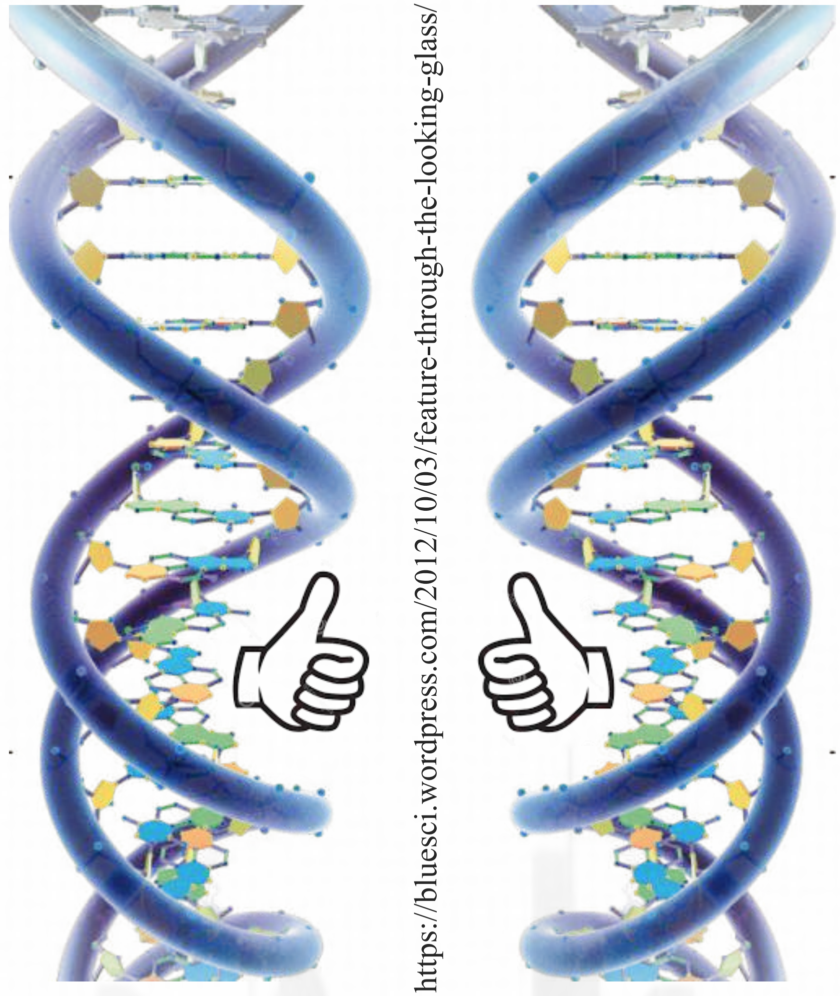
May 14, 2019

Dissipation of chiral magnetic wave

Igor Shovkovy
Arizona State University

Theoretical Physics Seminar
J. W. Goethe University, Frankfurt am Main

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]
[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]



CHIRAL MATTER

Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{\boldsymbol{p}}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{\boldsymbol{p}}}{|\vec{\boldsymbol{p}}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

For particles $(p_0 > 0)$: chirality = helicity

For antiparticles $(p_0 < 0)$: chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

Anomalous chiral matter

- Matter made of chiral fermions may allow $n_{\mathrm{L}} \neq n_{\mathrm{R}}$
- Unlike the electric charge $n_{\mathrm{R}} + n_{\mathrm{L}}$, the chiral charge $n_{\mathrm{R}} - n_{\mathrm{L}}$, is **not** conserved

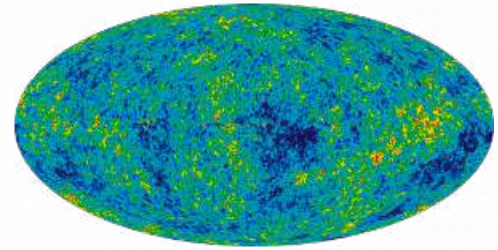
$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- Chiral anomaly, seen in the continuity equation, may have *macroscopic* signatures

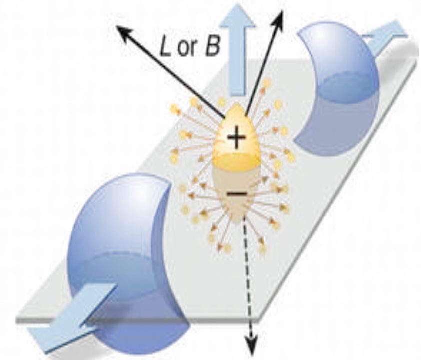
Chiral matter: Examples

- **Early Universe**, e.g.,
  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]



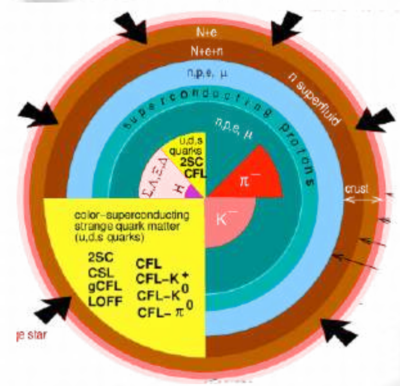
- **Heavy-ion collisions**, e.g.,

  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]



- **Super-dense matter in compact stars**, e.g.,

  [Yamamoto, Phys.Rev. D93, 065017 (2016)]

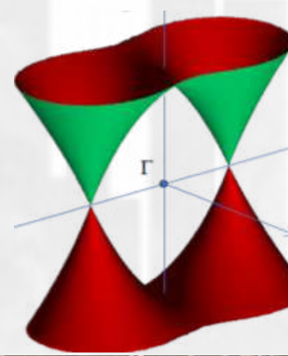


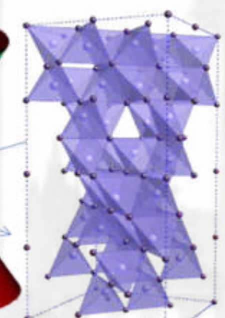
- **Superfluid ^{3}He-A**, e.g.,

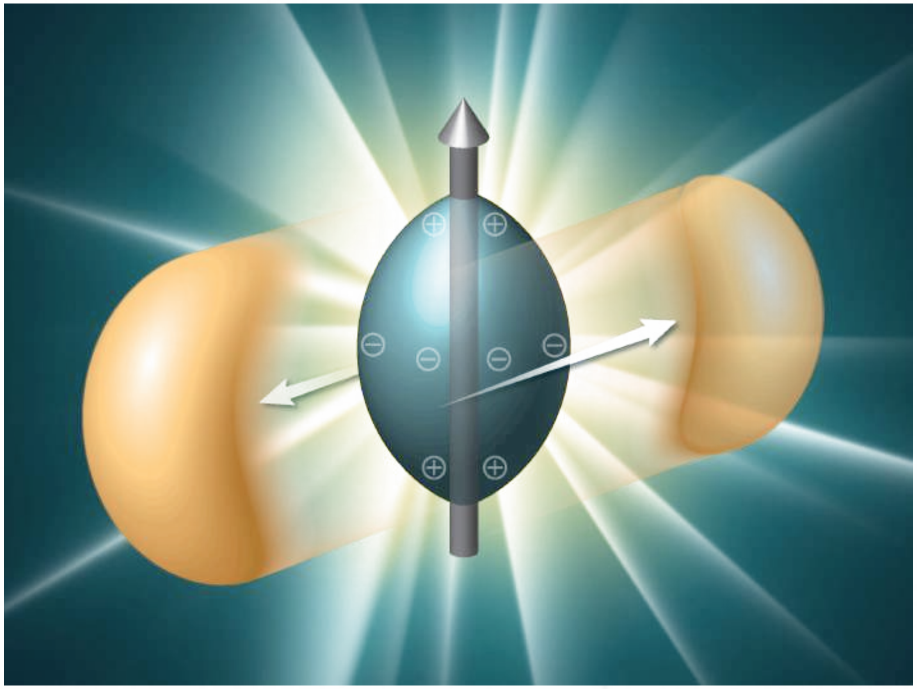
  [Volovik, JETP Lett. 105, 34 (2017)]

- **Dirac/Weyl (semi-)metals**, e.g.,

  [Li et. al. Nature Phys. 12, 550 (2016)]







https://www.bnl.gov/newsroom/news.php?a=25735

ANOMALOUS EFFECTS IN HEAVY-ION COLLISIONS

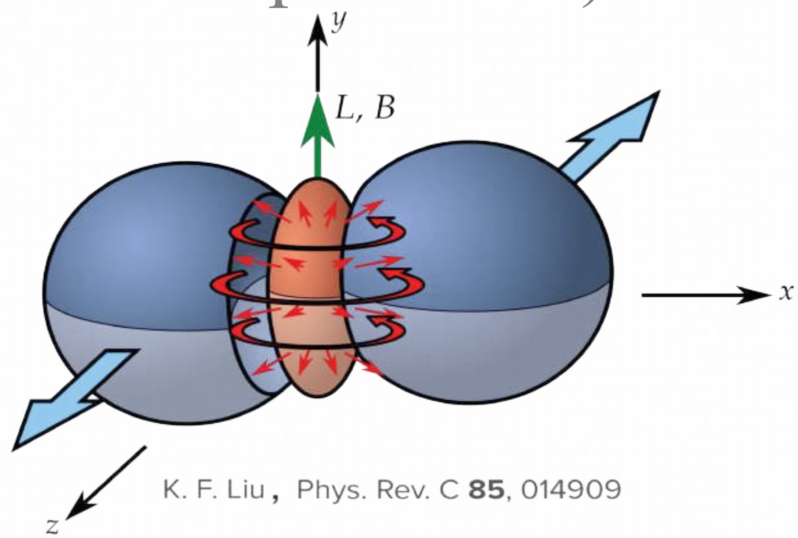
[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



K. F. Liu, Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak &. Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108], …

- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19}\text{ G } (\sim 100\text{ MeV})$$

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

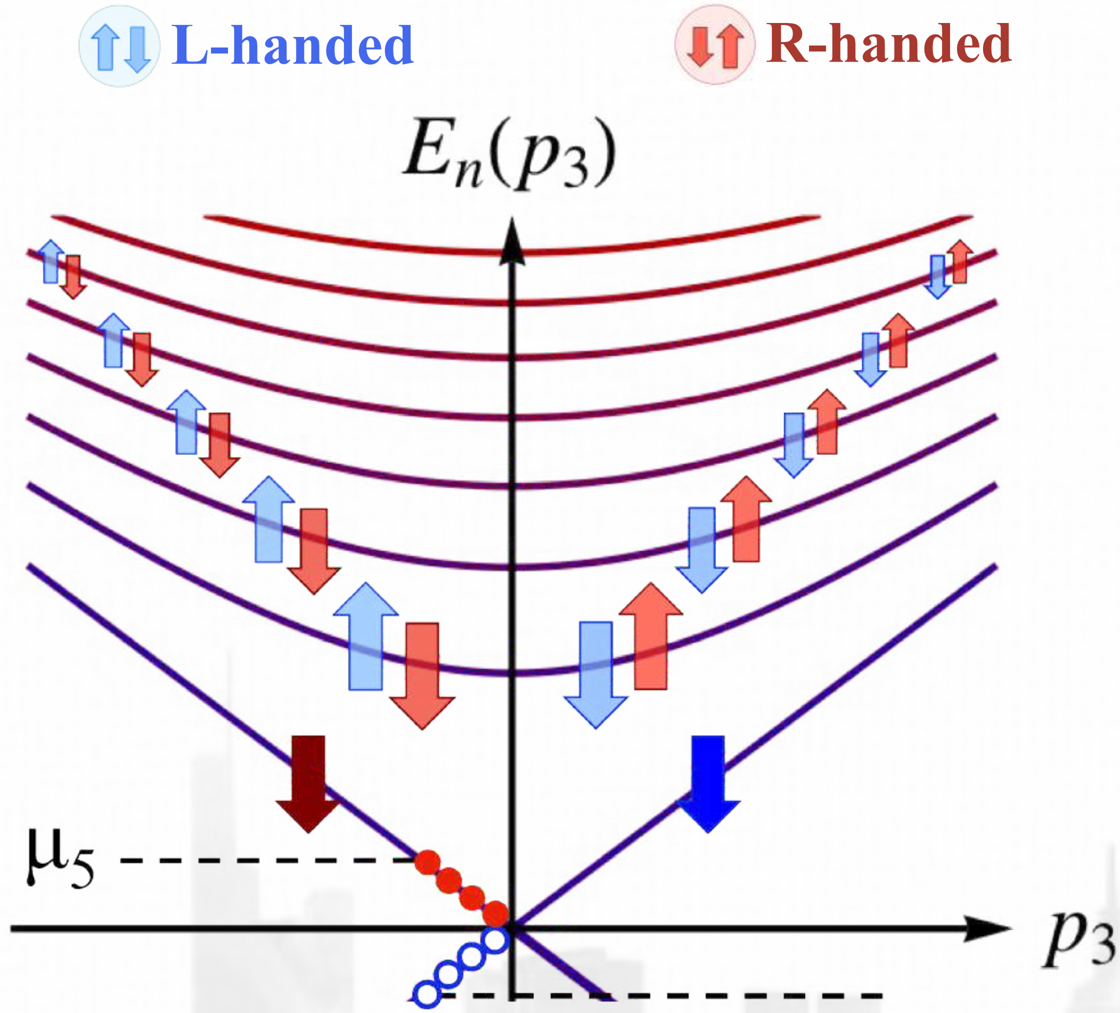
$$\omega \sim 9\times 10^{21} s^{-1}\ (\sim 10\text{ MeV})$$

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

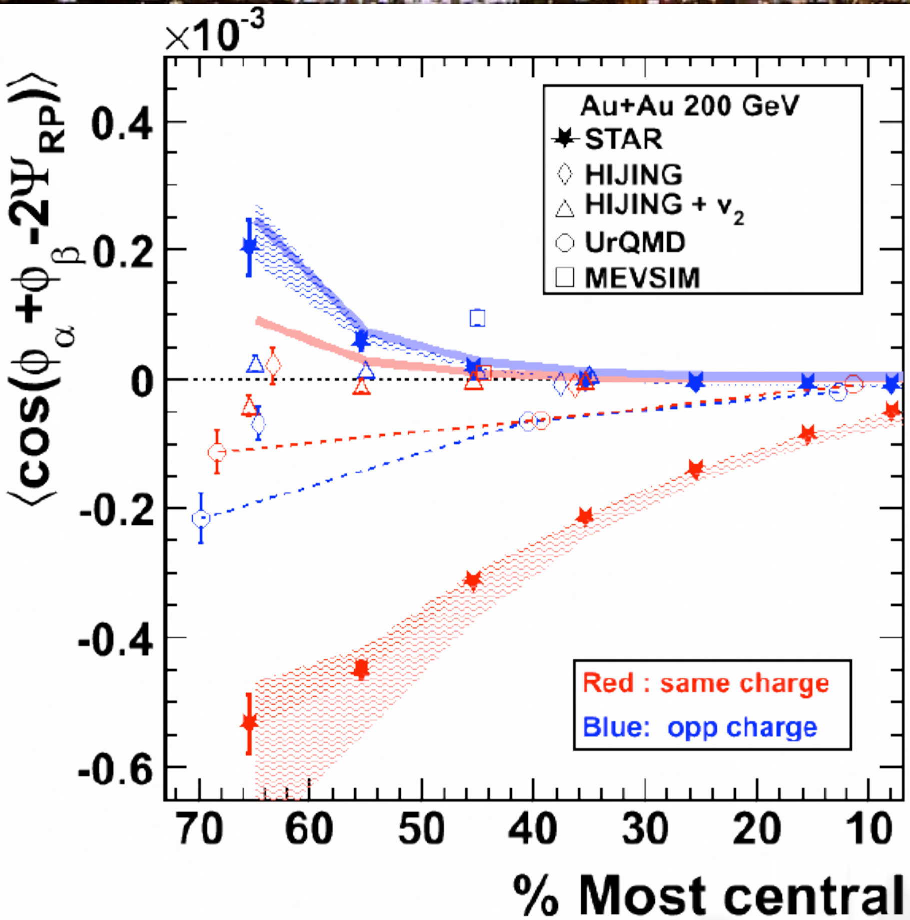
- R-handed states $p_3<0$ give current in $+z$ direction
- L-handed holes $p_3<0$ give current in $+z$ direction too!

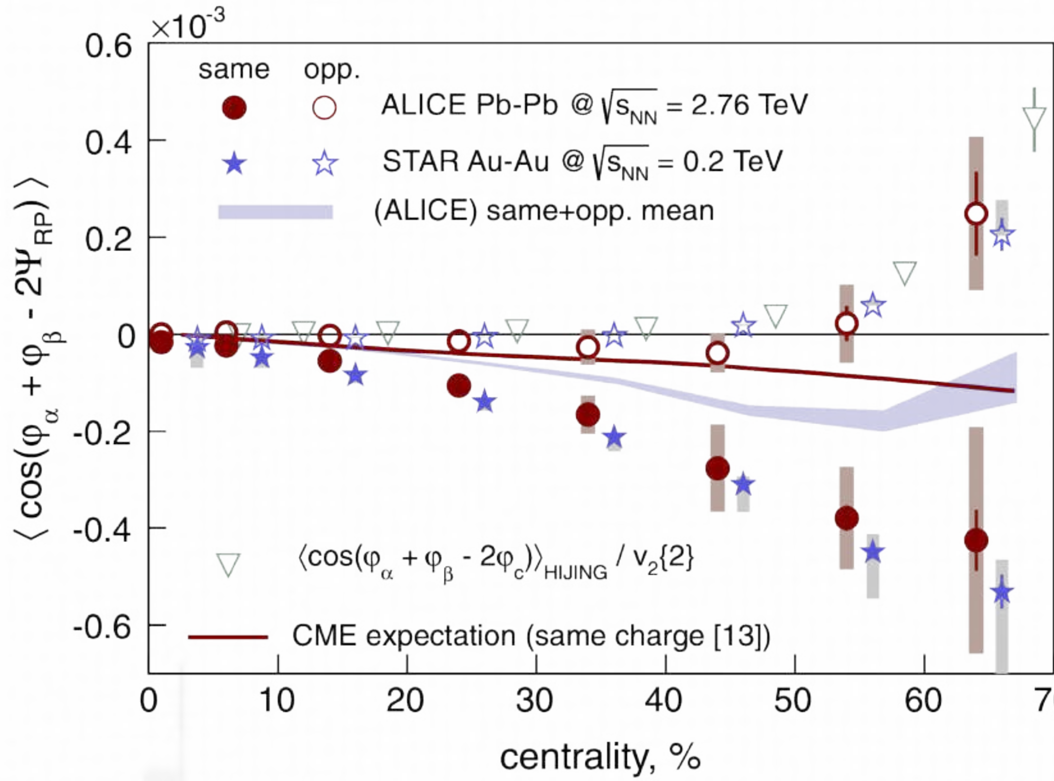


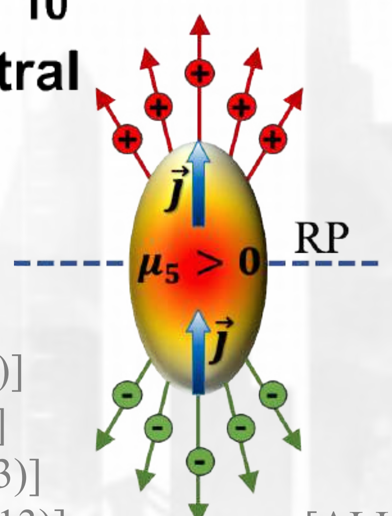
CME current: $$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence







Large background effects!

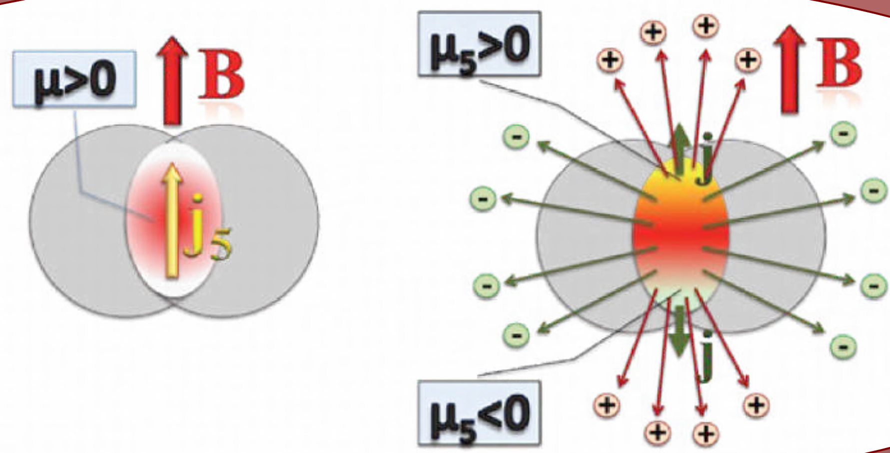
[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

[Belmont & Nagle, PRC **96**, 024901 (2017)]
[ALICE Collaboration, Phys. Lett. B**777**, 151 (2018)]

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW

$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

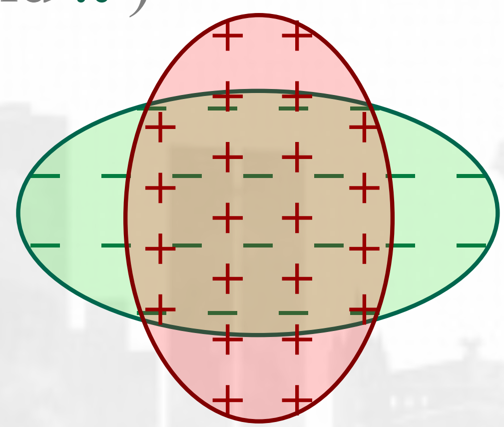


$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

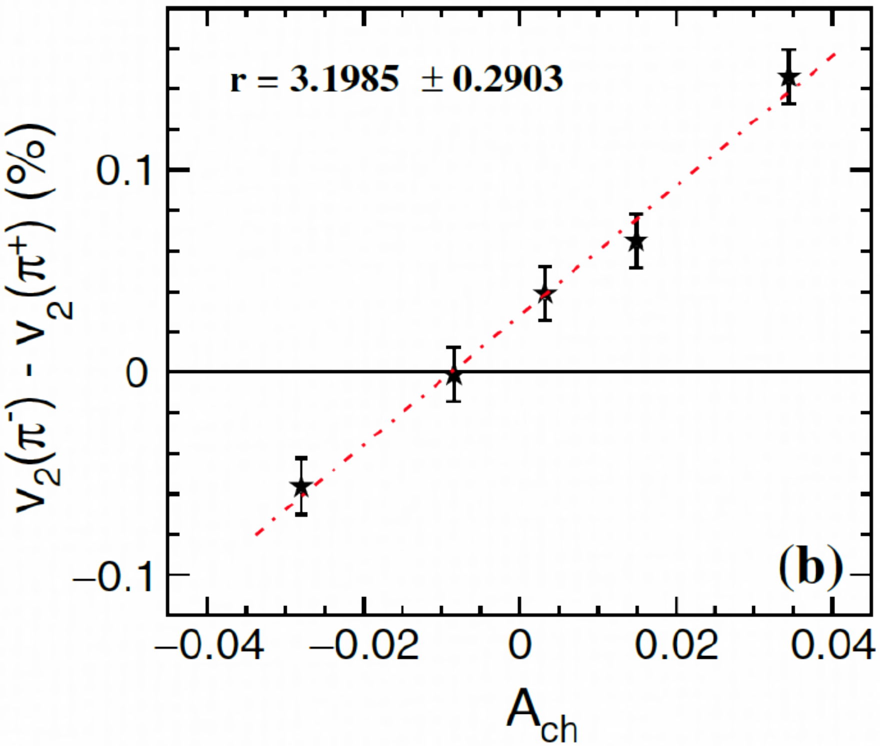
$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$

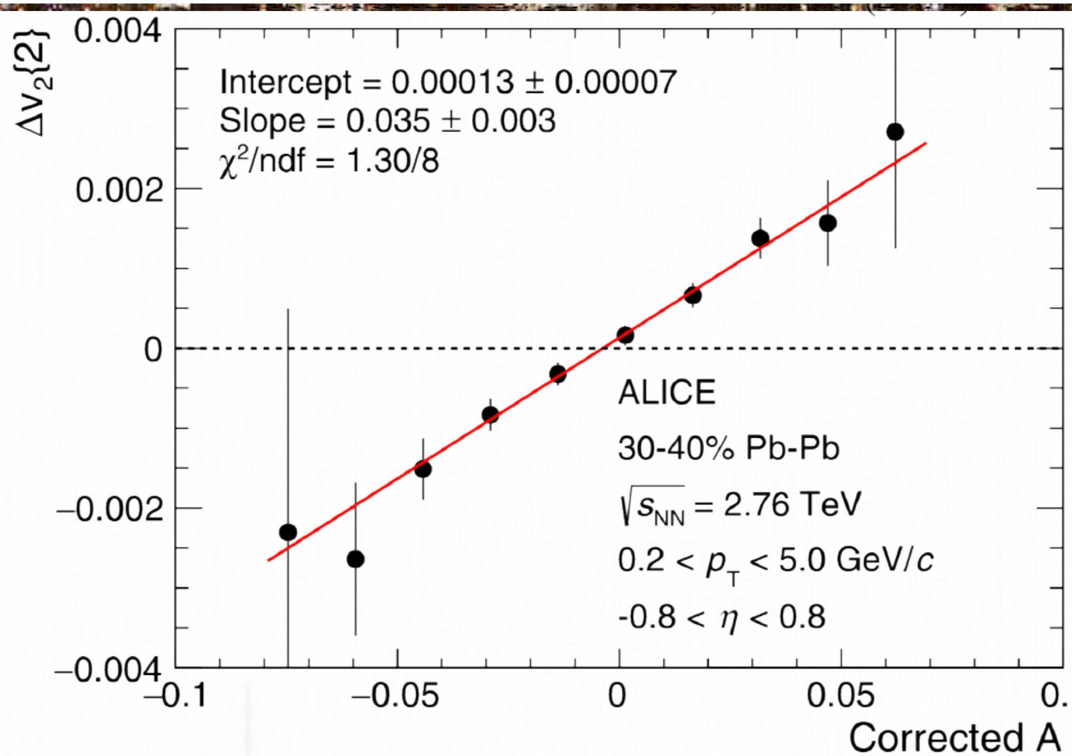


where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

CMW: Experimental evidence





[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

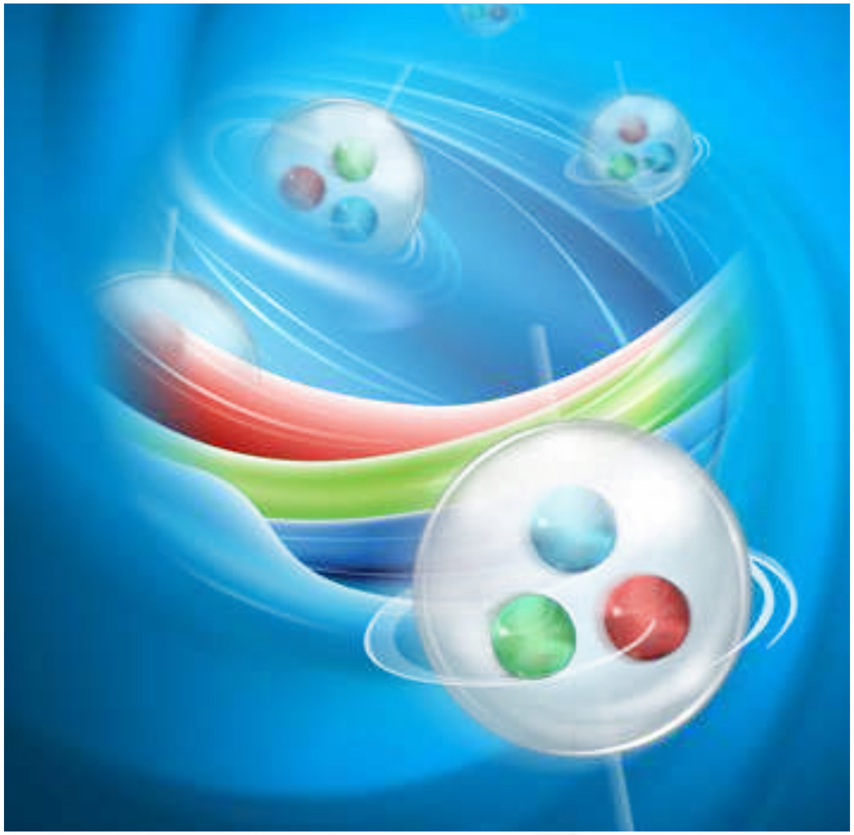
[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

This talk: the chiral magnetic wave is overdamped!



https://www.bnl.gov/newsroom/news.php?a=112068

FRAMEWORK: CHIRAL HYDRODYNAMICS

Chiral hydrodynamics

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)] [Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = eF^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = nu^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

Anomalous contributions

- Currents included new non-dissipative terms:

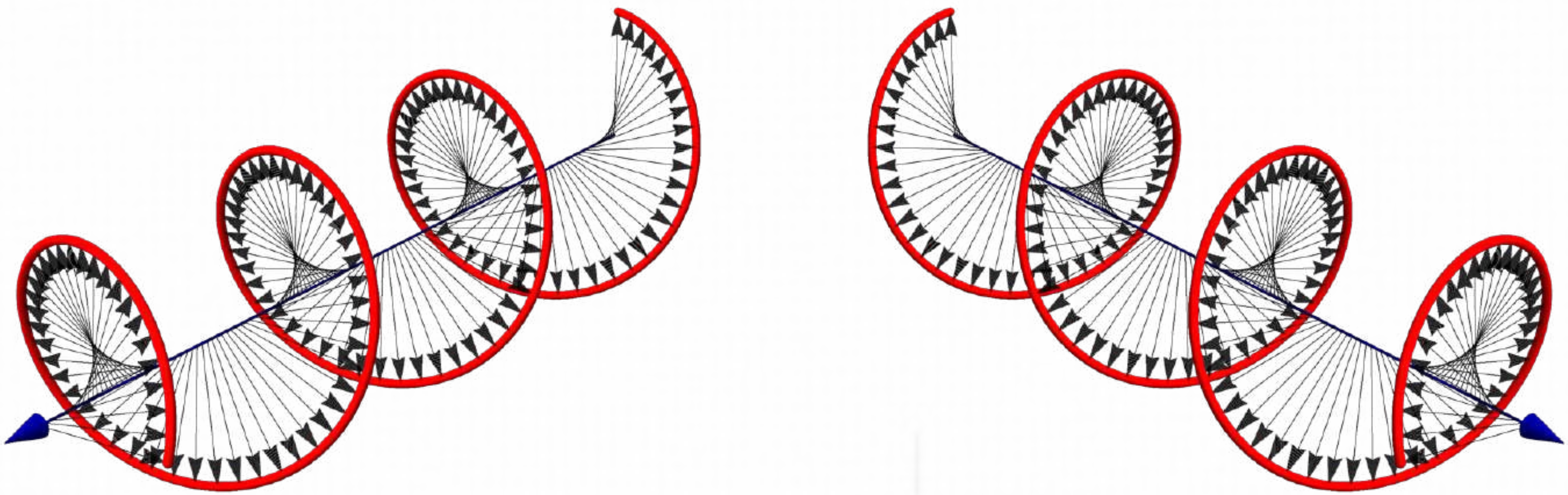
$$j^\mu = n u^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{6\pi^2\hbar^2}, \qquad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$

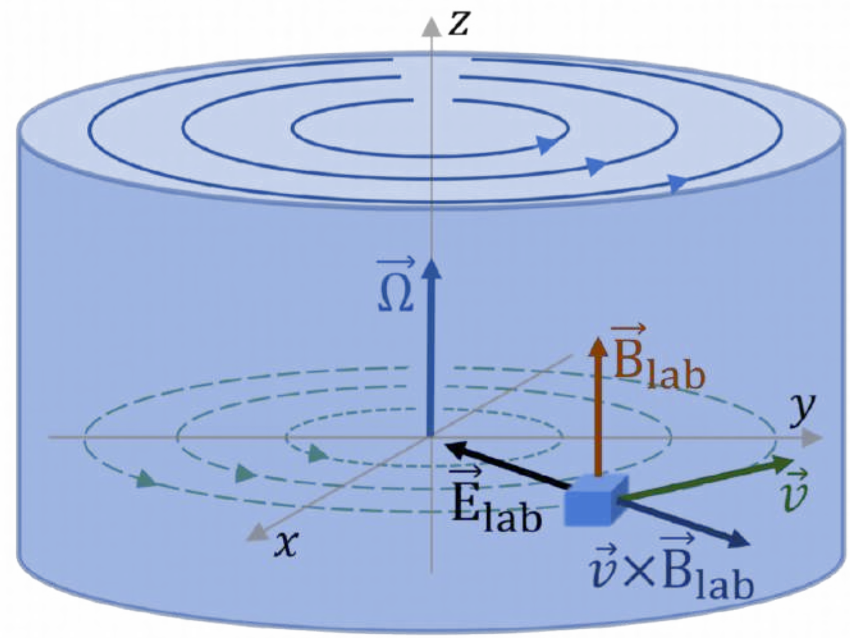


COLLECTIVE MODES IN CHIRAL MATTER

Rotating charged plasma

- Fluid velocity

$$\bar{u}^\nu = \gamma \begin{pmatrix} 1 \\ -\Omega y \\ \Omega x \\ 0 \end{pmatrix}$$



where $\gamma = 1/\sqrt{1-(\Omega\, r)^2}$

Vorticity: $\bar{\omega}^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{u}_\nu\partial_\alpha\bar{u}_\beta = \gamma^2\Omega\delta_3^\mu$

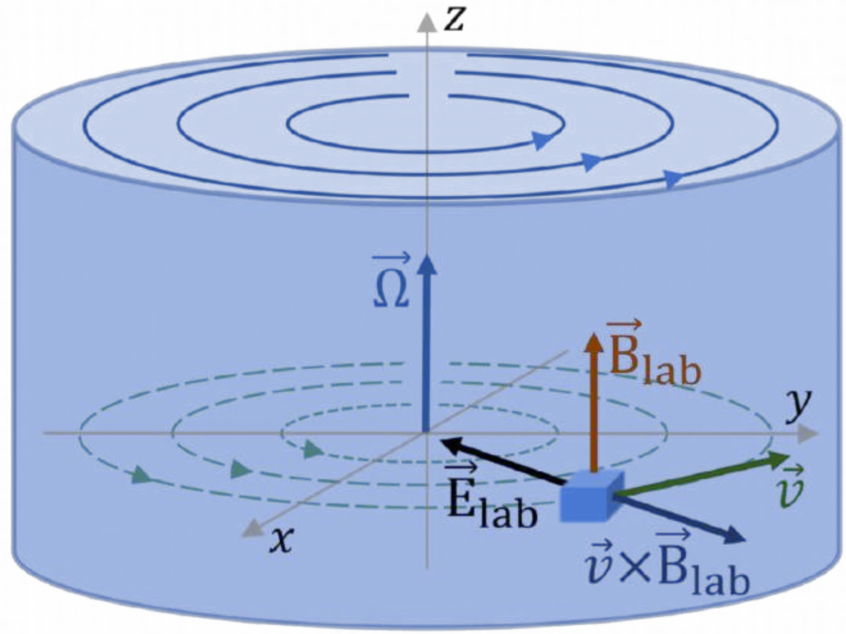
EM fields in lab frame: $\mathbf{B}_{\text{lab}} = \gamma B\hat{\mathbf{z}}$

$$\mathbf{E}_{\text{lab}} = -\gamma B\Omega\mathbf{r}_\perp$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Rotating charged plasma: equilibrium

- Maxwell equations:



$$\partial_\nu F^{\nu\mu} = enu^\mu + e\nu^\mu - en_{\mathrm{bg}} u^\mu_{\mathrm{bg}}$$

$$\partial_\nu \tilde{F}^{\nu\mu} = 0$$

where n_{bg} is the background

The solution is radially nonuniform:

$$B(r) = \gamma\left(B_0 - \frac{1}{2}en_{\mathrm{bg}}\Omega r^2\right) \simeq B_0 - \frac{1}{2}en_{\mathrm{bg}}\Omega r^2 + O\left(B_0 r^2 \Omega^2\right)$$

$$en_{\mathrm{eq}}(r) = \gamma^3\left(en_{\mathrm{bg}} - 2B_0\Omega\right) \simeq en_{\mathrm{bg}} - 2B_0\Omega + O\left(en_{\mathrm{bg}} r^2 \Omega^2\right)$$

(This is consistent with $\mu = \gamma\mu_0,\ \mu_5 = \gamma\mu_{5,0},\ T = \gamma T_0$.)

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Linearized equations, etc.

- Small perturbations ($\Omega \to 0$):

$$\delta s(x) = e^{-ik_0 t + ik_z z + im\theta} \delta s(r)$$
$$\delta v^3(x) = e^{-ik_0 t + ik_z z + im\theta} \delta v^3(r)$$
$$\delta v_\pm(x) = e^{-ik_0 t + ik_z z + i(m\pm 1)\theta} \delta v_\pm(r)$$

where

$$\delta s(r) = \delta s\, J_m(k_\perp r), \quad \text{for} \quad s = \mu, \mu_5, T$$
$$\delta v^3(r) = \delta v^3\, J_m(k_\perp r), \quad \text{for} \quad v^3 = u^3, B^3, E^3$$
$$\delta v_\pm(r) = \delta v_\pm\, J_{m\pm 1}(k_\perp r), \quad \text{for} \quad v_\pm = u_\pm, B_\pm, E_\pm$$

- Boundary conditions: $\delta s(R) = 0,\ \delta v^3(R) = 0$
- Transverse wave vectors: $k_\perp^{(i)} = \alpha_{m,i}/R$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Hierarchy of scales

- Mean free path $\ell_{\rm mfp} \simeq c\tau$, de Broglie wavelength $\ell_d = \hbar/T$, and typical wavelengths of modes $\lambda_k = 2\pi/k$:

$$\ell_d \ll \ell_{\rm mfp} \ll \lambda_k \lesssim R$$

- Magnetic length $\ell_B = \sqrt{\hbar/|eB|}$ $(\ell_d \ll \ell_B \text{ or } |eB| \ll T^2)$
  - Weak magnetic field: $\ell_B \gtrsim \ell_{\rm mfp}$
  - Moderately strong magnetic field: $\ell_B \lesssim \ell_{\rm mfp}$
- System size $R \lesssim \Omega^{-1}$
- In this work (with auxiliary parameter $\xi \simeq 0.01$)

$$\Omega\ell_{\rm mfp} \simeq \xi^2, \quad \xi^{3/2} \simeq \frac{\ell_{\rm mfp}}{R} \lesssim k\ell_{\rm mfp} \lesssim \xi^{1/2}, \quad \frac{\ell_{\rm mfp}}{\ell_B} \simeq \xi^{-1/4}, \quad \frac{\ell_{\rm mfp}}{\ell_d} \simeq \xi^{-1}$$

COLLECTIVE MODES IN HOT PLASMA

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Charged plasma at $\Omega = 0$

- Sound waves ($T \gg \mu$):

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = s_e \frac{3\sqrt{5}B\hbar^{3/2}k_z}{\sqrt{7}\pi T^2}\left(1 \pm \frac{\sqrt{5}e\mu}{2\sqrt{7}\pi\hbar^{3/2}k}\right) - \frac{1}{10} i\tau k^2.$$

where $k = \sqrt{k_z^2 + k_\perp^2}$ and $s_e = \pm 1.$

- There are also purely diffusive modes, e.g.,

$$k_0 = -\frac{e^2}{9\hbar^3} i\tau T^2$$

describing the charge diffusion (i.e., $\partial_t \mathbf{E} + e\sigma_E \mathbf{E} \approx 0.$)

Charged plasma at $\Omega \neq 0$

- Sound waves ($T \gg \mu$):

$$k_0 = \frac{sk}{\sqrt{3}} - \frac{2}{15} i\tau k^2 + m\Omega\left(\frac{2}{3} + \frac{5e^2\mu^2}{14\pi^2\hbar^3 k^2}\right)$$

- Alfven waves ($T \gg \mu$):

$$k_0^{(\pm)} = m\Omega + sk_z\sqrt{\frac{45\mathcal{B}_\pm^2\hbar^3}{7\pi^2T^4} + \left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm\mu}{14\pi^2T^2k}\right)^2} \pm k_z\left(\frac{\Omega}{k} - \frac{15e\mathcal{B}_\pm\mu}{14\pi^2T^2k}\right)$$

with a small imaginary part (not shown)

Here

$$\mathcal{B}_\pm = B - \frac{en_{\mathrm{eq}}\Omega}{6k_\perp^2}\left[2(m\pm 1)(m\pm 2) + k_\perp^2 R^2\right]$$

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]

Without dynamical fields

- Sound wave and two chiral waves ($T \gg \mu$):

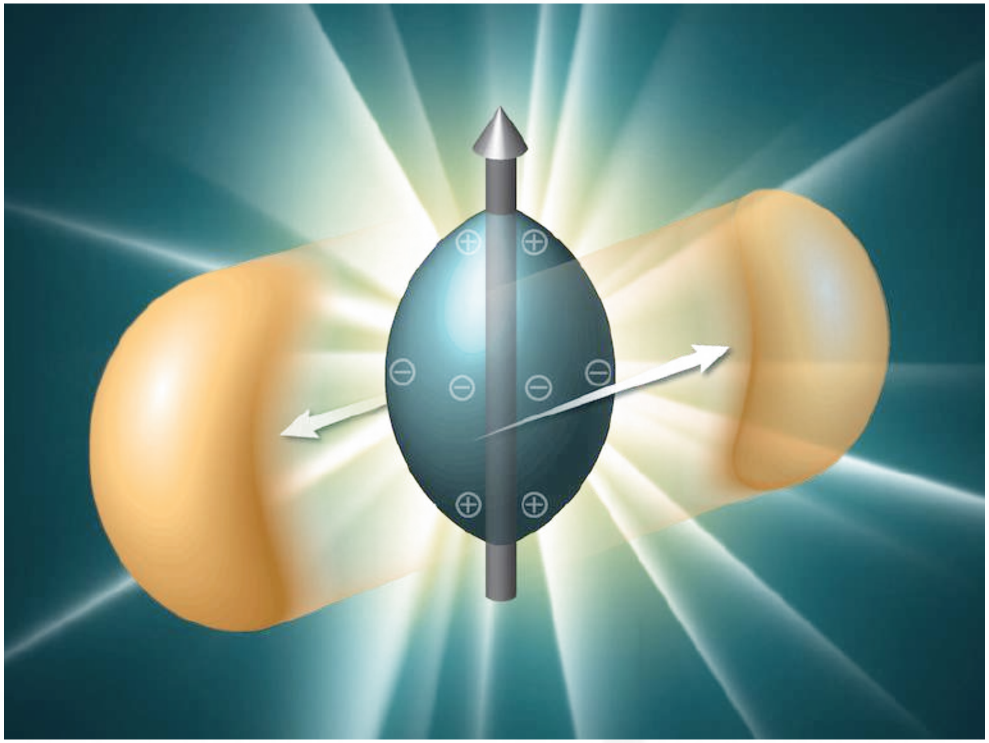
$$k_0 = \frac{s_e k}{\sqrt{3}} + \frac{2}{3} m\Omega - \frac{2}{15} i\tau k^2 \qquad \text{(sound)}$$

$$k_0 = m\Omega + s_e \frac{2k_z \Omega}{k} - \frac{1}{5} i k^2 \tau \qquad (\approx \text{CVW})$$

$$k_0 = m\Omega + s_e \frac{3e\mathcal{B}_0 \hbar k_z}{2\pi^2 T^2} - \frac{1}{3} i k^2 \tau \qquad (\approx \text{CMW})$$

- There are more propagating (fewer diffusive) modes
- CVW & CMW appear only in nondynamical regime

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]



https://www.bnl.gov/newsroom/news.php?a=25735

CHIRAL MAGNETIC WAVE

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]

$eB \ll T^2$: Diffusive CMW

- Simple 1-flavor model ($\mathbf{k} \parallel \mathbf{B}$):

$$k_0 \delta n - kB\delta\sigma_B + i\frac{\tau}{3}k^2\delta n - \frac{1}{e}\sigma_E k\delta E_z = 0$$

$$k_0 \delta n_5 - kB\delta\sigma_B^5 + i\frac{\tau}{3}k^2\delta n_5 - i\frac{e^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i\frac{\sigma_E}{2} \pm i\frac{\sigma_E}{2}\sqrt{1 - \left(\frac{3eB}{\pi^2 T^2 \sigma_E}\right)^2 \left(k^2 + \frac{e^2 T^2}{3}\right)} - i\frac{\tau}{3}k^2$$

- This is a completely diffusive mode when

$$\frac{3eB}{\pi^2 T^2 \sigma_E}\sqrt{k^2 + \frac{e^2 T^2}{3}} < 1$$

Naïve CMW ($\sigma_E \to 0$)

- In contrast, if Gauss's law is ignored, the mode is non-diffusive, i.e.,

$$k_0^{(\pm)} = \pm \frac{3eB}{2\pi^2 T^2}\sqrt{k^2 + \frac{e^2 T^2}{3}} - i\frac{\tau}{3}k^2, \quad \text{when} \quad \sigma_E \to 0.$$

- There is only a small dissipation due to the charge diffusion when $k \to 0$
- Notably, the CMW is gapped!
- The gap comes from the anomaly due to δE_z

$$-i\frac{e^2}{2\pi^2}B\delta E_z = -i\frac{e^2 B}{2\pi^2}\left(\frac{-ie\delta n}{k}\right)$$

CMW in HIC

- Model with two light u- and d-quarks:

$$k_0\delta n_f - \frac{eq_f Bk}{2\pi^2\chi_{f,5}}\delta n_{f,5} + iD_f k^2\delta n_f - \frac{1}{eq_f}\sigma_{E,f}k\delta E_z = 0$$

$$k_0\delta n_{f,5} - \frac{eq_f Bk}{2\pi^2\chi_f}\delta n_f + iD_f k^2\delta n_{f,5} - i\frac{e^2q_f^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\sum_f q_f\delta n_f = 0$$

where $f = u, d$, and $q_u = {}^2/_3$, $q_d = -{}^1/_3$

χ_f, D_f, and $\sigma_{E,f}$ are susceptibilities, diffusion coefficients and electrical conductivities for each quark flavor, respectively

Non-perturbative regime

Near-critical strongly coupled quark-gluon plasma

$$\sigma_E = \sum_f \sigma_{E,f} = c_\sigma C^\ell_{\text{em}} T$$

$$C^\ell_{\text{em}} = \left({}^5/_9\right) 4\pi\alpha_{\text{em}} \approx 0.051$$

$$\chi_f = c_\chi \chi_f^{(SB)}$$

$$D_f = \frac{c_D}{2\pi T}$$

Lattice data

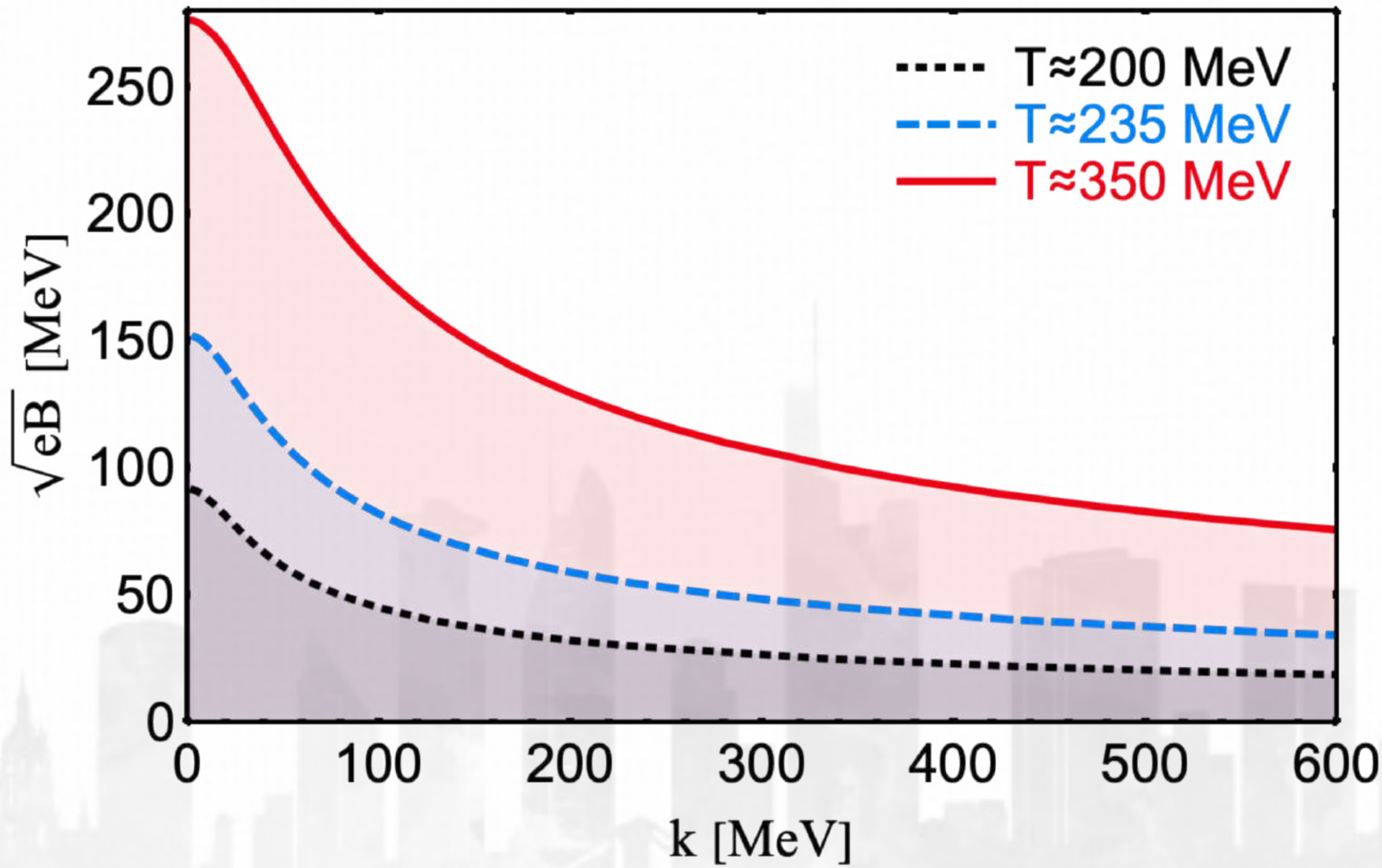
[Aarts, et. al. JHEP 1502, 186 (2015)]

	c_σ	c_χ	c_D
T=200 MeV	0.111	0.804	0.758
T=235 MeV	0.214	0.885	1.394
T=350 MeV	0.316	0.871	1.826

Results

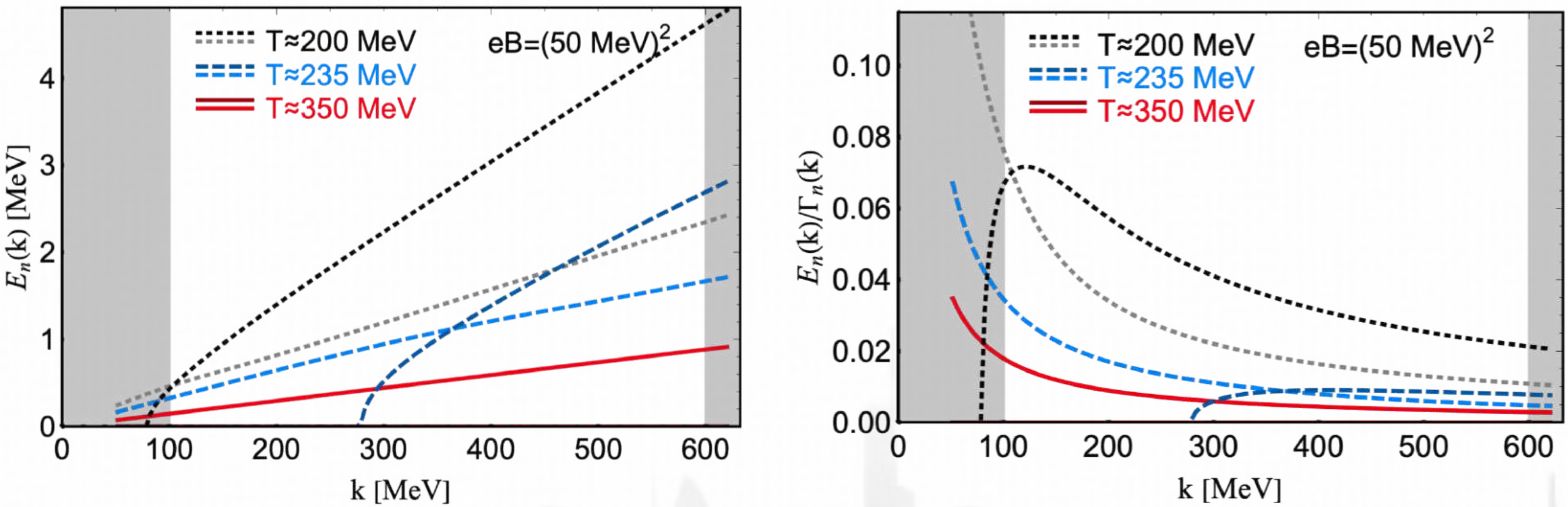
Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small eB & k:



Moderately strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



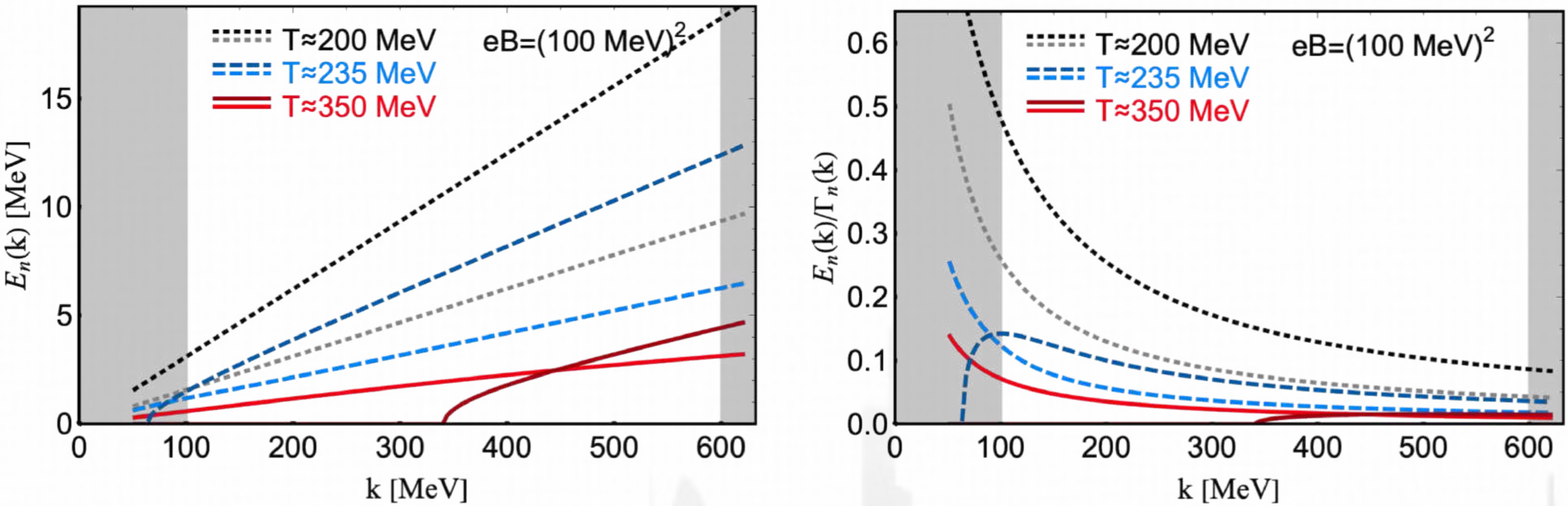
Allowed range of wave vectors:

$(50\ \text{MeV})\ 100\ \text{MeV} \lesssim k \lesssim 600\ \text{MeV}$

Wavelengths: $2\ \text{fm} \lesssim \lambda_k \lesssim 12\ \text{fm}\ (24\ \text{fm})$

Strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$

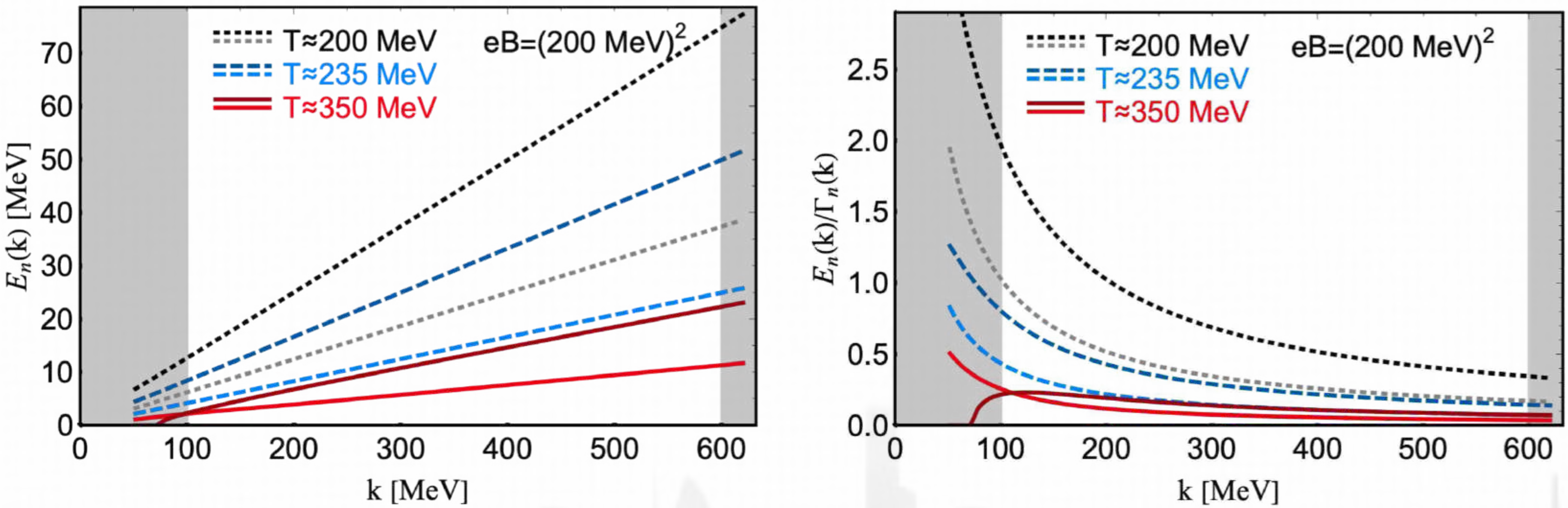


Even for such strong B-field, the CMW is strongly overdampled

Charge diffusion $iD_f k^2$ plays a big role $(k \gtrsim {}^{2\pi}/_{R})$

Very strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



The CMW may become a propagating mode only in extremely strong B-fields, $eB \gtrsim (200\text{ MeV})^2$

Otherwise, it is overdampled

Summary

- Macroscopic signatures of anomalous physics can be captured by hydrodynamics
- Propagating (not overdamped) hydro modes in charged rotating plasma are
  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density
- Dynamical electromagnetism plays a critical role
  - Electrical conductivity screens charge fluctuations
  - Charge diffusion is not negligible in finite-size systems
- Chiral magnetic wave in HIC is overdamped