

Collective modes in chiral plasmas with dynamical electromagnetism*

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*[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]



CHIRAL MATTER

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma} \cdot \vec{p}\right) \Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \Psi = \text{sign}(p_0) \gamma^5 \Psi$$

For particles ($p_0 > 0$): chirality = helicity

For antiparticles ($p_0 < 0$): chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime

– High temperature: $T \gg m$

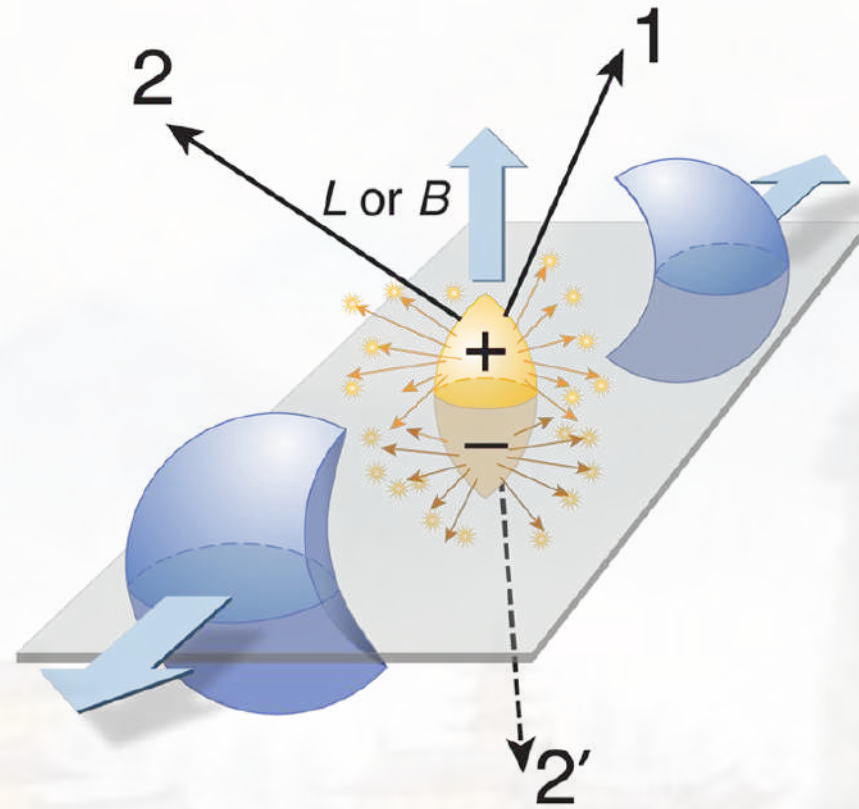
– High density: $\mu \gg m$

- Matter made of chiral fermions may allow $n_L \neq n_R$
- Unlike the electric charge $n_R + n_L$, the chiral charge $n_R - n_L$, is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c}$$

- The chiral anomaly can have *macroscopic* effects in chiral matter



<https://physics.aps.org/articles/v2/104>

ANOMALOUS EFFECTS IN HEAVY-ION COLLISIONS

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]

[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]

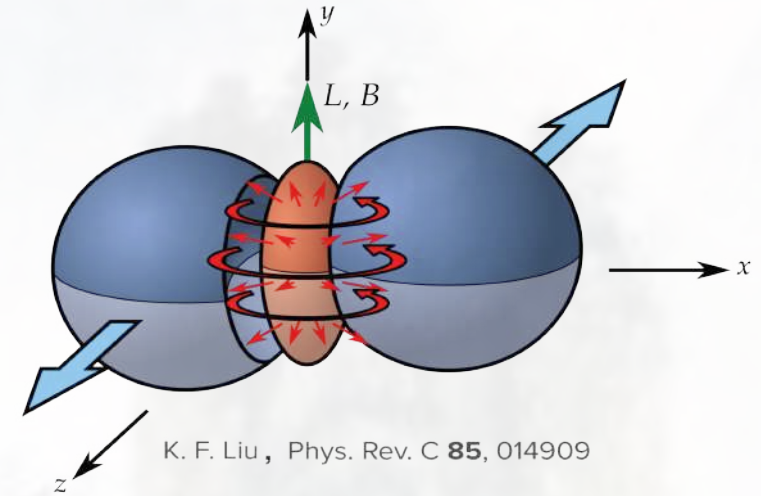
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

\vec{B} and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

$$e\mathbf{E}(t, \mathbf{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\mathbf{R}_n - R_n \mathbf{v}_n}{(R_n - \mathbf{R}_n \cdot \mathbf{v}_n)^3} (1 - v_n^2)$$

$$e\mathbf{B}(t, \mathbf{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n \cdot \mathbf{v}_n)^3} (1 - v_n^2)$$



[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak & Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108]

- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19} \text{ G } (\sim 100 \text{ MeV})$$

- Vorticity estimate:

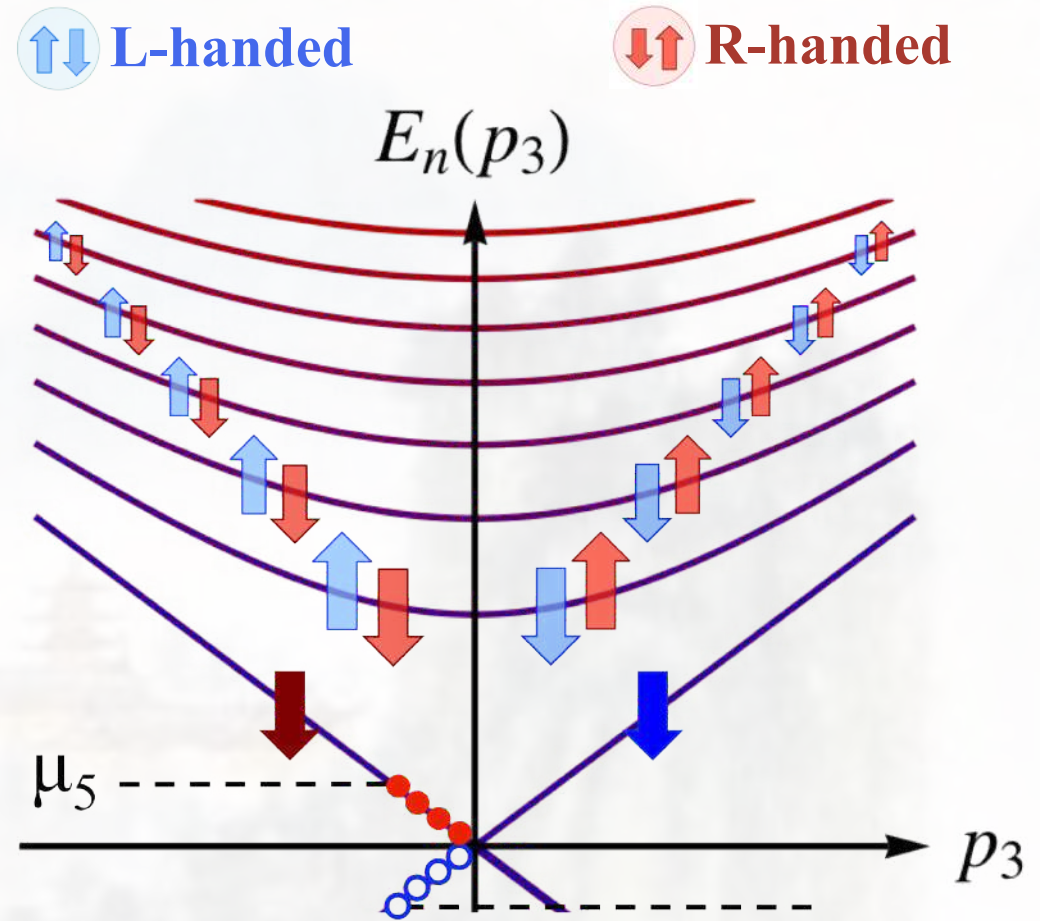
$$\omega \sim 10^{21} \text{ s}^{-1} (\sim 10 \text{ MeV})$$

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

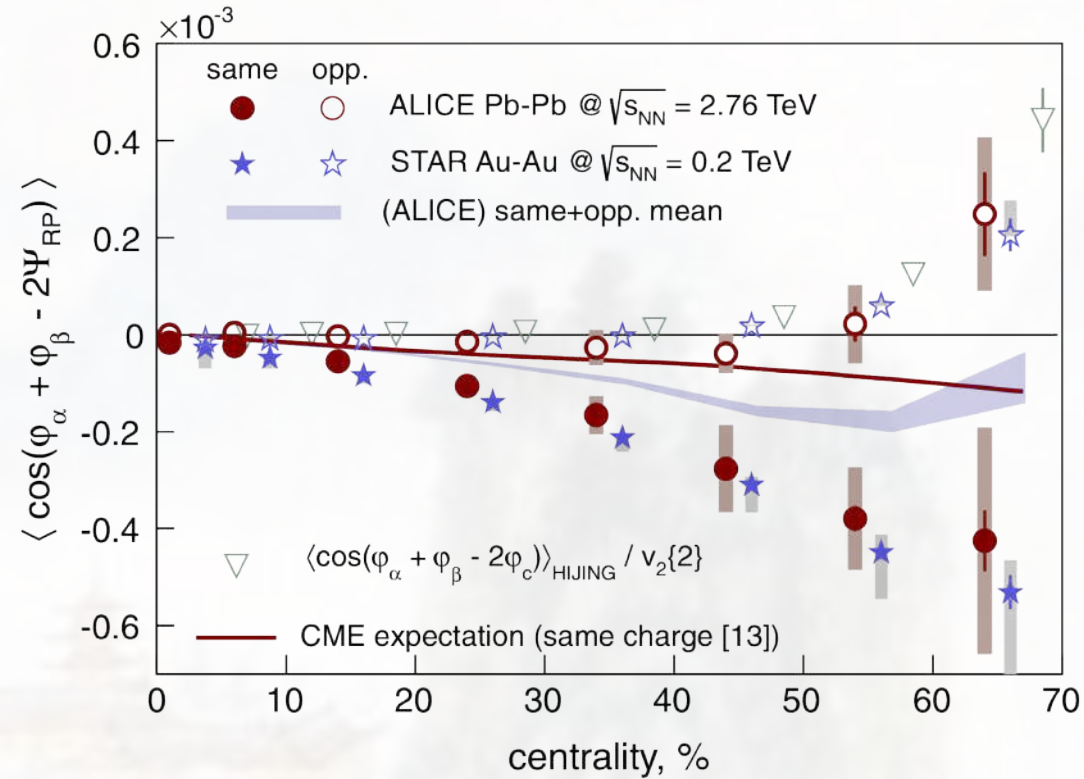
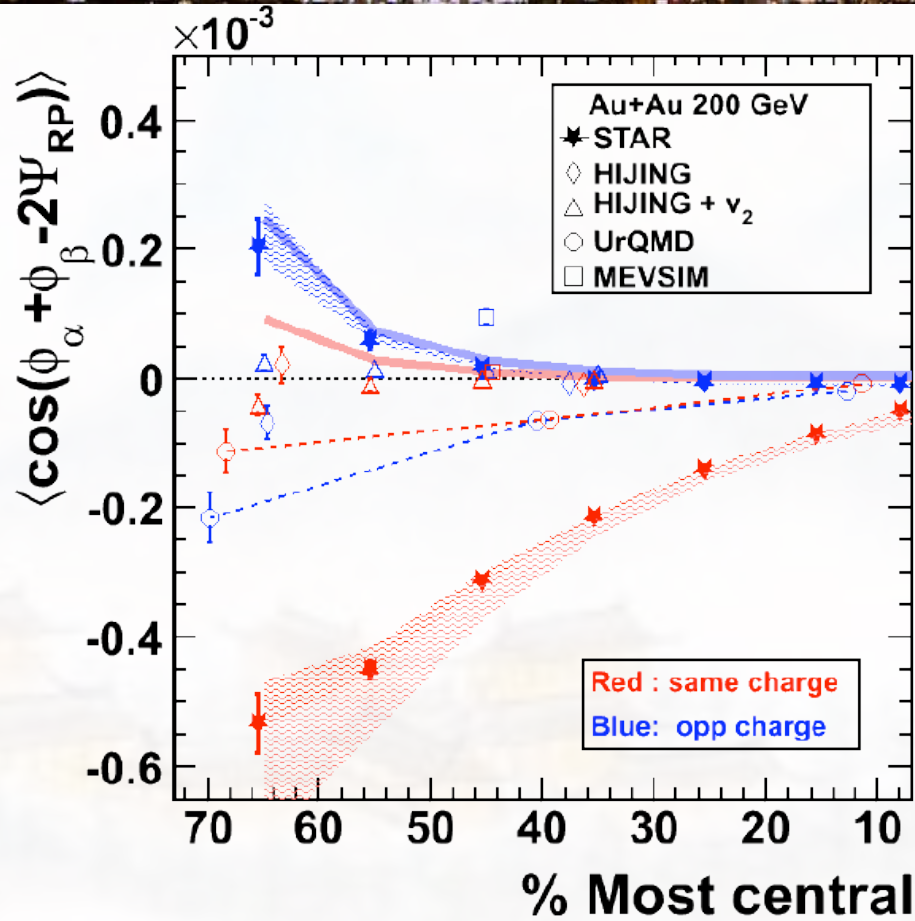
Spin polarized LLL ($s = \downarrow$ for particles of a *negative* charge):

- R-handed states $p_3 < 0$ give current in $+z$ direction
- L-handed holes $p_3 < 0$ give current in $+z$ direction too!



CME current:
$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]



Correlations of same & opposite charge particles: $\begin{cases} \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\mp - 2\Psi_{RP}) \rangle \\ \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}) \rangle \end{cases}$

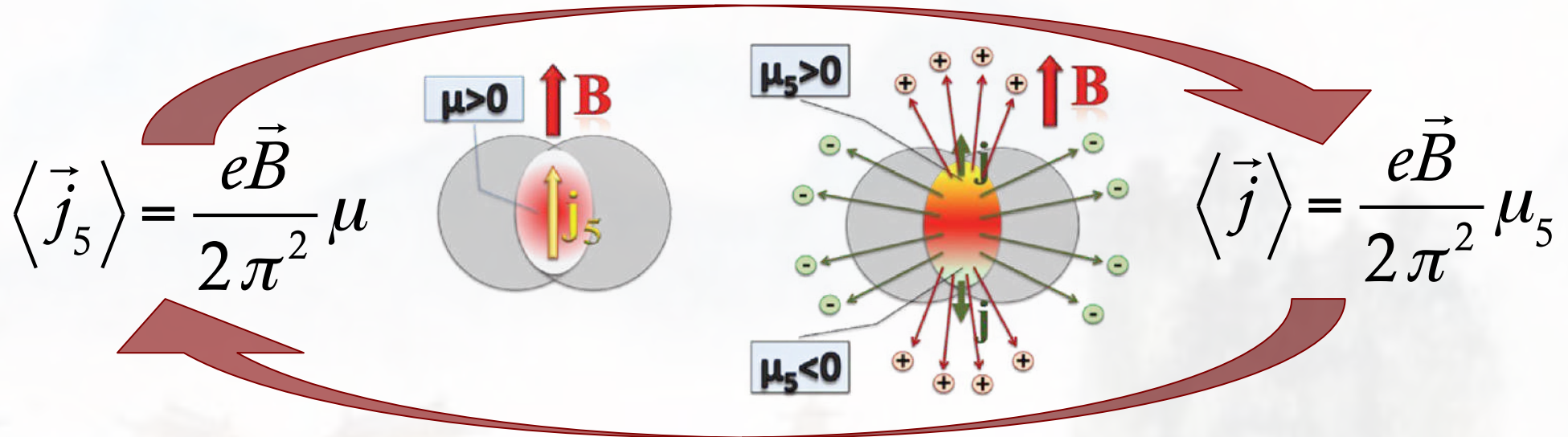
- [Abelev et al. (STAR), PRL **103**, 251601 (2009)]
- [Abelev et al. (STAR), PRC **81**, 054908 (2010)]
- [Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
- [Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)], [ALICE Collaboration, Phys. Lett. B **777**, 151 (2018)]

Chiral Magnetic Wave

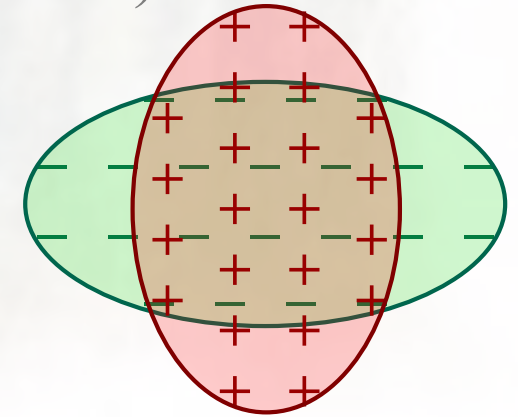
- Nonzero charge density @ $B \neq 0 \rightarrow$ CMW



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

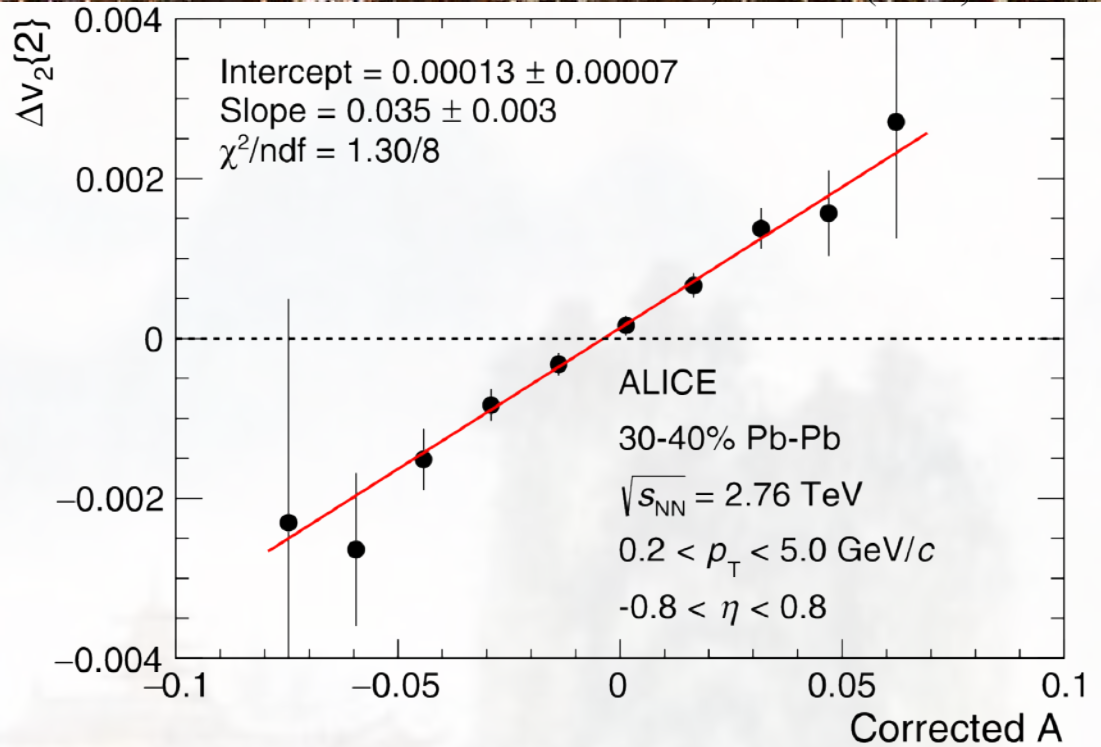
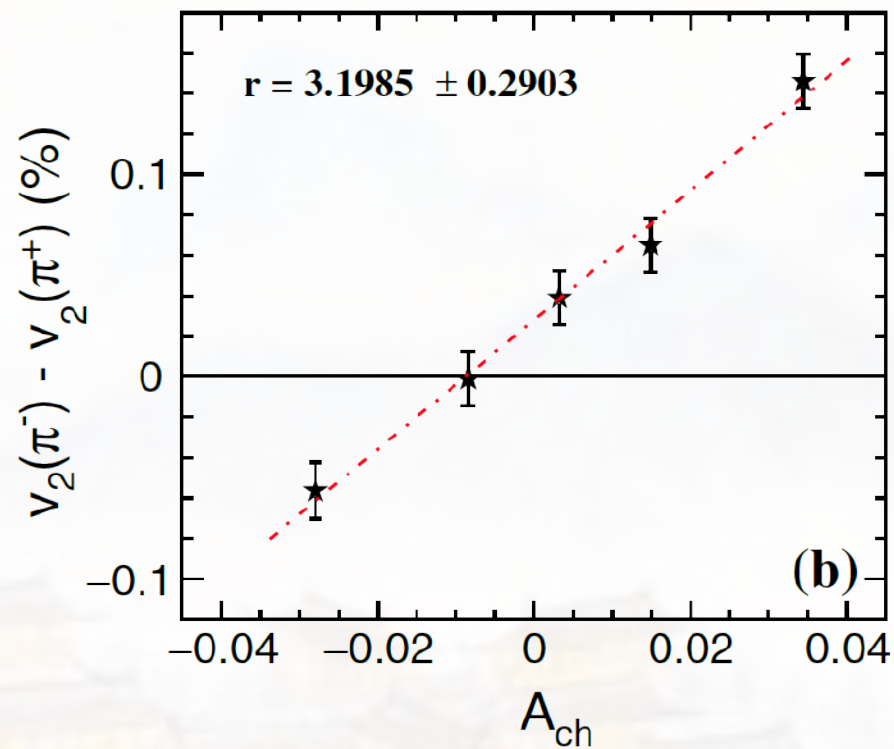
- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

$$\frac{dN_{\pm}}{d\phi} \approx \bar{N}_{\pm} [1 + 2v_2 \cos(2\phi) \mp A_{\pm} r \cos(2\phi)]$$



where A_{\pm} is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]



[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

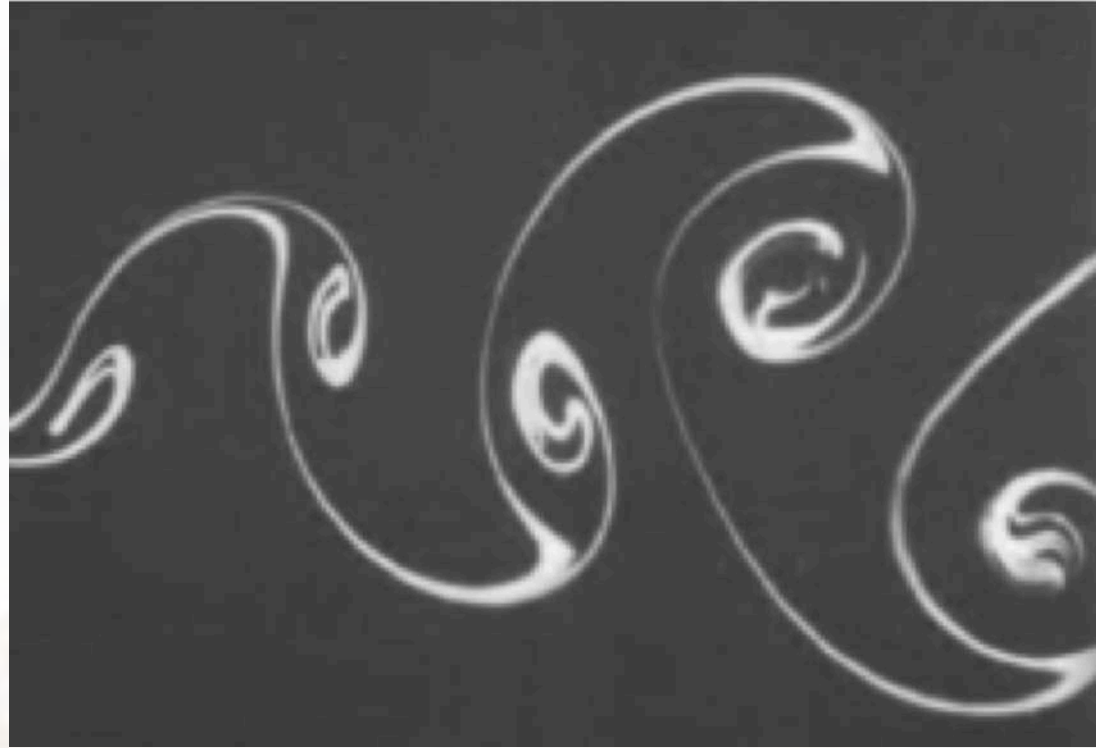
[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Higher harmonics of particle correlations are problematic...

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

In fact, the chiral magnetic wave might be overdamped...



CHIRAL HYDRODYNAMICS

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)]
[Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2 \hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = e F^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = n u^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

- Currents included new non-dissipative terms:

$$j^\mu = nu^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2 \hbar^2}, \quad \sigma_B = \frac{e\mu_5}{2\pi^2 \hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2 + \mu_5^2) + \pi^2 T^2}{6\pi^2 \hbar^2}, \quad \sigma_B^5 = \frac{e\mu}{2\pi^2 \hbar^2}$$

- Dissipative terms (first-order):

$$\nu^\mu = \nu_{\text{eq}}^\mu + \frac{\tau}{3} \nabla^\mu n - \tau \dot{u}^\mu n + \sigma_E E^\mu$$

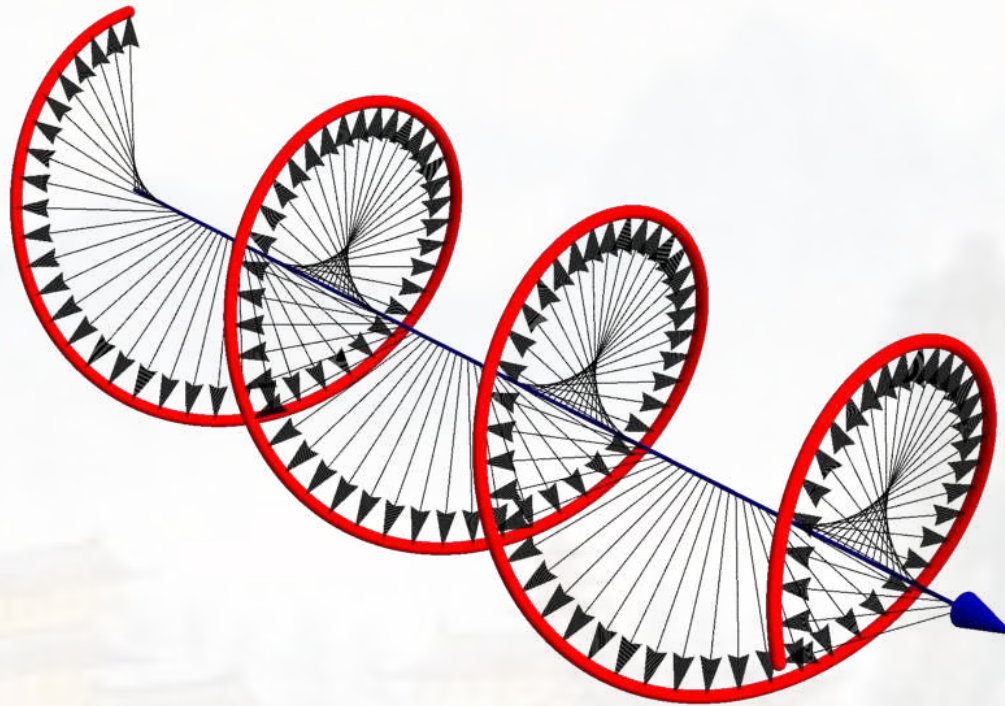
$$\nu_5^\mu = \nu_{5,\text{eq}}^\mu + \frac{\tau}{3} \nabla^\mu n_5 - \tau \dot{u}^\mu n_5 + \sigma_E^5 E^\mu$$

$$\pi^{\mu\nu} = \frac{8\tau\epsilon}{15} \Delta_{\alpha\beta}^{\mu\nu} (\partial^\alpha u^\beta)$$

where τ is the relaxation time and

$$\sigma_E = \tau e \frac{3(\mu^2 + \mu_5^2) + \pi^2 T^2}{9\pi^2 \hbar^3}$$

$$\sigma_E^5 = \tau e \frac{2\mu\mu_5}{3\pi^2 \hbar^3}$$



COLLECTIVE MODES IN CHIRAL MATTER

- Fluid velocity

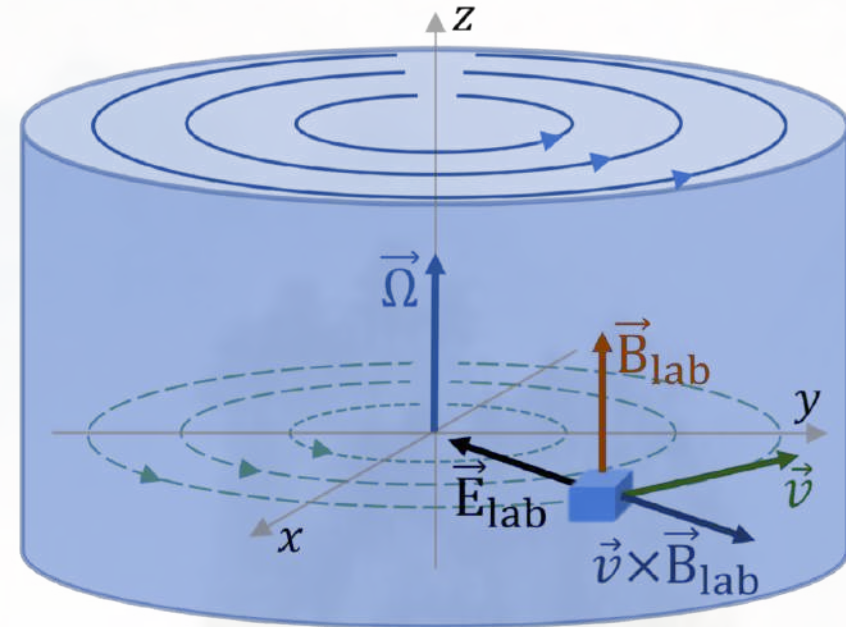
$$\bar{u}^\nu = \gamma \begin{pmatrix} 1 \\ -\Omega y \\ \Omega x \\ 0 \end{pmatrix}$$

where $\gamma = 1/\sqrt{1 - (\Omega r)^2}$

Vorticity: $\bar{\omega}^\mu = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} \bar{u}_\nu \partial_\alpha \bar{u}_\beta = \gamma^2 \Omega \delta_3^\mu$

EM fields in lab frame: $\mathbf{B}_{\text{lab}} = \gamma B \hat{\mathbf{z}}$

$$\mathbf{E}_{\text{lab}} = -\gamma B \Omega \mathbf{r}_\perp$$



[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

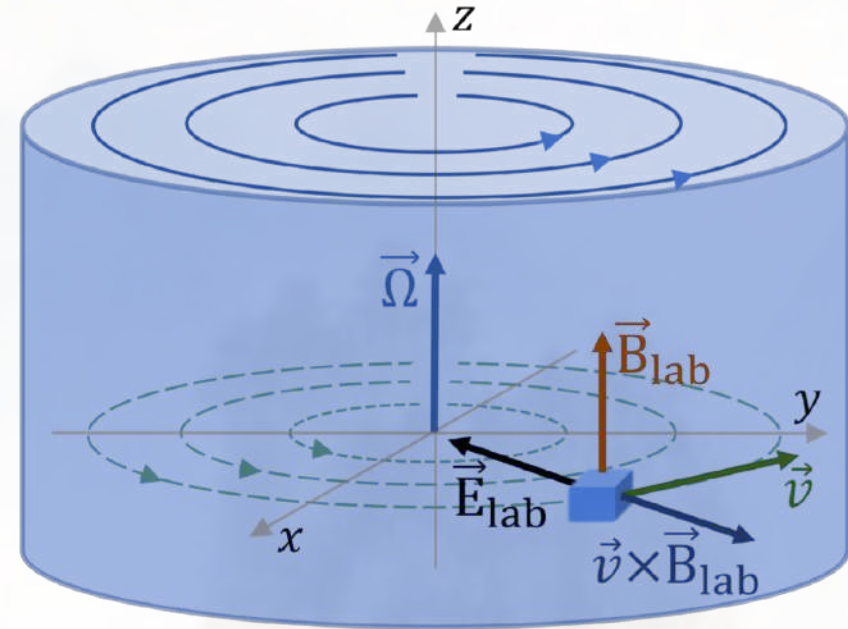
ASU Rotating charged plasma: equilibrium

- Maxwell equations:

$$\partial_\nu F^{\nu\mu} = enu^\mu + e\nu^\mu - en_{\text{bg}}u_{\text{bg}}^\mu$$

$$\partial_\nu \tilde{F}^{\nu\mu} = 0$$

where n_{bg} is the background



The solution is radially nonuniform:

$$B(r) = \gamma \left(B_0 - \frac{1}{2} en_{\text{bg}} \Omega r^2 \right) \simeq B_0 - \frac{1}{2} en_{\text{bg}} \Omega r^2 + O(B_0 r^2 \Omega^2)$$

$$en_{\text{eq}}(r) = \gamma^3 (en_{\text{bg}} - 2B_0 \Omega) \simeq en_{\text{bg}} - 2B_0 \Omega + O(en_{\text{bg}} r^2 \Omega^2)$$

(This is consistent with $\mu = \gamma\mu_0$, $\mu_5 = \gamma\mu_{5,0}$, $T = \gamma T_0$.)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Linearized equations, etc.

- Small perturbations ($\Omega \rightarrow 0$):

$$\delta s(x) = e^{-ik_0 t + ik_z z + im\theta} \delta s(r)$$

$$\delta v^3(x) = e^{-ik_0 t + ik_z z + im\theta} \delta v^3(r)$$

$$\delta v_{\pm}(x) = e^{-ik_0 t + ik_z z + i(m \pm 1)\theta} \delta v_{\pm}(r)$$

where

$$\delta s(r) = \delta s J_m(k_{\perp} r), \quad \text{for } s = \mu, \mu_5, T$$

$$\delta v^3(r) = \delta v^3 J_m(k_{\perp} r), \quad \text{for } v^3 = u^3, B^3, E^3$$

$$\delta v_{\pm}(r) = \delta v_{\pm} J_{m \pm 1}(k_{\perp} r), \quad \text{for } v_{\pm} = u_{\pm}, B_{\pm}, E_{\pm}$$

- Boundary conditions: $\delta s(R) = 0, \delta v^3(R) = 0$
- Transverse wave vectors: $k_{\perp}^{(i)} = \alpha_{m,i}/R$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Hierarchy of scales

- Mean free path $\ell_{\text{mfp}} \simeq c\tau$, de Broglie wavelength $\ell_d = \hbar/T$, and typical wavelengths of modes $\lambda_k = 2\pi/k$:

$$\ell_d \ll \ell_{\text{mfp}} \ll \lambda_k \lesssim R$$

- Magnetic length $\ell_B = \sqrt{\hbar/|eB|}$ ($\ell_d \ll \ell_B$ or $|eB| \ll T^2$)

– Weak magnetic field: $\ell_B \gtrsim \ell_{\text{mfp}}$

– Moderately strong magnetic field: $\ell_B \lesssim \ell_{\text{mfp}}$

- System size $R \lesssim \Omega^{-1}$

- In this work (with auxiliary parameter $\xi \simeq 0.01$)

$$\Omega \ell_{\text{mfp}} \simeq \xi^2, \quad \xi^{3/2} \simeq \frac{\ell_{\text{mfp}}}{R} \lesssim k \ell_{\text{mfp}} \lesssim \xi^{1/2}, \quad \frac{\ell_{\text{mfp}}}{\ell_B} \simeq \xi^{-1/4}, \quad \frac{\ell_{\text{mfp}}}{\ell_d} \simeq \xi^{-1}$$



COLLECTIVE MODES IN HOT PLASMA

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

- Sound waves ($T \gg \mu$): [Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15}i\tau k^2$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = s_e \frac{3\sqrt{5}B\hbar^{3/2}k_z}{\sqrt{7}\pi T^2} \left(1 \pm \frac{\sqrt{5}e\mu}{2\sqrt{7}\pi\hbar^{3/2}k} \right) - \frac{1}{10}i\tau k^2.$$

where $k = \sqrt{k_z^2 + k_\perp^2}$ and $s_e = \pm 1$.

- There are also purely diffusive modes, e.g.,

$$k_0 = -\frac{e^2}{9\hbar^3}i\tau T^2$$

describing the charge diffusion (i.e., $\partial_t \mathbf{E} + e\sigma_E \mathbf{E} \approx 0$.)

- Sound waves ($T \gg \mu$):

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15}i\tau k^2 + m\Omega \left(\frac{2}{3} + \frac{5e^2\mu^2}{14\pi^2\hbar^3 k^2} \right)$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = m\Omega + sk_z \sqrt{\frac{45\mathcal{B}_\pm^2 \hbar^3}{7\pi^2 T^4} + \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm \mu}{14\pi^2 T^2 k} \right)^2} \pm k_z \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm \mu}{14\pi^2 T^2 k} \right)$$

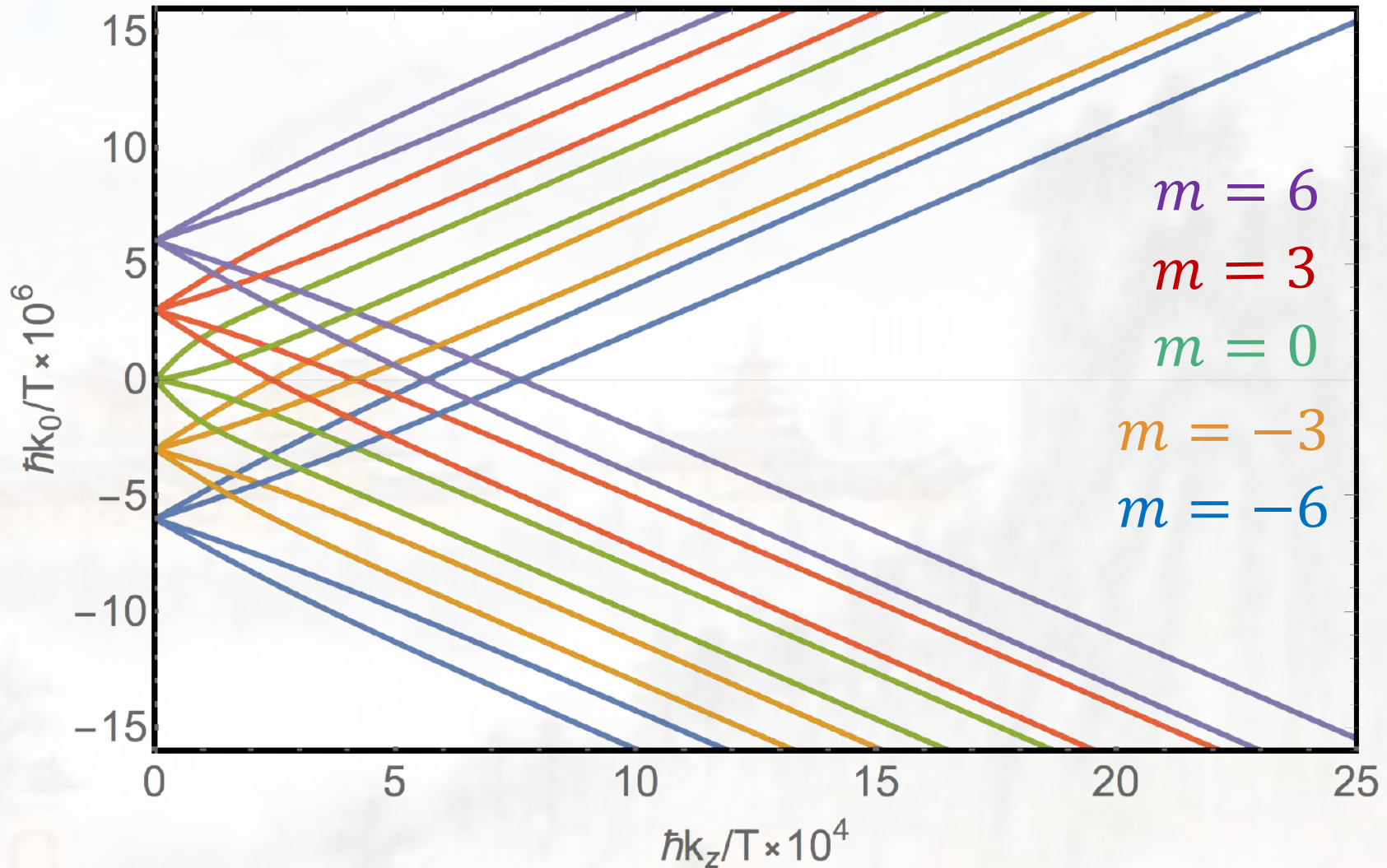
with a small imaginary part (not shown)

Here

$$\mathcal{B}_\pm = B - \frac{en_{\text{eq}}\Omega}{6k_\perp^2} [2(m \pm 1)(m \pm 2) + k_\perp^2 R^2]$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

- Alfven waves with angular momenta $m = 0, \pm 3, \pm 6$



[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

Without dynamical fields

- Sound wave and two chiral waves ($T \gg \mu$):

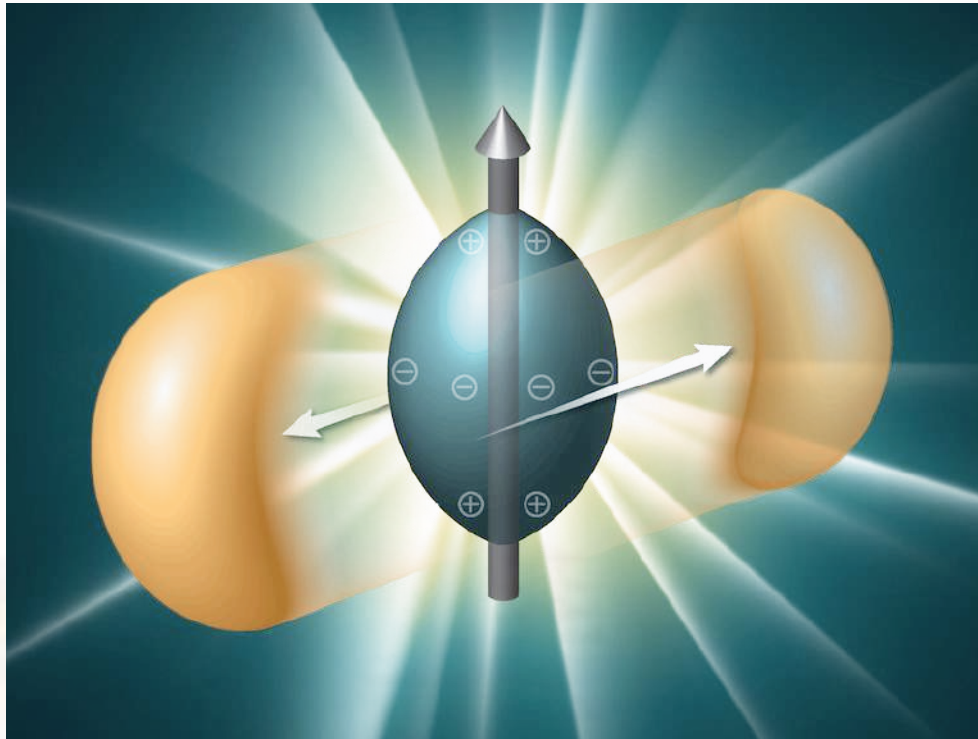
$$k_0 = \frac{s_e k}{\sqrt{3}} + \frac{2}{3} m \Omega - \frac{2}{15} i \tau k^2 \quad (\text{sound})$$

$$k_0 = m \Omega + s_e \frac{2k_z \Omega}{k} - \frac{1}{5} i k^2 \tau \quad (\text{CVW?})$$

$$k_0 = m \Omega + s_e \frac{3e\mathcal{B}_0 \hbar k_z}{2\pi^2 T^2} - \frac{1}{3} i k^2 \tau \quad (\text{CMW?})$$

- There are more propagating (fewer diffusive) modes
- CVW & CMW appear only in nondynamical regime

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]



<https://www.bnl.gov/newsroom/news.php?a=25735>

CHIRAL MAGNETIC WAVE

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

- Simple 1-flavor model ($\mathbf{k} \parallel \mathbf{B}$):

$$k_0 \delta n - kB \delta \sigma_B + i \frac{\tau}{3} k^2 \delta n = 0$$

$$k_0 \delta n_5 - kB \delta \sigma_B^5 + i \frac{\tau}{3} k^2 \delta n_5 = 0$$

$$k \delta E_z + ie \delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i \frac{\sigma_E}{2} \pm i \frac{\sigma_E}{2} \sqrt{1 - \left(\frac{3eB}{\pi^2 T^2 \sigma_E} \right)^2 \left(k^2 + \frac{e^2 T^2}{3} \right) - i \frac{\tau}{3} k^2}$$

- This is a completely diffusive mode when

$$\frac{3eB}{\pi^2 T^2 \sigma_E} \sqrt{k^2 + \frac{e^2 T^2}{3}} < 1$$

Naïve CMW ($\sigma_E \rightarrow 0$)

- In contrast, if Gauss's law is ignored, the mode is non-diffusive, i.e.,

$$k_0^{(\pm)} = \pm \frac{3eB}{2\pi^2 T^2} \sqrt{k^2 + \frac{e^2 T^2}{3}} - i \frac{\tau}{3} k^2, \quad \text{when } \sigma_E \rightarrow 0.$$

- There is only a small dissipation due to the charge diffusion when $k \rightarrow 0$
- Notably, the CMW is gapped!
- The gap comes from the anomaly due to δE_z

$$-i \frac{e^2}{2\pi^2} B \delta E_z = -i \frac{e^2 B}{2\pi^2} \left(\frac{-ie \delta n}{k} \right)$$

- Model with two light u- and d-quarks:

$$k_0 \delta n_f - \frac{eq_f Bk}{2\pi^2 \chi_{f,5}} \delta n_{f,5} + iD_f k^2 \delta n_f - \frac{1}{eq_f} \sigma_{E,f} k \delta E_z = 0$$

$$k_0 \delta n_{f,5} - \frac{eq_f Bk}{2\pi^2 \chi_f} \delta n_f + iD_f k^2 \delta n_{f,5} - i \frac{e^2 q_f^2}{2\pi^2} B \delta E_z = 0$$

$$k \delta E_z + ie \sum_f q_f \delta n_f = 0$$

where $f = u, d$, and $q_u = 2/3$, $q_d = -1/3$

χ_f , D_f , and $\sigma_{E,f}$ are susceptibilities, diffusion coefficients and electrical conductivities for each quark flavor, respectively

Near-critical strongly coupled quark-gluon plasma

$$\sigma_E = \sum_f \sigma_{E,f} = c_\sigma C_{\text{em}}^\ell T$$

$$\chi_f = c_\chi \chi_f^{(SB)}$$

$$D_f = \frac{c_D}{2\pi T}$$

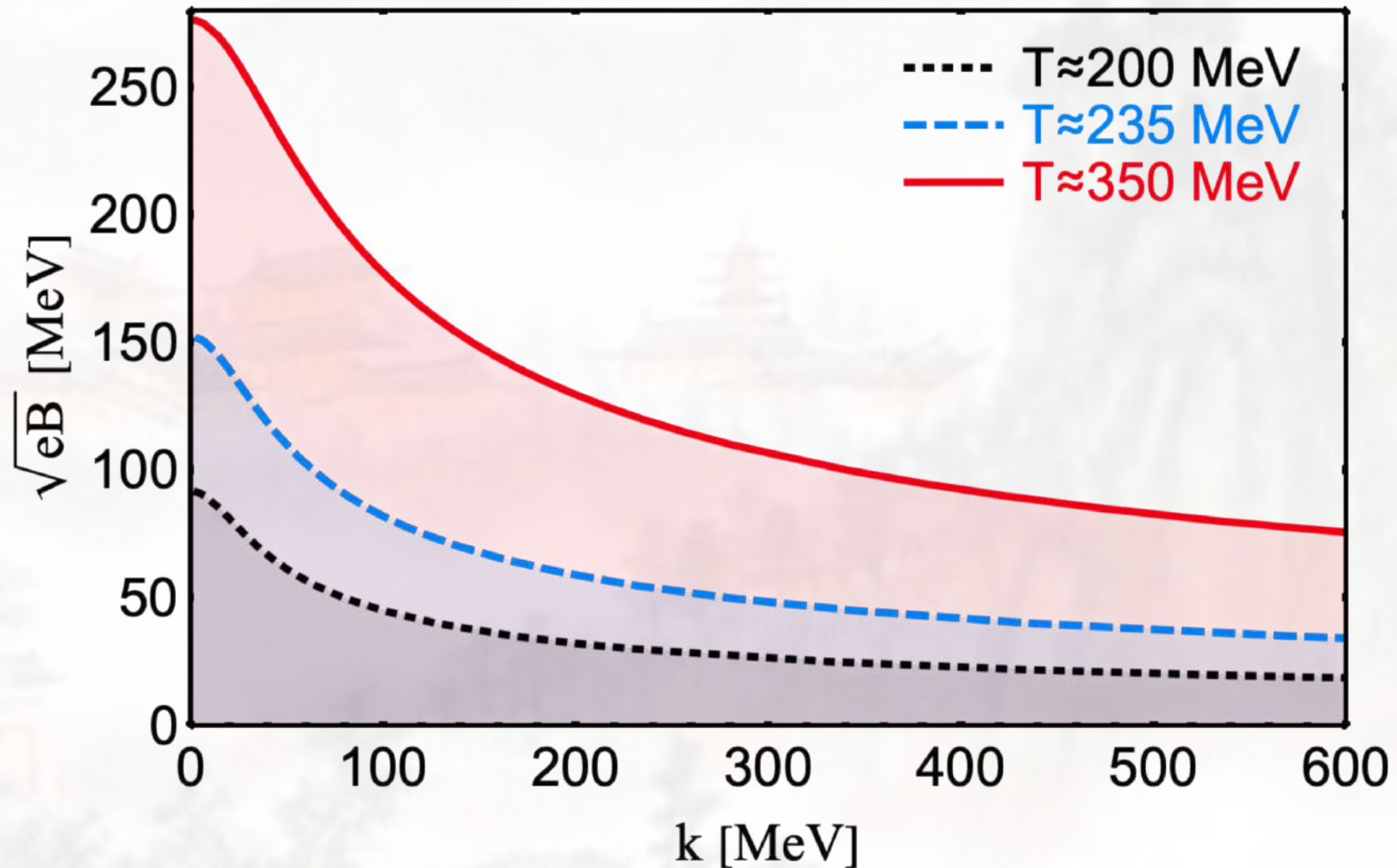
$$C_{\text{em}}^\ell = \left(\frac{5}{9}\right) 4\pi\alpha_{\text{em}} \approx 0.051$$

Lattice data [Aarts, et. al. JHEP 1502, 186 (2015)]

	c_σ	c_χ	c_D
T=200 MeV	0.111	0.804	0.758
T=235 MeV	0.214	0.885	1.394
T=350 MeV	0.316	0.871	1.826

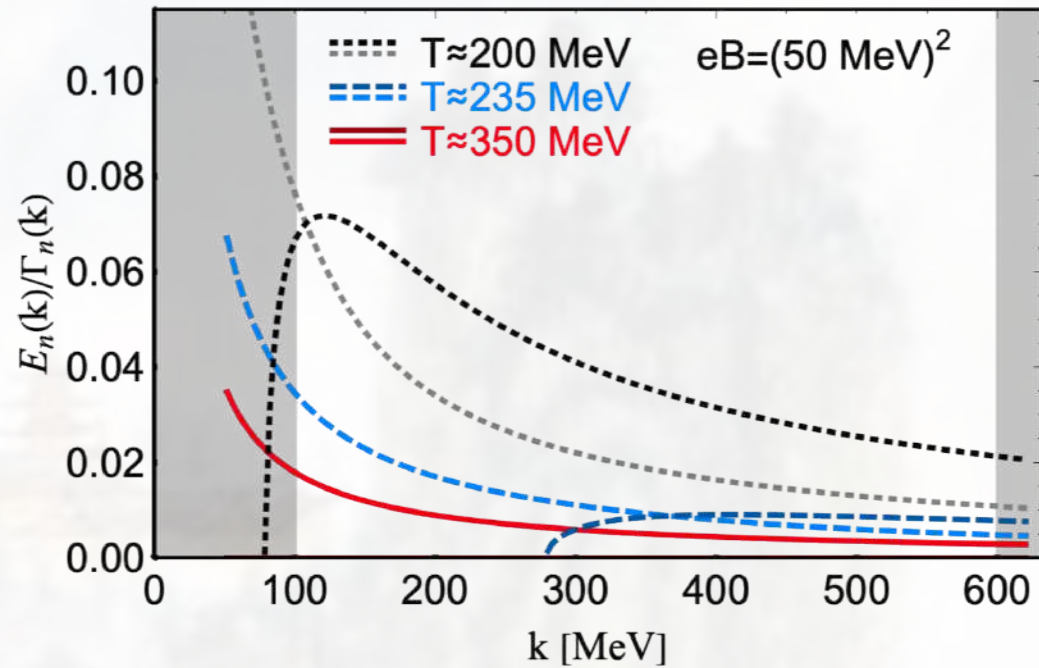
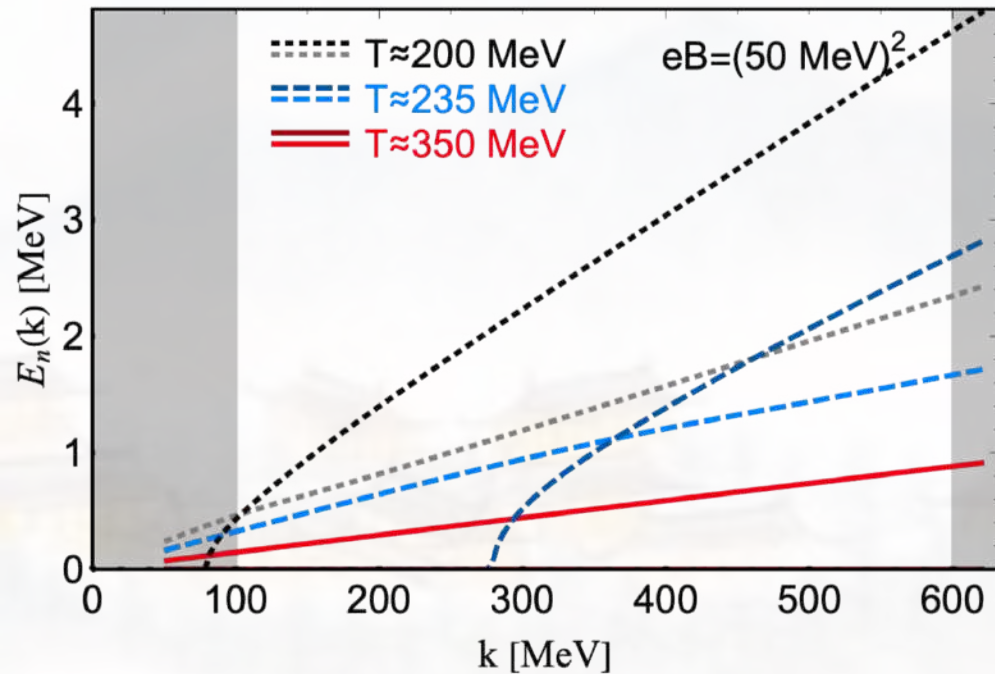
Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small eB & k :



Moderately strong B -field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



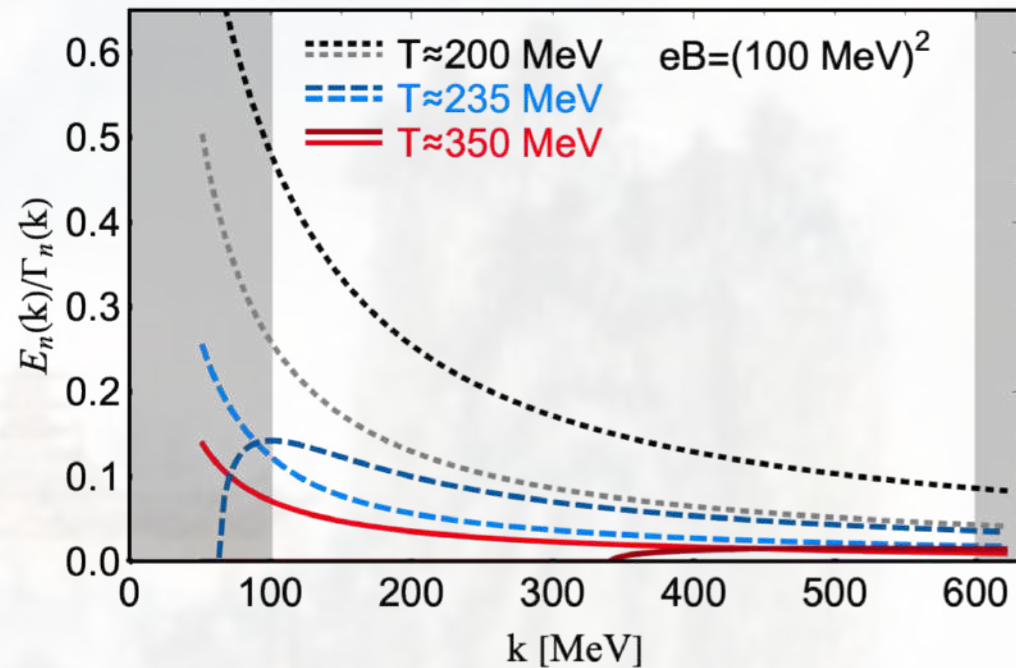
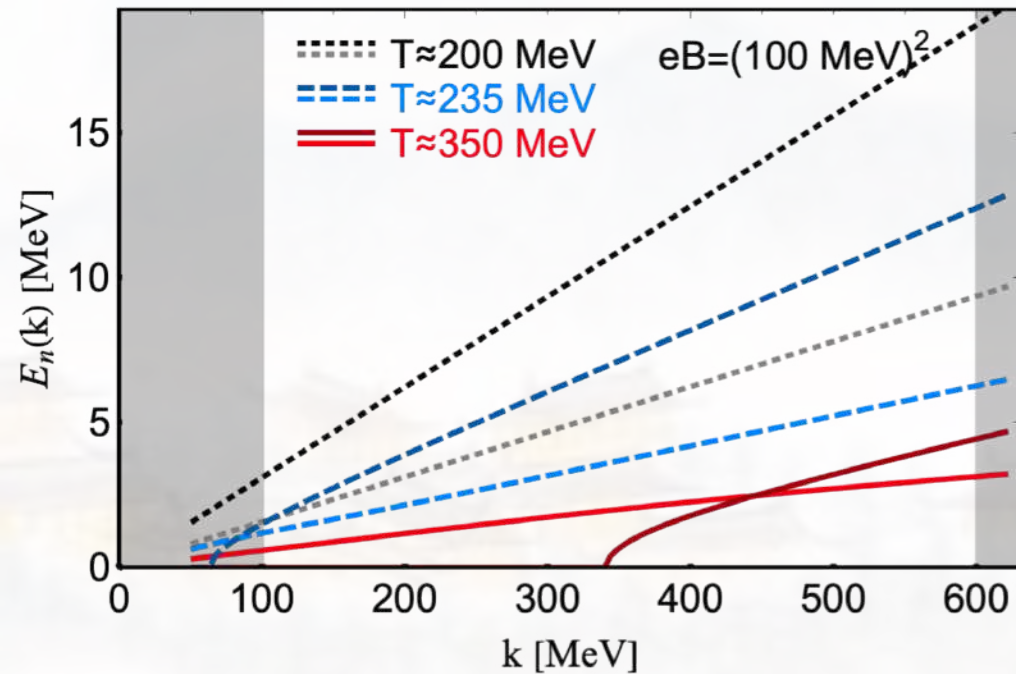
Allowed range of wave vectors:

$$(50 \text{ MeV}) \quad 100 \text{ MeV} \lesssim k \lesssim 600 \text{ MeV}$$

Wavelengths: $2 \text{ fm} \lesssim \lambda_k \lesssim 12 \text{ fm} \quad (24 \text{ fm})$

Strong B -field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$

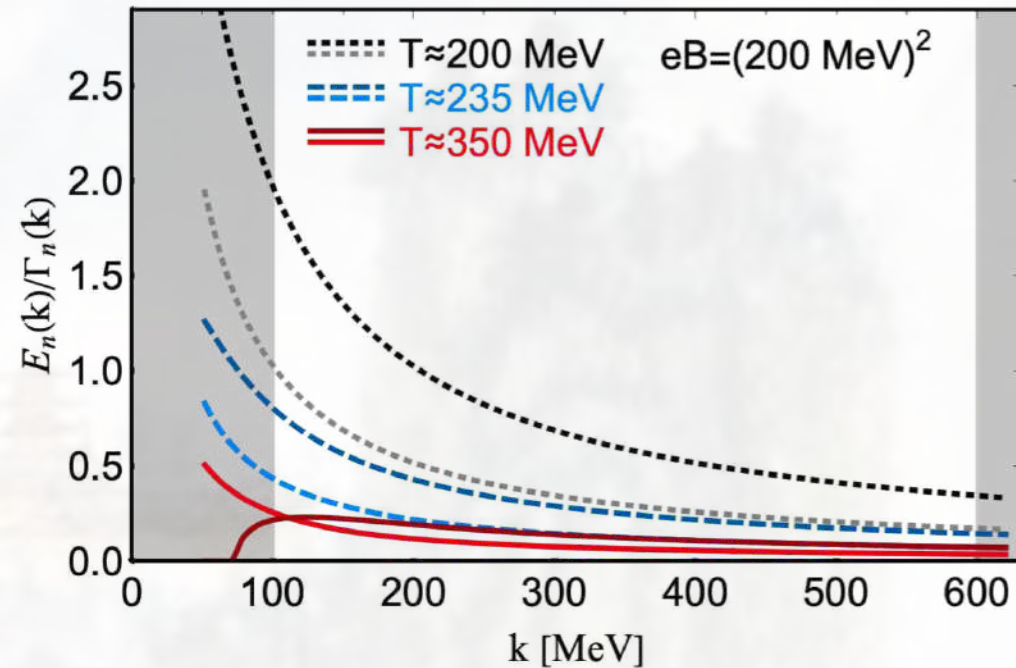
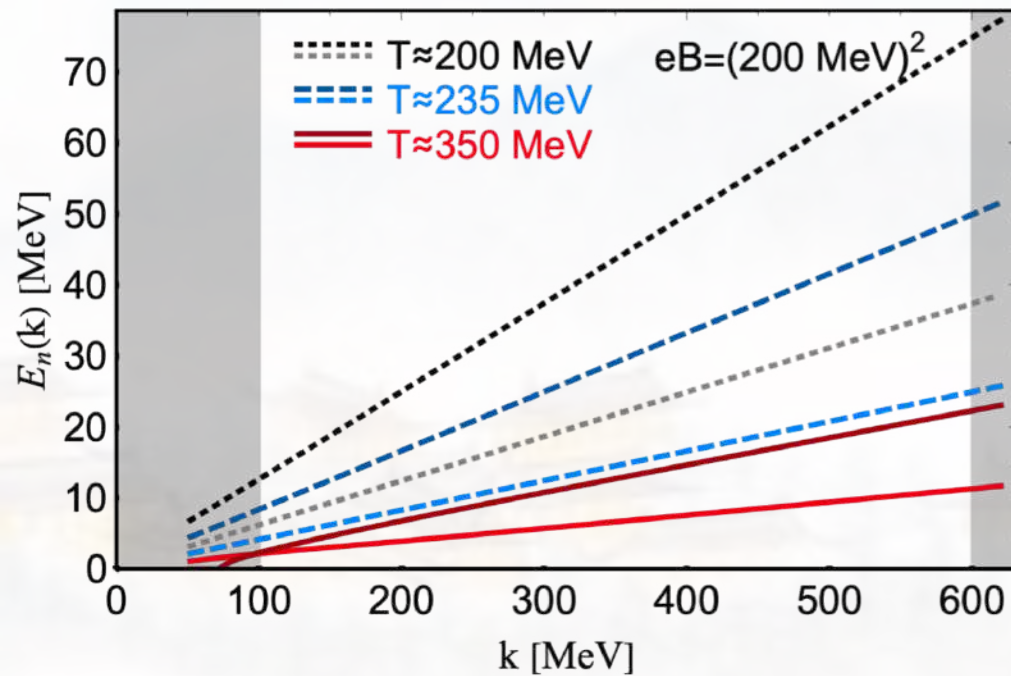


Even for such strong B -field, the CMW is strongly overdamped

Charge diffusion $iD_f k^2$ plays a big role ($k \gtrsim 2\pi/R$)

Very strong B -field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



The CMW may become a propagating mode only in extremely strong B -fields, $eB \gtrsim (200 \text{ MeV})^2$

Otherwise, it is overdamped

- Equilibrium state of charged rotating plasma is radially nonuniform
- Propagating (not overdamped) hydro modes in charged rotating plasma are
 - Sound and Alfvén waves @ high temperature
 - Plasmons and helicons @ high density
- Dynamical electromagnetism plays a crucial role
 - Electrical conductivity screens charge fluctuations
 - Charge diffusion is not negligible in finite-size systems
- Chiral magnetic wave in HIC is overdamped