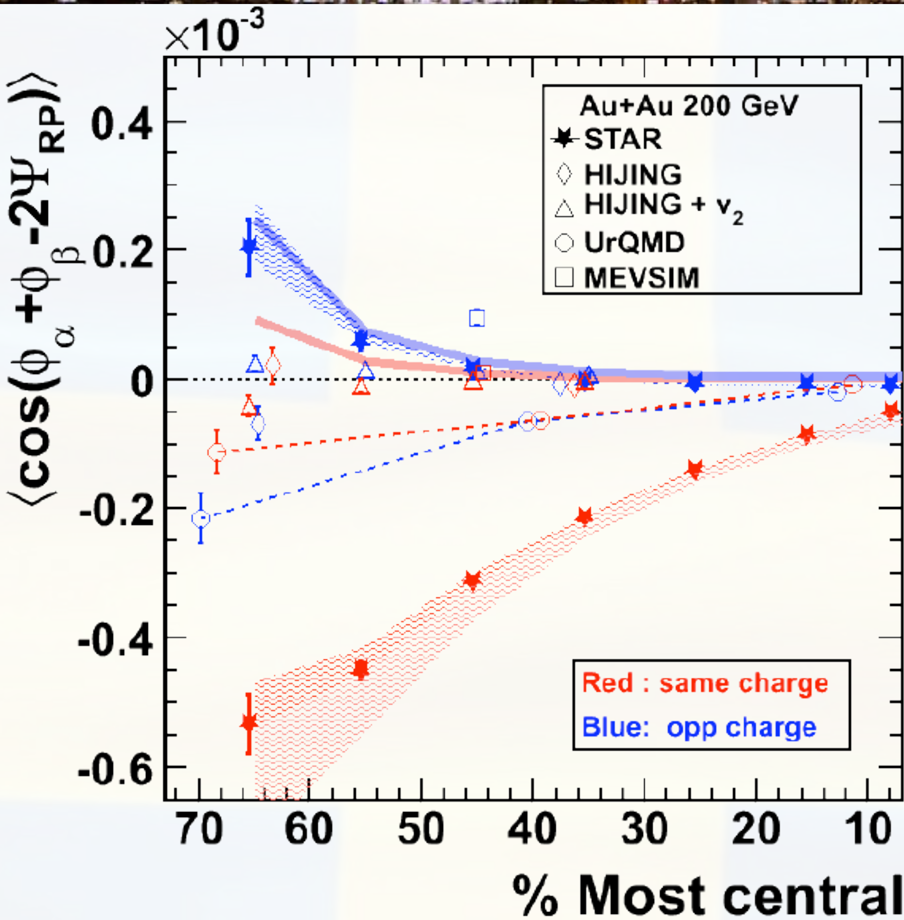
- R-handed states $p_3<0$ give current in $+z$ direction
- L-handed holes $p_3<0$ give current in $+z$ direction too!

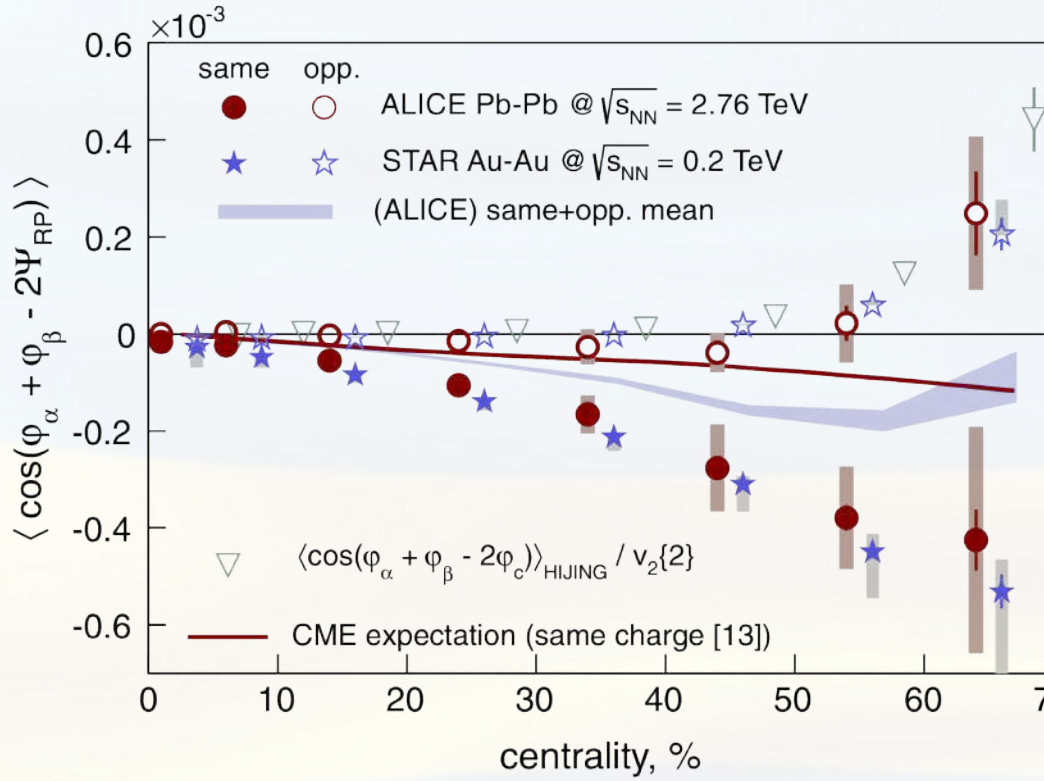


CME current: $$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence





Correlations of same & opposite charge particles:

$$\begin{cases} \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\mp} - 2\Psi_{RP}) \rangle \\ \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\pm} - 2\Psi_{RP}) \rangle \end{cases}$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)], [ALICE Collaboration, Phys. Lett. B777, 151 (2018)]

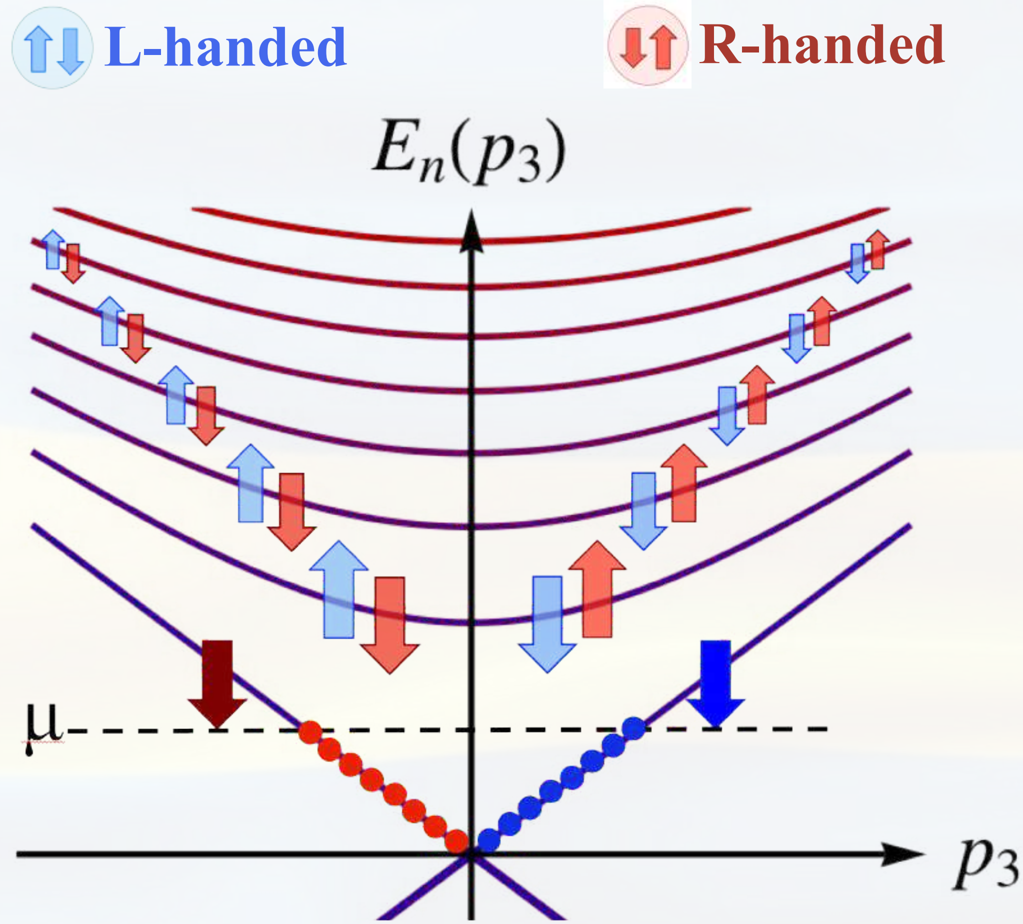
Chiral Separation Effect (μ≠0)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

- R-handed states $p_3<0$
- L-handed states $p_3>0$

This gives rise to a nonzero axial current density (CSE):

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$



[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

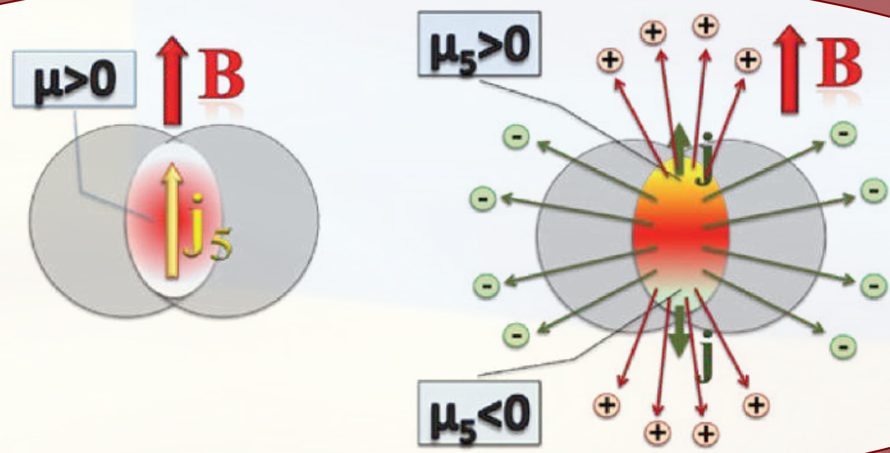
Note that the latter can be also interpreted as a spin density:

$$\langle \vec{j}_5 \rangle = \langle \psi^\dagger \vec{\Sigma} \psi \rangle, \quad \text{where} \quad \Sigma^k = \frac{i}{2}\varepsilon^{klm}[\gamma_l, \gamma_m]$$

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

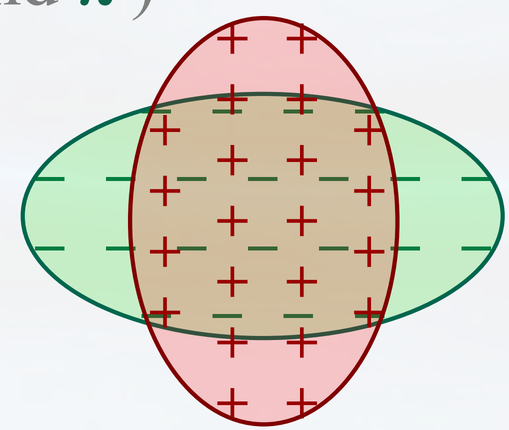


$$\left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

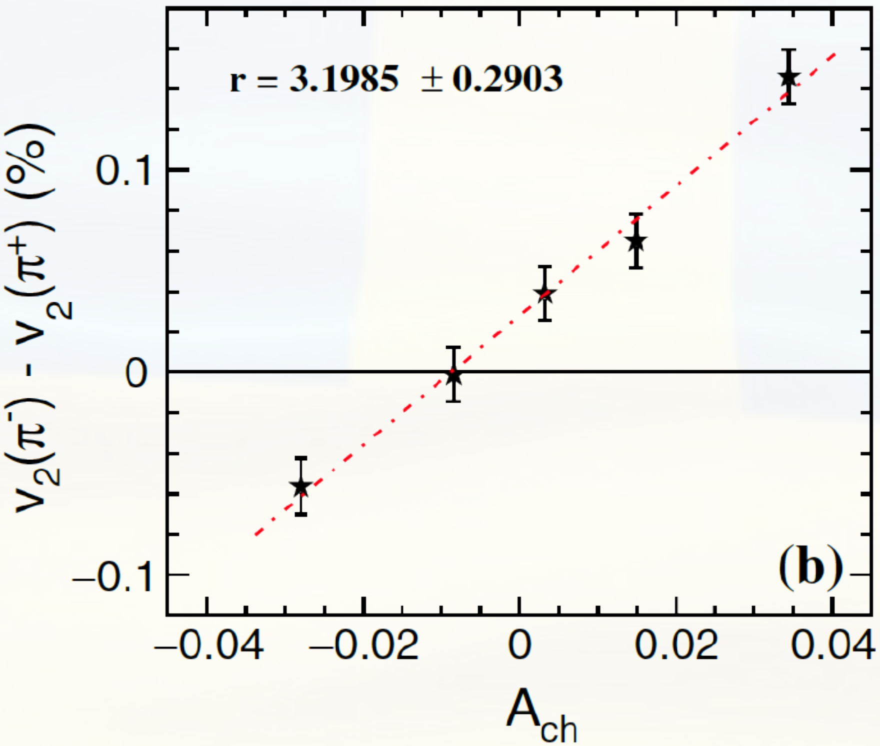
$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$

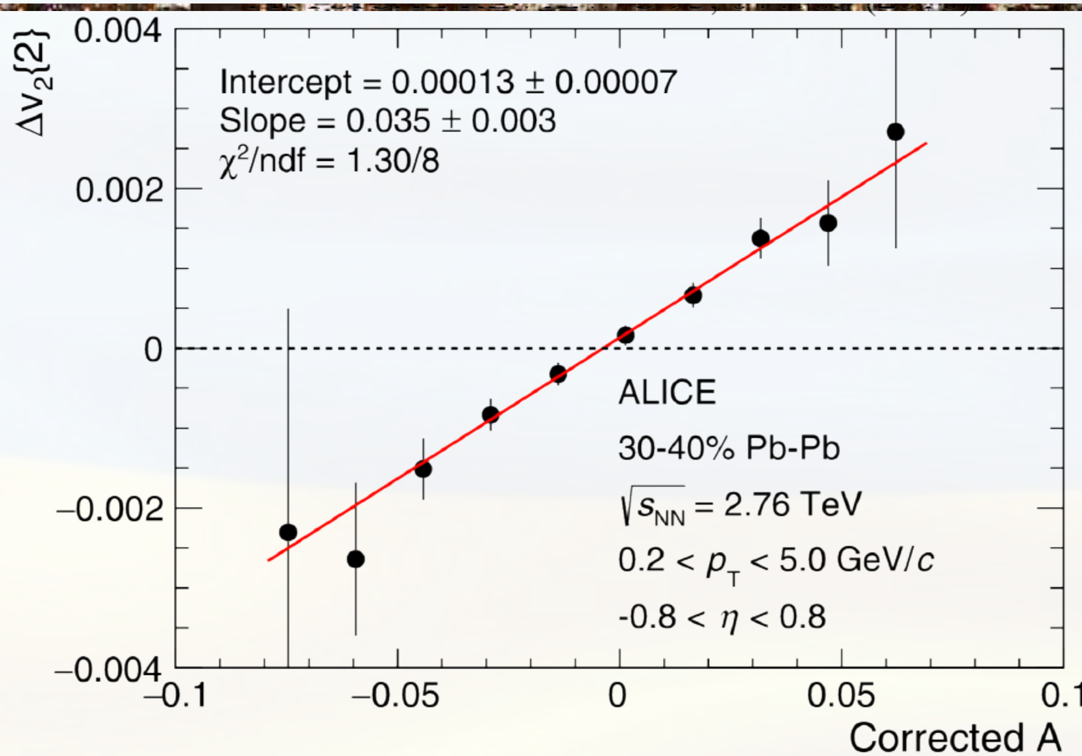


where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

CMW: Experimental evidence





[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

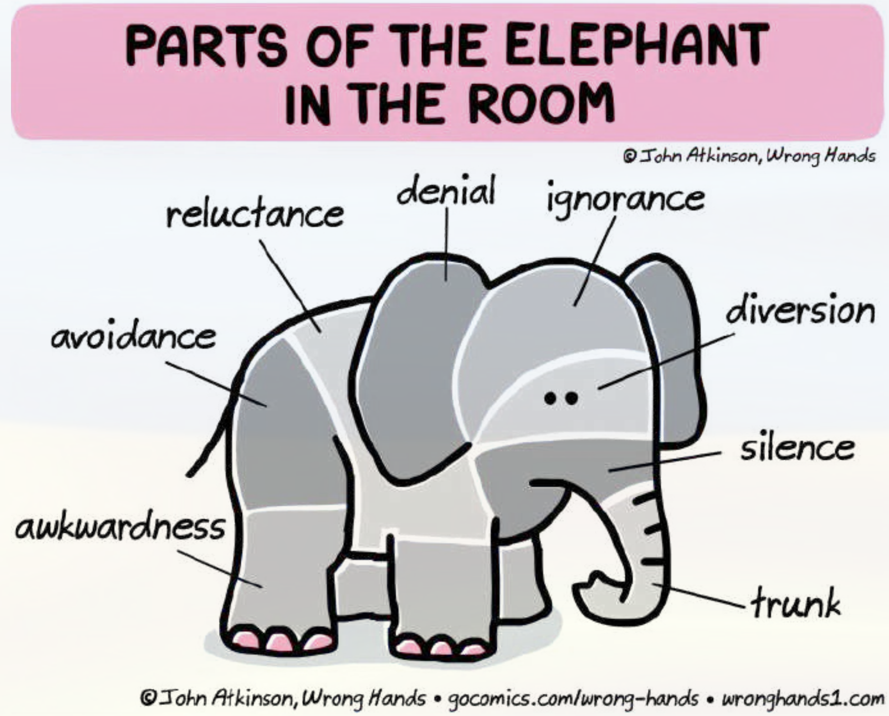
Higher harmonics of particle correlations are problematic…

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

Elephant in the room

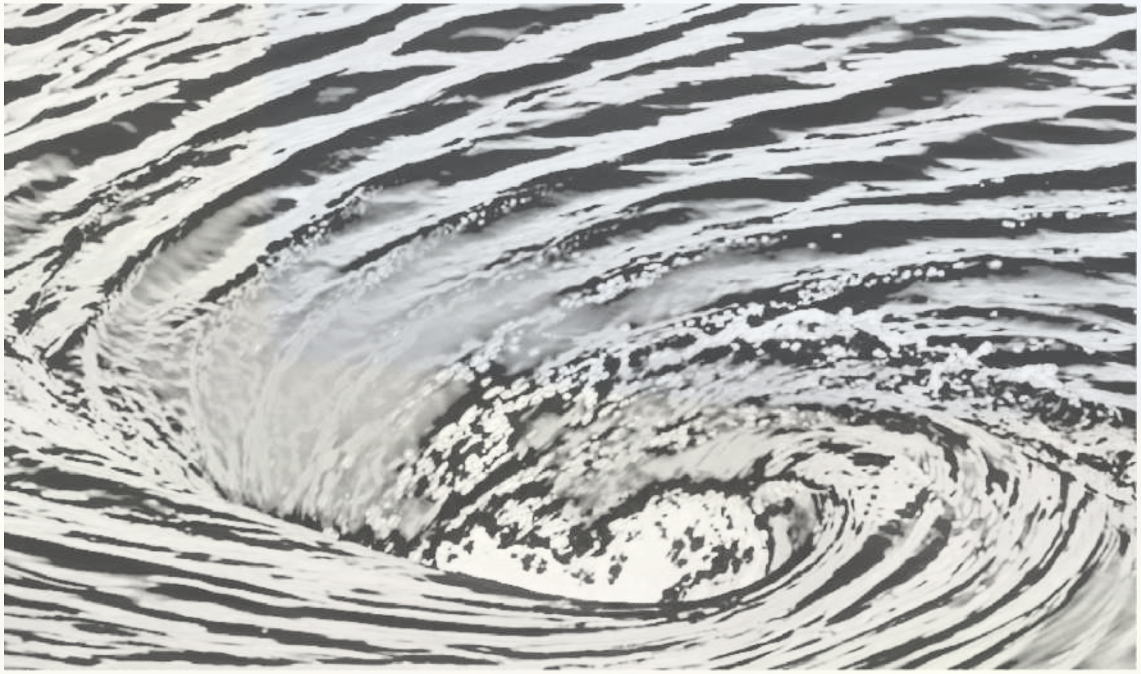
- What about *fluid flow* and *dynamical* electromagnetism?
- CMW: fluctuations of μ will induce electromagnetic fields!
- Damping rates of plasma flow and electromagnetic fields, i.e.,



$$\tau_{v\perp} \sim \frac{\epsilon + p}{(eB)^2 \sigma} \quad \text{and} \quad \tau_{EM} \sim \frac{1}{e^2 \sigma}$$

[Li, Yee, PRD 97, 056024 (2018)]

- When $\tau_{EM} \lesssim \tau_{v\perp}$ (i.e., $B^2 \lesssim \epsilon + p$), dynamical electro-magnetism is *essential*



CHIRAL HYDRODYNAMICS

Chiral hydrodynamics

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)] [Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = eF^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = nu^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

Anomalous contributions

- Currents included new non-dissipative terms:

$$j^\mu = nu^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{6\pi^2\hbar^2}, \qquad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$

Conceptual question

- **Can one derive chiral hydrodynamics from first principles?**
- Chiral kinetic theory (CKT) is a good starting point
- CKT itself can be derived from field theory
- Flashback: original (semi-rigorous) versions of CKT had several drawbacks, e.g.,
  - No explicit Lorentz covariance
  - Collisions are tricky (side-jumps, non-locality,…)

Chiral kinetic theory

- Kinetic equation:

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\mathbf{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)}$$
$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}$, $\mathbf{v} = \boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}$,

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right]$$

and $\mathbf{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Hydrodynamics from CKT

- Lorentz covariant formulation of CKT:

[Hidaka, Pu, Yang, Phys. Rev. D 95, 091901 (2017); Phys. Rev. D 97, 016004 (2018)]
See also [Carignano, Manuel, Torres-Rincon, arXiv:1806.01684]

$$\mathcal{D}_\mu W^\mu(p,x) = \delta(p^2) p\cdot C + \lambda\hbar e \tilde{F}^{\mu\nu} C_\mu p_\nu \delta'(p^2)$$

where $\mathcal{D}^\mu = \partial/\partial x^\mu - eF^{\mu\nu}\partial/\partial p^\nu$

$S^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta/(p\cdot u)$ is the spin tensor

C^μ is a collision operator

- Quasi-classical solution:

$$W^\mu(p,x) \equiv \underbrace{p^\mu \delta(p^2) f}_{O(1)} + \underbrace{\lambda\hbar S^{\mu\nu}\delta(p^2)(D_\nu f - C_\nu) + \lambda\hbar e\tilde{F}^{\mu\nu} p_\nu \delta'(p^2) f}_{O(\hbar)}$$

Approximations

- *Relaxation-time* approximation:

$$\mathcal{D}_\mu W^\mu = -\frac{u_\mu (W^\mu - W^\mu_{\text{eq}})}{\tau}$$

with W^μ_{eq} expressed in terms of the equilibrium distribution function,

$$f_{\text{eq}}(p,x) = \frac{1}{1 + e^{\text{sign}(p_0)(\varepsilon_p - \mu_\lambda)/T}}$$

where $\mu_\lambda \equiv \mu + \lambda\mu_5$, $\quad \varepsilon_p = \underbrace{u_\mu p^\mu}_{\text{O}(1)} + \underbrace{\frac{\lambda\hbar}{2}\frac{p\cdot\omega}{p\cdot u}}_{\text{O}(\hbar)}$

and $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu \partial_\alpha u_\beta$ is vorticity

Constitutive relations

- Conserved currents in CKT are moments of W^μ:

$$j^\mu = 2\sum_\lambda \int \frac{d^4p}{(2\pi)^3} W^\mu, \quad j_5^\mu = 2\sum_\lambda \lambda \int \frac{d^4p}{(2\pi)^3} W^\mu,$$

$$T^{\mu\nu} = \sum_\lambda \int \frac{d^4p}{(2\pi)^3} (W^\mu p^\nu + p^\mu W^\nu).$$

However, when using the CKT equation, we get

$$\partial_\mu j^\mu = -\frac{1}{\tau}(n - n_{\text{eq}})$$

$$\partial_\mu j_5^\mu + \frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu = -\frac{1}{\tau}(n_5 - n_{5,\text{eq}})$$

$$\underbrace{\partial_\nu T^{\mu\nu} - eF^{\mu\nu} j_\nu}_{\text{good}} = \underbrace{-\frac{u^\mu}{\tau}(\epsilon - \epsilon_{\text{eq}} + \cdots) - \frac{1}{\tau}\left(h^\mu - h^\mu_{\text{eq}} + \cdots\right)}_{\text{bad}}$$

Constitutive relations

- Conserved currents in CKT are moments of W^μ:

$$j^\mu = 2\sum_\lambda \int \frac{d^4p}{(2\pi)^3} W^\mu, \qquad j_5^\mu = 2\sum_\lambda \lambda \int \frac{d^4p}{(2\pi)^3} W^\mu,$$

$$T^{\mu\nu} = \sum_\lambda \int \frac{d^4p}{(2\pi)^3} (W^\mu p^\nu + p^\mu W^\nu).$$

However, when using the CKT equation, we get

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu + \frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu = 0$$

$$\partial_\nu T^{\mu\nu} - eF^{\mu\nu} j_\nu = 0$$

→ 6 constraints that define local equilibrium parameters T, μ, μ_5, u^μ

good — good

1^{st}-order dissipative hydro

- Dissipative terms (first-order):

$$\nu^\mu = \nu^\mu_{\rm eq} + \frac{\tau}{3}\nabla^\mu n - \tau \dot{u}^\mu n + \sigma_E E^\mu$$

$$\nu^\mu_5 = \nu^\mu_{5,\rm eq} + \frac{\tau}{3}\nabla^\mu n_5 - \tau \dot{u}^\mu n_5 + \sigma^5_E E^\mu$$

$$\pi^{\mu\nu} = \frac{8\tau\epsilon}{15}\Delta^{\mu\nu}_{\alpha\beta}(\partial^\alpha u^\beta)$$

where

$$\sigma_E = \tau e \frac{3(\mu^2 + \mu_5^2) + \pi^2 T^2}{9\pi^2 \hbar^3}$$

$$\sigma^5_E = \tau e \frac{2\mu\mu_5}{3\pi^2\hbar^3}$$

2^{nd}-order hydro (neutral)

- At this order, the constitutive relations are differential equations,

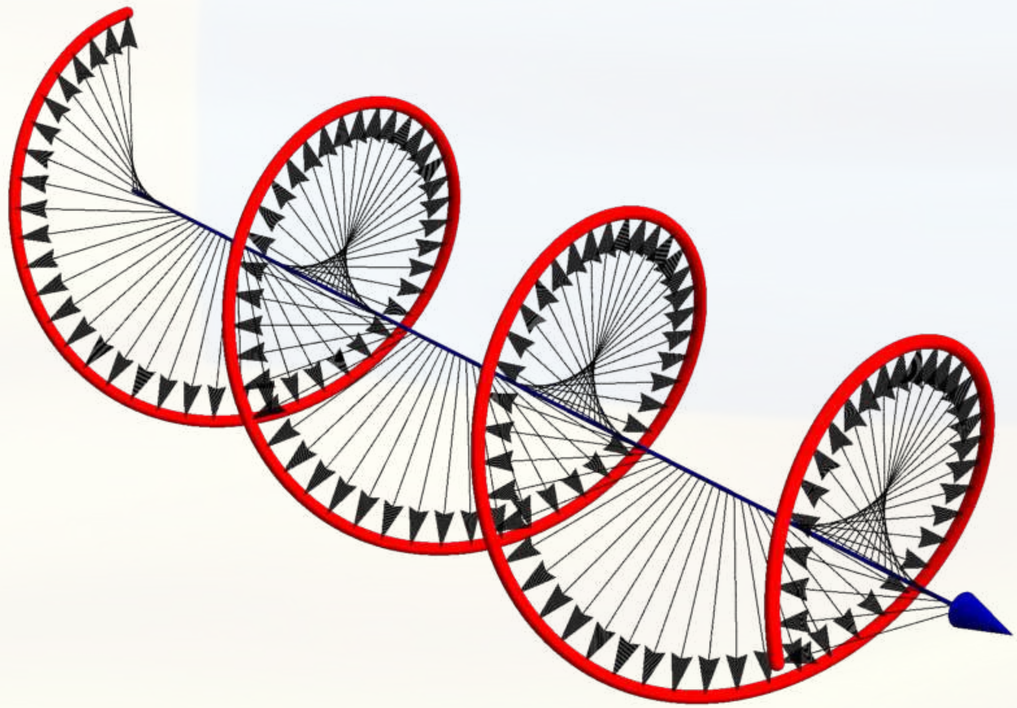
[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

$$\dot{\nu}^{\langle\mu\rangle} + \frac{\nu^\mu - \nu^\mu_{\rm eq}}{\tau} = -\dot{u}^\mu n + \frac{1}{3}\nabla^\mu n - \frac{n}{\epsilon + P}\Delta^{\mu\nu}\partial^\rho \pi_{\rho\nu} - \nu_\rho \omega^{\rho\mu} - (\partial\cdot u)\nu^\mu - \frac{9}{5}(\partial^{\langle\mu}u^{\rho\rangle})\nu_\rho + \ldots$$

$$\dot{\nu}_5^{\langle\mu\rangle} + \frac{\nu^\mu_5 - \nu^\mu_{5,\rm eq}}{\tau} = -\dot{u}^\mu n_5 + \frac{1}{3}\nabla^\mu n_5 - \frac{n_5}{\epsilon + P}\Delta^{\mu\nu}\partial^\rho \pi_{\rho\nu} - \nu_{5,\rho} \omega^{\rho\mu} - (\partial\cdot u)\nu^\mu_5 - \frac{9}{5}(\partial^{\langle\mu}u^{\rho\rangle})\nu_{5,\rho} + \ldots$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau} = -2h^{\langle\mu}\dot{u}^{\nu\rangle} + 2\pi^{\langle\mu}_\rho \omega^{\nu\rangle\rho} - \frac{10}{7}\pi^{\langle\mu}_\rho \sigma^{\nu\rangle\rho} - \frac{4}{3}\pi^{\mu\nu}\partial_\alpha u^\alpha + \frac{8}{15}(\partial^{\langle\mu}u^{\nu\rangle})\epsilon + \ldots$$

- Causality is Ok
- Stability is (probably) Ok



COLLECTIVE MODES IN NEUTRAL CHIRAL MATTER

Hydro modes in neutral plasma

- Sound waves

$$\Omega = \pm \frac{k_z}{\sqrt{3}} + \frac{3}{8}\hbar\bar{\omega}\frac{n_{5,\text{eq}}}{\epsilon_{\text{eq}}}k_z - \frac{2}{15}i\tau k_z^2$$

- Chiral vortical waves (CVW):

$$\Omega = \hbar\bar{\omega}v_1 k_z - \frac{1}{3}i\tau k_z^2, \qquad \Omega = \hbar\bar{\omega}v_2 k_z - \frac{1}{3}i\tau k_z^2$$

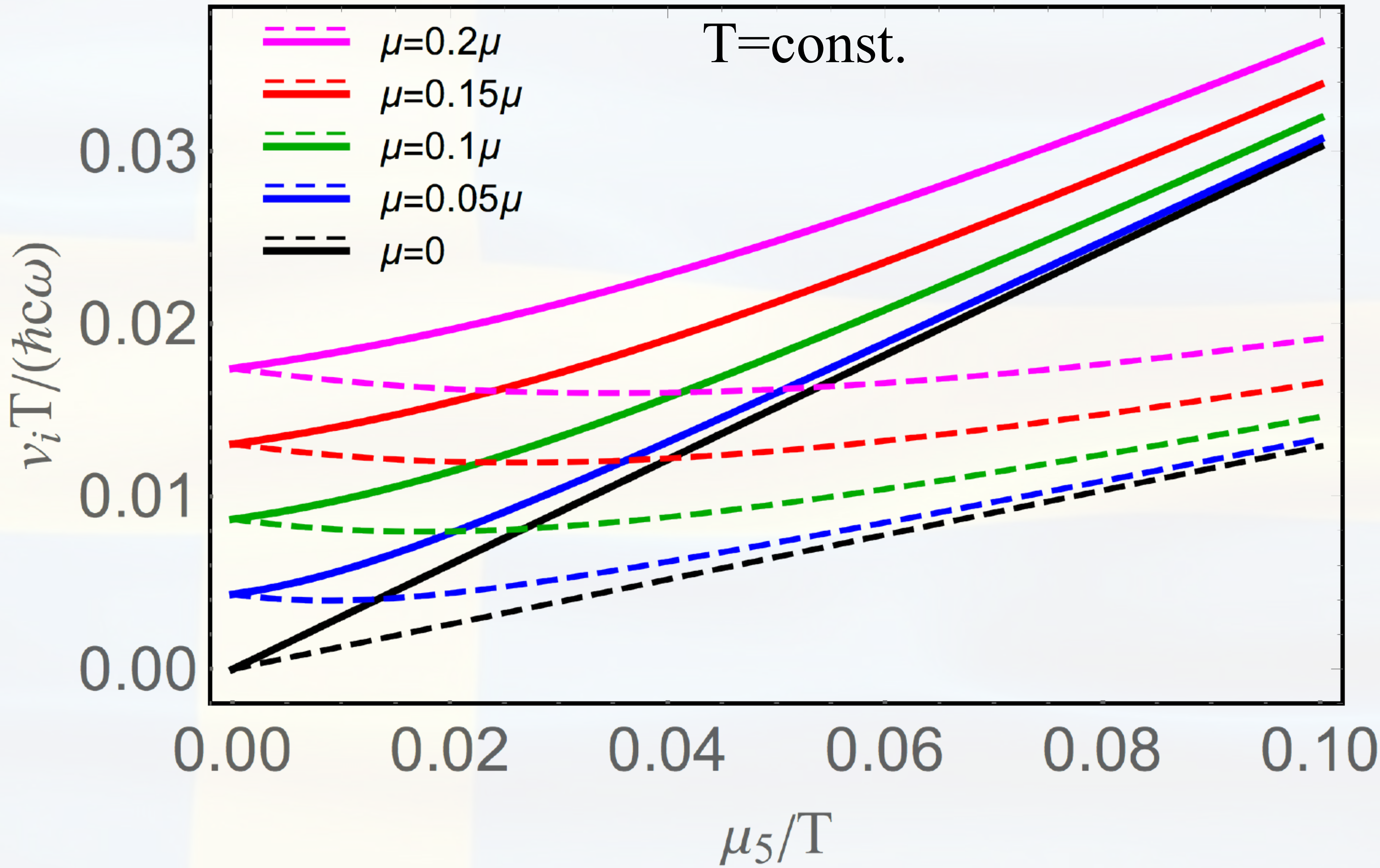
where $v_1 \neq v_2$ (along/against $\vec{\omega}$ direction)

- Oscillations of all thermodynamic parameters are important in CVW:

$$\delta\mu \neq 0,\ \delta\mu_5 \neq 0,\ \delta T \neq 0,\ \delta u^\mu \neq 0$$

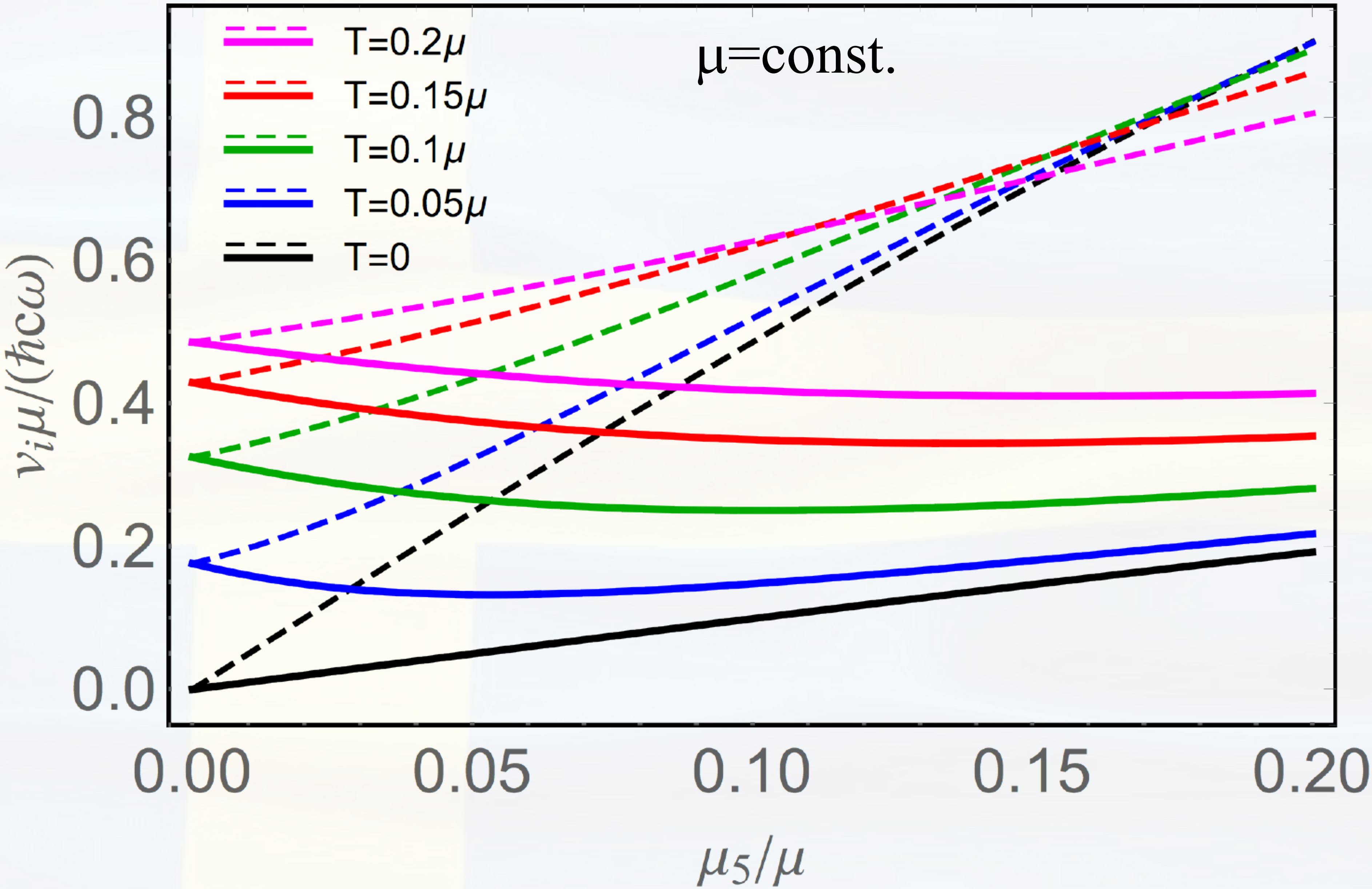
[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

CVW velocities @ high T



[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

CVW velocities @ large μ

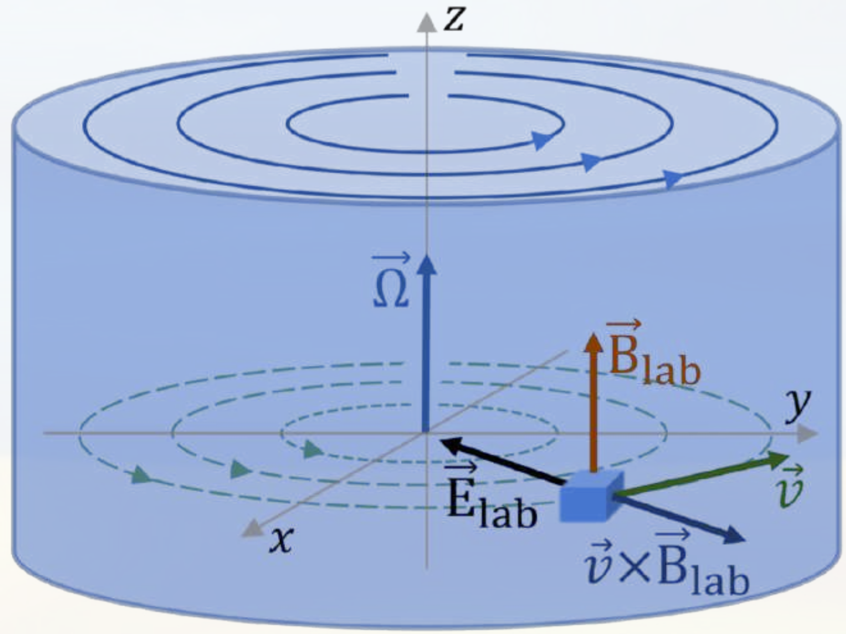


[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

Rotating charged plasma

- Fluid velocity

$$\bar{u}^{\nu} = \gamma \begin{pmatrix} 1 \\ -\Omega y \\ \Omega x \\ 0 \end{pmatrix}$$



where $\gamma = 1/\sqrt{1-(\Omega\, r)^2}$

Vorticity: $\bar{\omega}^{\mu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{u}_{\nu}\partial_{\alpha}\bar{u}_{\beta} = \gamma^2\Omega\delta_3^{\mu}$

EM fields in lab frame: $\mathbf{B}_{\mathrm{lab}} = \gamma B\hat{\mathbf{z}}$

$$\mathbf{E}_{\mathrm{lab}} = -\gamma B\Omega\mathbf{r}_{\perp}$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

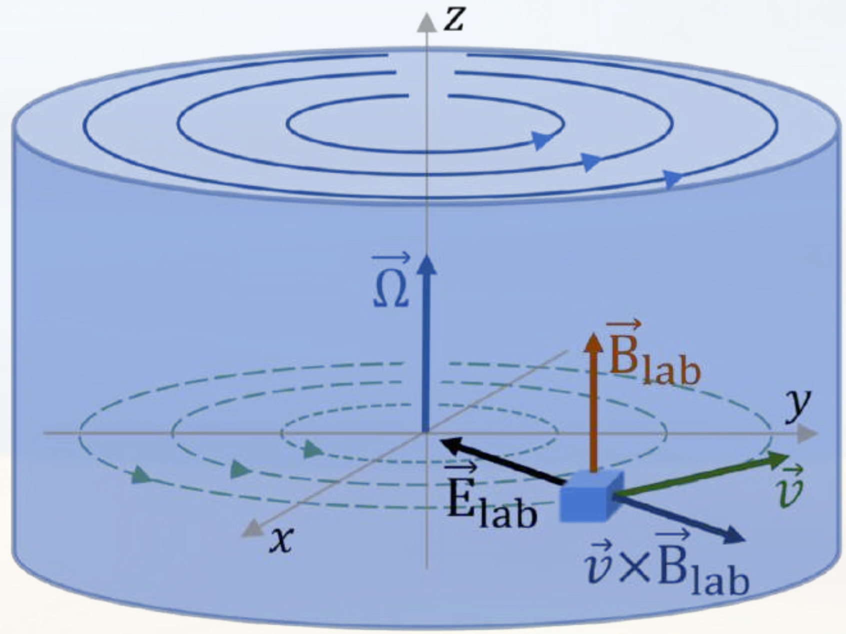
Rotating charged plasma: equilibrium

- Maxwell equations:

$$\partial_\nu F^{\nu\mu} = enu^\mu + e\nu^\mu - en_{\mathrm{bg}} u^\mu_{\mathrm{bg}}$$

$$\partial_\nu \tilde{F}^{\nu\mu} = 0$$

where n_{bg} is the background



The solution is radially nonuniform:

$$B(r) = \gamma\left(B_0 - \frac{1}{2} en_{\mathrm{bg}}\Omega r^2\right) \simeq B_0 - \frac{1}{2} en_{\mathrm{bg}}\Omega r^2 + O\left(B_0 r^2 \Omega^2\right)$$

$$en_{\mathrm{eq}}(r) = \gamma^3\left(en_{\mathrm{bg}} - 2B_0\Omega\right) \simeq en_{\mathrm{bg}} - 2B_0\Omega + O\left(en_{\mathrm{bg}} r^2 \Omega^2\right)$$

(This is consistent with $\mu = \gamma\mu_0$, $\mu_5 = \gamma\mu_{5,0}$, $T = \gamma T_0$.)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Linearized equations, etc.

- Small perturbations ($\Omega \to 0$):

$$\delta s(x) = e^{-ik_0 t + ik_z z + im\theta}\delta s(r)$$
$$\delta v^3(x) = e^{-ik_0 t + ik_z z + im\theta}\delta v^3(r)$$
$$\delta v_\pm(x) = e^{-ik_0 t + ik_z z + i(m\pm 1)\theta}\delta v_\pm(r)$$

where

$$\delta s(r) = \delta s\, J_m(k_\perp r), \quad \text{for} \quad s = \mu, \mu_5, T$$
$$\delta v^3(r) = \delta v^3\, J_m(k_\perp r), \quad \text{for} \quad v^3 = u^3, B^3, E^3$$
$$\delta v_\pm(r) = \delta v_\pm\, J_{m\pm 1}(k_\perp r), \quad \text{for} \quad v_\pm = u_\pm, B_\pm, E_\pm$$

- Boundary conditions: $\delta s(R) = 0,\ \delta v^3(R) = 0$
- Transverse wave vectors: $k_\perp^{(i)} = \alpha_{m,i}/R$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

COLLECTIVE MODES IN HOT PLASMA

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Hierarchy of scales

- Mean free path $\ell_{\mathrm{mfp}} \simeq c\tau$, de Broglie wavelength $\ell_d = \hbar/T$, and typical wavelengths of modes $\lambda_k = 2\pi/k$:

$$\ell_d \ll \ell_{\mathrm{mfp}} \ll \lambda_k \lesssim R$$

- Magnetic length $\ell_B = \sqrt{\hbar/|eB|}$ ($\ell_d \ll \ell_B$ or $|eB| \ll T^2$)
  - Weak magnetic field: $\ell_B \gtrsim \ell_{\mathrm{mfp}}$
  - Moderately strong magnetic field: $\ell_B \lesssim \ell_{\mathrm{mfp}}$
- System size $R \lesssim \Omega^{-1}$
- In this work (with auxiliary parameter $\xi \simeq 0.01$)

$$\Omega\ell_{\mathrm{mfp}} \simeq \xi^2, \quad \xi^{3/2} \simeq \frac{\ell_{\mathrm{mfp}}}{R} \lesssim k\ell_{\mathrm{mfp}} \lesssim \xi^{1/2}, \quad \frac{\ell_{\mathrm{mfp}}}{\ell_B} \simeq \xi^{-1/4}, \quad \frac{\ell_{\mathrm{mfp}}}{\ell_d} \simeq \xi^{-1}$$

Charged plasma at $\Omega = 0$

- Sound waves ($T \gg \mu$): [Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = s_e \frac{3\sqrt{5}B\hbar^{3/2}k_z}{\sqrt{7}\pi T^2}\left(1 \pm \frac{\sqrt{5}e\mu}{2\sqrt{7}\pi\hbar^{3/2}k}\right) - \frac{1}{10} i\tau k^2.$$

where $k = \sqrt{k_z^2 + k_\perp^2}$ and $s_e = \pm 1$.

- There are also purely diffusive modes, e.g.,

$$k_0 = -\frac{e^2}{9\hbar^3} i\tau T^2$$

describing the charge diffusion (i.e., $\partial_t \mathbf{E} + e\sigma_E \mathbf{E} \approx 0$.)

Charged plasma at $\Omega \neq 0$

- Sound waves ($T \gg \mu$):

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2 + m\Omega\left(\frac{2}{3} + \frac{5e^2\mu^2}{14\pi^2\hbar^3 k^2}\right)$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = m\Omega + sk_z\sqrt{\frac{45\mathcal{B}_\pm^2\hbar^3}{7\pi^2T^4} + \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm\mu}{14\pi^2T^2k}\right)^2} \pm k_z\left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm\mu}{14\pi^2T^2k}\right)$$

with a small imaginary part (not shown)

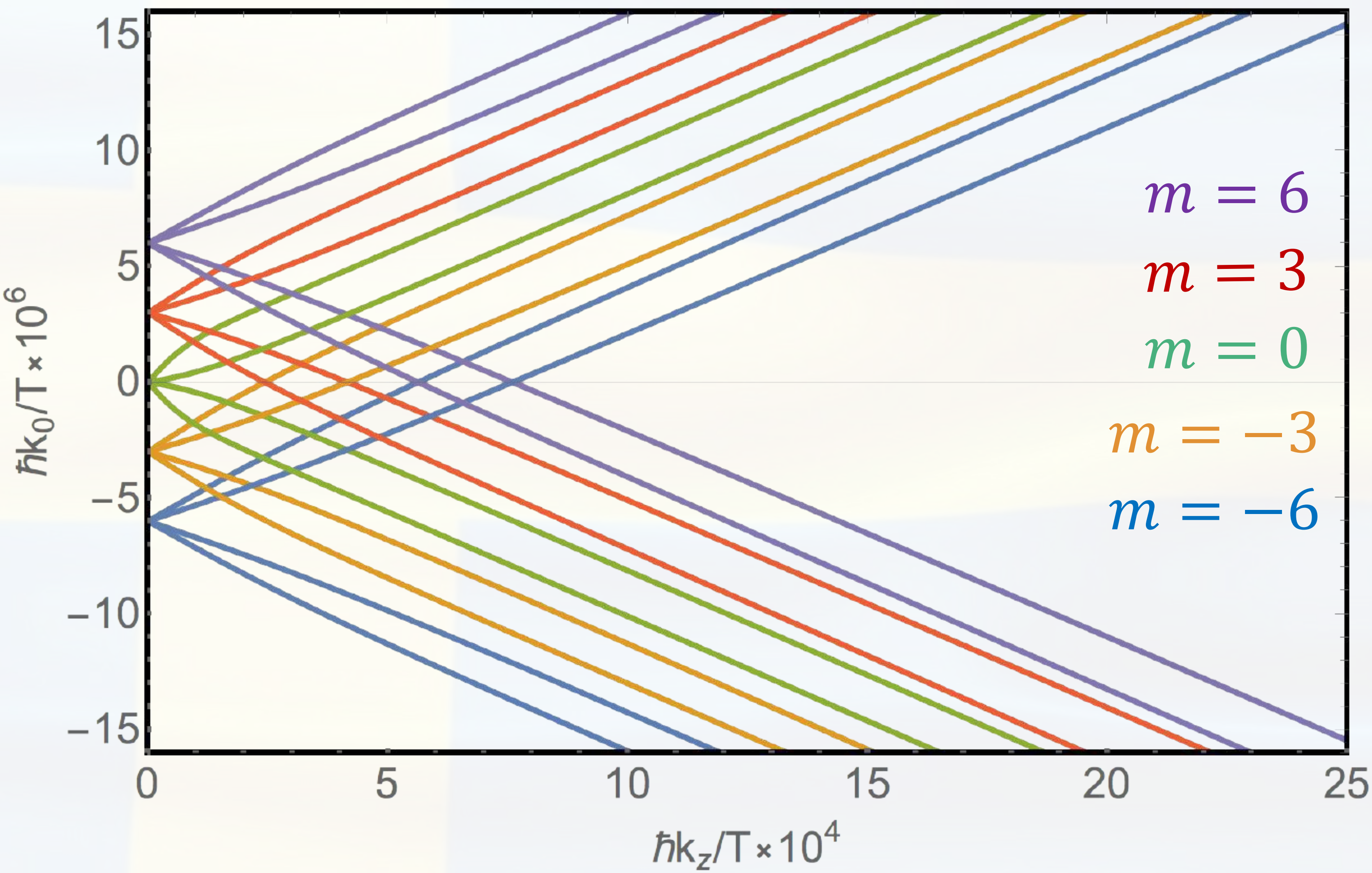
Here

$$\mathcal{B}_\pm = B - \frac{en_{\text{eq}}\Omega}{6k_\perp^2}\left[2(m\pm 1)(m\pm 2) + k_\perp^2 R^2\right]$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Alfven waves ($T \gg \mu$)

- Alfven waves with angular momenta $m = 0, \pm 3, \pm 6$



[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Without dynamical fields

- Sound wave and two chiral waves ($T \gg \mu$):

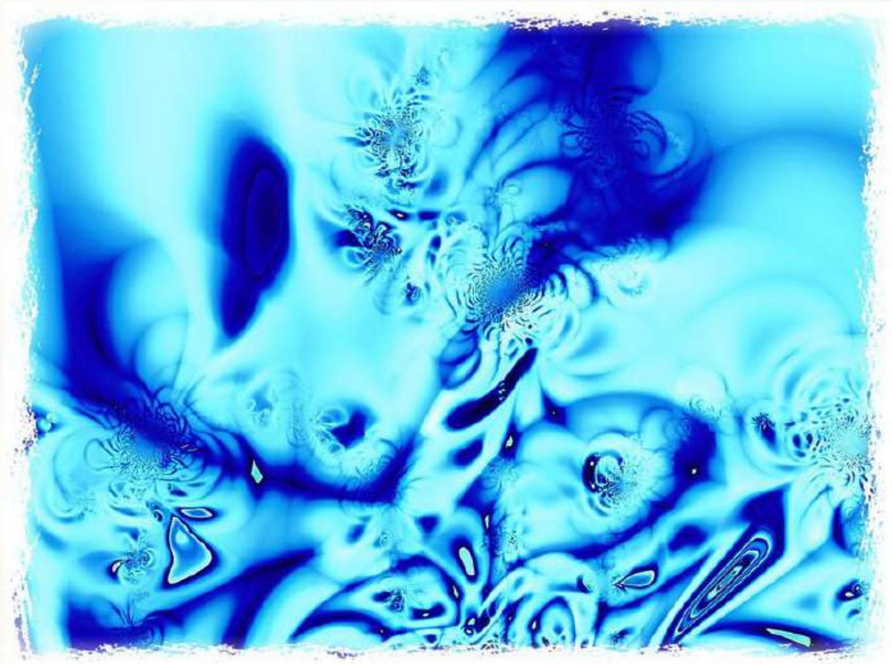
$$k_0 = \frac{s_e k}{\sqrt{3}} + \frac{2}{3} m\Omega - \frac{2}{15} i\tau k^2 \qquad \text{(sound)}$$

$$k_0 = m\Omega + s_e \frac{2k_z\Omega}{k} - \frac{1}{5} ik^2\tau \qquad \text{(CVW?)}$$

$$k_0 = m\Omega + s_e \frac{3e\mathcal{B}_0 \hbar k_z}{2\pi^2 T^2} - \frac{1}{3} ik^2\tau \qquad \text{(CMW?)}$$

- There are more propagating (fewer diffusive) modes
- CVW & CMW appear only in nondynamical regime

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]



COLLECTIVE MODES IN COLD PLASMA

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Charged plasma at $\Omega = 0$

- Plasmon modes ($\mu \gg T$):

$$k_0^{(s)} = s_e \frac{e\mu}{\sqrt{3}\pi\hbar^{3/2}} + s\frac{eB}{2\mu} - \frac{ie^2\tau T^2}{18\hbar^3} - \frac{1}{10} i\tau k^2$$

where $s_e = \pm 1$ and $s = -1, 0, 1.$

- Helicon ($\mu \gg T$):

$$k_0 = s_e \frac{3\pi^2 eBkk_z\hbar^3}{e^2\mu^3} - \frac{3\pi^2\hbar^3}{5e^2\mu^2} i\tau k^4$$

which are good propagating modes at moderately strong magnetic fields

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Charged plasma at $\Omega \neq 0$

- Plasmon modes ($\mu \gg T$):

$$k_0^{(s)} = \pm \frac{e\mu}{\sqrt{3}\pi\hbar^{3/2}} + s\frac{e\mathcal{B}_s}{2\mu} - \frac{ie^2\tau T^2}{18\hbar^3} - \frac{1}{10}i\tau k^2$$

(not affected by Ω at linear order)

- Helicon ($\mu \gg T$):

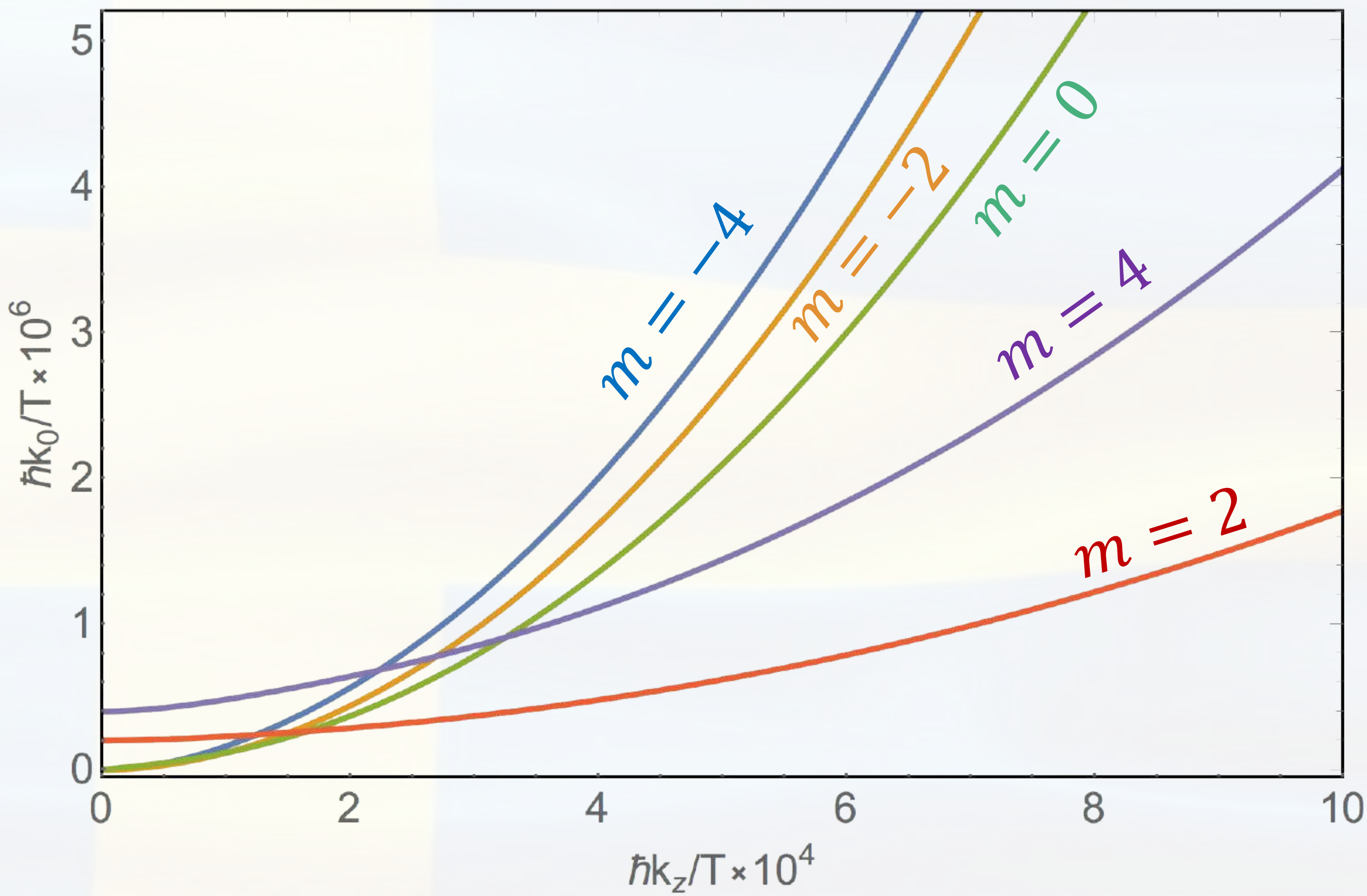
$$k_0 = m\Omega\left(\frac{1}{2} - \frac{k_z^2}{k_\perp^2}\right) + s\sqrt{\frac{m^2\Omega^2}{4} + \frac{9\pi^4(\mathcal{B}_+ + \mathcal{B}_-)^2 k^2 k_z^2 \hbar^5}{4\mu^6}}$$

which also has a small imaginary part (not shown)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Helicons ($\mu \gg T$)

- Helicons with angular momenta $m = 0, \pm 2, \pm 4$:



- Note that the modes are gapless for all $m \leq 0$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

No dynamical fields, $\Omega \neq 0$

- Sound modes instead of plasmons ($\mu \gg T$):

$$k_0 = \frac{s_e k}{\sqrt{3}} + \frac{2}{3} m\Omega - \frac{2}{15} i\tau k^2$$

(as expected when Gauss's law is turned off)

- There is a helicon mode, but its spectrum is strongly modified ($\mu \gg T$):

$$k_0 = m\Omega + \frac{s_e}{5\mu} e\mathcal{B}_s k k_z \tau^2 - \frac{1}{5} i\tau k^2$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Summary

- Neutral chiral plasma has chiral vortical waves
  - Speeds for opposite direction waves differ @ $\mu_5 \neq 0$
- Equilibrium state of charged rotating plasma is radially nonuniform
- Propagating (not overdamped) hydro modes in charged rotating plasma are
  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density
- Dynamical electromagnetism plays a crucial role
- Many interesting hydro modes in Dirac/Weyl materials