University of Stavanger
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Norway

Anomalous chiral matter:
from quark-gluon plasma to novel materials

Igor Shovkovy
Arizona State University

Chiral forms of matter

- **Early Universe**, e.g.,

  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]

- **Heavy-ion collisions**, e.g.,

  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

- **Super-dense matter in compact stars**, e.g.,

  [Yamamoto, Phys.Rev. D93, 065017 (2016)]

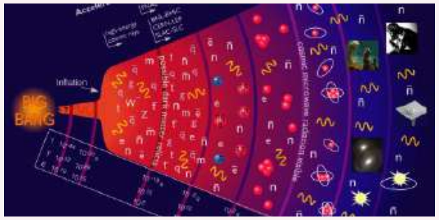
- **Ultra-relativistic jets from black holes**

- **Superfluid ^{3}He-A**, e.g.,

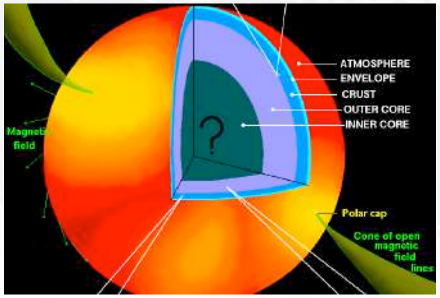
  [Volovik, JETP Lett. 105, 34 (2017)]

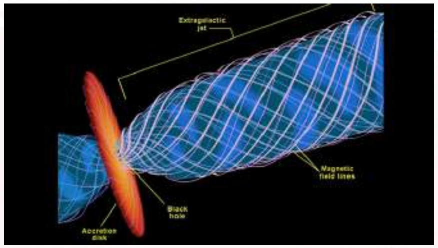
- **Dirac/Weyl (semi-)metals**, e.g.,

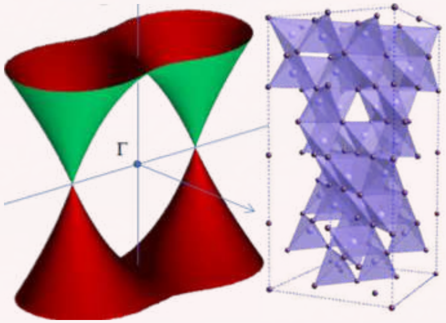
  [Li et. al. Nature Phys. 12, 550 (2016)]




Credit: Brookhaven National Laboratory







Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{\boldsymbol{p}}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{\boldsymbol{p}}}{|\vec{\boldsymbol{p}}|}\Psi = \text{sign}(p_0)\gamma^5\Psi$$

For particles ($p_0 > 0$): chirality = helicity

For antiparticles ($p_0 < 0$): chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

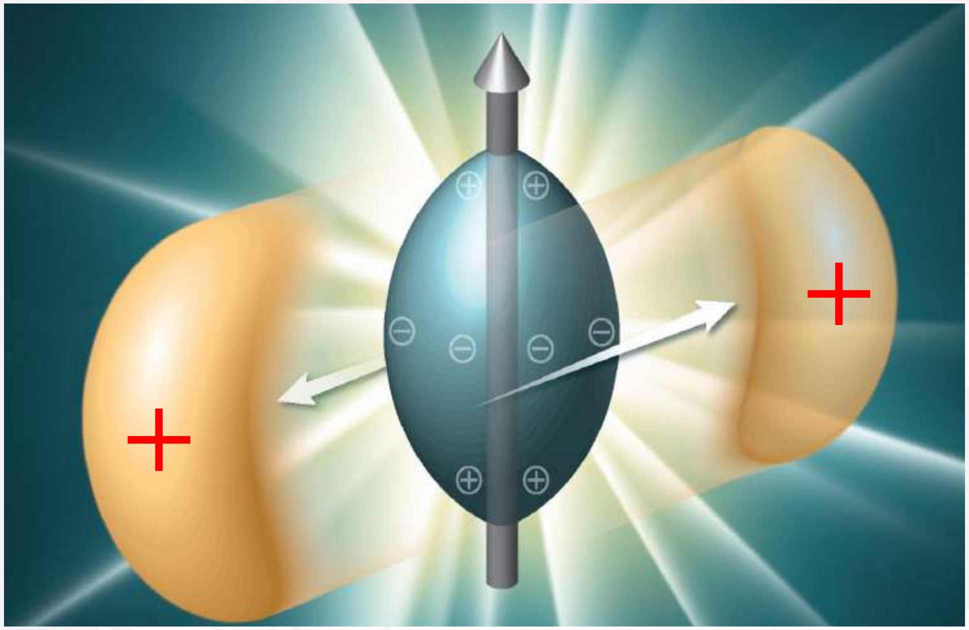
Anomalous chiral matter

- Matter made of chiral fermions may allow $n_L \neq n_R$
- The chiral charge ($n_R - n_L$), unlike the electric charge ($n_R + n_L$), is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral anomaly can have *macroscopic* implications in chiral matter



ANOMALOUS EFFECTS IN HEAVY-ION COLLISIONS

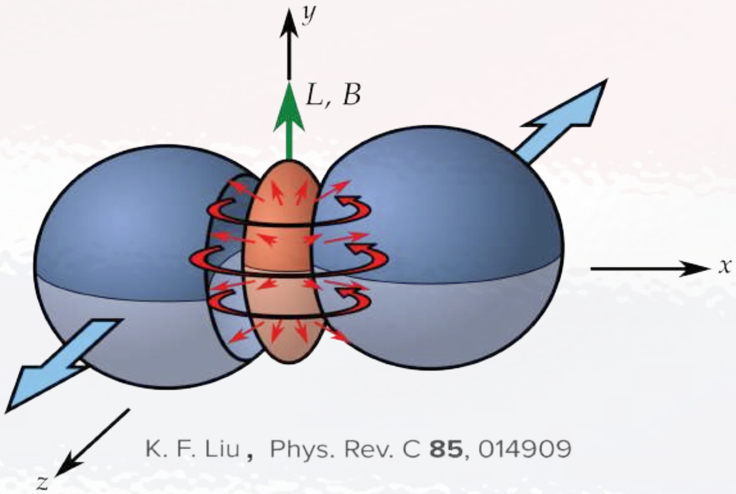
[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



K. F. Liu, Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak &. Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108]

- Magnetic field estimate:

$$\mathrm{B} \sim 10^{18} \text{ to } 10^{19}\ \mathrm{G}\ (\sim 100\ \mathrm{MeV})$$

- Vorticity estimate:

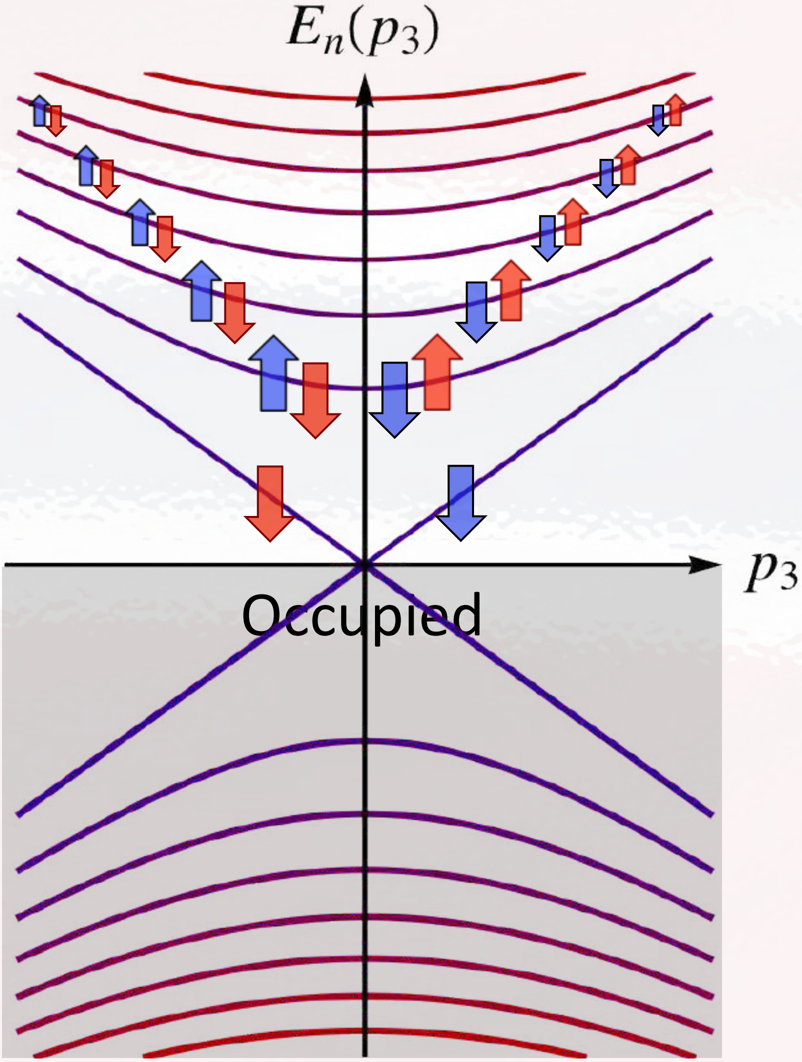
$$\omega \sim 10^{21} s^{-1}\ (\sim 10\ \mathrm{MeV})$$

Effect of magnetic field

- Dirac equation with massless fermions

$$\left[i\gamma^{0}\partial_{0} - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum



$$E_{n}^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^{2}}$$

where

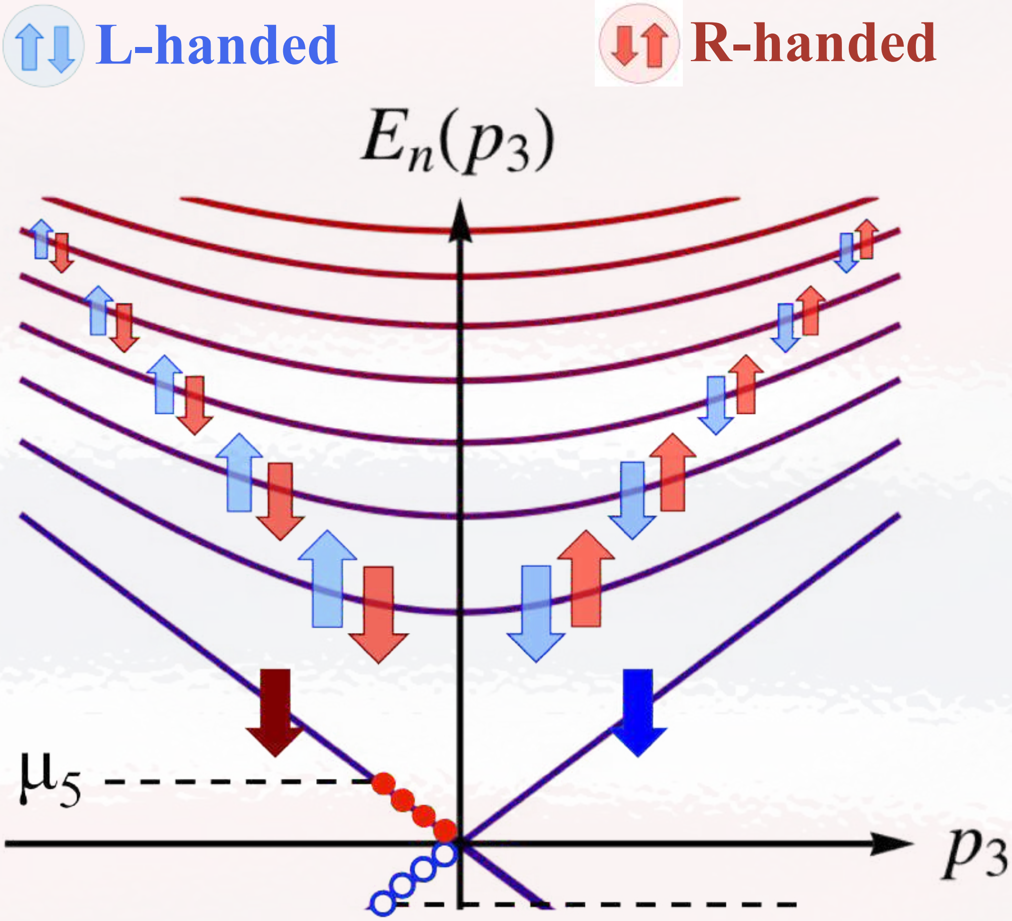
$$n = \overbrace{s}^{\boxed{s = \pm 1/2 \quad \text{(spin)}}} + \underbrace{k + \frac{1}{2}}_{\boxed{k = 0,\ 1,\ 2,\ldots \text{(orbital)}}}$$

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

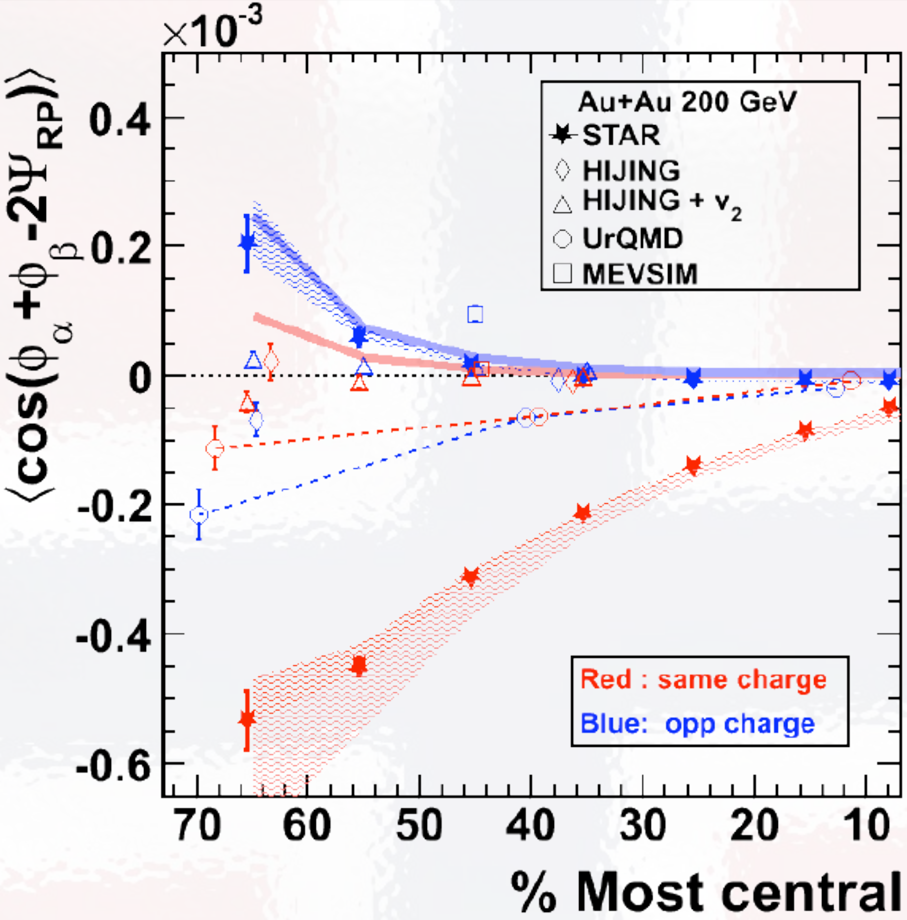
- R-handed states $p_3<0$ give current in +*z* direction
- L-handed holes $p_3<0$ give current in +*z* direction too!

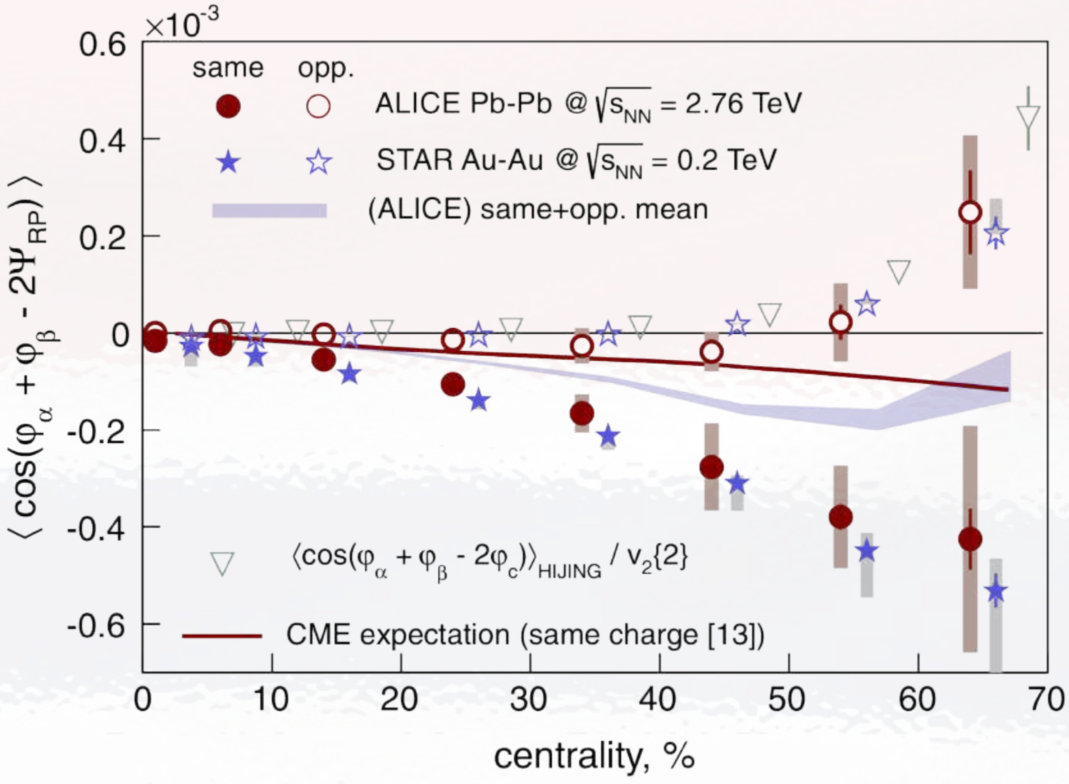


CME current: $$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence





Correlations of same & opposite charge particles:

$$\left\{ \begin{array}{l} \langle\cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\mp} - 2\Psi_{\mathrm{RP}})\rangle \\ \langle\cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\pm} - 2\Psi_{\mathrm{RP}})\rangle \end{array} \right.$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)]
[ALICE Collaboration, Phys. Lett. B777, 151 (2018)]

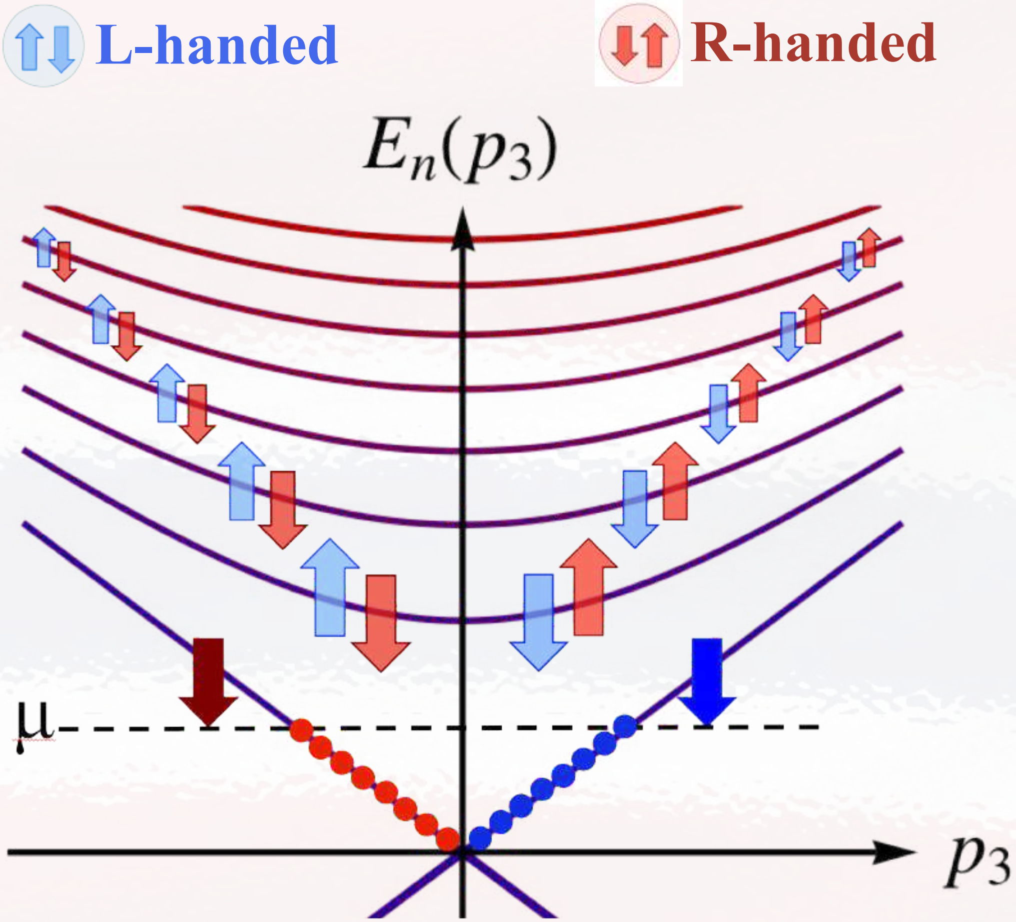
Chiral Separation Effect (μ≠0)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

- R-handed states $p_3<0$
- L-handed states $p_3>0$

This gives rise to a nonzero axial current density (CSE):

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$



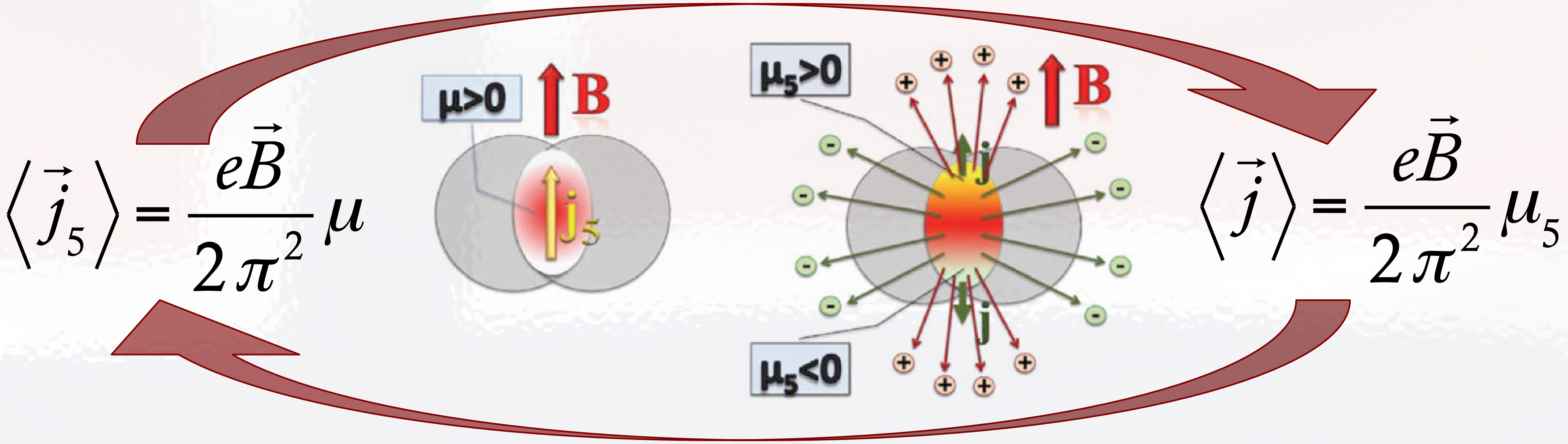
[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Note that the latter can be also interpreted as a spin density:

$$\langle \vec{j}_5 \rangle = \langle \psi^\dagger \vec{\Sigma} \psi \rangle, \quad \text{where} \quad \Sigma^k = \frac{i}{2}\varepsilon^{klm}\left[\gamma_l, \gamma_m\right]$$

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW



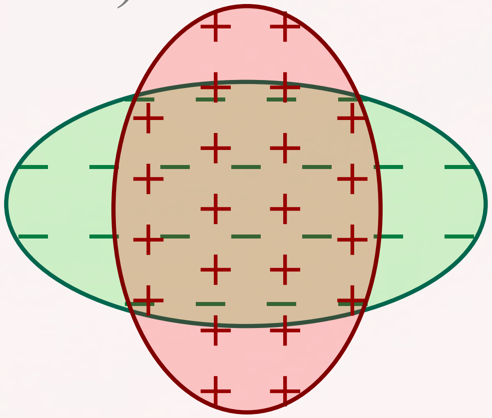
$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

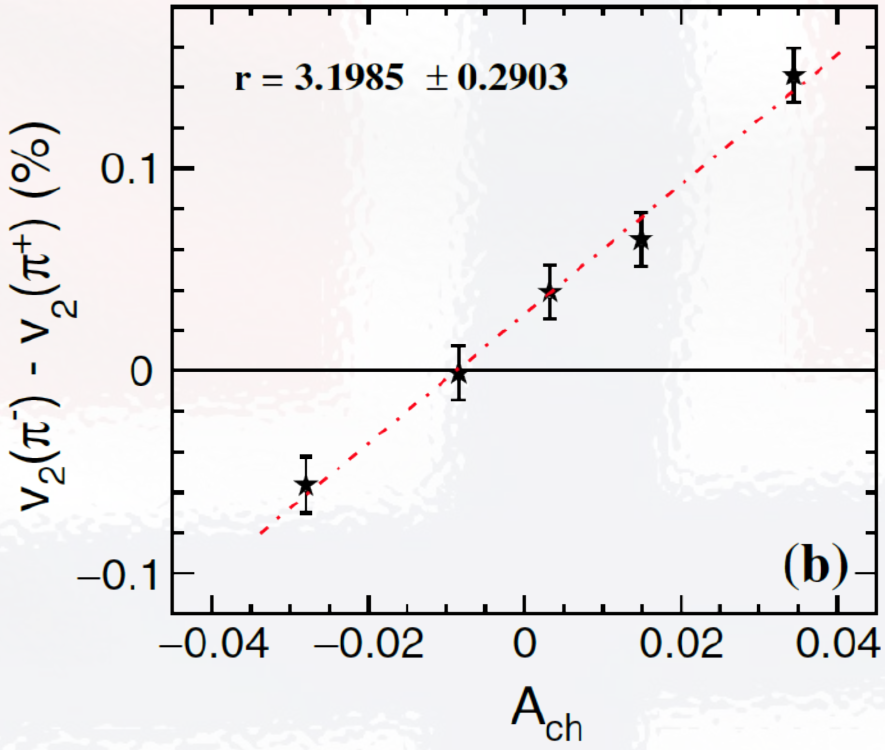
$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$

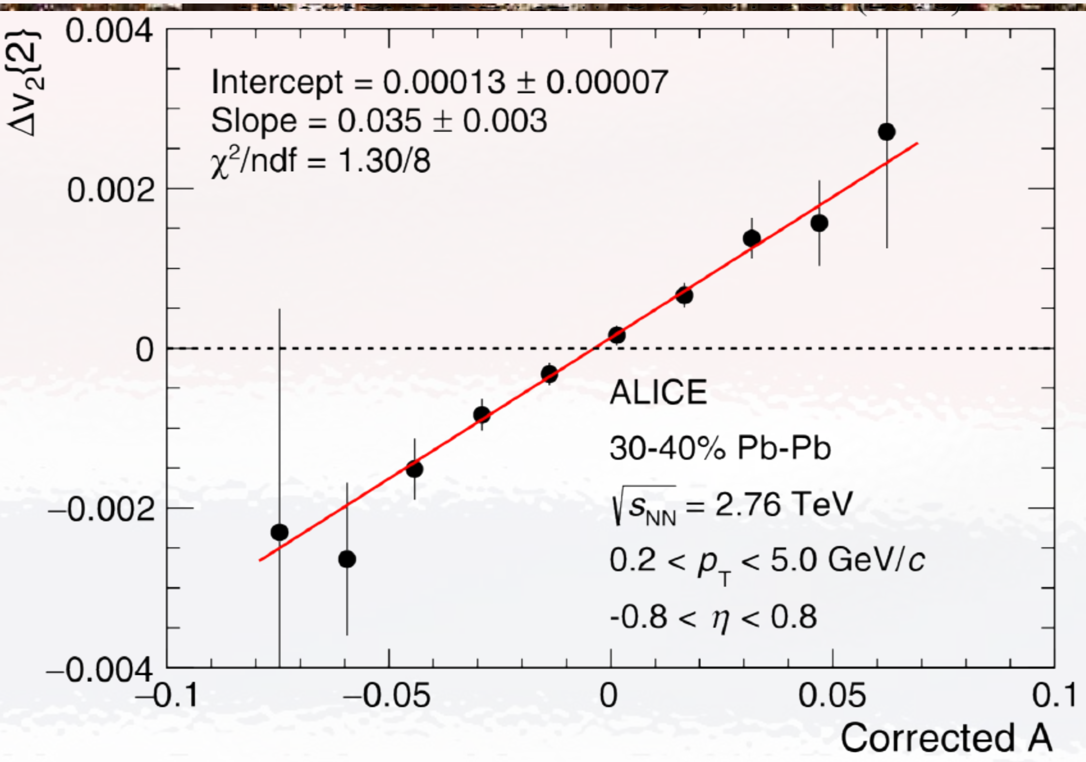


where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

CMW: Experimental evidence





[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Higher harmonics of particle correlations are problematic…

Background effects may dominate over the signal!

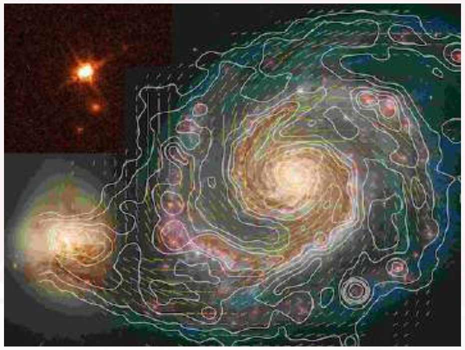
[CMS Collaboration, arXiv:1708.08901]

There are theoretical reasons that CMW is overdamped

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Anomalous plasmas elsewhere?

- Inverse magnetic cascade may produce seeds of helical magnetic fields in the early Universe



[Vilenkin, Phys. Rev. D22, 3080 (1980)],
[Joyce & Shaposhnikov, astro-ph/9703005],
[Giovannini & Shaposhnikov, hep-ph/9710234]

- Eigenmodes of long wavelength and fixed helicity grow:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma}\left(4\pi C_5\mu_5 - ck\right)B_k$$

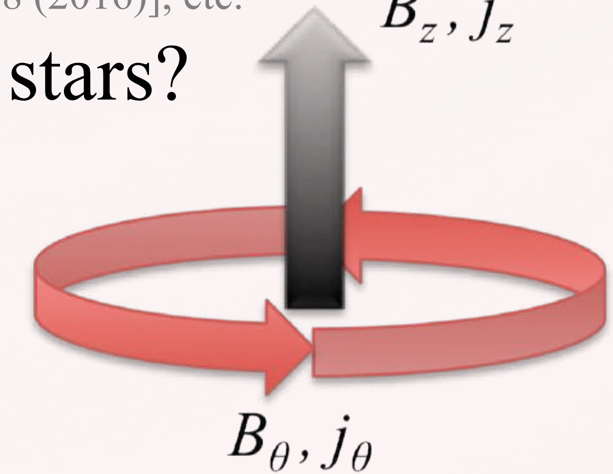
[Boyarsky et al., PRL **108**, 031301 (2012)], [Tashiro et al., PRD **86**, 105033 (2012)],
[Manuel et al., PRD **92**, 074018 (2015)], [Hirono et al., PRD **92**, 125031 (2015)],
[Buividovich et al., PRD **94**, 025009 (2016)], [Gorbar et al., PRD **94**, 103528 (2016)], etc.

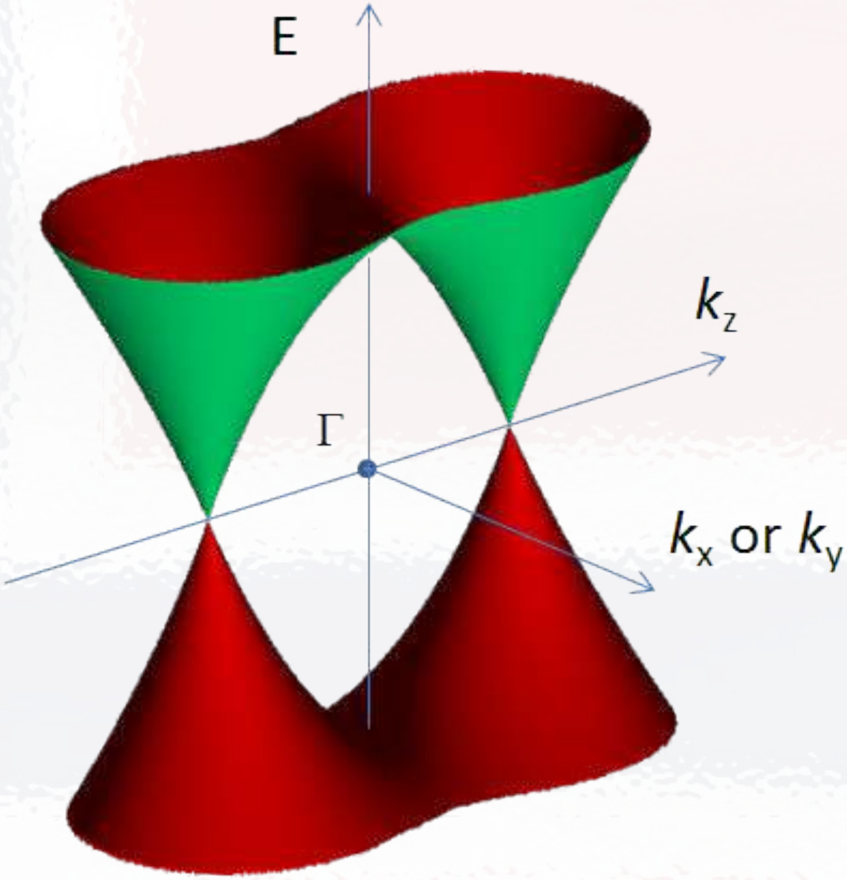
- Strong helical magnetic field in compact stars?

[Ohnishi, Yamamoto, arXiv:1402.4760]
[Yamamoto, Phys. Rev. D **93**, 065017 (2016)]
[Dvornikov, J. Exp. Theor. Phys. **123** 967 (2016)]

- Perhaps, not…

[Grabowska, Kaplan, Reddy, Phys. Rev. D **91**, 085035 (2015)]





Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL MATERIALS

[Liu et al., Science **343**, 864 (2014)], [Neupane et al., Nature Commun. **5**, 3786 (2014)]
[Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)], [Li et al., Nature Physics **12**, 550 (2016)]
[S.-Y. Xu et al., Science **349**, 613 (2015)], [B. Q. Lv et al., Phys. Rev. X **5**, 031013 (2015)]
[S.-Y. Xu et al., Nature Physics **11**, 748 (2015)], [S.-Y. Xu et al., Science Adv. **1**, 1501092 (2015)]
[F. Y. Bruno et al., Phys. Rev. B **94**, 121112 (2016)]

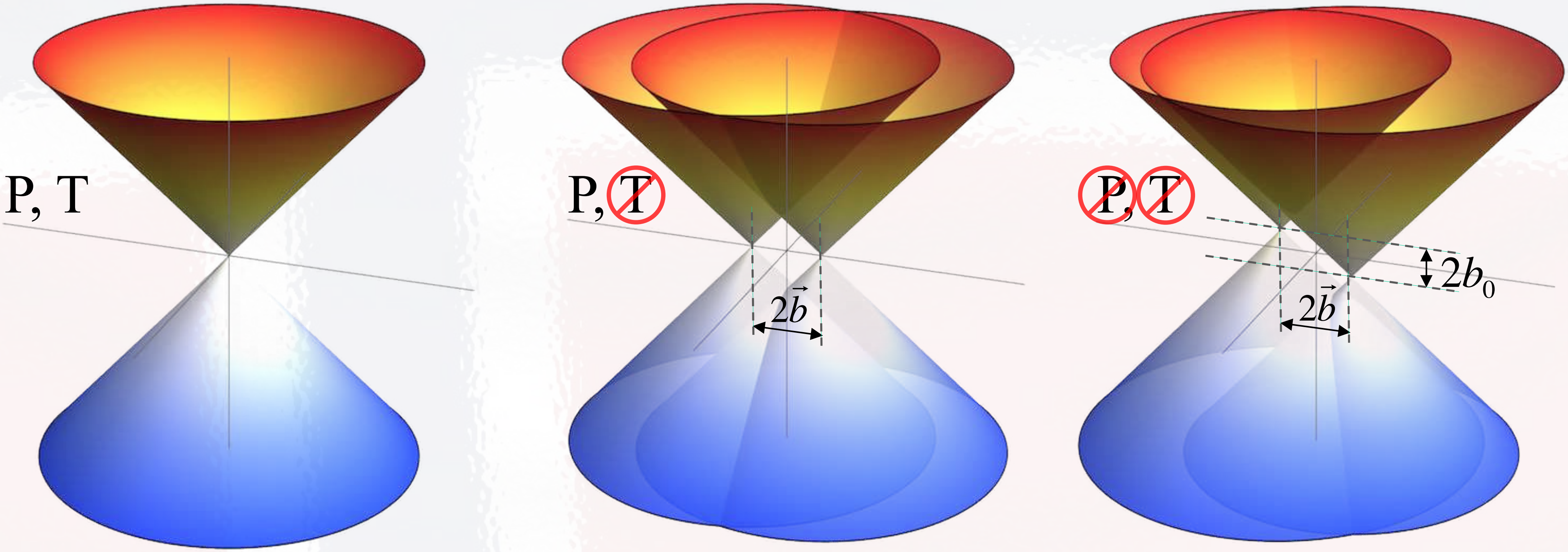
Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \overset{\cancel{T}}{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5} + \overset{\cancel{P}}{b_0\gamma^0\gamma^5}\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$)

Weyl (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Weyl fermions on a lattice

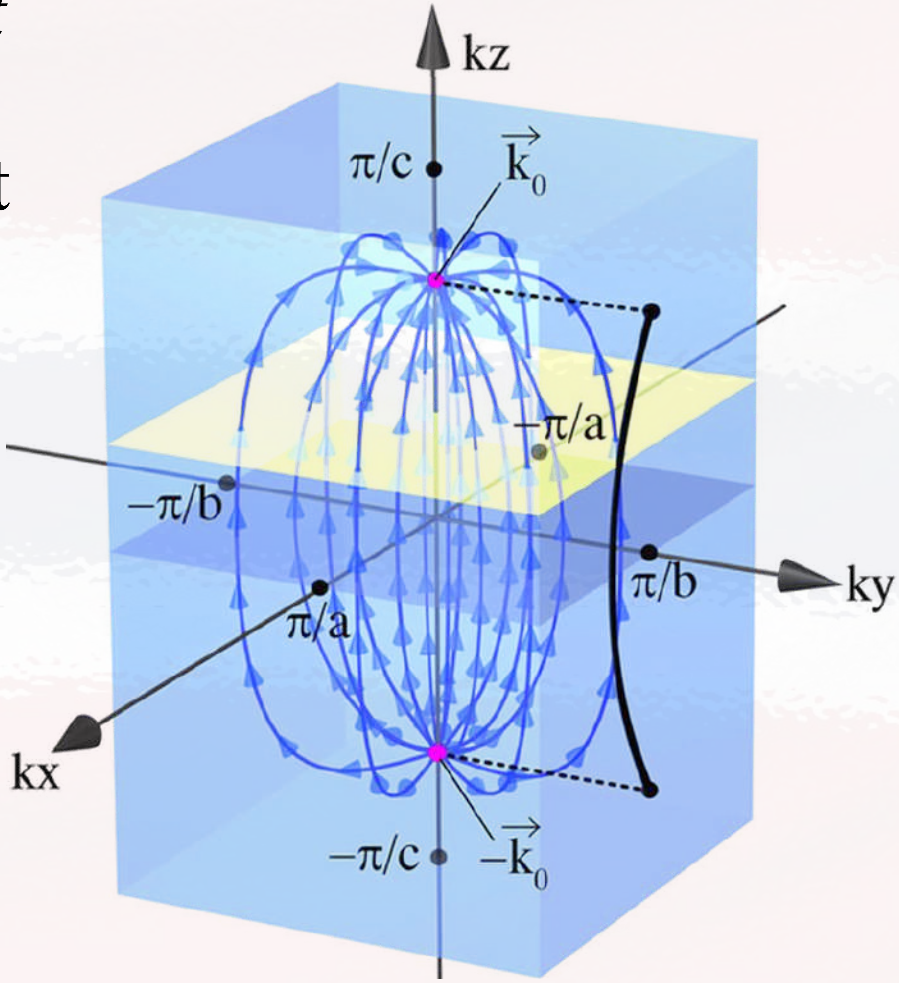
- In solid state physics, the momentum space (Brillouin zone) is *compact*

- *Berry curvature* is nonzero at Weyl nodes

$$\vec{\boldsymbol{\Omega}}_k = \lambda \frac{\vec{k}}{2k^3}$$

- Weyl nodes carry nonzero *topological charges*

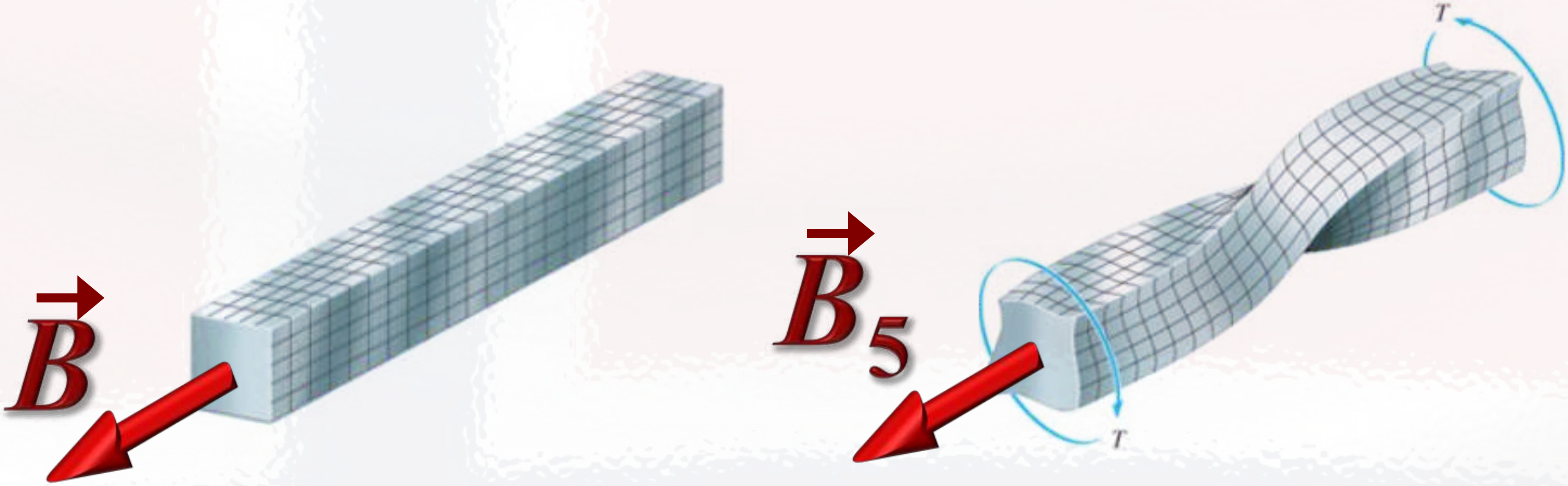
$$C = \frac{1}{2\pi} \oint \vec{\boldsymbol{\Omega}}_k \cdot d\vec{\boldsymbol{S}} = \lambda = \pm 1$$



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

- Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]



PSEUDO-ELECTROMAGNETIC FIELDS

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]
[Grushin, Venderbos,Vishwanath, Ilan, Phys. Rev. X 6, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B 94, 241405 (2016)]
[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Strain in Weyl materials

- Strains in the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

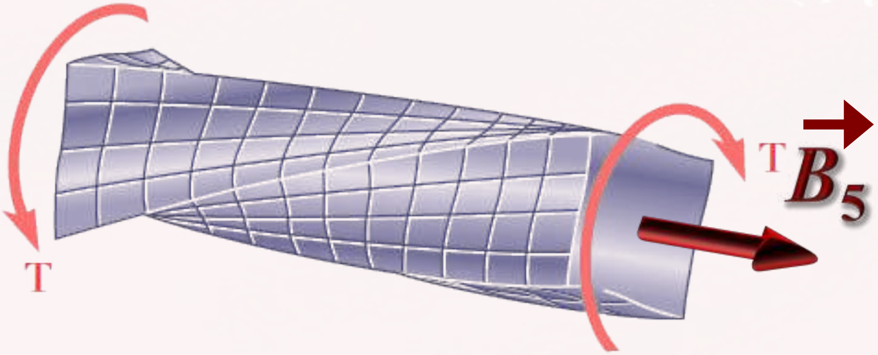
where the chiral gauge fields are

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
[Grushin, Venderbos,Vishwanath, Ilan, PRX **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$



leading to the pseudo-EM fields

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$

CONSISTENT CHIRAL KINETIC THEORY

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral kinetic theory

- Kinetic equation:

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t}+\frac{\left[e\tilde{\mathbf{E}}_\lambda+\frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda)+\frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}}f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$

$$+\frac{\left[\mathbf{v}+e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda)+\frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}}f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}=0$$

where $\tilde{\mathbf{E}}_\lambda=\mathbf{E}_\lambda-(1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}$, $\mathbf{v}=\boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}$, and

$$\epsilon_{\mathbf{p}}=v_F p\left[1-\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda=\lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Current and chiral anomaly

- The definitions of density and current are

$$\begin{aligned}
\rho_\lambda &= e \int \frac{d^3p}{(2\pi\hbar)^3} \left[1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)\right] f_\lambda, \\
\mathbf{j}_\lambda &= e \int \frac{d^3p}{(2\pi\hbar)^3} \left[\mathbf{v} + \frac{e}{c}(\mathbf{v} \cdot \mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda \times \mathbf{\Omega}_\lambda)\right] f_\lambda \\
&\quad + e\mathbf{\nabla} \times \int \frac{d^3p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}} \mathbf{\Omega}_\lambda,
\end{aligned}$$

They satisfy the following anomalous relations:

$$\frac{\partial \rho_5}{\partial t} + \mathbf{\nabla} \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5)\Big]$$ ✔

$$\frac{\partial \rho}{\partial t} + \mathbf{\nabla} \cdot \mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B})\Big]$$ ✘

Consistent definition of current

- Bardeen-Zumino (Chern-Simons) term is needed,

$$\delta j^{\mu} = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda}A^5_{\nu}F_{\rho\lambda}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]

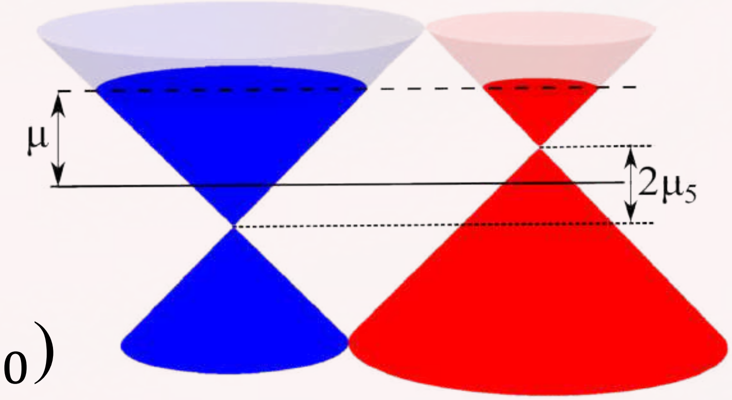
or

$$\delta\rho = -\frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{b}\cdot\mathbf{B})$$

$$\delta\mathbf{j} = -\frac{e^3}{2\pi^2\hbar^2 c}b_0\mathbf{B} + \frac{e^3}{2\pi^2\hbar^2 c}[\mathbf{b}\times\mathbf{E}]$$

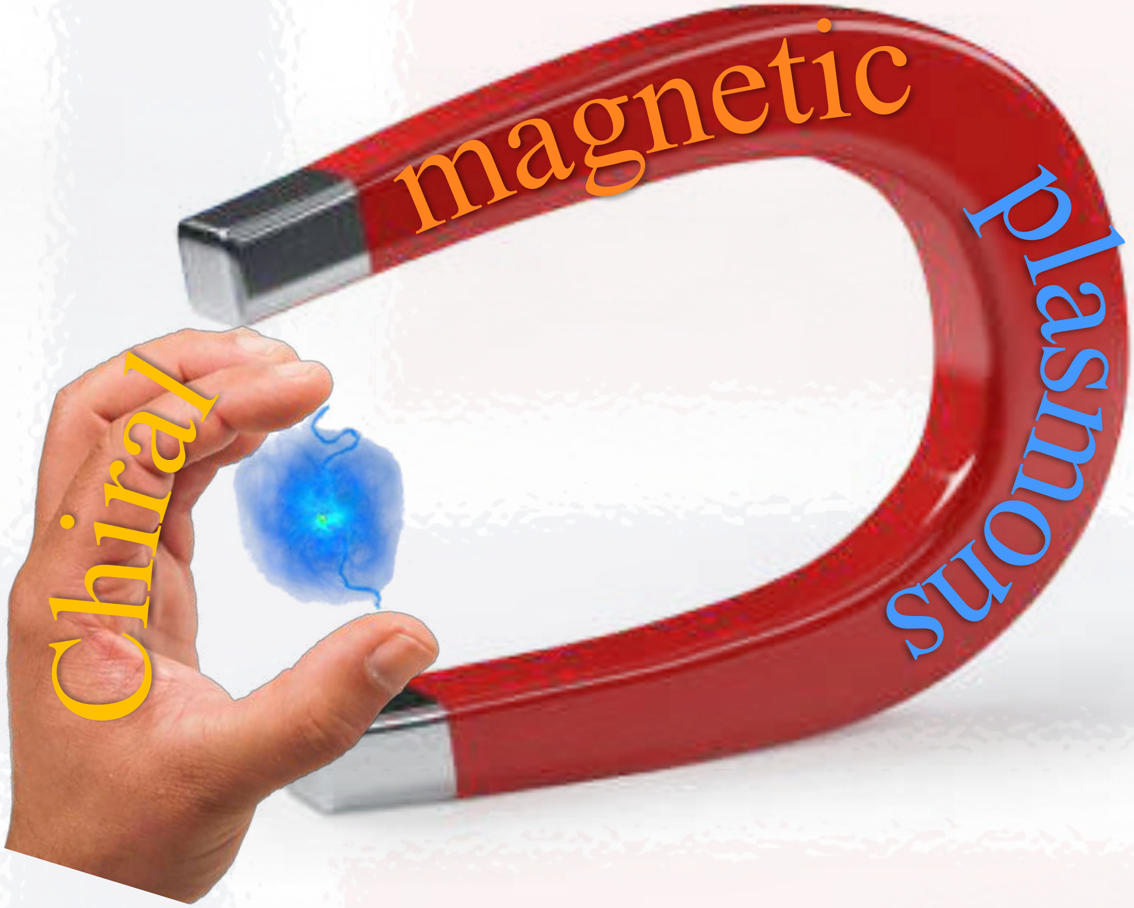
[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **96**, 085130 (2017)]

- Its role and implications:
  - Electric charge is conserved ($\partial_{\mu}J^{\mu} = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)



Chiral effects in Weyl materials

- Any observable properties of Weyl materials directly sensitive to b_0 and $\vec{b}$? Chiral anomaly? Topology?
- Potentially, there are many observable effects:
  - Anomalous Hall effect
  - Negative magnetoresistance
  - Strain/torsion induced CME
  - Quantum oscillations of the density of states
  - Strain/torsion dependent resistance
  - Unusual features of collective modes
  - Anomalous electric/chiral/thermal transport
  - Unusual features of nonlocal transport



CHIRAL MAGNETIC PLASMONS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\mathrm{tr}}^{\pm} = \Omega_e\sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

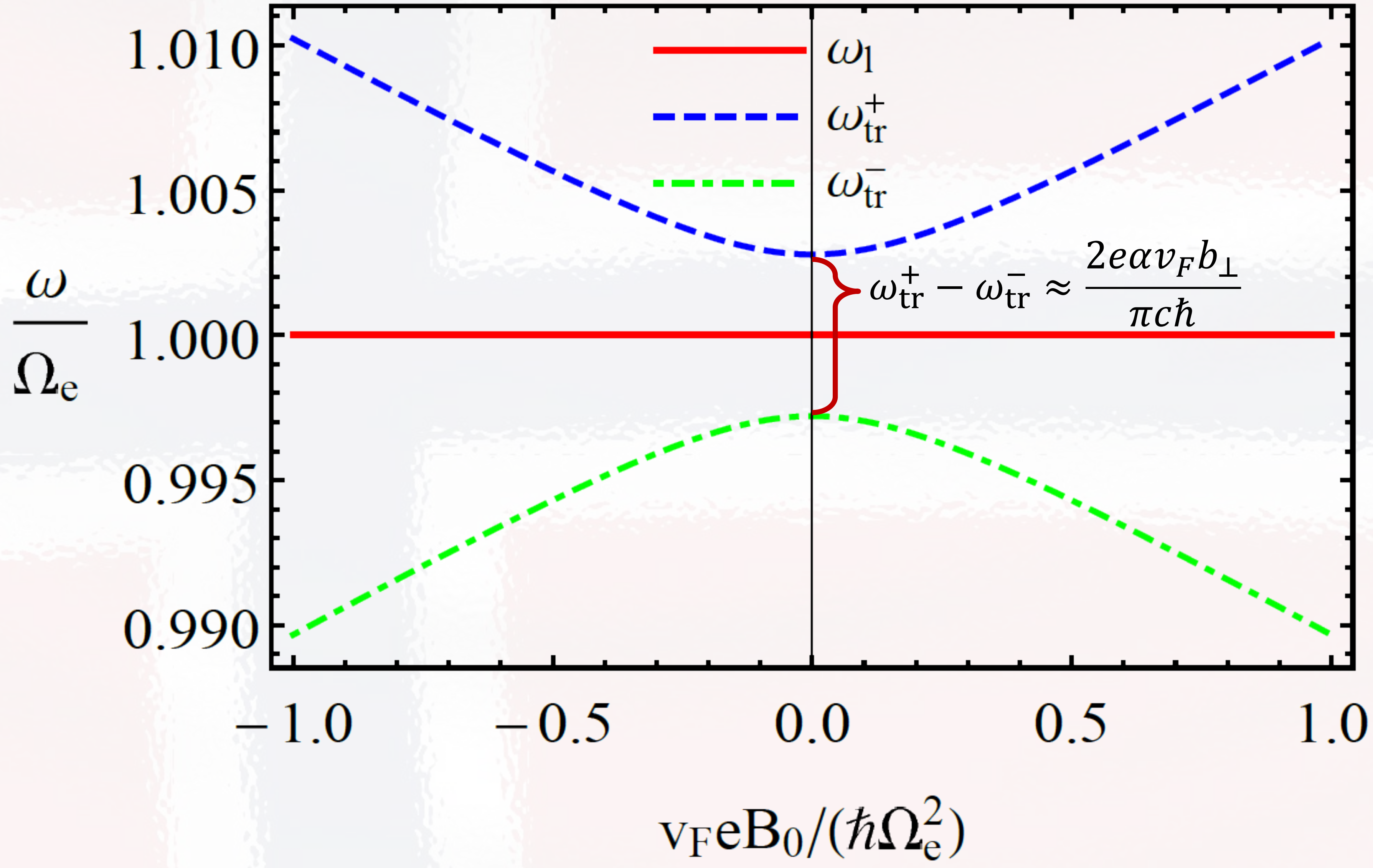
and

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\left\{9\hbar^2 b_\perp^2 + \left[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_\parallel - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda}F\left(\frac{\mu_\lambda}{T}\right)\right]^2\right\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

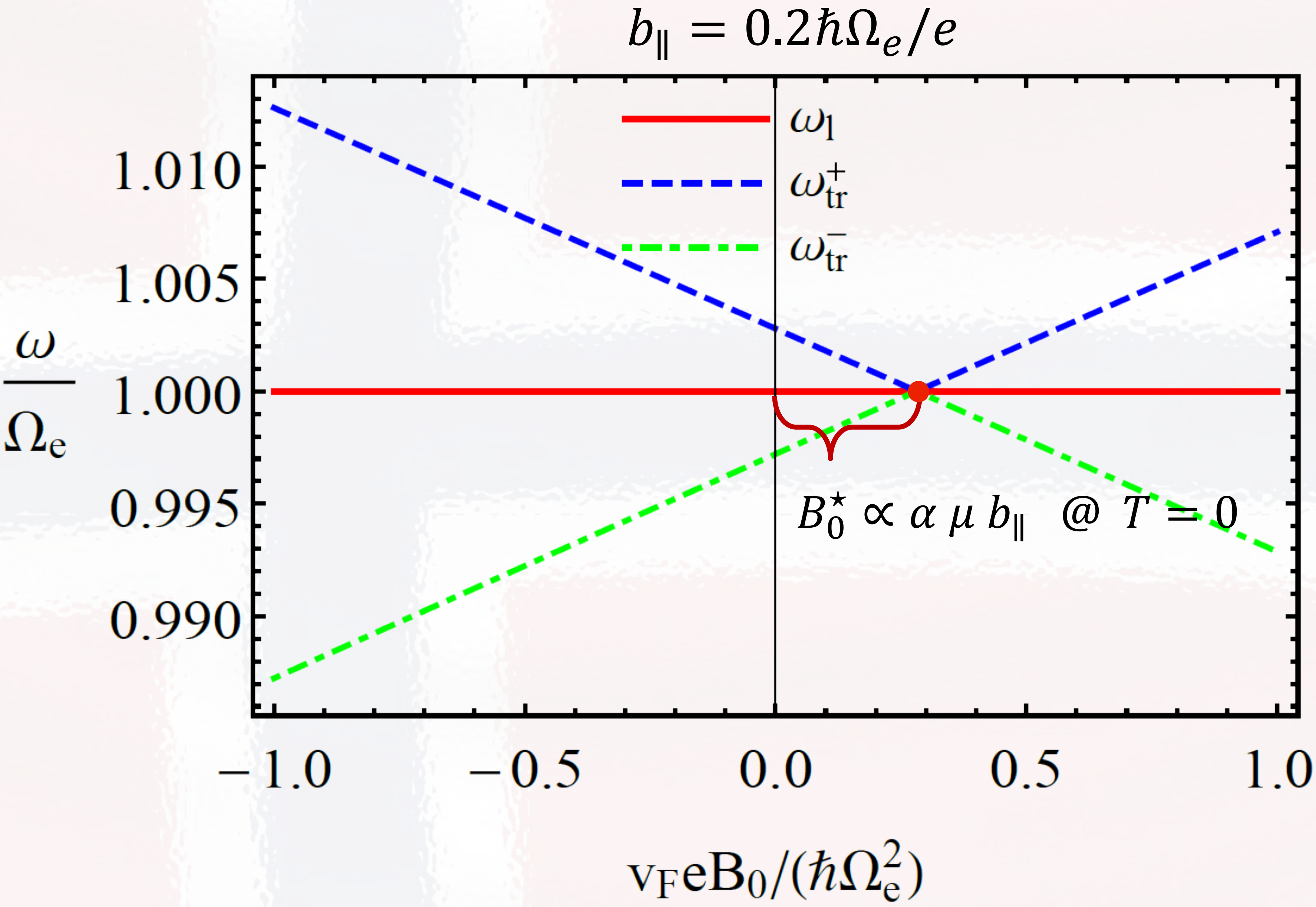
Plasmon frequencies, $\vec{B} \perp \vec{b}$

$b_\perp = 0.2\hbar\Omega_e/e$

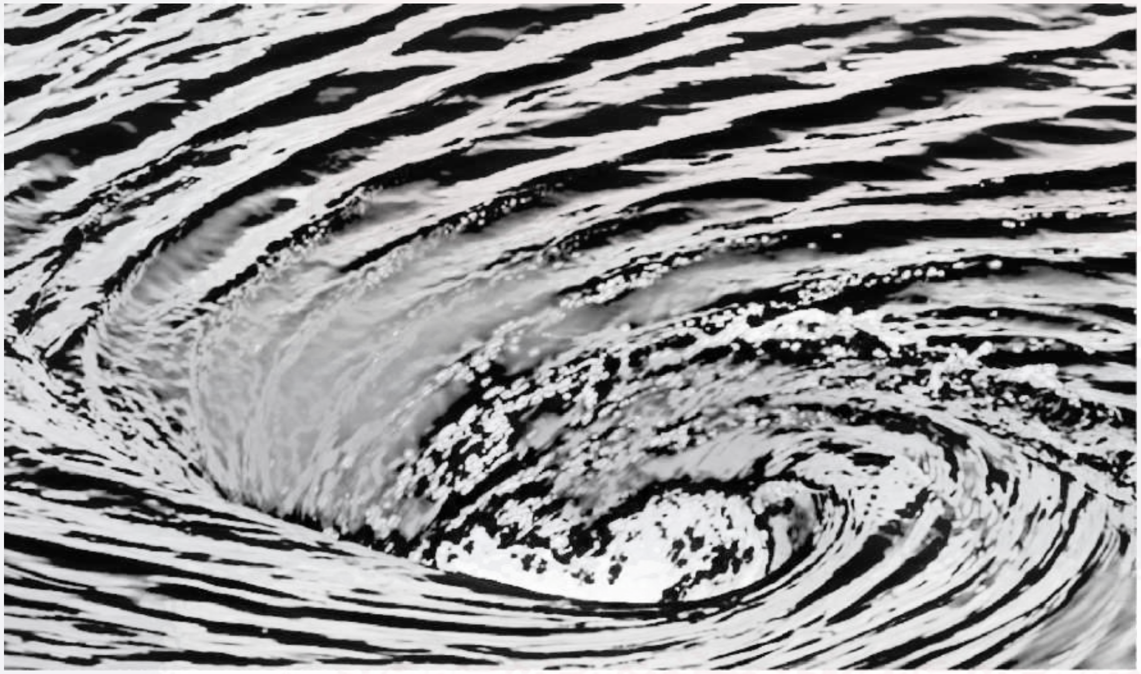


[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmon frequencies, $\vec{B} \parallel \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]



ELECTRON TRANSPORT IN HYDRODYNAMIC REGIME

Hydrodynamics in Weyl metals

The Euler equation for electron fluid: [Gurzhi, JETP **17**, 521 (1963)]

$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right) = -en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_\mathbf{r})$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **97**, 121105(R) (2018)]

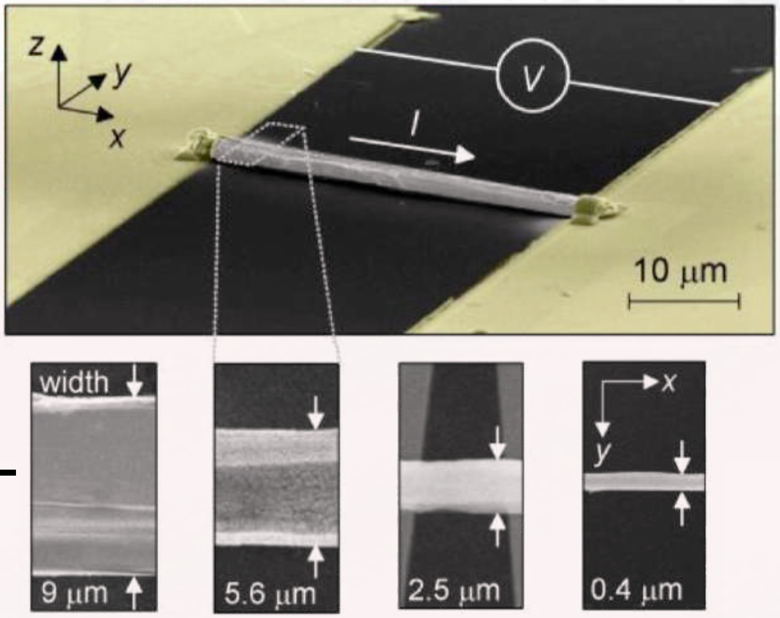
The energy conservation

$$\partial_t\epsilon = -\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_\mathbf{r})$$

+ Maxwell equations with the Chern-Simons currents

$$\rho_{\mathrm{CS}} = -\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\mathrm{CS}} = -\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$



Experimental evidence in tungsten diphosphide (WP_2) [Gooth et al., Nature Commun. **9**, 4093 (2018)]

Rich spectrum of hydro modes

- Magneto-acoustic wave ($\rho = 0$):

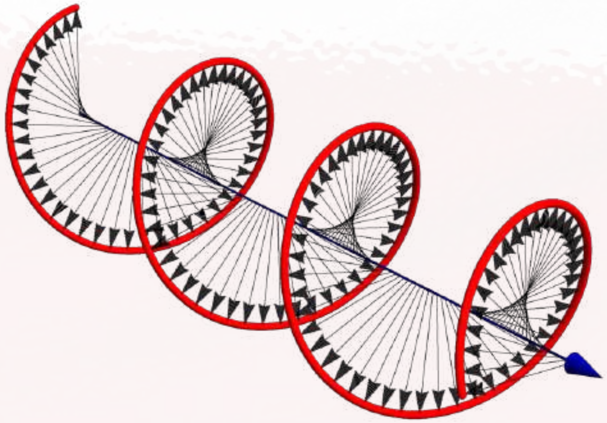
$$\omega_{\mathrm{s},\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau}\sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)}\left[2|\mathbf{k}|^2 B_0^2 - (\mathbf{k}\cdot\mathbf{B}_0)\right]}{3w_0}}$$

- *Gapped* chiral magnetic wave ($\rho = 0$):

$$\omega_{\mathrm{gCMW},\pm} = \pm \frac{eB_0\sqrt{3v_F^3\left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_\parallel^2\right)}}{2\pi^2 T_0^2 c\sqrt{\varepsilon_e \hbar}}$$

- Helicons ($\rho \neq 0$):

$$\omega_{\mathrm{h},\pm} \approx \mp \frac{c k_\parallel^2 B_0}{4\pi \mu_m \rho_0} - \frac{i}{\tau}\frac{c^2 k_\parallel^2 w_0}{4\pi \mu_m v_F^2 \rho_0^2} + O(k_\parallel^3)$$



- New anomalous Hall waves at $\vec{b} \neq 0$, etc.

[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

Some lessons (for high energy)

- **Neutral** chiral plasma has chiral vortical waves
  - Speeds depend on the direction @ $\mu_5 \neq 0$

[Gorbar, Rybalka, Shovkovy, Phys. Rev. D 95, 096010 (2017)]

(might be important for neutrino flow in supernovas/protoneutron stars)

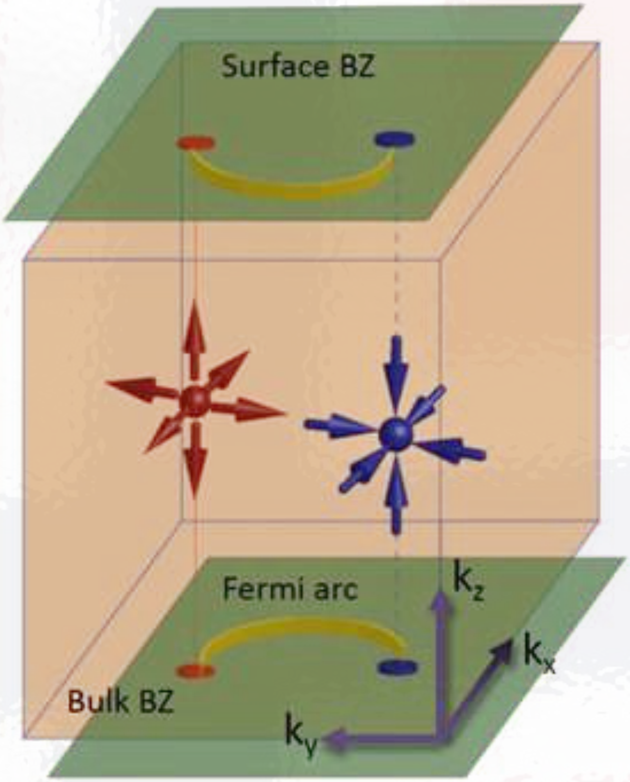
- Propagating (not overdamped) hydrodynamic modes in **charged** chiral plasma are

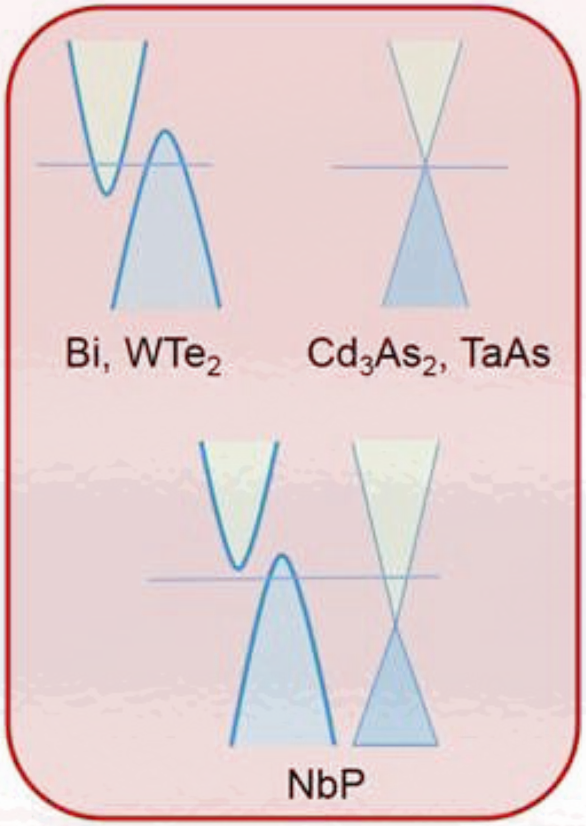
[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density
- **Dynamical electromagnetism** plays a crucial role
  - no chiral magnetic waves, special chiral Alfven waves, or other exotic modes obtained in background-field approximation
  - chiral magnetic wave predictions for HIC should be revised

Summary

- Anomalous *macroscopic* effects are expected in many forms of chiral plasmas
- Experimental search for anomalous signatures in high-energy physics is extremely *difficult*
- Low-energy *chiral fermions* can be realized in Dirac/Weyl materials
- Fundamental anomalous physics could be tested in *table-top experiments*
- Chirality and anomaly could be valuable in *applied* research

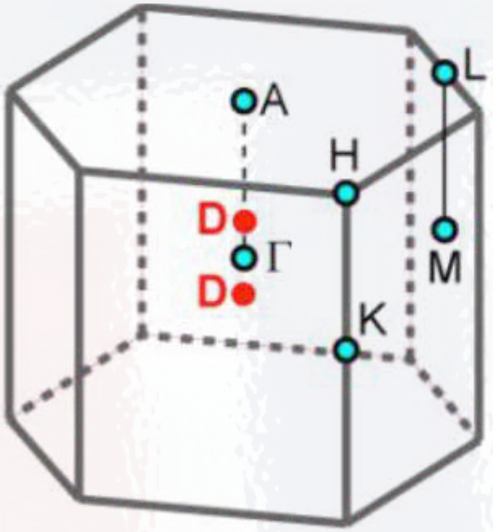


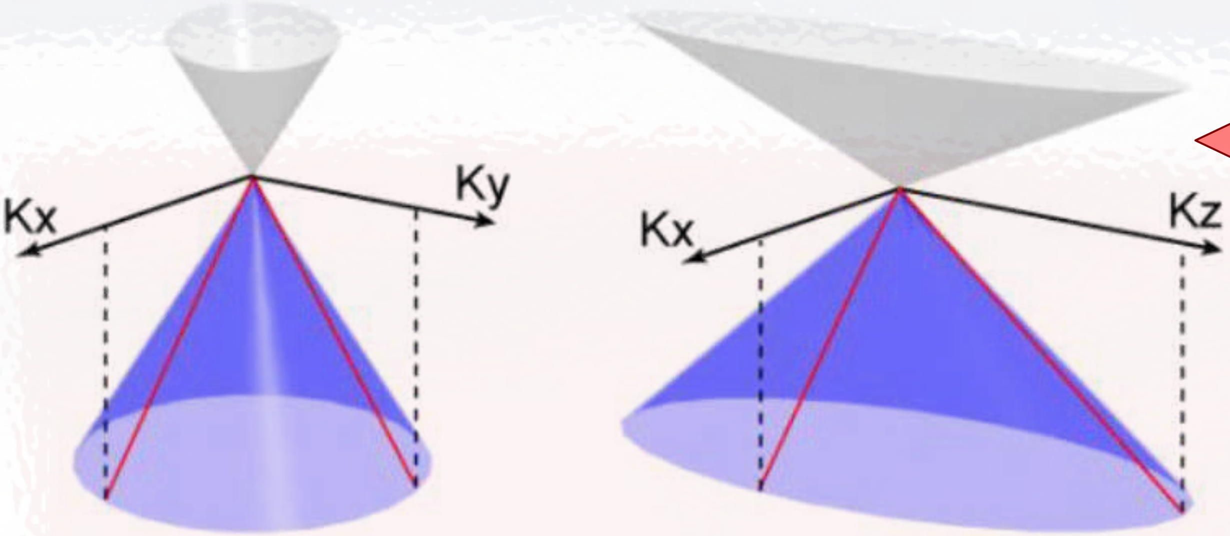


EXTRAS

Dirac materials

- $Bi_{1\text{-}x}Sb_x$ alloy (at $x \approx 4\%$)
- Na_3Bi [Liu et al., Science **343**, 864 (2014)]
- Cd_3As_2 [Neupane et al., Nature Commun. **5**, 3786 (2014)]
  [Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
- $ZrTe_5$ [Li et al., Nature Physics **12**, 550 (2016)]





$$E = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + v_z^2 k_z^2}, \quad v_x \approx v_y \approx 3.74\times10^5\ m/s, \quad v_y \approx 2.89\times10^4\ m/s$$

Weyl materials

- TaAs (tantalum arsenide) [S.-Y. Xu et al., Science 349, 613 (2015)]
  [B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
- NbAs (niobium arsenide) [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
- TaP (tantalum phosphide) [S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
- NbP (niobium phosphide) [I. Belopolski et al. arXiv:1509.07465]
- WTe_2 (tungsten telluride) [F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]