ARIZONA STATE UNIVERSITY

EFFECTIVE THEORIES ETC.
May 6, 2019
NORDITA

Downfall of chiral magnetic wave

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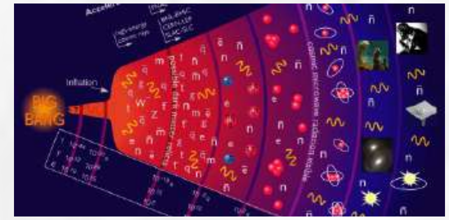
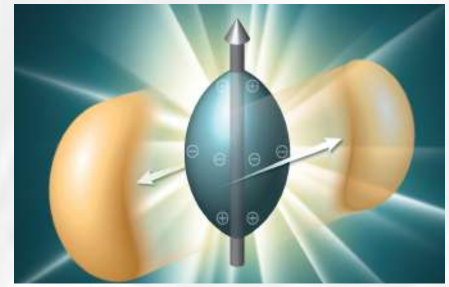
[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]
[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]
[Gorbar, Miransky, Shovkovy, Sukhachov, arXiv: 1712.08947, Low Temp. Phys. 44, 487 (2018)]

Chiral forms of matter

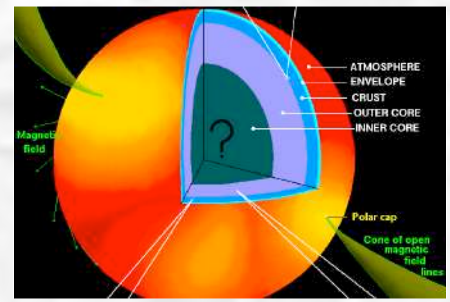
- **Early Universe**, e.g.,
  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]



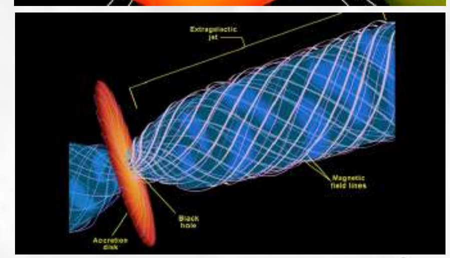
- **Heavy-ion collisions**, e.g.,
  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]



- **Super-dense matter in compact stars**, e.g.,
  [Yamamoto, Phys.Rev. D93, 065017 (2016)]

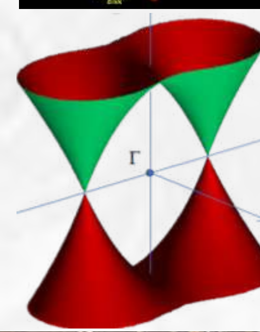


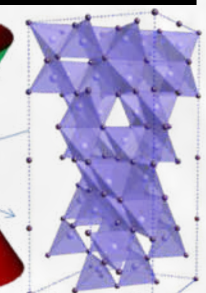
- **Ultra-relativistic jets from black holes**



- **Superfluid ^{3}He-A**, e.g.,
  [Volovik, JETP Lett. 105, 34 (2017)]

- **Dirac/Weyl (semi-)metals**, e.g.,
  [Li et. al. Nature Phys. 12, 550 (2016)]





Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{p}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

For particles $(p_0 > 0)$: chirality = helicity

For antiparticles $(p_0 < 0)$: chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

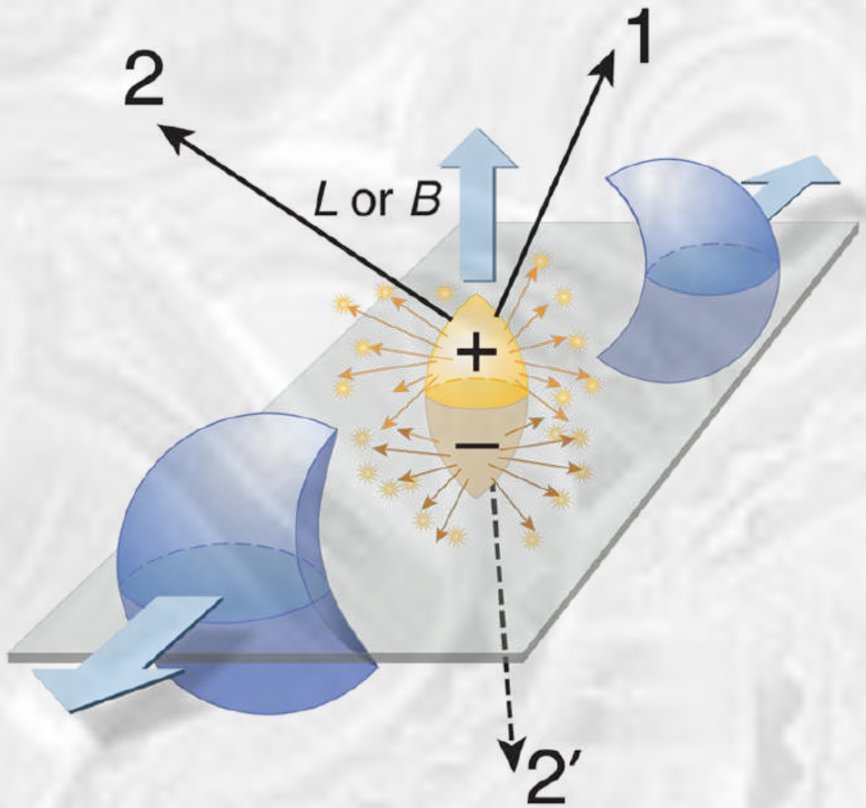
Anomalous chiral matter

- Matter made of chiral fermions may allow $n_{\mathrm{L}} \neq n_{\mathrm{R}}$
- Unlike the electric charge $n_{\mathrm{R}} + n_{\mathrm{L}}$, the chiral charge $n_{\mathrm{R}} - n_{\mathrm{L}}$, is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral anomaly can have *macroscopic* effects in chiral matter



https://physics.aps.org/articles/v2/104

CHIRAL MAGNETIC WAVE

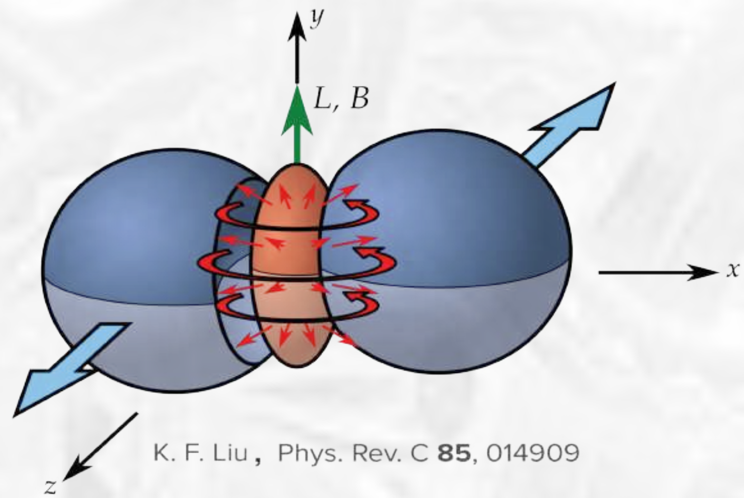
[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



K. F. Liu, Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak &. Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108], …

- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19}\ \text{G}\ (\sim 100\ \text{MeV})$$

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

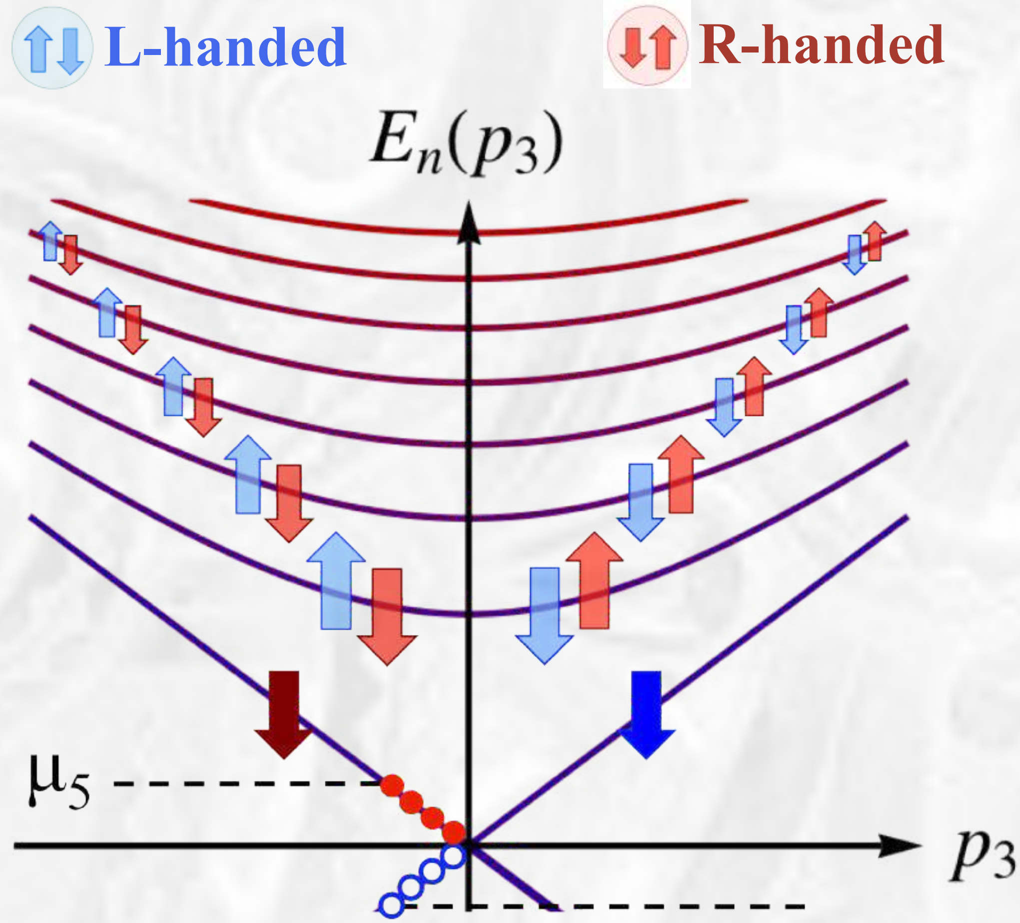
$$\omega \sim 9\times 10^{21} s^{-1}\ (\sim 10\ \text{MeV})$$

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

- R-handed states $p_3<0$ give current in $+z$ direction
- L-handed holes $p_3<0$ give current in $+z$ direction too!



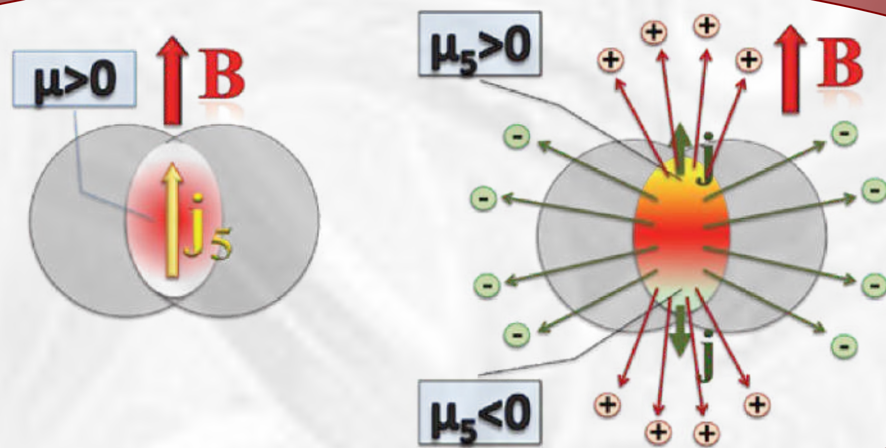
CME current: $$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

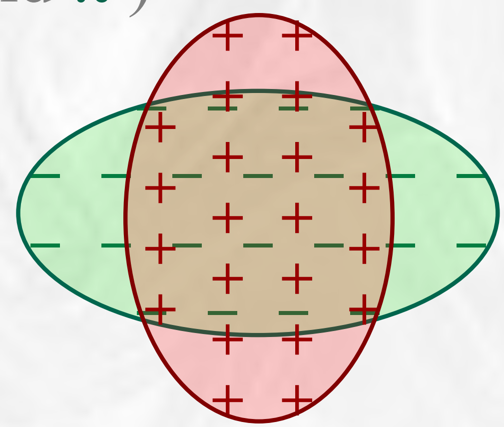


$$\left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

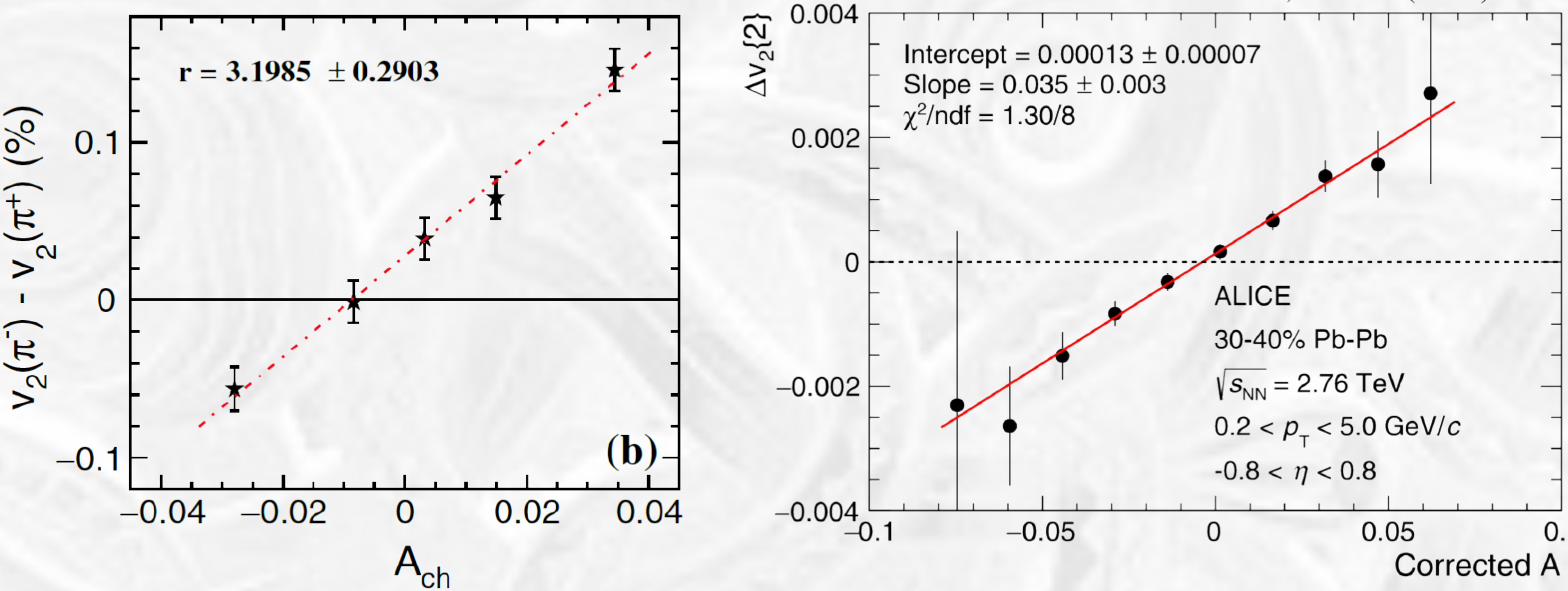
$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$



where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

CMW: Experimental evidence



[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

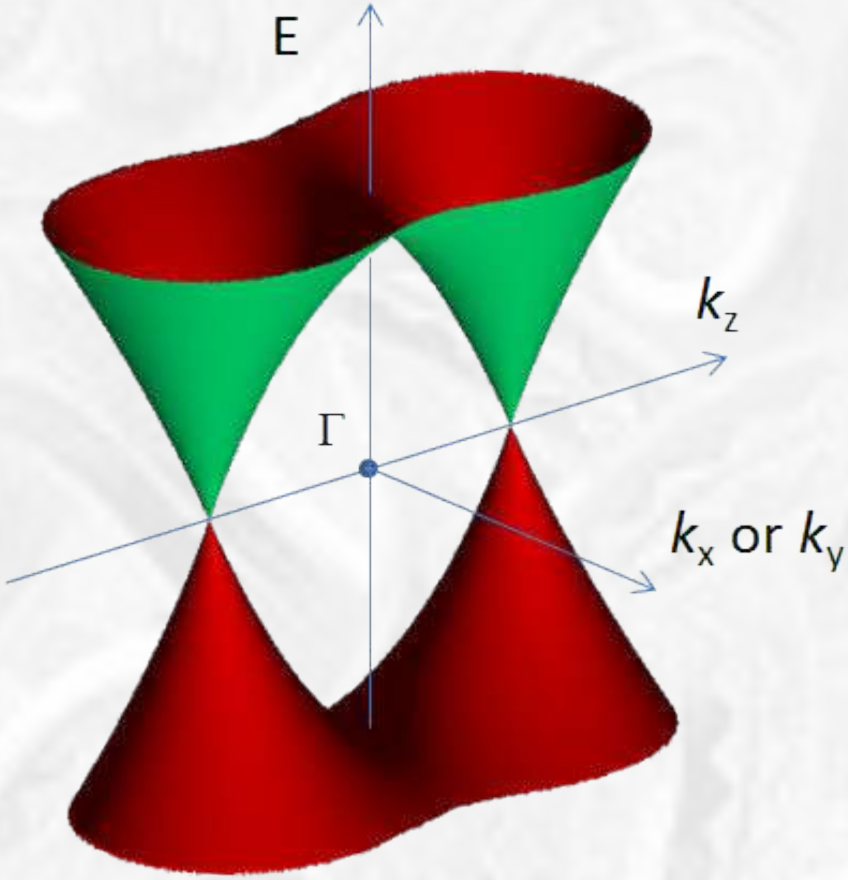
[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Higher harmonics of particle correlations are problematic…

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

In fact, the chiral magnetic wave might be overdapmed…



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

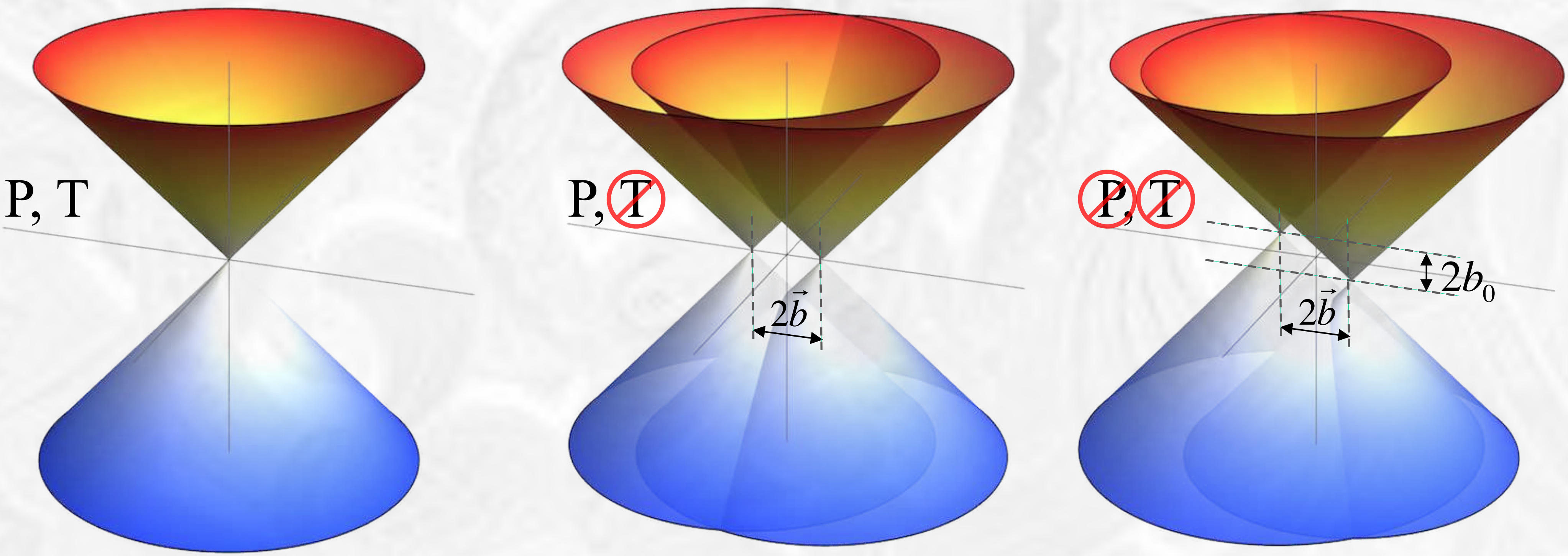
ANOMALOUS PLASMA IN DIRAC & WEYL MATERIALS

Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \overset{\cancel{\mathrm{T}}}{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5} + \overset{\cancel{\mathrm{P}}}{b_0\gamma^0\gamma^5}\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) **Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Strain in Weyl materials

- Strains in the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

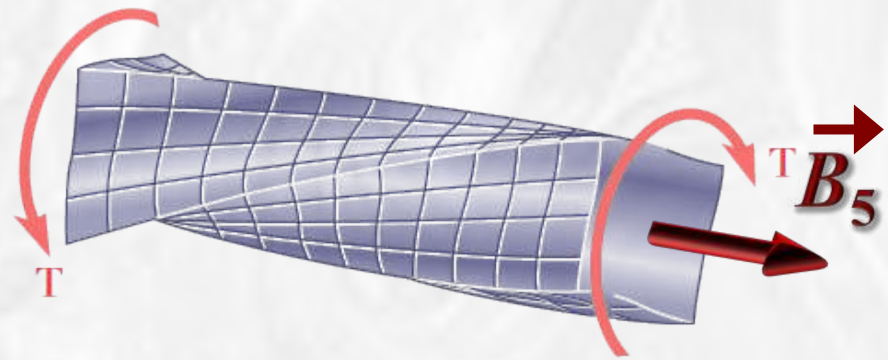
where the chiral gauge fields are

[Cortijo, Ferreiros, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]
[Zubkov, Annals Phys. **360**, 655 (2015)]
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
[Grushin, Venderbos,Vishwanath, Ilan, PRX **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

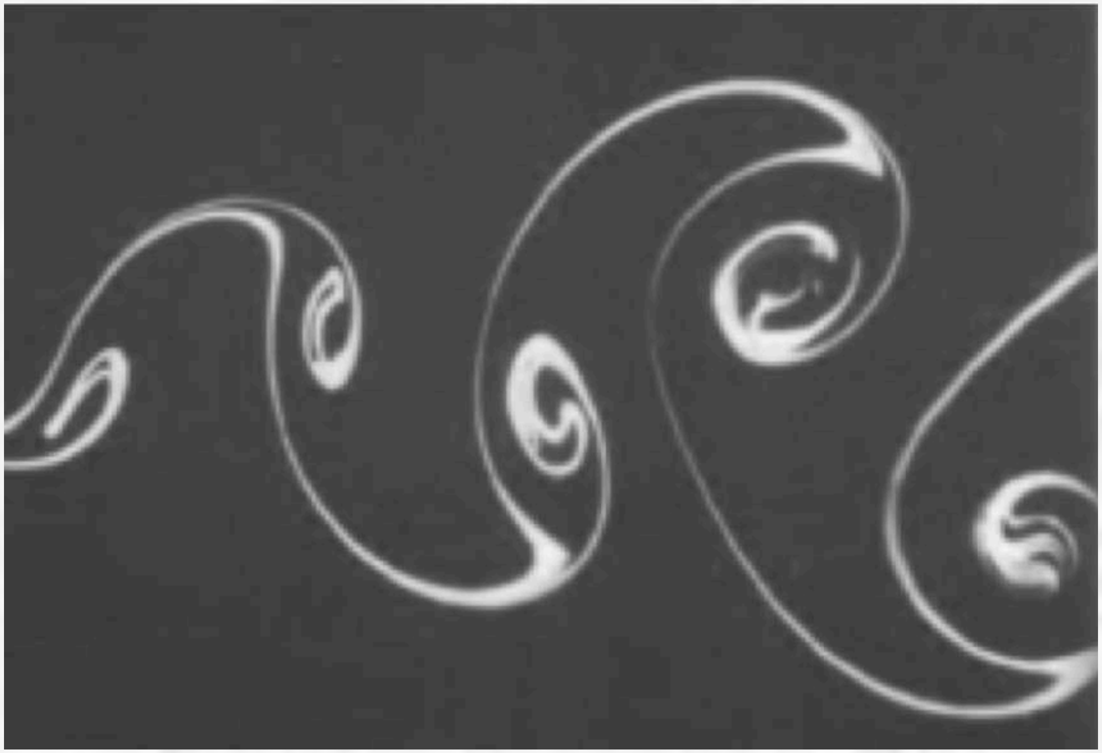
$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$



leading to the pseudo-EM fields

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$



CHIRAL HYDRODYNAMICS

Chiral hydrodynamics

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)] [Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = eF^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = n u^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

Anomalous contributions

- Currents included new non-dissipative terms:

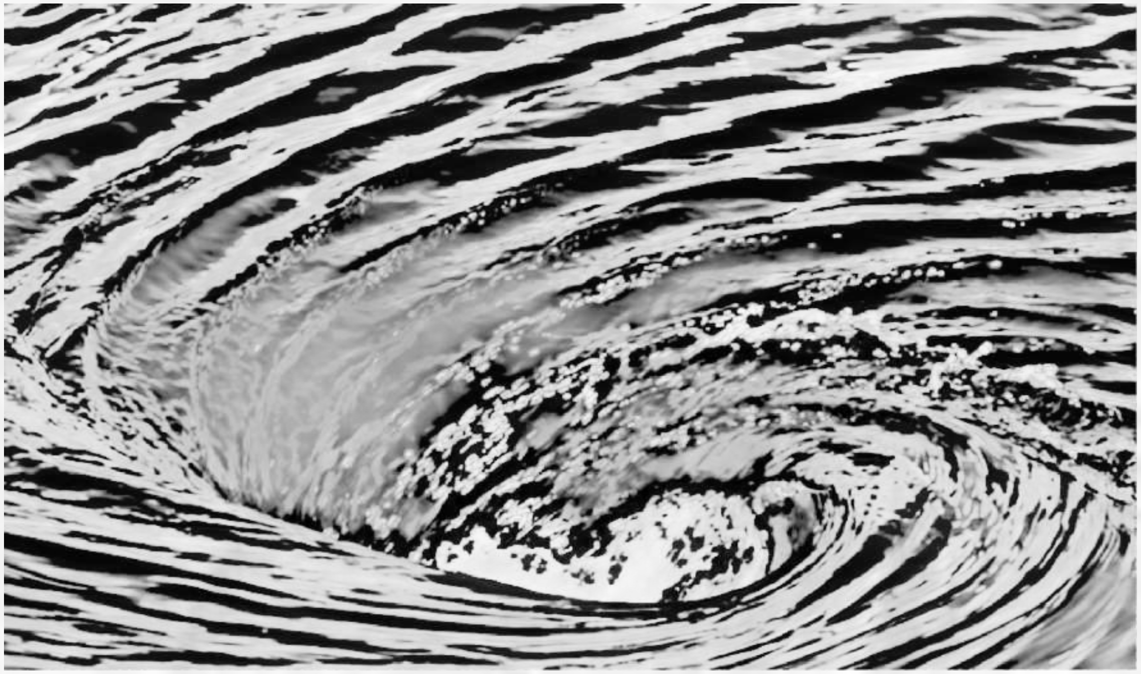
$$j^\mu = nu^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{6\pi^2\hbar^2}, \quad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$



ELECTRON TRANSPORT IN HYDRODYNAMIC REGIME

Hydrodynamics in Weyl metals

The Euler equation for electron fluid: [Gurzhi, JETP 17, 521 (1963)]

$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right)=-en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_{\mathbf{r}})$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

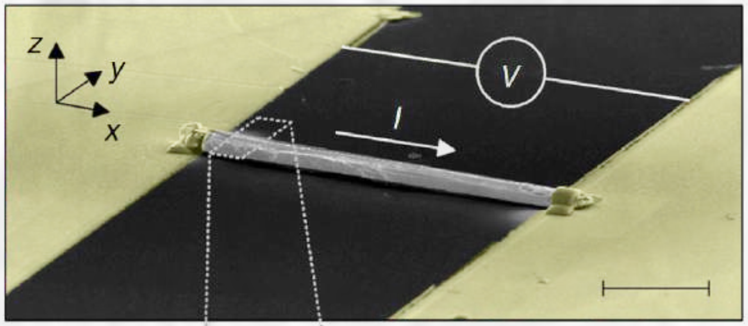
The energy conservation

$$\partial_t\epsilon=-\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_{\mathbf{r}})$$

\+ Maxwell equations with the Chern-Simons currents

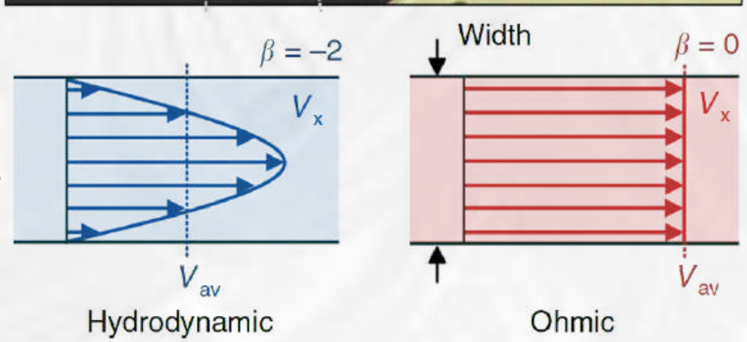
$$\rho_{\text{CS}}=-\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\text{CS}}=-\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$



Experimental evidence in tungsten diphosphide (WP_2): $\rho=\rho_0+\rho_1 w^{\beta}$

[Gooth et al., Nature Commun. **9**, 4093 (2018)]



Rich spectrum of hydro modes

- Magneto-acoustic wave ($\rho = 0$):

$$\omega_{\mathrm{s},\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau}\sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)}\left[2|\mathbf{k}|^2 B_0^2 - (\mathbf{k}\cdot\mathbf{B}_0)\right]}{3w_0}}$$

- *Gapped* chiral magnetic wave ($\rho = 0$):

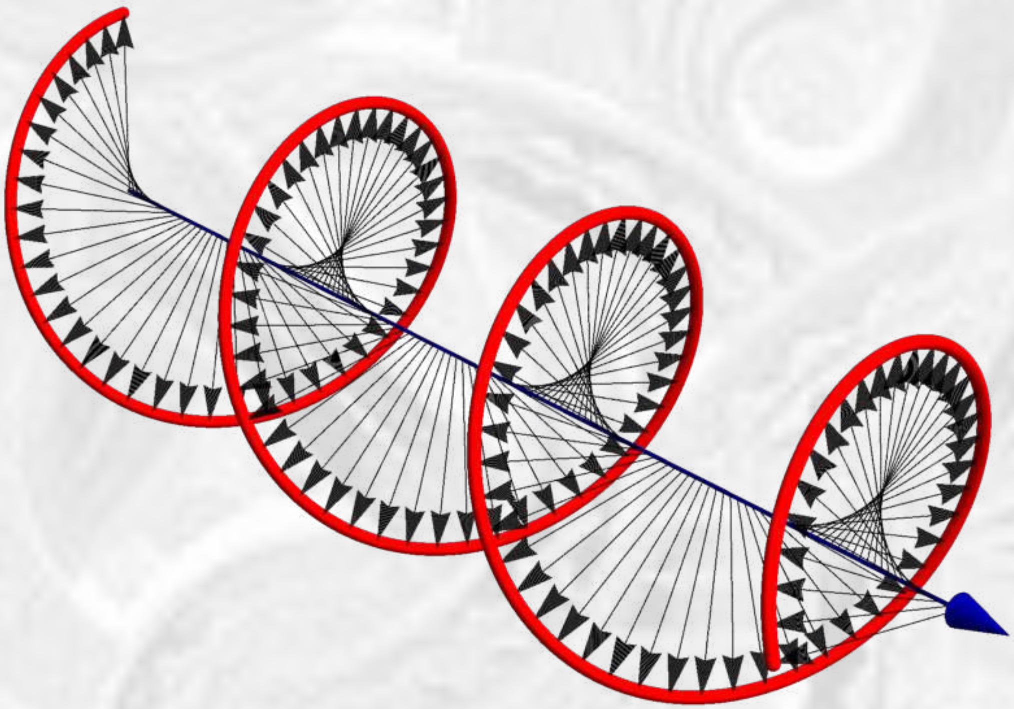
$$\omega_{\mathrm{gCMW},\pm} = \pm \frac{eB_0\sqrt{3v_F^3\left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_\parallel^2\right)}}{2\pi^2 T_0^2 c\sqrt{\varepsilon_e \hbar}}$$

- Helicons ($\rho \neq 0$):

$$\omega_{\mathrm{h},\pm} \approx \mp \frac{c k_\parallel^2 B_0}{4\pi \mu_m \rho_0} - \frac{i}{\tau}\frac{c^2 k_\parallel^2 w_0}{4\pi \mu_m v_F^2 \rho_0^2} + O(k_\parallel^3)$$

- New anomalous Hall waves at $\vec{b} \neq 0$, etc.

[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

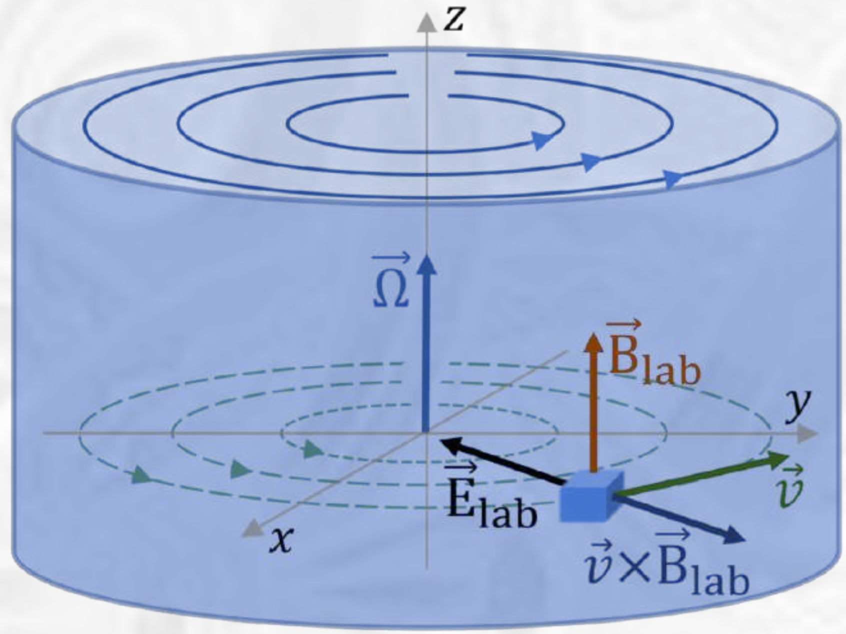


COLLECTIVE MODES IN RELATIVISTIC CHIRAL MATTER

Rotating charged plasma

- Fluid velocity

$$\bar{u}^{\nu} = \gamma \begin{pmatrix} 1 \\ -\Omega y \\ \Omega x \\ 0 \end{pmatrix}$$



where $\gamma = 1/\sqrt{1-(\Omega\, r)^2}$

Vorticity: $\bar{\omega}^{\mu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{u}_{\nu}\partial_{\alpha}\bar{u}_{\beta} = \gamma^2\Omega\delta_3^{\mu}$

EM fields in lab frame: $\mathbf{B}_{\rm lab} = \gamma B\hat{\mathbf{z}}$

$$\mathbf{E}_{\rm lab} = -\gamma B\Omega\mathbf{r}_{\perp}$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

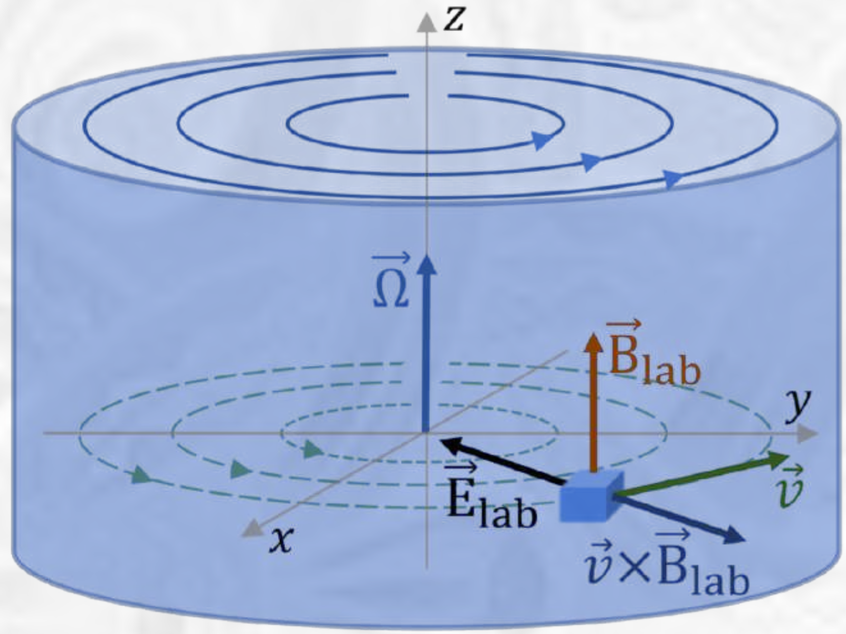
Rotating charged plasma: equilibrium

- Maxwell equations:

$$\partial_\nu F^{\nu\mu} = enu^\mu + e\nu^\mu - en_{\rm bg}u^\mu_{\rm bg}$$

$$\partial_\nu \tilde{F}^{\nu\mu} = 0$$

where $n_{\rm bg}$ is the background



The solution is radially nonuniform:

$$B(r) = \gamma\left(B_0 - \frac{1}{2}en_{\rm bg}\Omega r^2\right) \simeq B_0 - \frac{1}{2}en_{\rm bg}\Omega r^2 + O\left(B_0 r^2\Omega^2\right)$$

$$en_{\rm eq}(r) = \gamma^3\left(en_{\rm bg} - 2B_0\Omega\right) \simeq en_{\rm bg} - 2B_0\Omega + O\left(en_{\rm bg}r^2\Omega^2\right)$$

(This is consistent with $\mu = \gamma\mu_0$, $\mu_5 = \gamma\mu_{5,0}$, $T = \gamma T_0$.)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Linearized equations, etc.

- Small perturbations ($\Omega \to 0$):

$$\delta s(x) = e^{-ik_0 t + ik_z z + im\theta} \delta s(r)$$
$$\delta v^3(x) = e^{-ik_0 t + ik_z z + im\theta} \delta v^3(r)$$
$$\delta v_\pm(x) = e^{-ik_0 t + ik_z z + i(m\pm 1)\theta} \delta v_\pm(r)$$

where

$$\delta s(r) = \delta s \, J_m(k_\perp r), \quad \text{for} \quad s = \mu, \mu_5, T$$
$$\delta v^3(r) = \delta v^3 \, J_m(k_\perp r), \quad \text{for} \quad v^3 = u^3, B^3, E^3$$
$$\delta v_\pm(r) = \delta v_\pm \, J_{m\pm 1}(k_\perp r), \quad \text{for} \quad v_\pm = u_\pm, B_\pm, E_\pm$$

- Boundary conditions: $\delta s(R) = 0, \ \delta v^3(R) = 0$

- Transverse wave vectors: $k_\perp^{(i)} = \alpha_{m,i}/R$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Hierarchy of scales

- Mean free path $\ell_{\rm mfp} \simeq c\tau$, de Broglie wavelength $\ell_d = \hbar/T$, and typical wavelengths of modes $\lambda_k = 2\pi/k$:

$$\ell_d \ll \ell_{\rm mfp} \ll \lambda_k \lesssim R$$

- Magnetic length $\ell_B = \sqrt{\hbar/|eB|}$ ($\ell_d \ll \ell_B$ or $|eB| \ll T^2$)
  - Weak magnetic field: $\ell_B \gtrsim \ell_{\rm mfp}$
  - Moderately strong magnetic field: $\ell_B \lesssim \ell_{\rm mfp}$
- System size $R \lesssim \Omega^{-1}$
- In this work (with auxiliary parameter $\xi \simeq 0.01$)

$$\Omega\ell_{\rm mfp} \simeq \xi^2, \quad \xi^{3/2} \simeq \frac{\ell_{\rm mfp}}{R} \lesssim k\ell_{\rm mfp} \lesssim \xi^{1/2}, \quad \frac{\ell_{\rm mfp}}{\ell_B} \simeq \xi^{-1/4}, \quad \frac{\ell_{\rm mfp}}{\ell_d} \simeq \xi^{-1}$$

COLLECTIVE MODES IN HOT PLASMA

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Charged plasma at $\Omega = 0$

- Sound waves ($T \gg \mu$): [Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = s_e \frac{3\sqrt{5}B\hbar^{3/2}k_z}{\sqrt{7}\pi T^2}\left(1 \pm \frac{\sqrt{5}e\mu}{2\sqrt{7}\pi\hbar^{3/2}k}\right) - \frac{1}{10} i\tau k^2 .$$

where $k = \sqrt{k_z^2 + k_\perp^2}$ and $s_e = \pm 1$.

- There are also purely diffusive modes, e.g.,

$$k_0 = -\frac{e^2}{9\hbar^3} i\tau T^2$$

describing the charge diffusion (i.e., $\partial_t \mathbf{E} + e\sigma_E \mathbf{E} \approx 0$.)

Charged plasma at $\Omega \neq 0$

- Sound waves ($T \gg \mu$):

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2 + m\Omega \left(\frac{2}{3} + \frac{5e^2\mu^2}{14\pi^2\hbar^3 k^2} \right)$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = m\Omega + sk_z \sqrt{\frac{45\mathcal{B}_\pm^2\hbar^3}{7\pi^2 T^4} + \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm\mu}{14\pi^2 T^2 k} \right)^2} \pm k_z \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm\mu}{14\pi^2 T^2 k} \right)$$

with a small imaginary part (not shown)

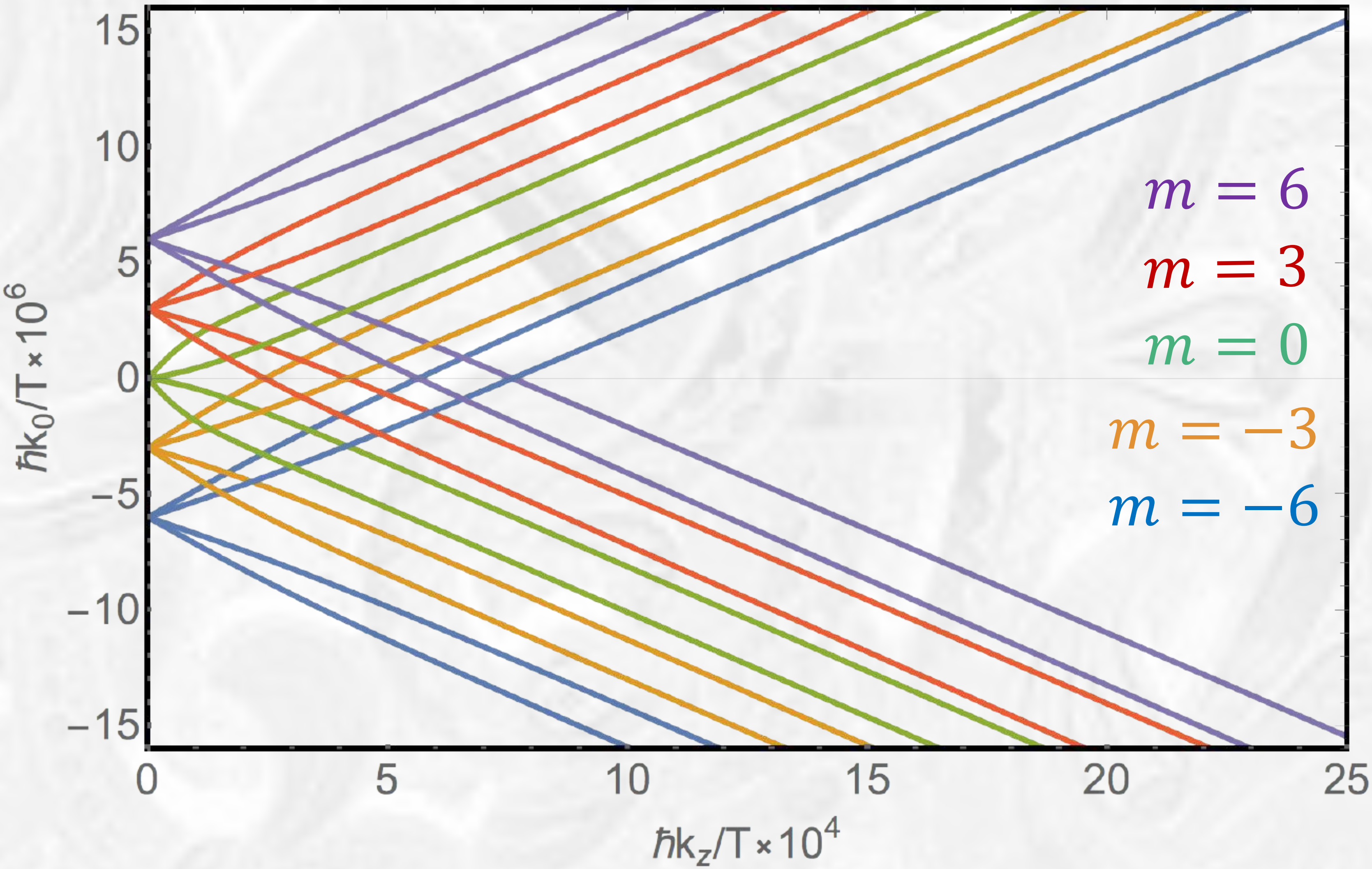
Here

$$\mathcal{B}_\pm = B - \frac{en_{\rm eq}\Omega}{6k_\perp^2}\left[2(m\pm 1)(m\pm 2) + k_\perp^2 R^2\right]$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Alfven waves ($T \gg \mu$)

- Alfven waves with angular momenta $m = 0, \pm 3, \pm 6$



[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Without dynamical fields

- Sound wave and two chiral waves ($T \gg \mu$):

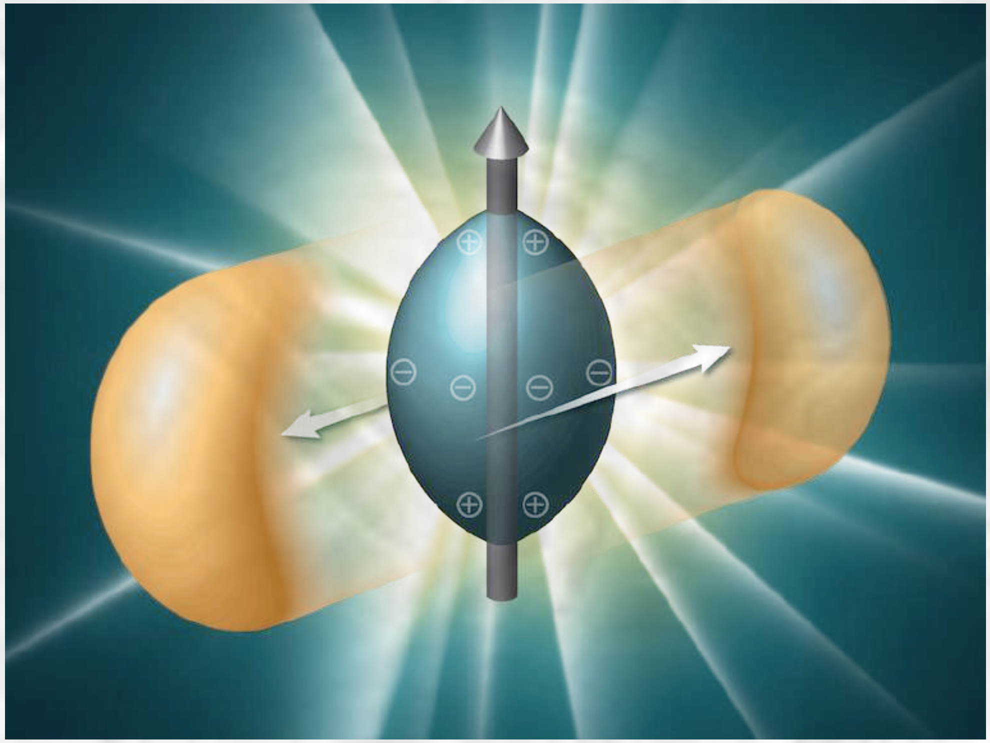
$$k_0 = \frac{s_e k}{\sqrt{3}} + \frac{2}{3} m\Omega - \frac{2}{15} i\tau k^2 \qquad \text{(sound)}$$

$$k_0 = m\Omega + s_e \frac{2k_z\Omega}{k} - \frac{1}{5} ik^2\tau \qquad \text{(CVW?)}$$

$$k_0 = m\Omega + s_e \frac{3e\mathcal{B}_0\hbar k_z}{2\pi^2 T^2} - \frac{1}{3} ik^2\tau \qquad \text{(CMW?)}$$

- There are more propagating (fewer diffusive) modes
- CVW & CMW appear only in nondynamical regime

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]



https://www.bnl.gov/newsroom/news.php?a=25735

CHIRAL MAGNETIC WAVE

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]

$eB \ll T^2$: Diffusive CMW

- Simple 1-flavor model ($\mathbf{k} \parallel \mathbf{B}$):

$$k_0 \delta n - kB\delta\sigma_B + i\frac{\tau}{3}k^2\delta n - \frac{1}{e}\sigma_E k \delta E_z = 0$$

$$k_0 \delta n_5 - kB\delta\sigma_B^5 + i\frac{\tau}{3}k^2\delta n_5 - i\frac{e^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i\frac{\sigma_E}{2} \pm i\frac{\sigma_E}{2}\sqrt{1 - \left(\frac{3eB}{\pi^2 T^2 \sigma_E}\right)^2 \left(k^2 + \frac{e^2T^2}{3}\right)} - i\frac{\tau}{3}k^2$$

- This is a completely diffusive mode when

$$\frac{3eB}{\pi^2 T^2 \sigma_E}\sqrt{k^2 + \frac{e^2T^2}{3}} < 1$$

Naïve CMW ($\sigma_E \to 0$)

- In contrast, if Gauss's law is ignored, the mode is non-diffusive, i.e.,

$$k_0^{(\pm)} = \pm\frac{3eB}{2\pi^2T^2}\sqrt{k^2+\frac{e^2T^2}{3}} - i\frac{\tau}{3}k^2, \quad \text{when} \quad \sigma_E \to 0.$$

- There is only a small dissipation due to the charge diffusion when $k \to 0$
- Notably, the CMW is gapped!
- The gap comes from the anomaly due to δE_z

$$-i\frac{e^2}{2\pi^2}B\delta E_z = -i\frac{e^2B}{2\pi^2}\left(\frac{-ie\delta n}{k}\right)$$

CMW in HIC

- Model with two light u- and d-quarks:

$$k_0 \delta n_f - \frac{e q_f B k}{2\pi^2 \chi_{f,5}} \delta n_{f,5} + i D_f k^2 \delta n_f - \frac{1}{e q_f} \sigma_{E,f} k \delta E_z = 0$$

$$k_0 \delta n_{f,5} - \frac{e q_f B k}{2\pi^2 \chi_f} \delta n_f + i D_f k^2 \delta n_{f,5} - i \frac{e^2 q_f^2}{2\pi^2} B \delta E_z = 0$$

$$k \delta E_z + i e \sum_f q_f \delta n_f = 0$$

where $f = u, d$, and $q_u = {}^2/_3$, $q_d = -{}^1/_3$

χ_f, D_f, and $\sigma_{E,f}$ are susceptibilities, diffusion coefficients and electrical conductivities for each quark flavor, respectively

Non-perturbative regime

Near-critical strongly coupled quark-gluon plasma

$$\sigma_E = \sum_f \sigma_{E,f} = c_\sigma C_{\rm em}^\ell T$$

$$\chi_f = c_\chi \chi_f^{(SB)}$$

$$D_f = \frac{c_D}{2\pi T}$$

$$\boxed{C_{\rm em}^\ell = \left({}^5/_9\right) 4\pi\alpha_{\rm em} \approx 0.051}$$

Lattice data

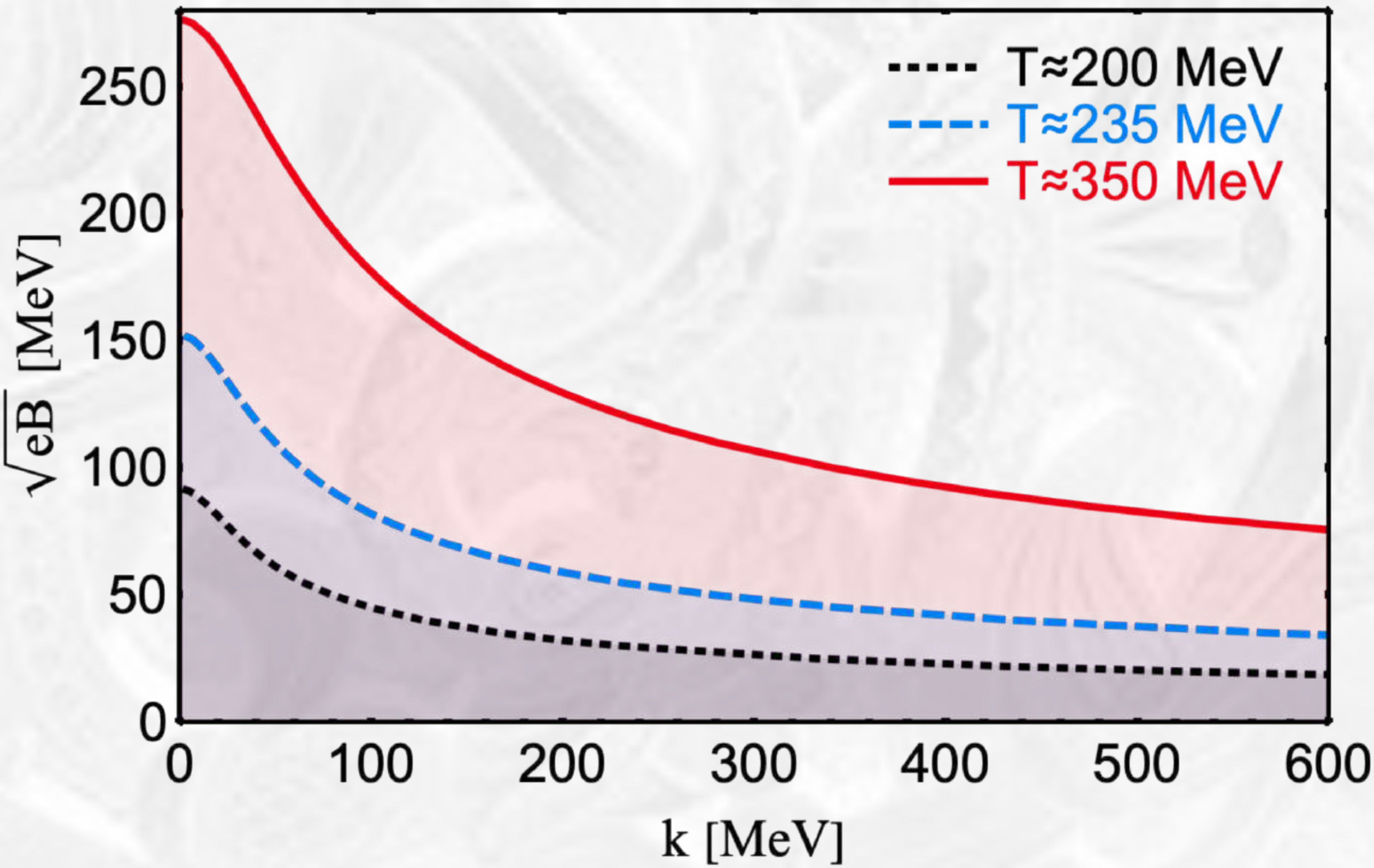
[Aarts, et. al. JHEP 1502, 186 (2015)]

	c_σ	c_χ	c_D
T=200 MeV	0.111	0.804	0.758
T=235 MeV	0.214	0.885	1.394
T=350 MeV	0.316	0.871	1.826

Results

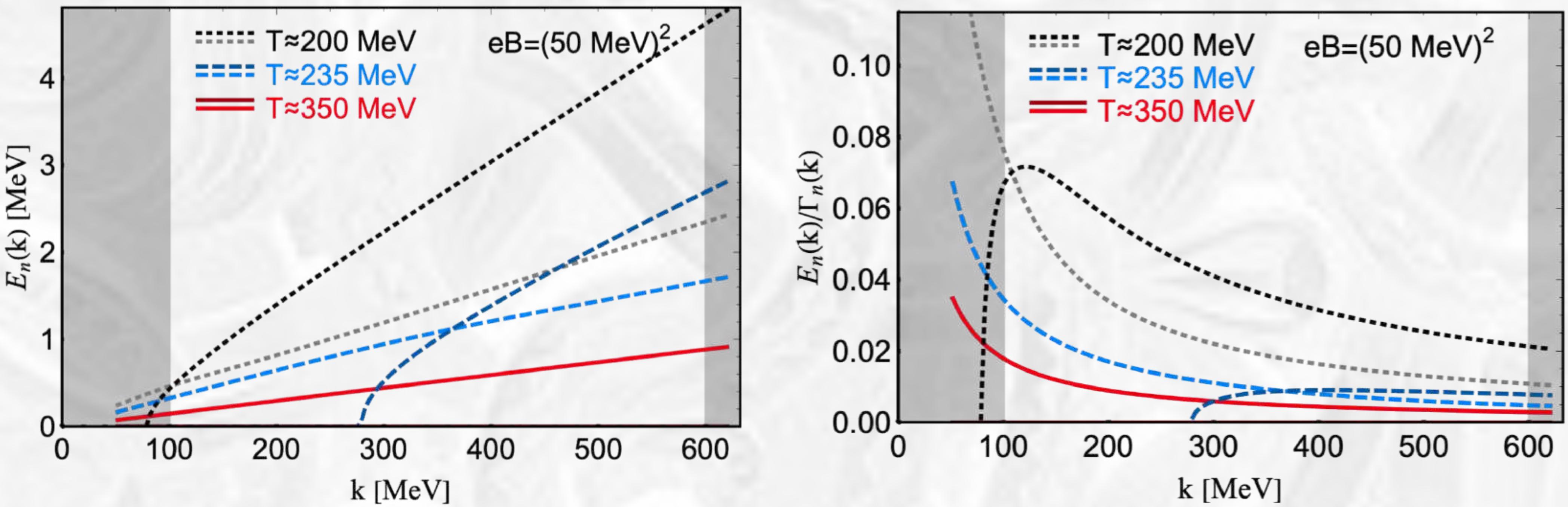
Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small *eB* & *k*:



Moderately strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



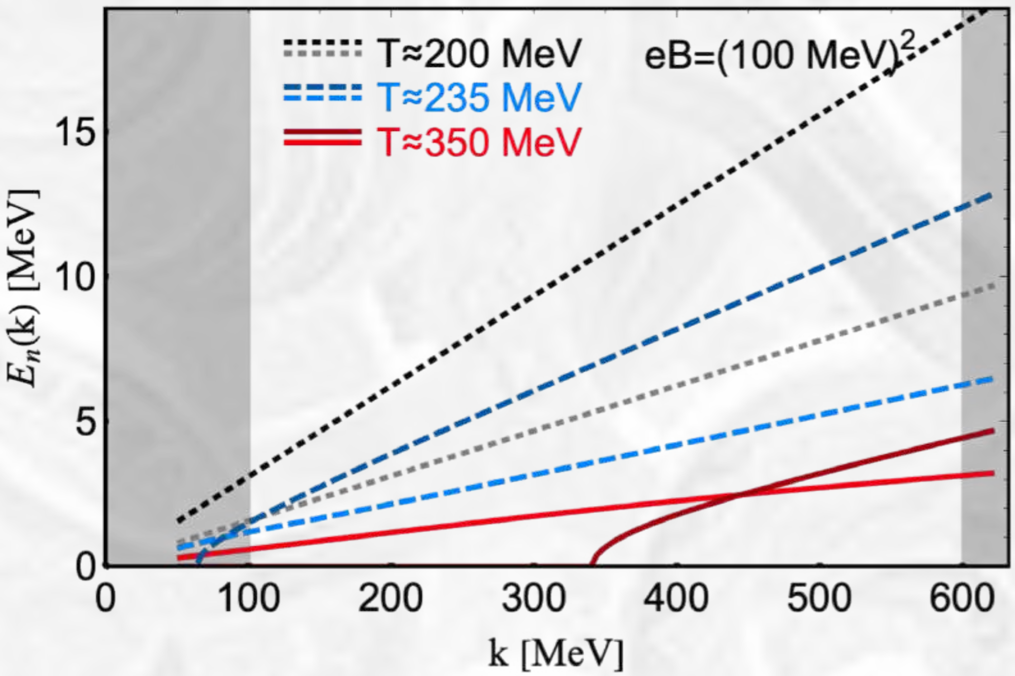
Allowed range of wave vectors:

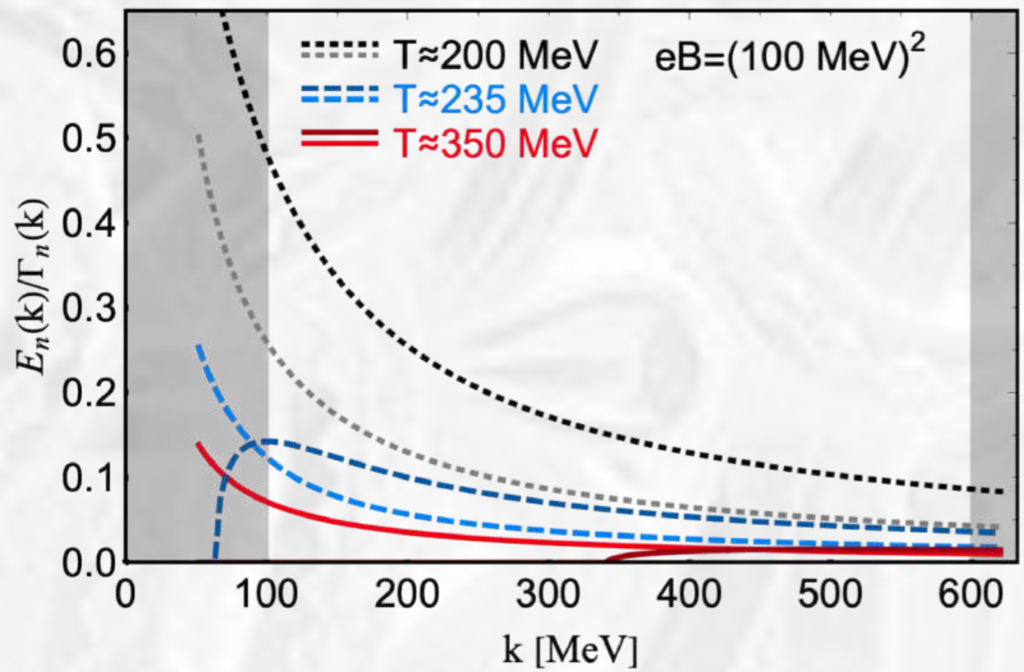
$$(50\text{ MeV})\ 100\text{ MeV} \lesssim k \lesssim 600\text{ MeV}$$

Wavelengths: $2\text{ fm} \lesssim \lambda_k \lesssim 12\text{ fm}\ (24\text{ fm})$

Strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



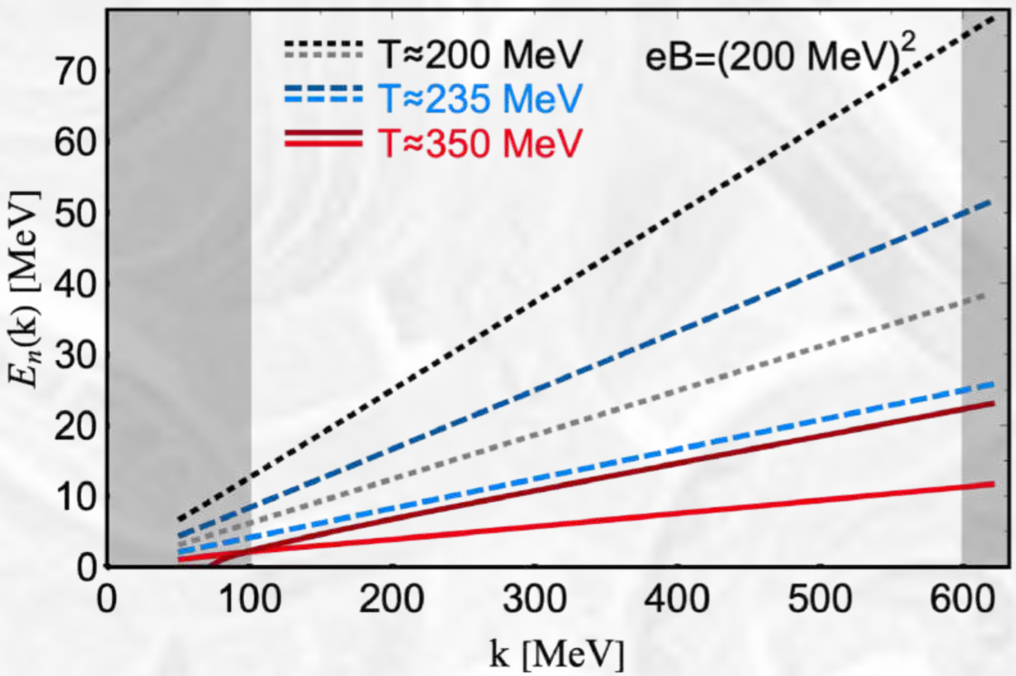


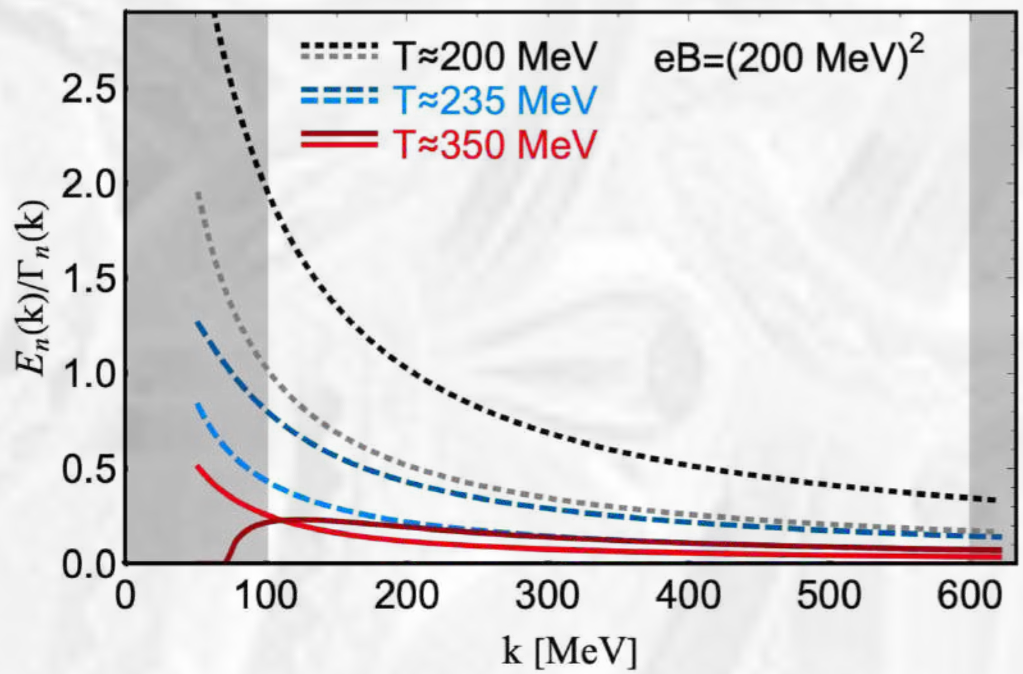
Even for such strong B-field, the CMW is strongly overdampled

Charge diffusion $iD_f k^2$ plays a big role $(k \gtrsim {}^{2\pi}/_{R})$

Very strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$





The CMW may become a propagating mode only in extremely strong B-fields, $eB \gtrsim (200\ \mathrm{MeV})^2$

Otherwise, it is overdampled

Summary

- Anomalous physics can be captured in hydrodynamic regime
- Propagating (not overdamped) hydro modes in charged rotating plasma are
  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density
- Dynamical electromagnetism plays a crucial role
  - Electrical conductivity screens charge fluctuations
  - Charge diffusion is not negligible in finite-size systems
- Chiral magnetic wave in HIC is overdamped