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POLYTECHNIC CAMPUS

Physics Opportunities . . .
AUG 7, 2019
SLAC

DIMENSIONAL REDUCTION & CATALYSIS OF DYNAMICAL SYMMETRY BREAKING BY A MAGNETIC FIELD

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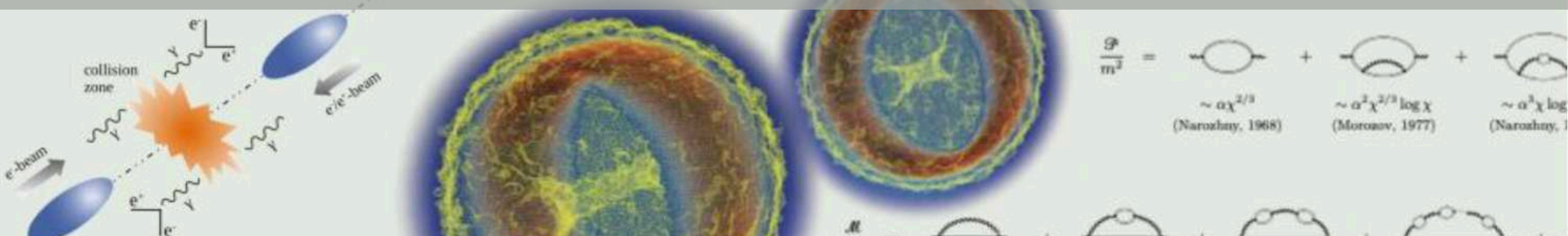
Physics Opportunities at a Lepton Collider in the Fully Nonperturbative QED Regime

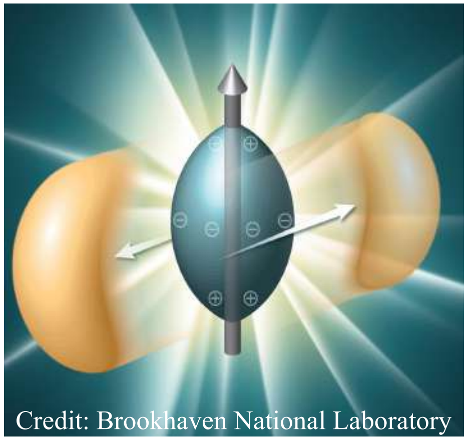


SLAC National Accelerator Laboratory   August 7-9, 2019

QCD in magnetic Fields

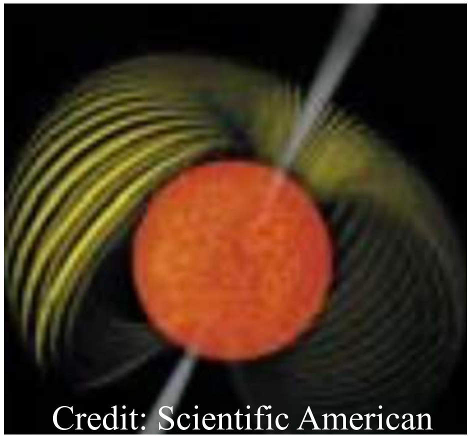
- Relativistic collisions of *heavy ions* produce quark-gluon plasma & strong magnetic fields

10^{18} - 10^{19} Gauss ($\sqrt{|eB|} \sim$ **100 MeV**)



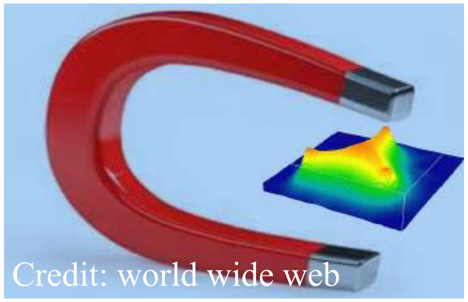
- Quark matter may form inside *magnetars*

10^{14} - 10^{16} Gauss ($\sqrt{|eB|} \sim$ **1 MeV to 10 MeV**)



- Strong magnetic field is a *theoretical tool* to probe the confinement dynamics in QCD at short distance scales, $\ell \sim 1/\sqrt{|eB|}$

$\gtrsim 10^{19}$ Gauss ($\sqrt{|eB|} \gtrsim$ **240 MeV**)



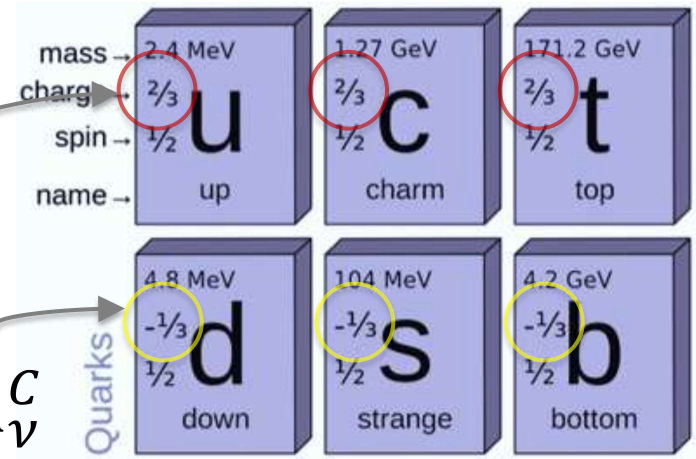
Set the stage

- Lagrangian density of QCD in an external magnetic field

$$\mathcal{L} = -\frac{1}{2} F_A^{\mu\nu} F_{\mu\nu}^A + \bar{\psi}_f (i\gamma^\mu D_\mu)\psi_f$$

where $D_\mu = \partial_\mu + igA_\mu^A \lambda^A/2 + ie_f A_\mu^{\text{ext}}$

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + gf^{ABC} A_\mu^B A_\nu^C$$



- The global chiral symmetry of the model

$$\underbrace{SU_L(N_u)\times SU_R(N_u)}_{\text{chiral symmetry of up-flavors}}\times\underbrace{SU_L(N_d)\times SU_R(N_d)}_{\text{chiral symmetry of down-flavors}}\times\underbrace{U_A^{(-)}(1)}_{\text{anomaly-free combination of } U_A^{(u)}(1) \text{ and } U_A^{(d)}(1)}$$

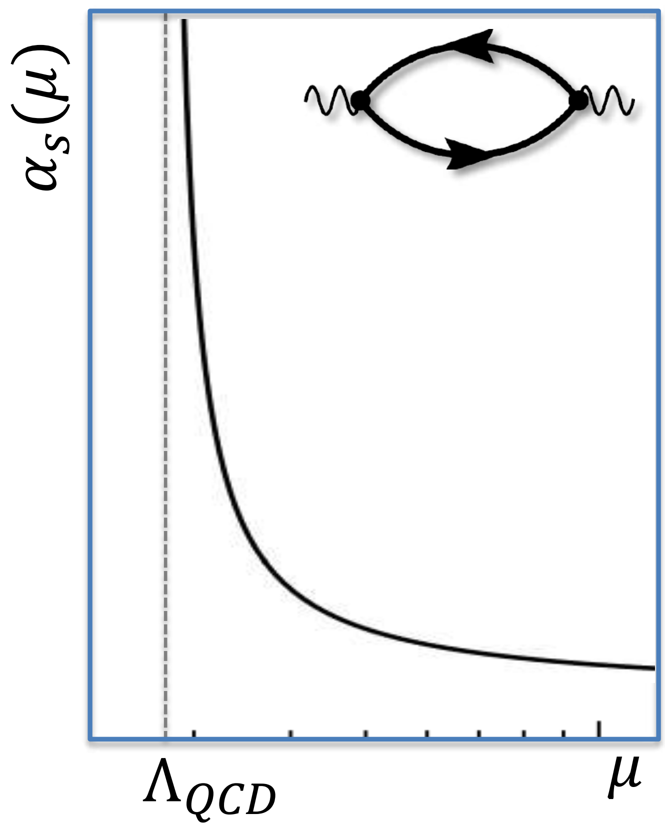
- Quark masses $m_u \neq m_d \neq 0$ break the symmetry down to

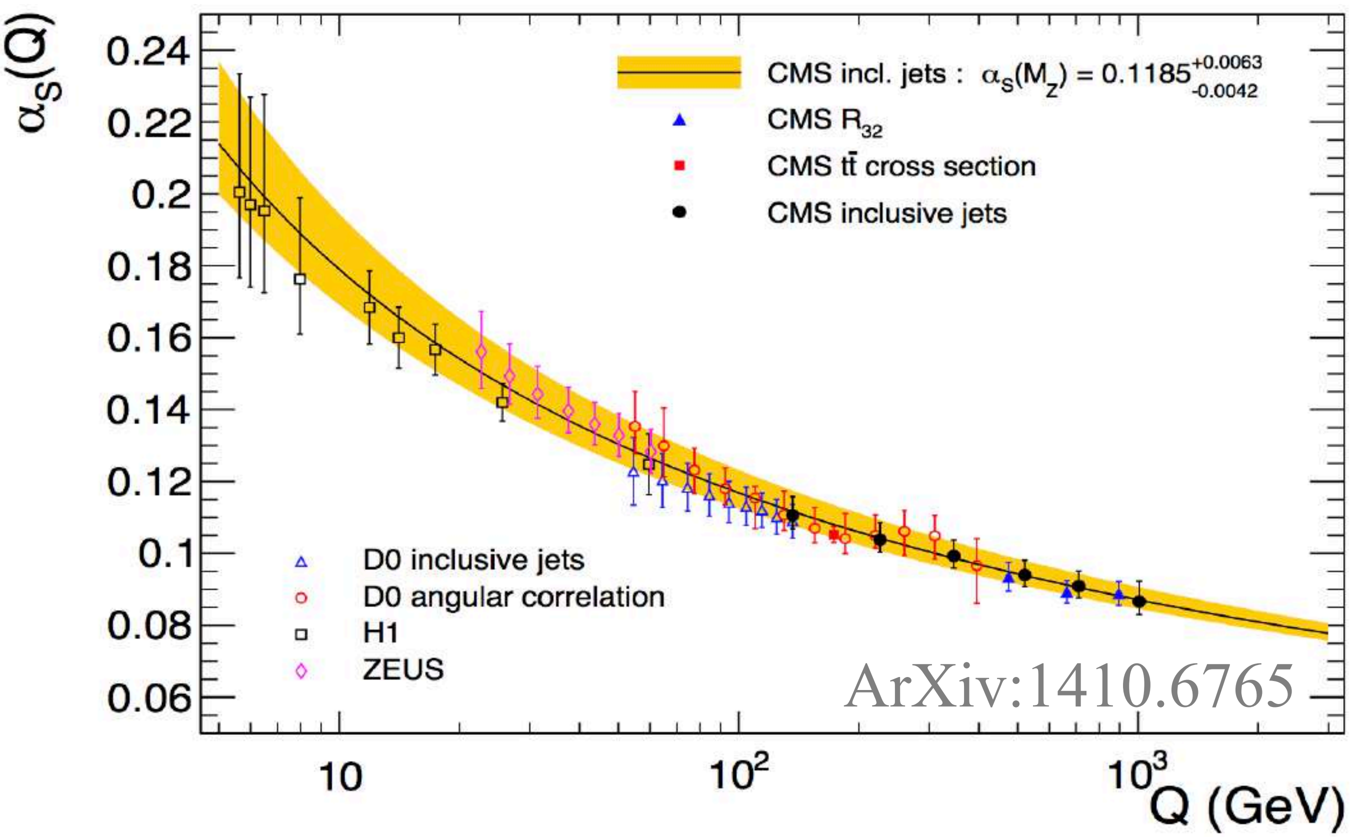
$$SU_V(N_u)\times SU_V(N_d)$$

RUNNING COUPLING & CONFINMENT

- Coupling constant in QCD runs with the energy scale,

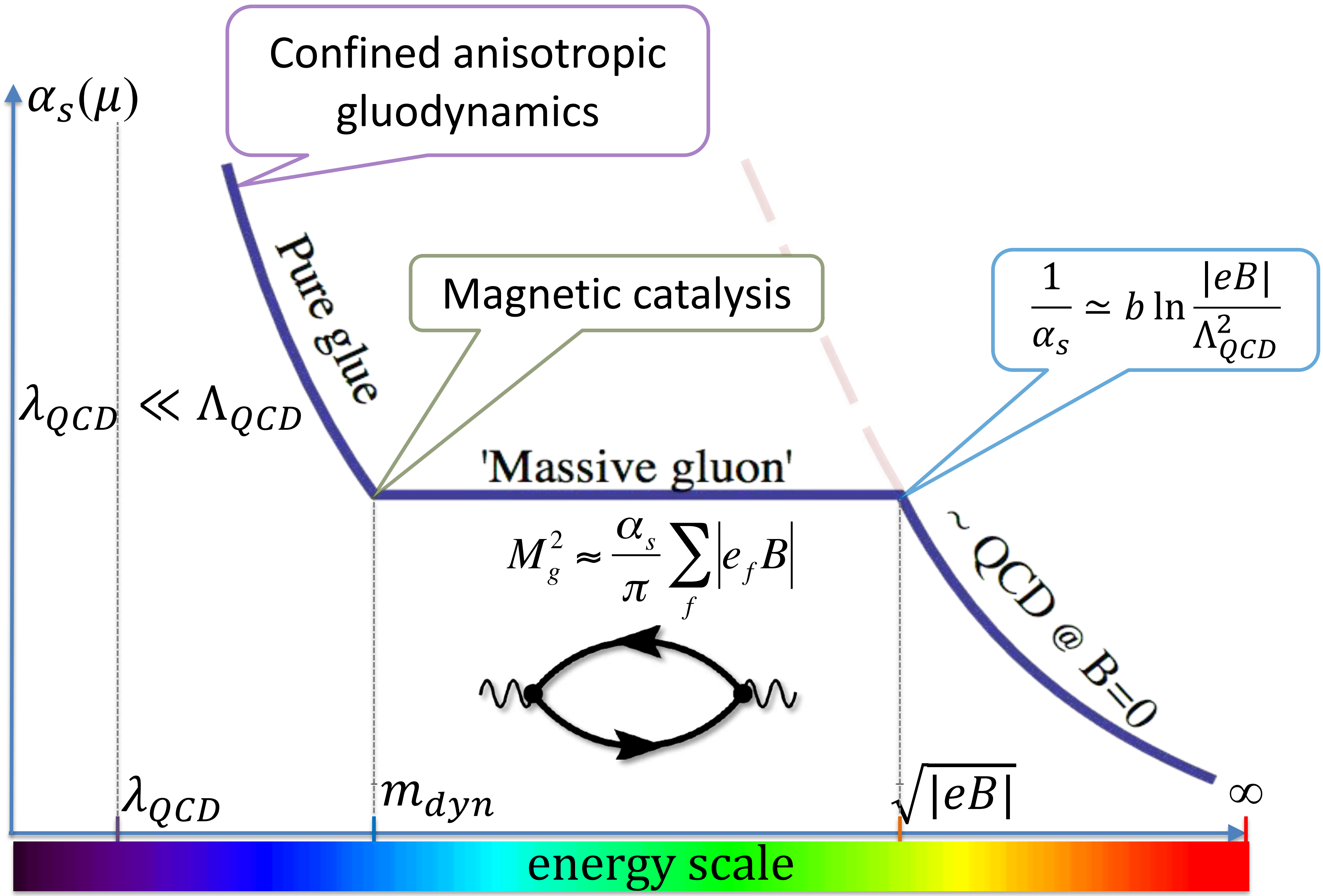
$$\frac{1}{\alpha_s(\mu)} \simeq b \ln \frac{\mu^2}{\Lambda_{QCD}^2}, \quad \text{where} \quad b = \frac{11\,N_c - 2N_f}{12\pi}$$





- The question is: What happens in a strong magnetic field?

RUNNING α_s IN QCD AT STRONG B



FREE DIRAC FERMIONS, B≠0

- Dirac equation:

$$(i\gamma^\mu D_\mu - m)\psi = 0$$

where $A_\mu = (A_0, -\vec{A})$ and the Landau gauge $\vec{A} = (-By, 0,0)$ is used

- Solutions take the form $\psi = (i\gamma^\mu D_\mu + m)\phi$, where

$$\phi_{k,\pm} \propto \frac{1 \pm i\gamma^1\gamma^2}{2}\varphi_k(y)e^{-i\omega t + ip_x x + ip_z z}$$

- Here φ_k are harmonic oscillator wave functions,

$$\varphi_k \propto H_k(\xi)e^{-\frac{\xi^2}{2}}, \quad \xi = \frac{y}{l} + p_x l \quad \text{and} \quad l = \frac{1}{\sqrt{|eB|}}$$

p_x defines the center of Landau orbits in y-direction

- The Landau level energies are

$$\omega = E_n^\pm = \pm\sqrt{2n|eB| + p_z^2 + m^2}$$

where $n = \underbrace{k + \frac{1}{2}}_{\text{orbital}} + \underbrace{s_z}_{\text{spin}}$ and $s_z = \pm\frac{1}{2}$ is an eigenvalue of $\frac{i}{2}\gamma^1\gamma^2$

LANDAU ENERGY SPECTRUM

- Landau energy levels at $m = 0$

$$E_n^{\pm} = \pm\sqrt{2n|eB| + p_z^2}$$

where $n = \underbrace{k + \frac{1}{2}}_{\text{orbital}} + \underbrace{s_z}_{\text{spin}}$

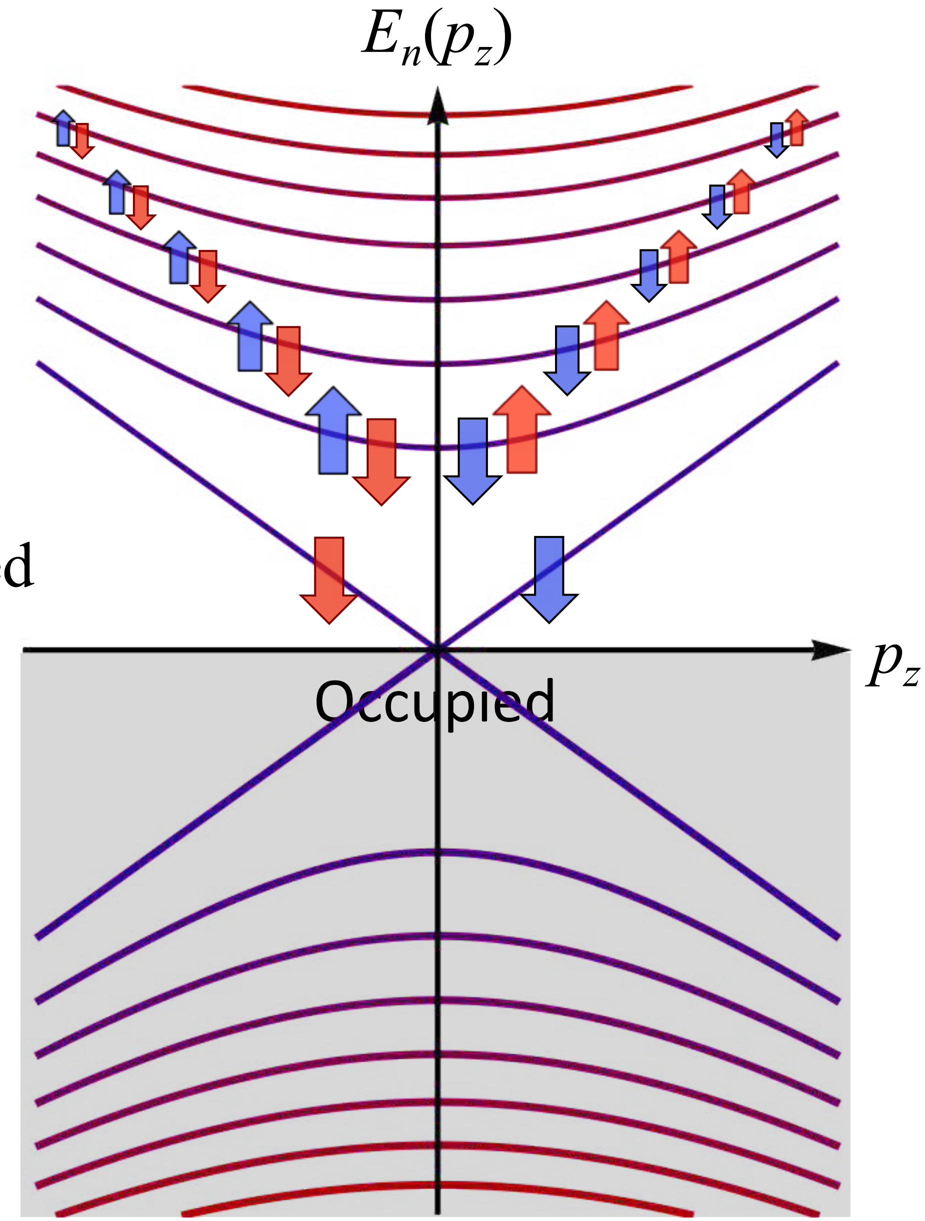
- Lowest Landau level is spin polarized

$$E_0^{\pm} = \pm p_z \qquad \left(k = 0,\ s_z = -\frac{1}{2}\right)$$

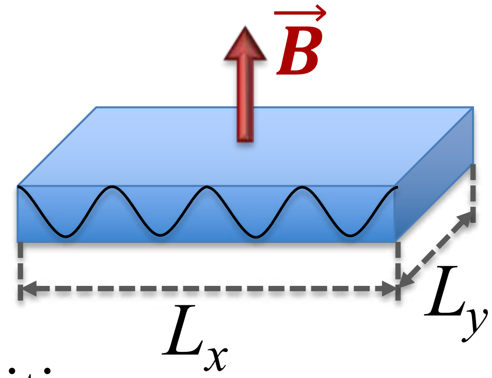
- Higher Landau levels ($n \geq 1$) are twice as degenerate:

(i) $k = n$ & $s = -\frac{1}{2}$

(ii) $k = n - 1$ & $s = +\frac{1}{2}$



Degeneracy of Landau levels



- The Landau level energies are independent of p_x

$$E_n^{\pm} = \pm\sqrt{2n|eB| + p_z^2 + m^2}$$

- Consider a finite box with periodic boundary conditions
- The wave function $\psi(x) \propto e^{ip_x x}$ satisfies $\psi(0) = \psi(L_x)$, i.e.,

$$e^{ip_x L_x} = 1 \quad \Longrightarrow \quad p_x = \frac{2\pi n}{L_x}, \quad n = 1,2, \ldots, N_{\max}$$

- Since p_x defines the center of Landau orbits in y-direction:

$$y_{c,\max} \approx -p_{x,\max} l^2 \lesssim L_y \quad \Longrightarrow \quad \frac{2\pi N_{\max}}{L_x} \frac{1}{|eB|} \approx L_y$$

- Thus, the degeneracy is

$$N_{\max} \approx \frac{|eB|}{2\pi} L_x L_y$$

DIRAC PROPAGATOR AT B≠0

- By definition,

$$G(r,r') = i\left\langle r\middle|\left(i\gamma^\mu D_\mu - m\right)^{-1}\middle|r'\right\rangle$$

$$\mathbb{I} = \sum |k,p_z,s_z\rangle\langle k,p_z,s_z|$$

$$= i\left(i\gamma^\mu D_\mu + m\right)_r\left\langle r\middle|\left[-D^\mu D_\mu + i\gamma^1\gamma^2 eB - m^2\right]^{-1}\middle|r'\right\rangle$$

$$= i\left(i\gamma^\mu D_\mu + m\right)_r \sum\langle r|k,p_z,s_z\rangle\,(\omega^2 - E_n^2)^{-1}\langle k,p_z,s_z|r'\rangle$$

- Note that the explicit form of the wave functions is the same as before

$$\psi_{k,p_z,s_z}(r) = \langle r|k,p_z,s_z\rangle \propto H_k(\xi)e^{-\frac{\xi^2}{2}}e^{-i\omega t+ip_z z}\,U_{s_z},\quad \text{where }\ \xi = \frac{y}{l} + p_x l$$

- Then, the propagator has the form

$$G(\omega,p_z;\vec{r}_\perp,\vec{r}_\perp') = e^{i\Phi(\vec{r}_\perp,\vec{r}_\perp')}\tilde{G}(\omega,p_z;\vec{r}_\perp - \vec{r}_\perp')$$

where $\Phi(\vec{r}_\perp,\vec{r}_\perp') = -\mathrm{e}\int_{\vec{r}_\perp'}^{\vec{r}_\perp} A_\nu dr^\nu$ is the Schwinger phase, and

$$\tilde{G}(\omega,p_z;\vec{r}_\perp - \vec{r}_\perp') = \int \frac{d^2\vec{p}_\perp}{(2\pi)^2}\,e^{i\vec{p}_\perp\cdot(\vec{r}_\perp - \vec{r}_\perp')}\,\tilde{G}(\omega,\vec{p})$$

Dirac Propagator at B≠0

- The Fourier transform of the translation invariant part reads

$$\tilde{G}(\omega,\vec{p}) = i e^{-\vec{p}_\perp^2 l^2} \sum_{n=0}^{\infty} \frac{(-1)^n D_n(\omega,\vec{p})}{\omega^2 - E_n^2}$$

where

$$D_n(\omega,\vec{p}) = 2(\omega\gamma^0 - p_z\gamma^3 + m)[\mathcal{P}_- L_n(2\vec{p}_\perp^2 l^2) - \mathcal{P}_+ L_{n-1}(2\vec{p}_\perp^2 l^2)] + 4(\vec{p}_\perp \cdot \vec{\gamma}_\perp) L_{n-1}^1 (2\vec{p}_\perp^2 l^2)$$

Laguerre polynomials

and the following notation for the spin projectors is used

$$\mathcal{P}_\pm = \frac{1 \pm i\gamma^1\gamma^2}{2}$$

- Similarly, in momentum-coordinate space representation:

$$\tilde{G}(\omega, p_z; \vec{r}_\perp) = i \frac{e^{-\vec{r}_\perp^2/(4l^2)}}{2\pi l^2} \sum_{n=0}^{\infty} \frac{F_n(\omega, p_z; \vec{r}_\perp)}{\omega^2 - E_n^2}$$

where $$F_n(\omega, p_z; \vec{r}_\perp) = 2(\omega\gamma^0 - p_z\gamma^3 + m)\left[\mathcal{P}_- L_n\left(\frac{\vec{r}_\perp^2}{2l^2}\right) - \mathcal{P}_+ L_{n-1}\left(\frac{\vec{r}_\perp^2}{2l^2}\right)\right] - \frac{i}{l^2}(\vec{r}_\perp \cdot \vec{\gamma}_\perp) L_{n-1}^1\left(\frac{\vec{r}_\perp^2}{2l^2}\right)$$

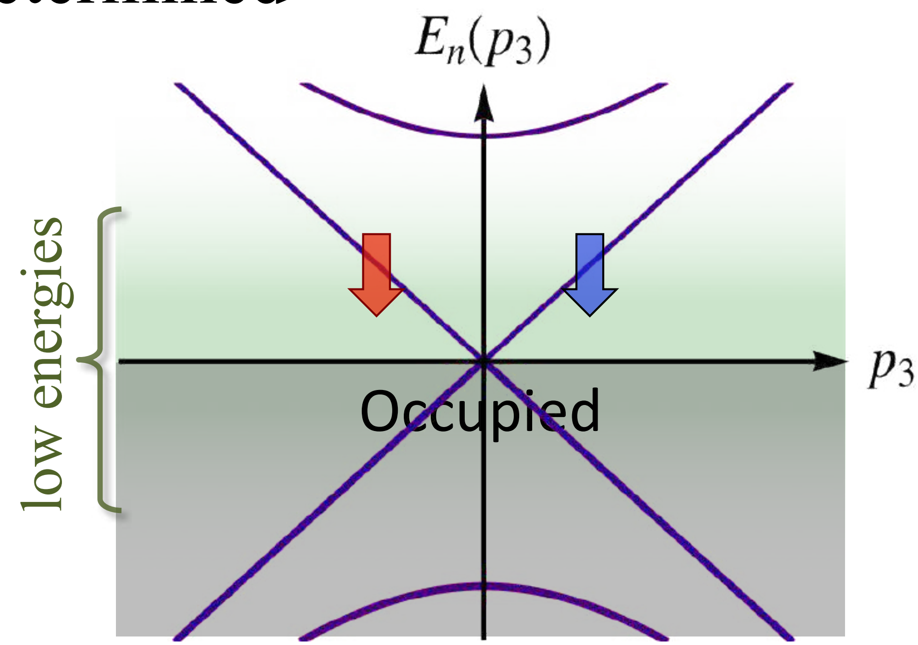
DIMENSIONAL REDUCTION

- The low-energy dynamics is determined by the lowest Landau level (n=0)

$$E_0^{\pm} = \pm p_z$$

- This is a (1+1)D spectrum!

- Propagator is also (1+1)D:



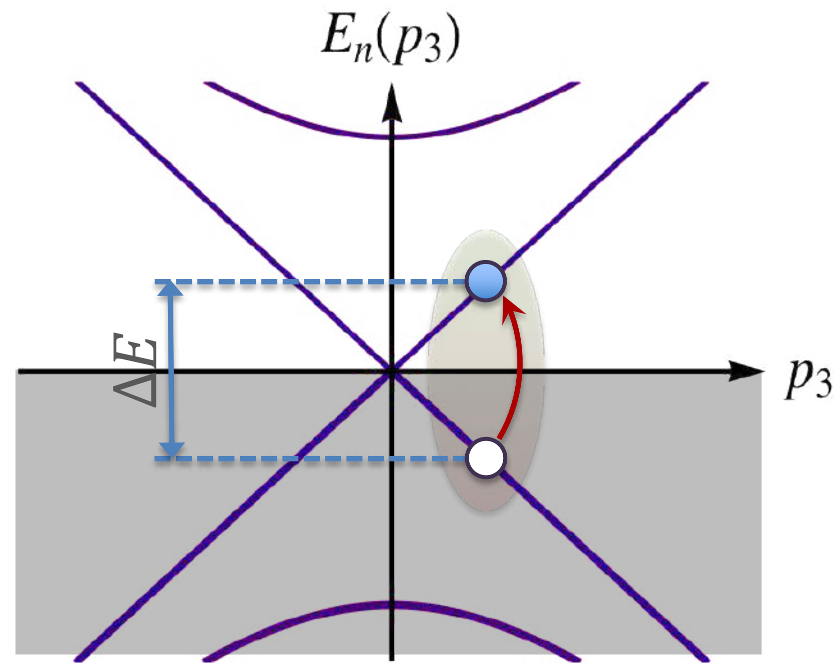
$$\tilde{G}_{LLL}(\omega, \vec{p}) = 2ie^{-\vec{p}_\perp^2 l^2} \frac{\omega\gamma^0 - p_z\gamma^3}{\omega^2 - p_z^2} \frac{1 - i\gamma^1\gamma^2}{2}$$

- In addition, there is a nonzero density of states at E=0:

$$\left.\frac{dn}{dE}\right|_{E=0} = \frac{1}{\delta E}\left(\frac{N_{\max}}{L_x L_y}\right)\left(\int_0^{\delta E} \frac{dp_z}{2\pi}\right) = \frac{|eB|}{4\pi^2}$$

PAIRING INSTABILITY

- Thought experiment:
  - Create a particle-antiparticle pair (energy price: ΔE)
  - The pair can form a bosonic bound state (energy gain: $-\epsilon_b$)
  - If $\epsilon_b > \Delta E$, copious formation of bound states is beneficial



  - Note, ΔE can be arbitrarily small when $m = 0$ (!)
  - The bound states of fermions are bosons
  - Bosons can (and will) occupy the lowest energy state ($\vec{P} = 0$), and thus form a Bose condensate $\langle \bar{\psi}\psi \rangle \neq 0$
  - Ground state (vacuum) changes its properties (e.g., chiral symmetry breaks down, an energy gap opens in spectrum)

Do bound states always form in 3D?

- Consider a 3D potential well in quantum mechanics
  [Landau-Lifshitz, Quantum Mechanics]

$$U(r) = \begin{cases} -g\dfrac{\pi^2\hbar^2}{8m_*a^2} & \text{for} \quad r \le a \\ 0 & \text{for} \quad r > a \end{cases}$$

- Bound states form only when the well is deep enough (namely, $g > 1$):

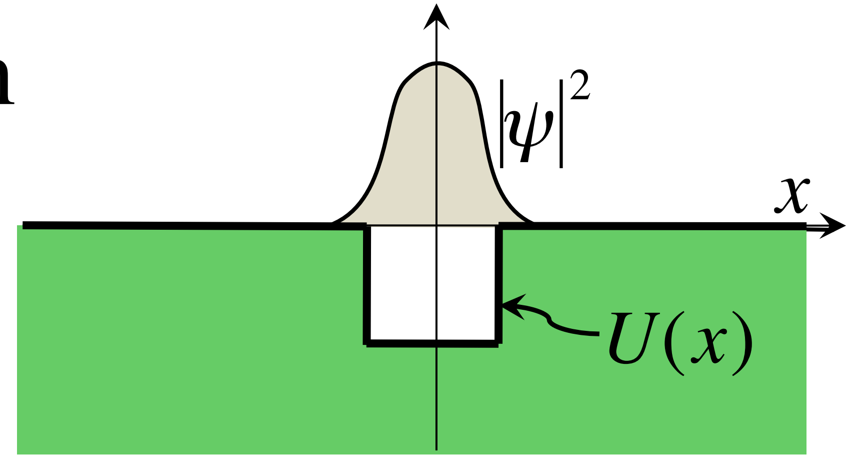
$$|E_{3D}| \approx \frac{\pi^4\hbar^2}{2^7 a^2 m_*}(g-1)^2\,, \quad \text{assuming} \quad 0 < g-1 << 1$$

- There are no bound states when $g < 1$, i.e., when the well is not deep enough (in other words, when the coupling constant is not strong enough)

COMPARE: BOUND STATES IN 1D

- Bound states always form

$$\left|E_{1D}\right| \approx \frac{m_*}{2\hbar^2}\left(-\int_{-\infty}^{+\infty} U(x)\,dx\right)^2$$



- This is a perturbative result (!)

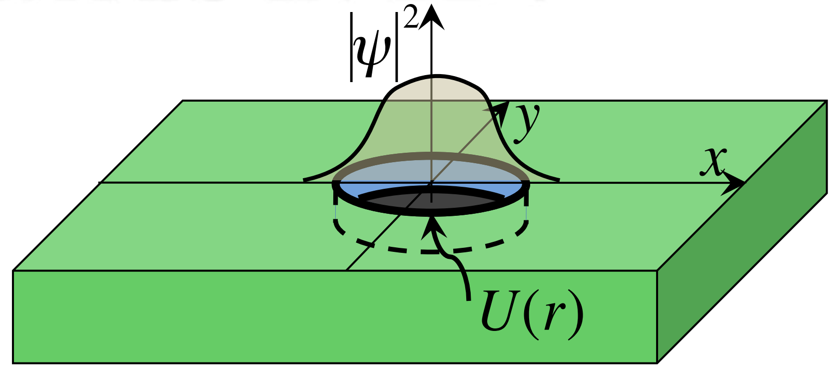
$$\left|E_{1D}\right| \propto g^2\,, \quad \text{when} \quad U(x) \rightarrow gU(x)$$

- Rigorous statement: at least one bound state exists if

$$\int \left(1+|x|\right)\left|U(x)\right| dx < \infty \quad \& \quad \int U(x)\,dx \leq 0$$

[B. Simon, Annals Phys. 97 (1976) 279]

HOW ABOUT BOUND STATES IN 2D?



- Bound states always form

$$\left|E_{2D}\right| \approx \frac{\hbar^2}{a^2 m_*} \exp\left(-\frac{\hbar^2}{m_*} \left| \int_0^\infty r U(r) dr \right|^{-1} \right)$$

- This is a nonperturbative result

$$\left|E_{2D}\right| \propto \exp\left(-\frac{C}{g} \right), \quad \text{when} \quad U(x) \to gU(x)$$

- Rigorous statement: at least one bound state exists if

$$\int \left|U(x)\right|^{1+\varepsilon} d^2x < \infty, \quad \int (1+x^2)^{\varepsilon} \left|U(x)\right| d^2x < \infty \quad \& \quad \int U(x)\, d^2x \leq 0$$

[B. Simon, Annals Phys. 97 (1976) 279]

UNIVERSAL MAGNETIC CATALYSIS

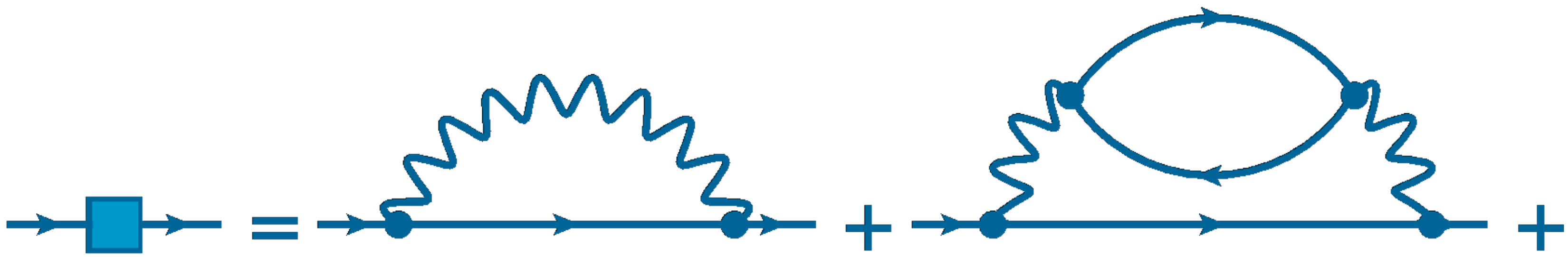
- Quantum field theory of charged fermions (m=0) at $\vec{\boldsymbol{B}} \neq \mathbf{0}$
  - Dimensional reduction (caused by a nonzero $\vec{\boldsymbol{B}}$)
  - Nonzero density of states ($\propto |eB|$) at E=0
  - Attraction between particles and antiparticles
- Universal outcome:
  - Spontaneous rearrangement of the ground state
  - Breakdown of chiral symmetry
  - Opening a nonzero gap in the Dirac spectrum

[Gusynin, Miransky, Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994)]
[Shovkovy, Lect. Notes Phys. **871**, 13 (2013)]

- This is similar to superconductivity in metals due to Cooper pairing of electrons

SYMMETRY BREAKING: METHOD I

- The Schwinger-Dyson equation for the fermion self-energy/ propagator



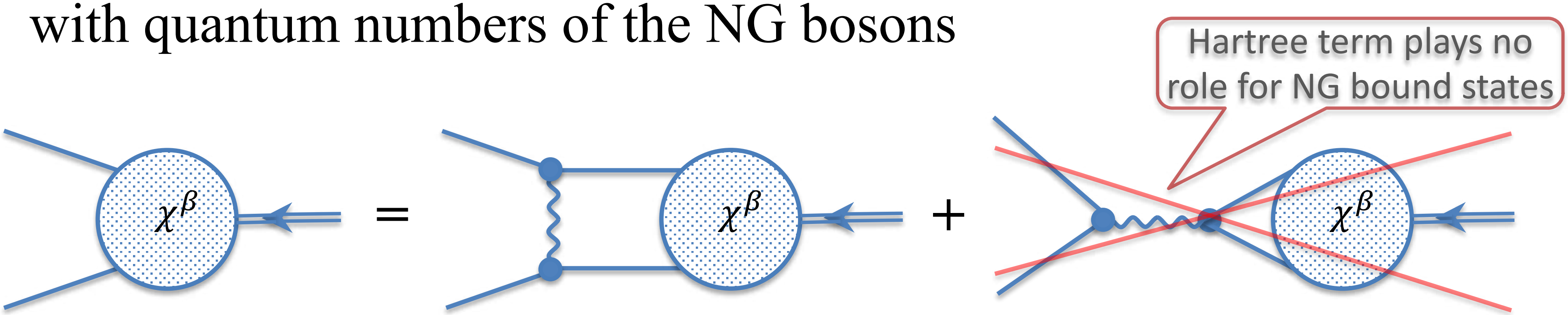
- In coordinate space, e.g.,

$$G^{-1}(x,y) = G_0^{-1}(x,y) + 4\pi\alpha_s\, \gamma^\mu T^A\, G(x,y)\, \gamma^\nu T^B\, \mathcal{D}_{\mu\nu}^{AB}(y-x)$$

- Note: both the propagator $G(x,y)$ and its inverse $G^{-1}(x,y)$ have the same Schwinger phase $e^{i\Phi(\vec{r}_\perp,\vec{r}'_\perp)}$
- Like in a metal, a large density of states at E=0 implies that screening effects are important

SYMMETRY BREAKING: METHOD II

- Homogeneous Bethe-Salpeter equation for a *massless* bound states with quantum numbers of the NG bosons



$$\chi^\beta_{AB}(u,u';P) = -i\int d^4u_1 d^4u_1' d^4u_2 d^4u_2' G_{AA_1}(u,u_1) K_{A_1B_1;A_2B_2}(u_1u_1', u_2u_2')\chi^\beta_{A_2B_2}(u_2,u_2';P) G_{B_1B}(u_2',u')$$

The NG wave function is defined by $\chi^\beta_{AB} = \langle 0|T\psi_A(u)\bar{\psi}_B(u')|P;\beta\rangle$

and (the ladder approximation) kernel in QED is

$$K_{A_1B_1;A_2B_2}(u_1u_1', u_2, u_2') = -4\pi i\alpha\delta_{a_1a_2}\delta_{b_2b_1}\gamma^\mu_{n_1n_2}\gamma^\nu_{m_2m_1}\mathcal{D}_{\mu\nu}(u_2'-u_2)\delta(u_1-u_2)\delta(u_1'-u_2')$$
$$+\cancel{4\pi i\alpha\delta_{a_1b_1}\delta_{b_2a_2}\gamma^\mu_{n_1m_1}\gamma^\nu_{m_2n_2}\mathcal{D}_{\mu\nu}(u_1-u_2)\delta(u_1-u_1')\delta(u_2-u_2')}$$

QED IN STRONG MAGNETIC FIELD

- The NG-boson wave function ($r_\mu = u_\mu - u'_\mu$):

$$\chi^\beta_{AB}(u,u';P) = \lambda^\beta_{ab} e^{-iPR} \exp\left[-ier^\mu A^{\text{ext}}_\mu(R)\right] \tilde{\chi}_{nm}(R,r;P)$$

- In the LLL approximation,

$$\varphi(p_\parallel) = \frac{\pi\alpha}{(2\pi)^4}\int d^2k_\parallel \left(1-i\gamma^1\gamma^2\right)\gamma^\mu \frac{\hat{k}_\parallel + m_{\text{dyn}}}{k^2_\parallel - m^2_{\text{dyn}}}\varphi(k_\parallel)\frac{\hat{k}_\parallel + m_{\text{dyn}}}{k^2_\parallel - m^2_{\text{dyn}}}\gamma^\nu \left(1-i\gamma^1\gamma^2\right) D^\parallel_{\mu\nu}(k_\parallel - p_\parallel)$$

where we introduced $(\hat{p}_\parallel - m_{\text{dyn}})\tilde{\chi}(p)(\hat{p}_\parallel - m_{\text{dyn}}) = \exp(-l^2\mathbf{p}^2_\perp)\varphi(p_\parallel)$

and

$$D^\parallel_{\mu\nu}(k_\parallel - p_\parallel) = i\pi\delta_{\mu\nu}\int_0^\infty \frac{dx\exp(-l^2x/2)}{(k_\parallel - p_\parallel)^2 + x}$$

- The LLL solution has the Dirac structure

$$\varphi(p_\parallel) = A\gamma_5\left(1 - i\gamma_1\gamma_2\right)$$

- The equation for $A(p_\parallel)$ reads

$$A(p_\parallel) = \frac{\alpha}{2\pi^2}\int \frac{A(k_\parallel)d^2k_\parallel}{k^2_\parallel + m^2_{dyn}} \int_0^\infty dx \frac{e^{-xl^2/2}}{x + (k_\parallel - p_\parallel)^2}$$

Effective Schrodinger problem

- Rewrite the problem in terms of

$$\Psi(\mathbf{r}) = \int \frac{d^2 k_\parallel}{(2\pi)^2} \frac{e^{i\mathbf{r}\cdot k_\parallel}}{k_\parallel^2 + m_{dyn}^2} A(k_\parallel)$$

- Function $\Psi(\mathbf{r})$ satisfies the following 2D Schrodinger equation:

$$\left[-\nabla_{\mathbf{r}}^2 + m_{dyn}^2 + V(\mathbf{r})\right]\Psi(\mathbf{r}) = 0$$

where the effective potential is long-ranged

$$V(\mathbf{r}) = -\frac{\alpha}{2\pi^2}\int d^2 p e^{i\mathbf{p}\cdot\mathbf{r}} \int_0^\infty \frac{dx \exp(-x/2)}{l^2 p^2 + x} \simeq -\frac{2\alpha}{\pi}\frac{1}{r^2}, \quad r \to \infty$$

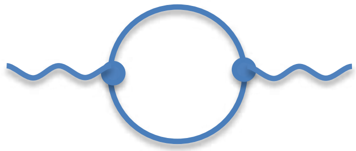
- The lowest energy bound state gives

$$m_{dyn} \simeq C\sqrt{|eB|}\exp\left[-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right]$$ (LLL & weak coupling)

$\exp(-C/\sqrt{\alpha})$ is due to a long-range interaction

Screening effects

- Photon exchange interaction is screened in a strong B-field

$$\Pi_{\mu\nu} \equiv$$  $$\simeq \left(q_\mu^\parallel q_\nu^\parallel - q_\parallel^2 g_{\mu\nu}^\parallel\right) e^{-q_\perp^2 l^2} \Pi\left(q_\parallel^2\right)$$

- The screened photon propagator reads

$$\mathcal{D}_{\mu\nu}(q) = -i\left[\frac{1}{q^2} g_{\mu\nu}^\perp + \frac{q_\mu^\parallel q_\nu^\parallel}{q^2 q_\parallel^2} + \frac{1}{q^2 + q_\parallel^2 \Pi(q_\perp^2, q_\parallel^2)}\left(g_{\mu\nu}^\parallel - \frac{q_\mu^\parallel q_\nu^\parallel}{q_\parallel^2}\right) - \frac{\lambda}{q^2}\frac{q_\mu q_\nu}{q^2}\right]$$

where the polarization function has the asymptotes

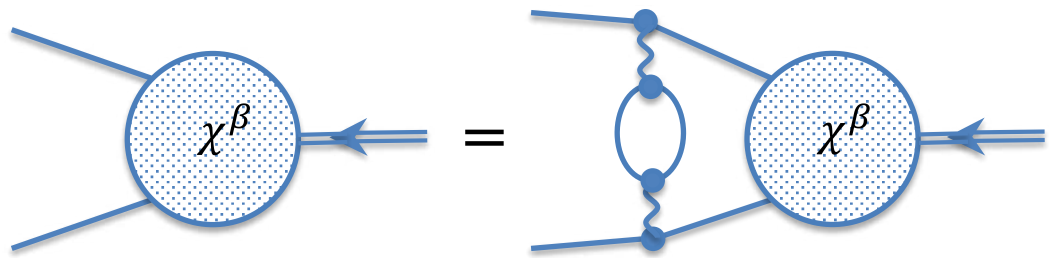
$$\Pi(q_\parallel^2) \simeq \frac{\bar{\alpha}}{3\pi}\frac{|eB|}{m_{\text{dyn}}^2}, \quad \text{as } |q_\parallel^2| \ll m_{\text{dyn}}^2$$ (extremely narrow range in $q_\parallel^2$)

$$\Pi(q_\parallel^2) \simeq -\frac{2\bar{\alpha}}{\pi}\frac{|eB|}{q_\parallel^2} \quad \text{as } |q_\parallel^2| \gg m_{\text{dyn}}^2 \quad \Longrightarrow \quad \frac{1}{q^2 + q_\parallel^2 \Pi(q_\perp^2, q_\parallel^2)} \simeq \frac{1}{q^2 - M_\gamma^2}$$

where the effective photon screening mass is $M_\gamma^2 = \frac{2\bar{\alpha}}{\pi}|eB|$

Improved Ladder Approximation

- With screening effects included,

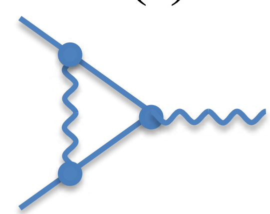


- The result is similar up to the replacement $\alpha \to \alpha/2$

$$m_{dyn} \simeq C\sqrt{|eB|}\,\exp\left[-\frac{\pi}{2}\left(\frac{\pi}{\alpha}\right)^{1/2}\right]$$

$\alpha \to \alpha/2$ changes the result *dramatically*

- The improved ladder approximation is *not* reliable either (!)
  - The vertex corrections are important too
  - More singularities $\sim \ln\left(|eB|/m_{dyn}^2\right) \sim 1/\sqrt{\alpha}$ at higher orders



- Re-summation of infinitely many diagrams is needed (!)

Toward exact result

- QED in a strong B-field looks like (1+1)D Schwinger model
- Use same strategy: fix a gauge in which all (singular) vertex corrections vanish!
- Such a (non-local) gauge exists

$$D_{\mu\nu}(q) = -i\frac{1}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - id(q_\perp^2, q_\parallel^2)\frac{q_\mu^\parallel q_\nu^\parallel}{q^2 q_\parallel^2}$$

where

$$d = -q_\parallel^2 \Pi/[q^2 + q_\parallel^2 \Pi] + q_\parallel^2/q^2$$

- Then, the full photon propagator

$$\mathcal{D}_{\mu\nu}(q) = -i\frac{g_{\mu\nu}^\parallel}{q^2 + q_\parallel^2 \Pi(q_\perp^2, q_\parallel^2)} - i\frac{g_{\mu\nu}^\perp}{q^2} + i\frac{q_\mu^\perp q_\nu^\perp + q_\mu^\perp q_\nu^\parallel + q_\mu^\parallel q_\nu^\perp}{(q^2)^2}$$

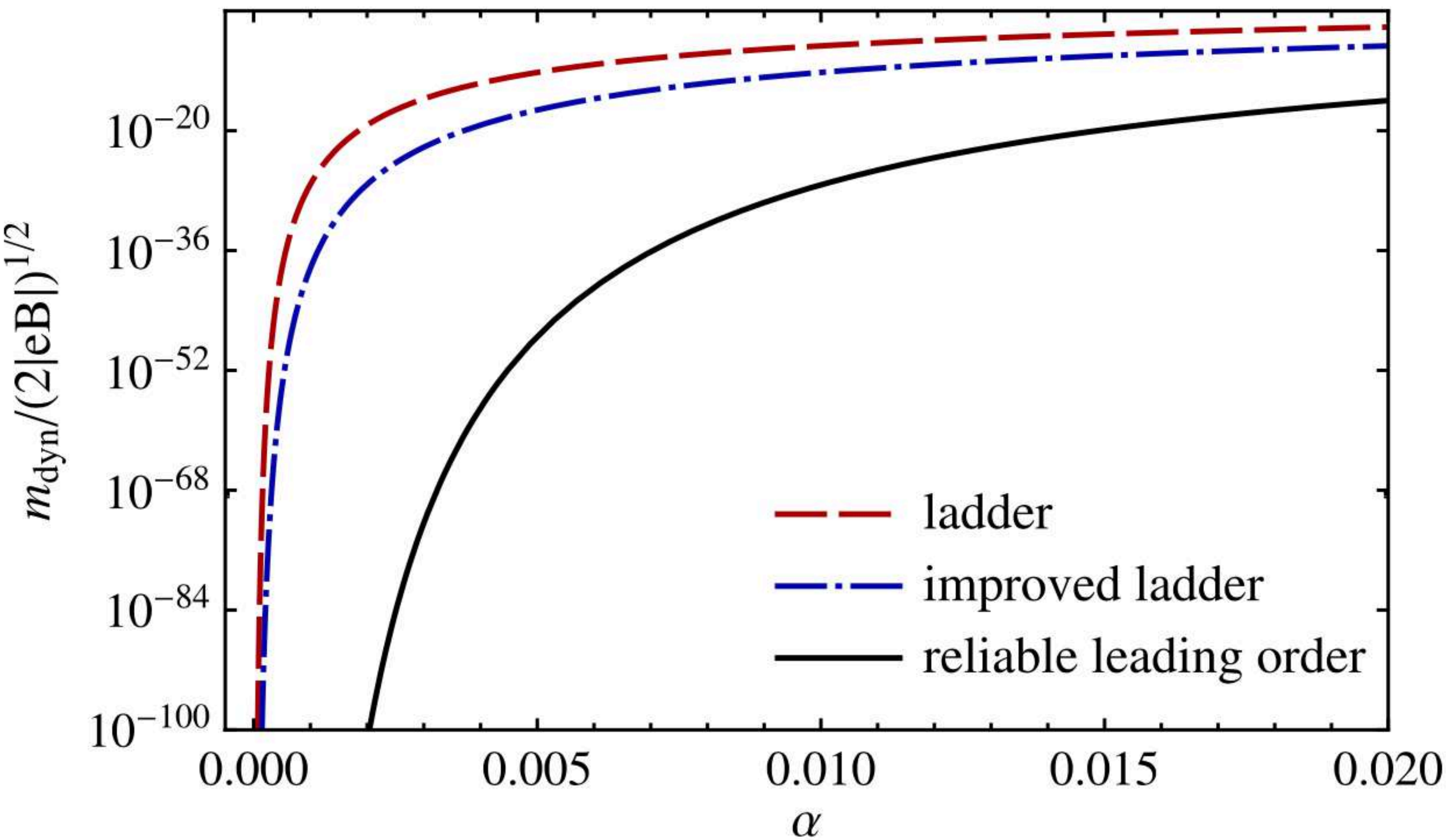
- No dangerous infrared singularities appear because

$$\mathcal{P}_-\gamma_\mu\mathcal{P}_- = \gamma_{\parallel,\mu} \quad \text{and} \quad \gamma_{\parallel,\alpha}\,\gamma_{\parallel,\mu_1}\,\gamma_{\parallel,\mu_2}\cdots\gamma_{\parallel,\mu_{2n+1}}\,\gamma_\parallel^\alpha = 0$$

Dynamical mass in QED

- In such a nonlocal gauge, the dynamical mass is reliable

$$m_{dyn} \simeq \sqrt{2|eB|}\,(\alpha N_f)^{1/3} \exp\left[-\frac{\pi}{\alpha \ln\frac{C_1}{\alpha N_f}}\right], \quad C_1 \approx 1.82 \pm 0.06$$



QCD IN STRONG MAGNETIC FIELD

- The Schwinger-Dyson equation

$$G^{-1}(x,y) = G_0^{-1}(x,y) + 4\pi\alpha_s\, \gamma^\mu T^A\, G(x,y)\, \gamma^\nu T^B\, \mathcal{D}_{\mu\nu}^{AB}(y-x)$$

- Non-Abelian structure ($T^A T^A = C_2$): $\alpha \to \frac{N_c^2-1}{2N_c}\alpha_s$

- Screening effects in the strong field limit ($\sqrt{|eB|} \gg \Lambda_{QCD}$)

$$\mathcal{P}^{AB,\mu\nu} \simeq \frac{\alpha_s}{6\pi}\delta^{AB}\left(k_\parallel^\mu k_\parallel^\nu - k_\parallel^2 g_\parallel^{\mu\nu}\right)\sum_{q=1}^{N_f}\frac{|e_q B|}{m_q^2}, \quad \text{for } |k_\parallel^2| \ll m_q^2$$

$$\mathcal{P}^{AB,\mu\nu} \simeq -\frac{\alpha_s}{\pi}\delta^{AB}\left(k_\parallel^\mu k_\parallel^\nu - k_\parallel^2 g_\parallel^{\mu\nu}\right)\sum_{q=1}^{N_f}\frac{|e_q B|}{k_\parallel^2}, \quad \text{for } m_q^2 \ll |k_\parallel^2| \ll |eB|$$

Expression for dynamical mass

- The gluon effective mass reads ($m_{dyn}^2 \ll |k_\parallel^2| \ll |eB|$)

$$M_g^2 \simeq \frac{\alpha_s}{\pi}\sum_f |e_f B| = \frac{\alpha_s}{3\pi}(2N_u + N_d)\,|eB|$$

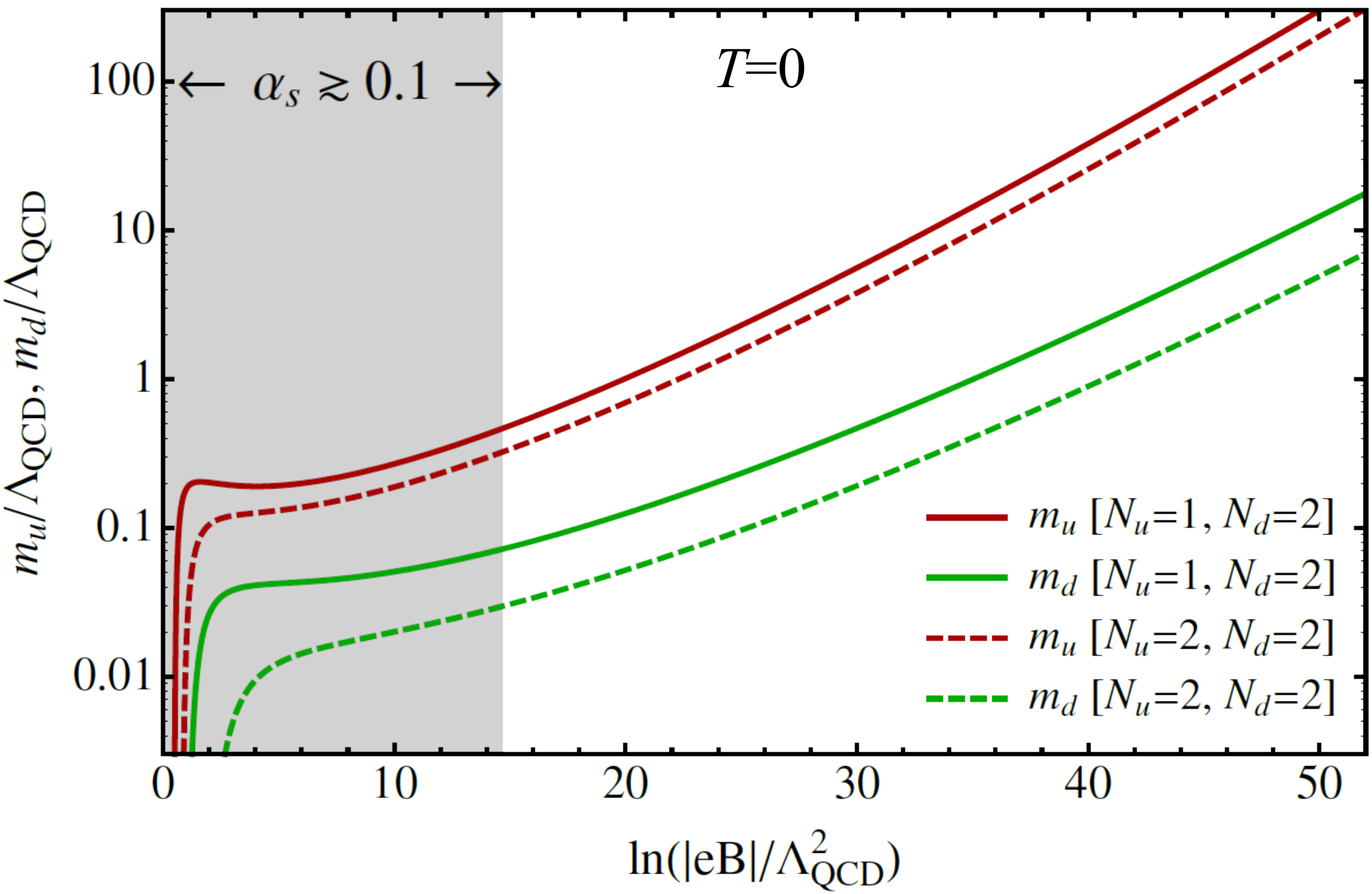
- As in QED, the non-local gauge prevents singular infrared corrections in higher-order diagrams
- Compared to QED: $\alpha \to \frac{N_c^2-1}{2N_c}\alpha_s$ and $M_\gamma^2 \to M_g^2$
- Then,

$$m_q^2 \simeq 2C_1|e_qB|\left(c_q\alpha_s\right)^{2/3}\exp\left[-\frac{4N_c\pi}{\alpha_s(N_c^2-1)\ln(C_2/c_q\alpha_s)}\right]$$

where $C_1 \simeq C_2 \simeq 1$ and $c_q \simeq (2N_u + N_d)|e|/(6\pi|e_q|)$

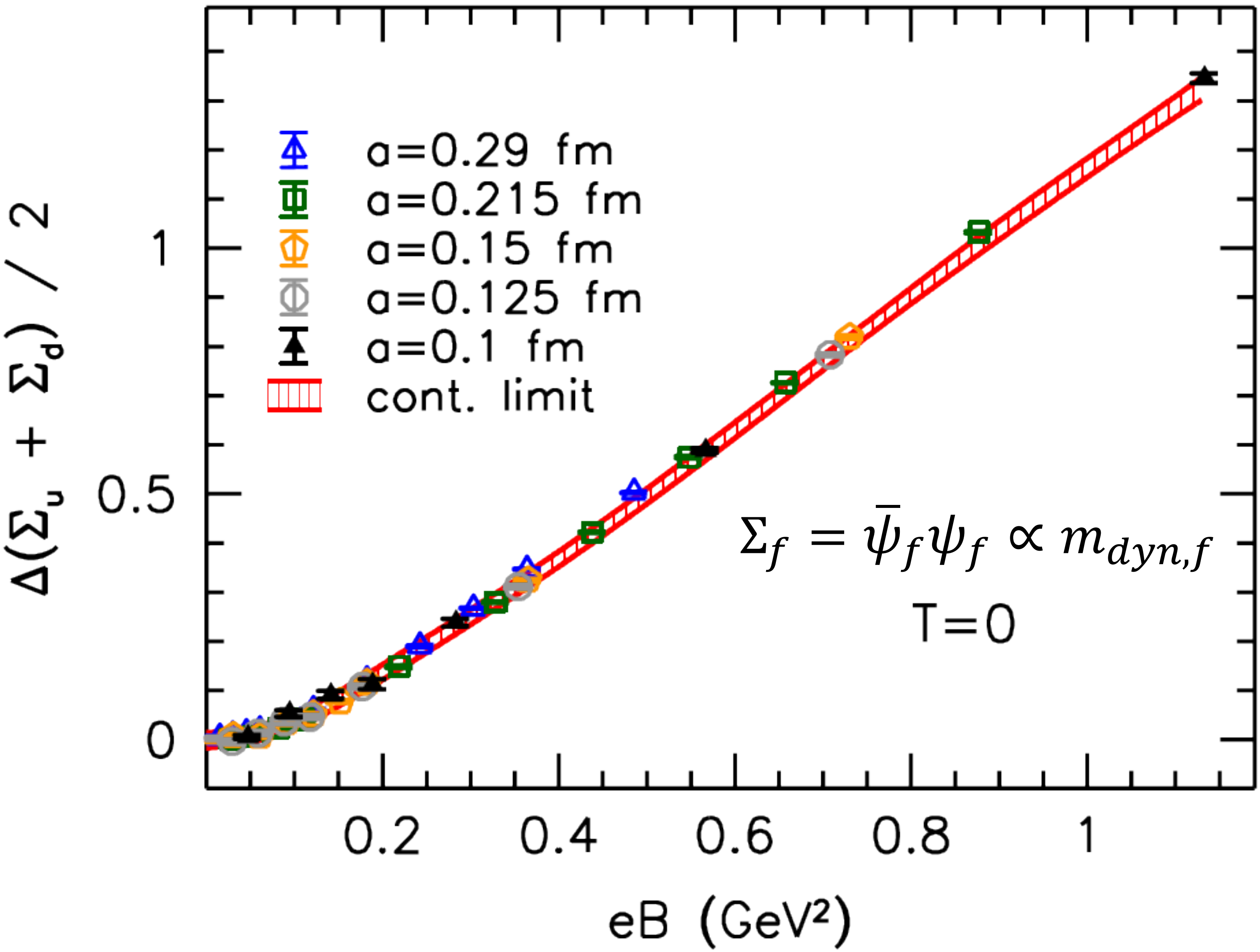
QUARK MASS VS. B

- Quantitative dependence on the field ($\sqrt{|eB|} \gg \Lambda_{\text{QCD}}$)



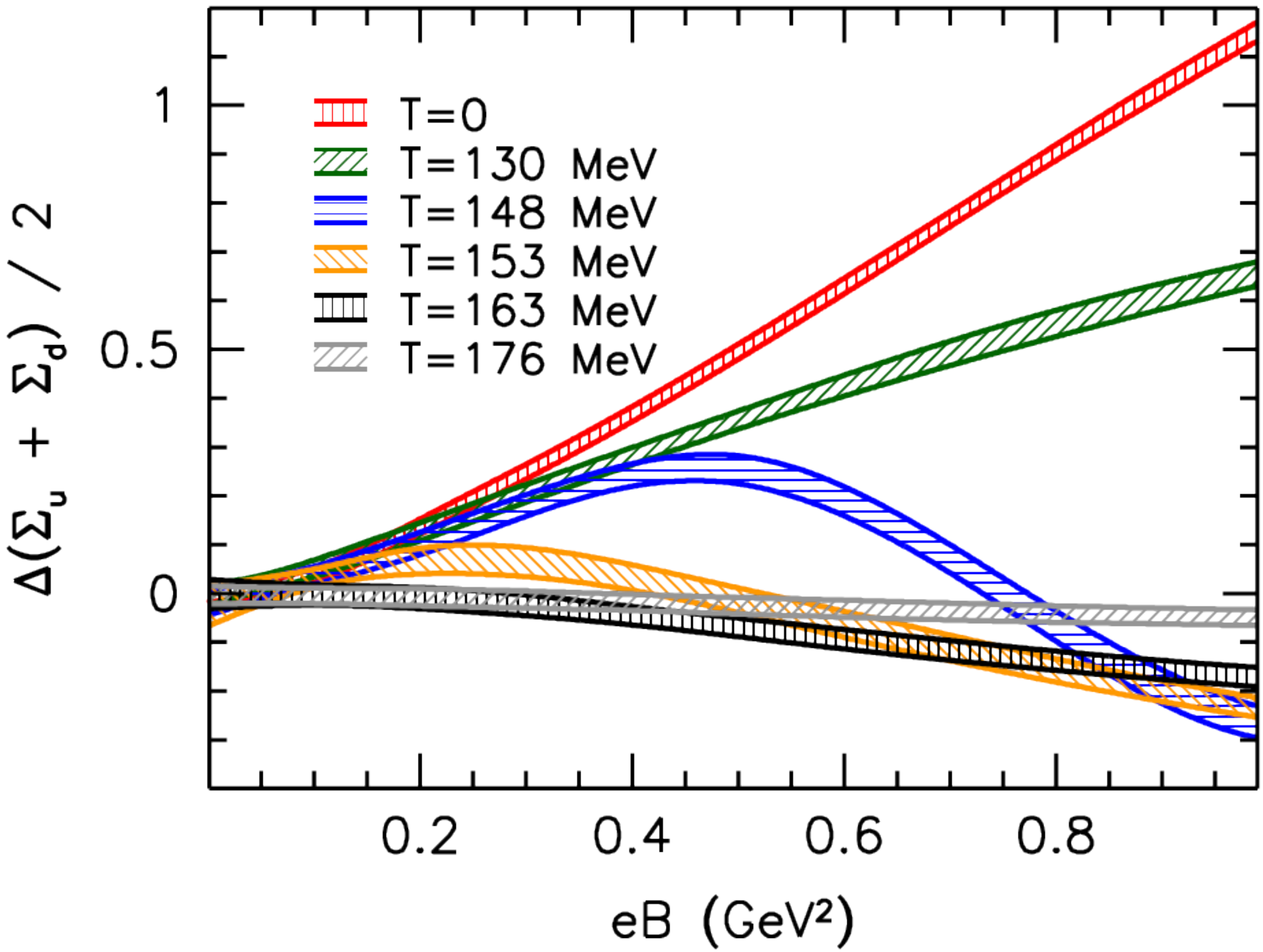
[Miransky & Shovkovy, Phys. Rev. D **66** (2002) 045006]

Chiral condensate in lattice QCD



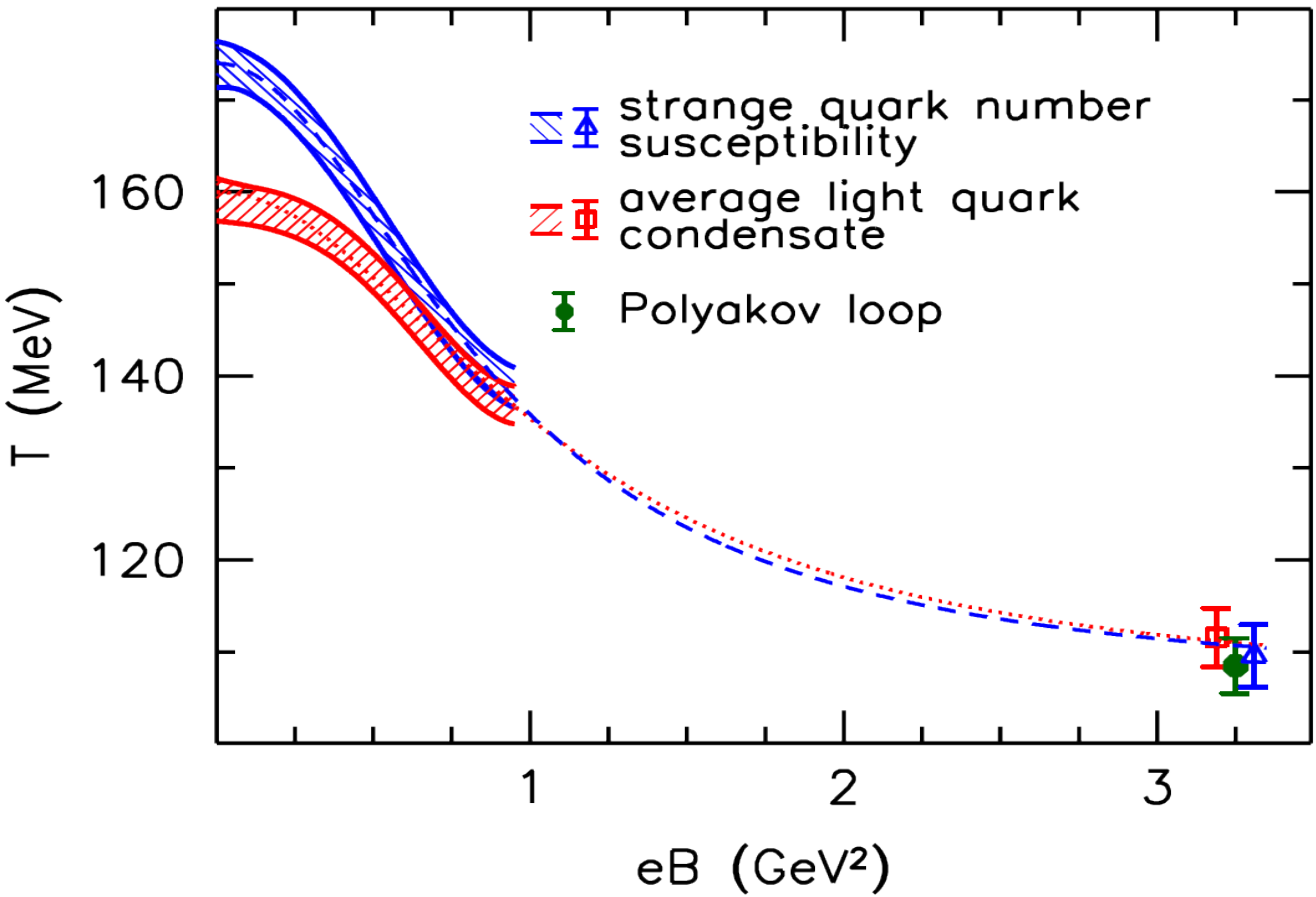
[Bali et al., Phys. Rev. D**86**, 071502 (2012)]

Catalysis vs. Inverse Catalysis



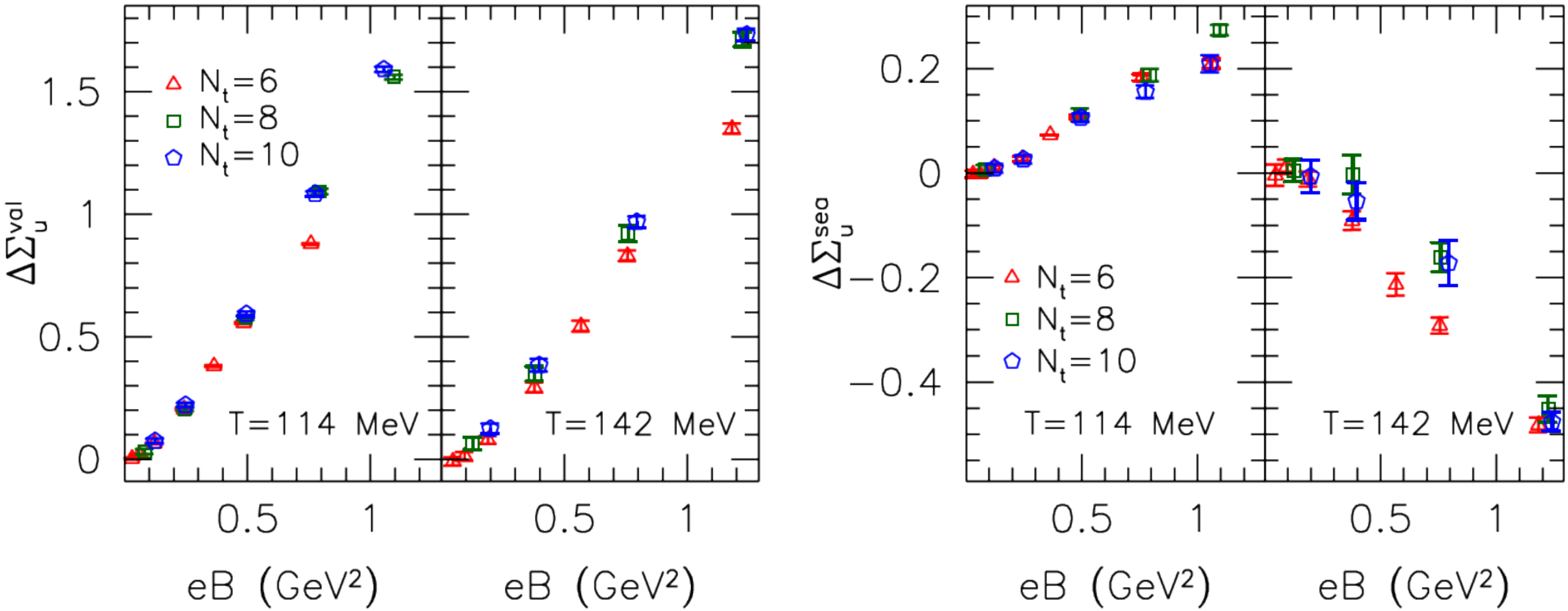
[Bali et al., Phys. Rev. D**86**, 071502 (2012)]

INVERSE CATALYSIS (T_C VS. B)



[Bali et al., JHEP 02, 044 (29012)], [Bali et al., PRD**86**, 071502 (2012)], [G. Endrodi, JHEP 1507 (2015) 173]

VALENCE VS. SEA

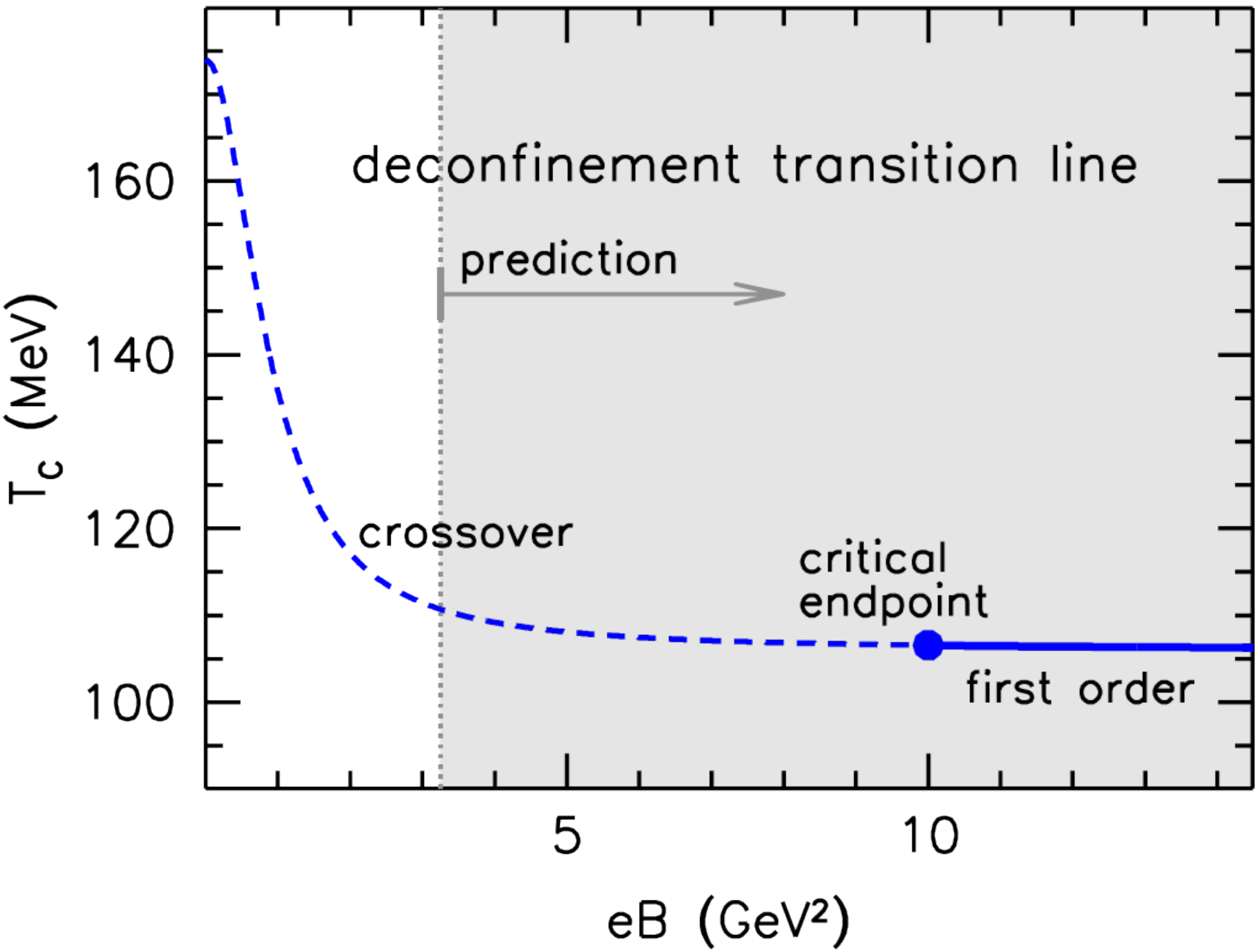


[Bruckmann, G. Endrodi, T. G. Kovacs, JHEP 04 (2013) 112]

- Gluon screening (?)
- Polyakov loops (?)

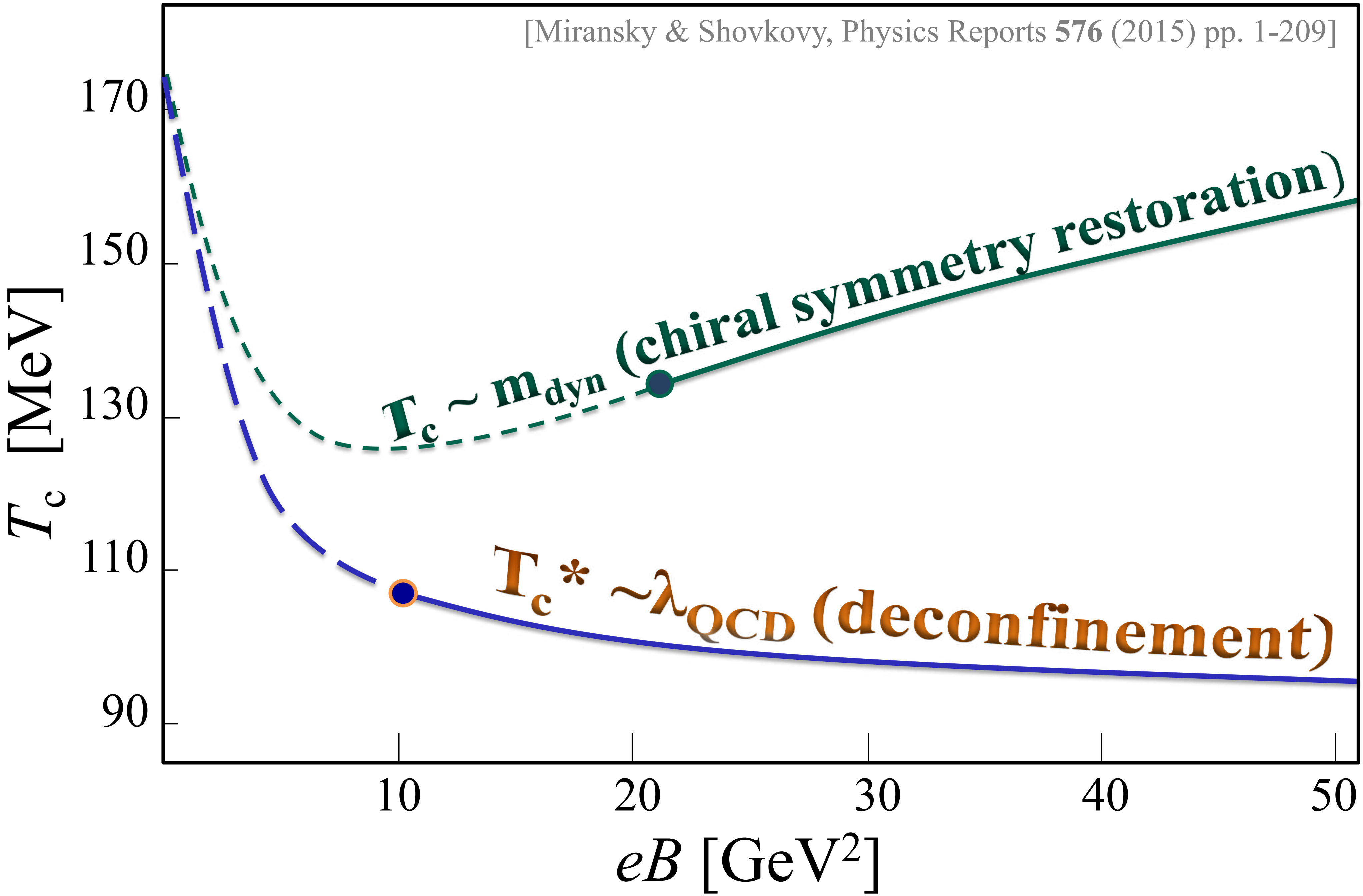
} or, perhaps, something else (?)

SUPER-STRONG *B*: PREDICTION



[Cohen & Yamamoto, PRD**89**, 054029 (2014)], [G. Endrodi, JHEP 1507 (2015) 173]

PREDICTED PHASE DIAGRAM



SUMMARY

- Strong magnetic field effects:
  - dimensional reduction
  - nonzero density of states at E=0
  - enhanced particle-antiparticle pairing dynamics
- Even weakly coupled regime is nonperturbative
- Magnetic catalysis in QED is hidden behind the “large” electron mass
- Strong magnetic field effects are testable in QED-like Dirac materials