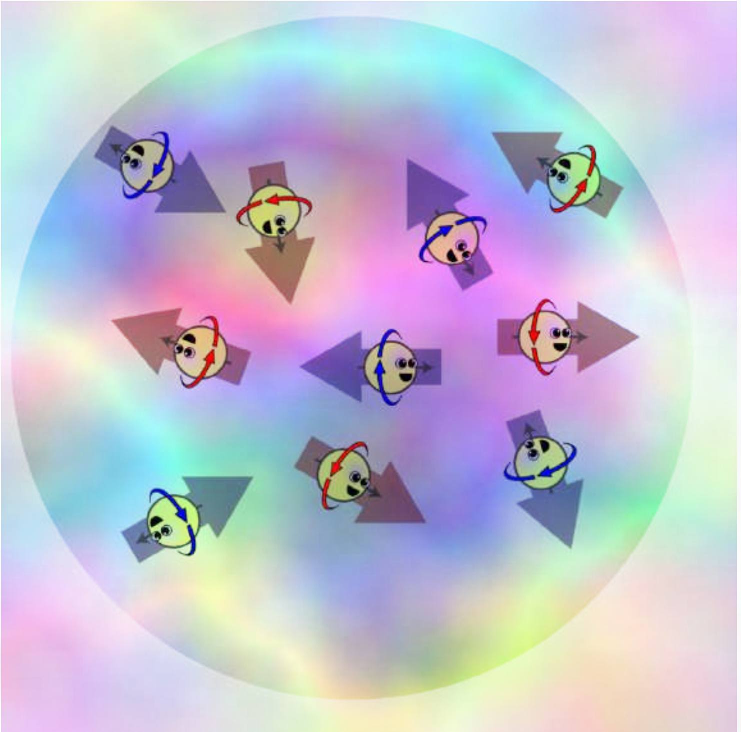
ARIZONA STATE UNIVERSITY

QUANTUM KINETIC THEORIES
Dec 9, 2019
Kyoto, Japan

Applications of chiral kinetic theory in Dirac and Weyl semimetals

Igor Shovkovy
Arizona State University



CHIRAL MATTER

Relativistic Matter

- **Nonrelativistic**
  - Particles move much slower than the speed of light
  - Kinetic energies are much smaller than the rest energy

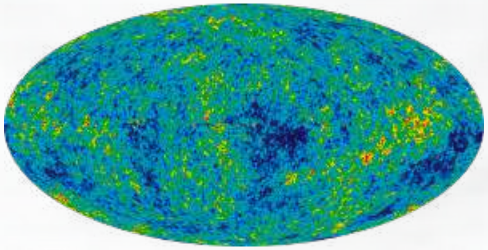
$$E_{\text{kin}} << E_{\text{rest}} : \quad E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

- **Relativistic**
  - Particle velocities approach the speed of light
  - Kinetic energies are comparable to, or larger than E_{rest}

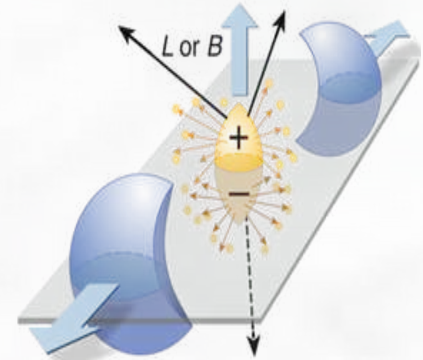
$$E_{\text{kin}} \geq E_{\text{rest}} : \quad E = c\sqrt{p^2 + m^2c^2} \approx c\,p$$

Where is chiral matter?

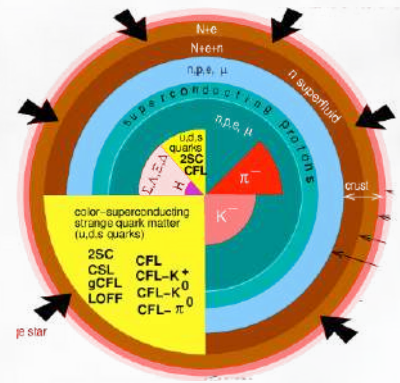
- **Early Universe**, e.g.,
  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]



- **Heavy-ion collisions**, e.g.,
  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

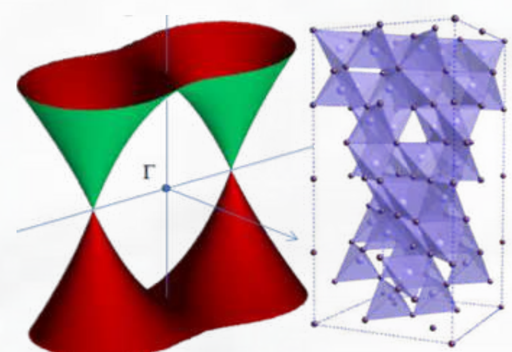


- **Super-dense matter in compact stars**, e.g.,
  [Yamamoto, Phys.Rev. D93, 065017 (2016)]



- **Superfluid ^{3}He-A**, e.g.,
  [Volovik, JETP Lett. 46, 98 (1987), JETP Lett. 105, 34 (2017)]

- **Dirac/Weyl (semi-)metals**, e.g.,
  [Li et. al. Nature Phys. 12, 550 (2016)]

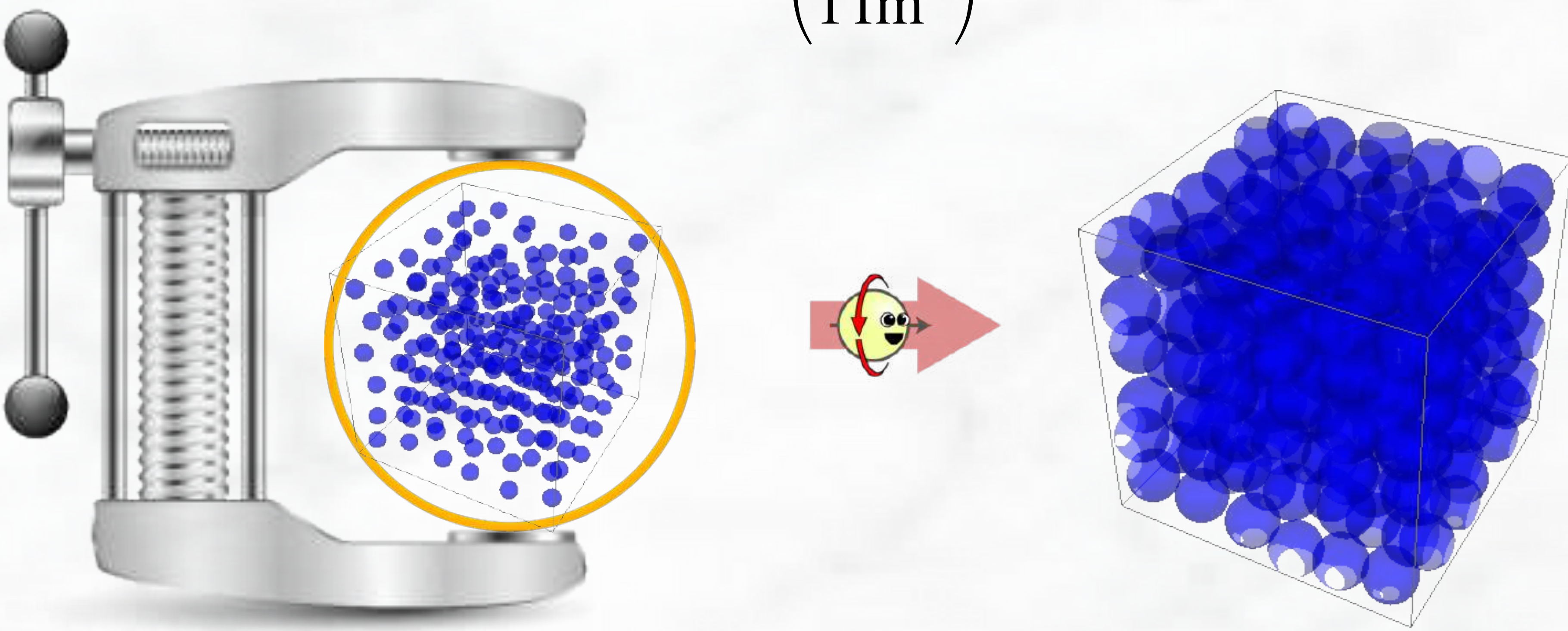


Superdense Matter

- **What happens when you squeeze matter to very high density?** (e.g., neutrons inside neutron stars)

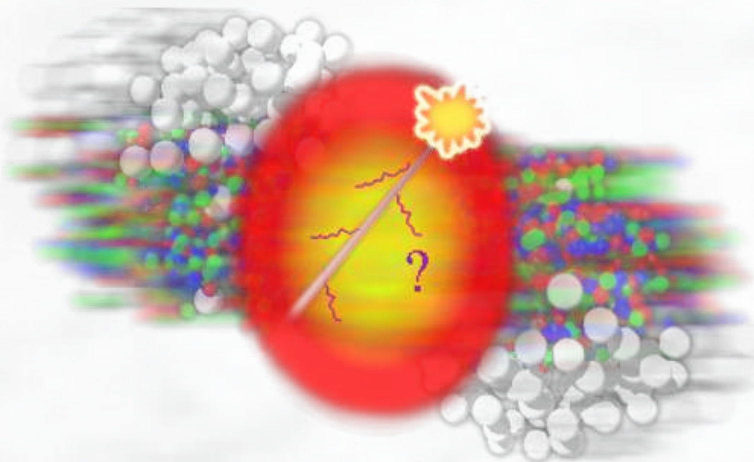
Pauli exclusion principle: fermions fill out quantum states with momenta from $p_{\min} \approx 0$ to $p_{\max} \propto \hbar n^{1/3}$

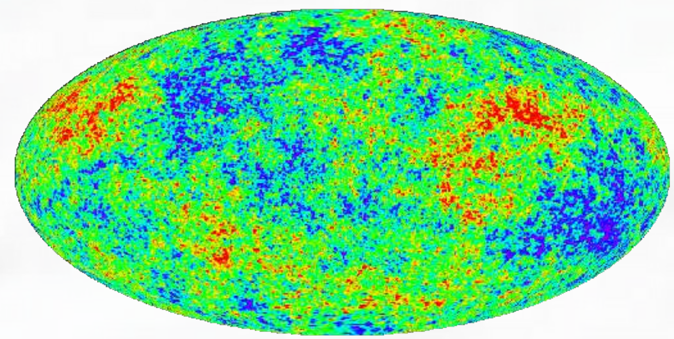
$$p_{\max} \propto 200 \left(\frac{n}{1\,\mathrm{fm}^3} \right)^{1/3} \mathrm{MeV/c}$$



Superhot Matter

- **What happens when you heat matter to very high temperature?** (e.g., matter in heavy ion collisions)

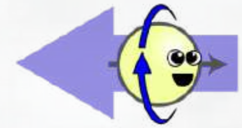




Particles carry large **kinetic energy**: typical value of energy/momentum is proportional to temperature:

$$p \propto k_B T / c \sim 200 \left(\frac{k_B T}{200 \text{ MeV}} \right) \text{MeV/c}$$

Chiral fermions



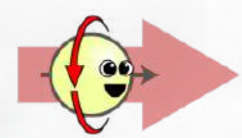
- Note: **chirality** is *quantum* and *relativistic* concept
- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{p}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}\,|}\Psi = \mathrm{sign}(p_0)\gamma^5\,\Psi$$

For particles ($p_0 > 0$): chirality = helicity

For antiparticles ($p_0 < 0$): chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T \gg m$
  - High density: $\mu \gg m$



Chiral anomaly

- For a *local* transformation $\delta\psi = i\alpha(x)\gamma^5 T\psi$

$$\delta S = \int d^4x\, j_5^\mu \partial_\mu \alpha(x)$$

- The path integral integration measure [Fujikawa, Phys. Rev. Lett. 42, 1195 (1979)]

$$[d\psi][d\bar{\psi}] \to \exp\left[i\int d^4x\, \mathcal{A}(x)\alpha(x)\right][d\psi][d\bar{\psi}]$$

$$\mathcal{A}(x) = -\frac{1}{16\pi^2}\varepsilon_{\mu\nu\lambda\kappa}F_\alpha^{\mu\nu}F_\beta^{\lambda\kappa}\mathrm{Tr}\left[T^\alpha T^\beta T\right]$$

- Since a change of integration variables cannot change the result

$$\delta\int[d\psi][d\bar{\psi}]e^{iS} = i\int d^4x\int[d\psi][d\bar{\psi}]\,\left[\mathcal{A}(x)\alpha(x) + j_5^\mu\partial_\mu\alpha(x)\right]e^{iS} = 0$$

- One obtains the following anomaly relation (in QED):

$$\left\langle \partial_\mu j_5^\mu \right\rangle_A = \mathcal{A}(x) = -\frac{1}{16\pi^2}\varepsilon_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa}$$

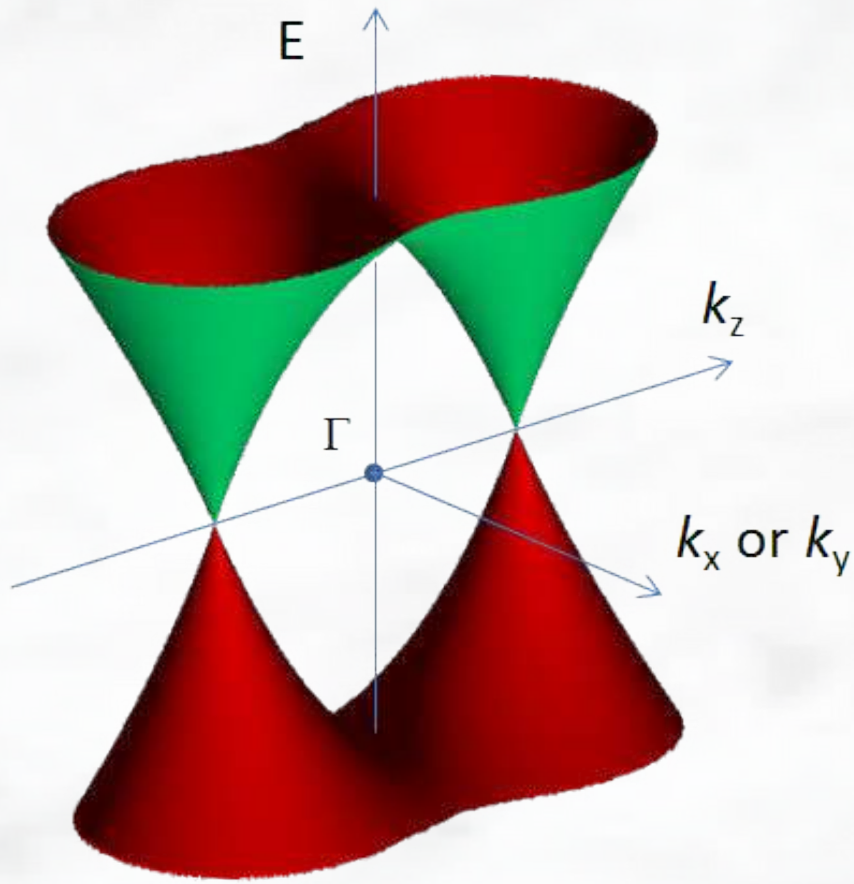
Chiral matter

- Matter made of chiral fermions with $n_\mathrm{L} \neq n_\mathrm{R}$
- Unlike the electric charge ($n_\mathrm{R} + n_\mathrm{L}$), the chiral charge ($n_\mathrm{R}$ - n_L) is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral anomaly may affect the properties of matter

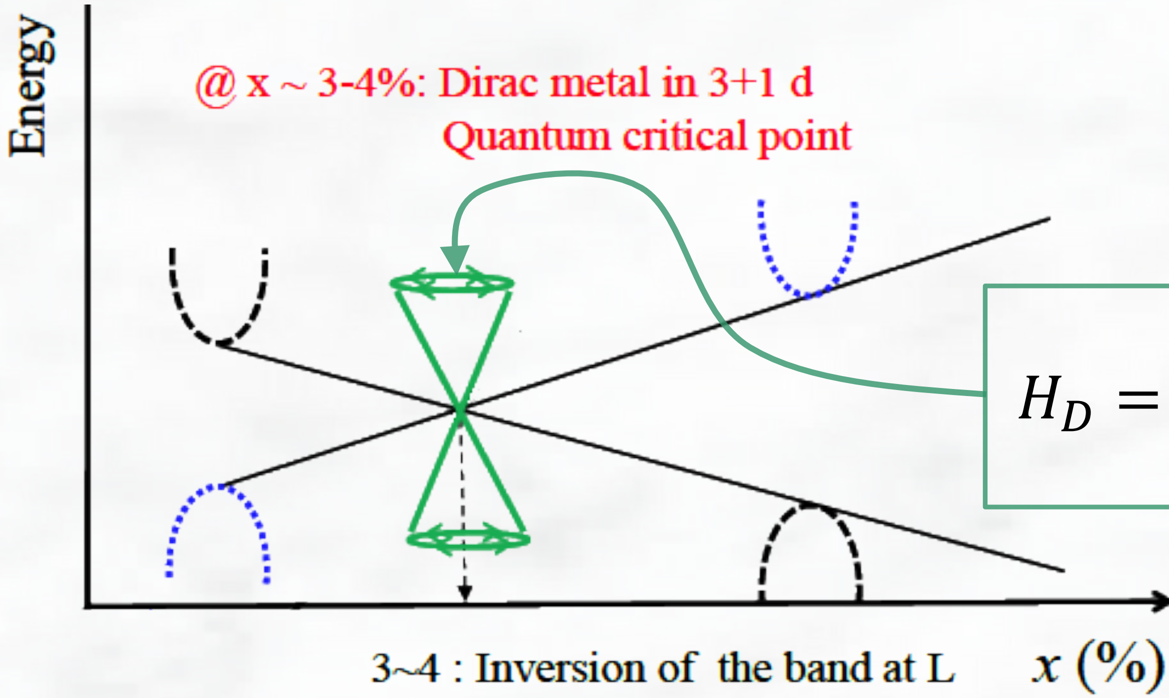


Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL MATERIALS

Dirac semimetals

- Solid state materials with Dirac quasiparticles:
  - $Bi_{1\text{-}x}Sb_x$ alloy



$$H_D = \begin{pmatrix} v_F(\vec{\boldsymbol{k}}\cdot\vec{\sigma}) & 0 \\ 0 & -v_F(\vec{\boldsymbol{k}}\cdot\vec{\sigma}) \end{pmatrix}$$

- “New” 3D Dirac materials (ARPES):
  - Na_3Bi (Potassium bismuthide) [Liu et al., Science **343**, 864 (2014)]
  - Cd_3As_2 (Cadmium arsenide) [Neupane et al., Nature Commun. **5**, 3786 (2014)] [Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

Dirac materials

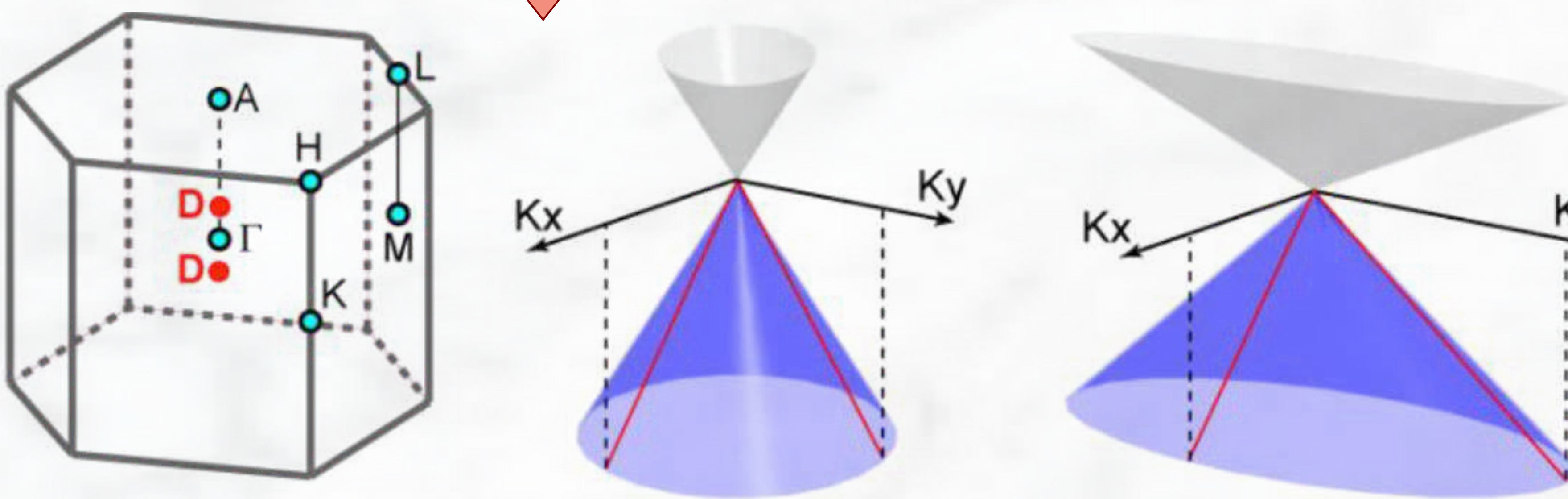
- $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
- Na_3Bi
- Cd_3As_2
- $ZrTe_5$

[Liu et al., Science **343**, 864 (2014)]

[Neupane et al., Nature Commun. **5**, 3786 (2014)]

[Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

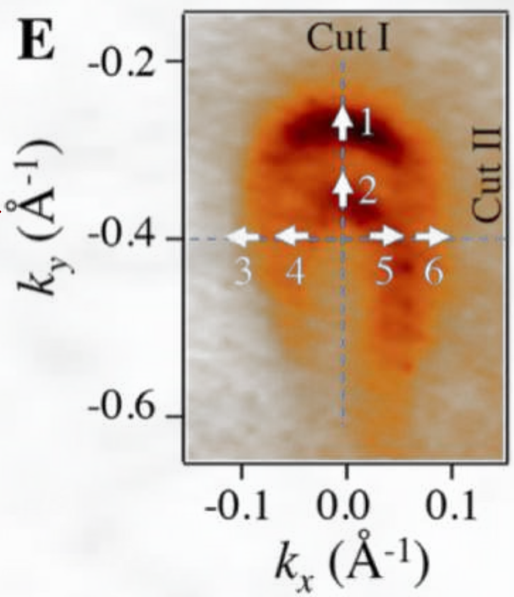
[Li et al., Nature Physics **12**, 550 (2016)]

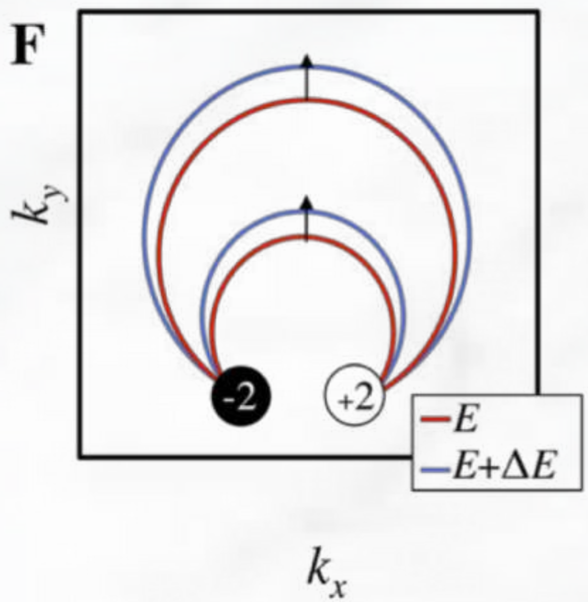


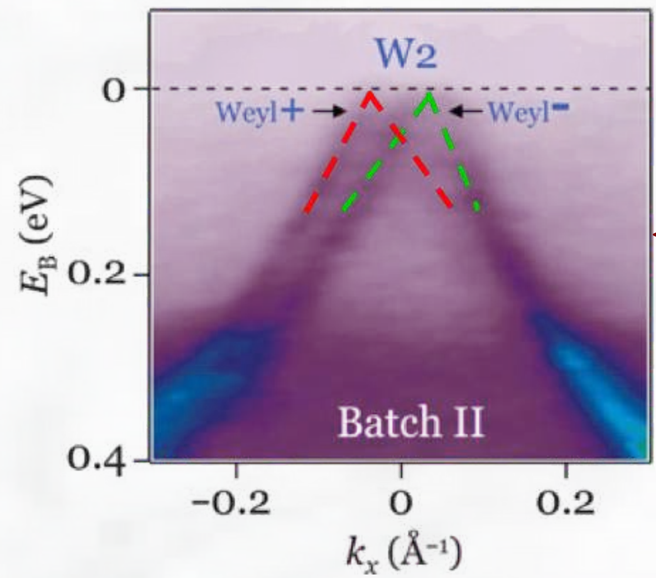
$$E = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + v_z^2 k_z^2}, \quad v_x \approx v_y \approx 3.74\times 10^5\ m/s, \quad v_y \approx 2.89\times 10^4\ m/s$$

Weyl materials

- TaAs (tantalum arsenide) [S.-Y. Xu et al., Science 349, 613 (2015)] [B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
- NbAs (niobium arsenide) [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
- TaP (tantalum phosphide) [S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
- NbP (niobium phosphide) [I. Belopolski et al. arXiv:1509.07465]
- WTe_2 (tungsten telluride) [F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]

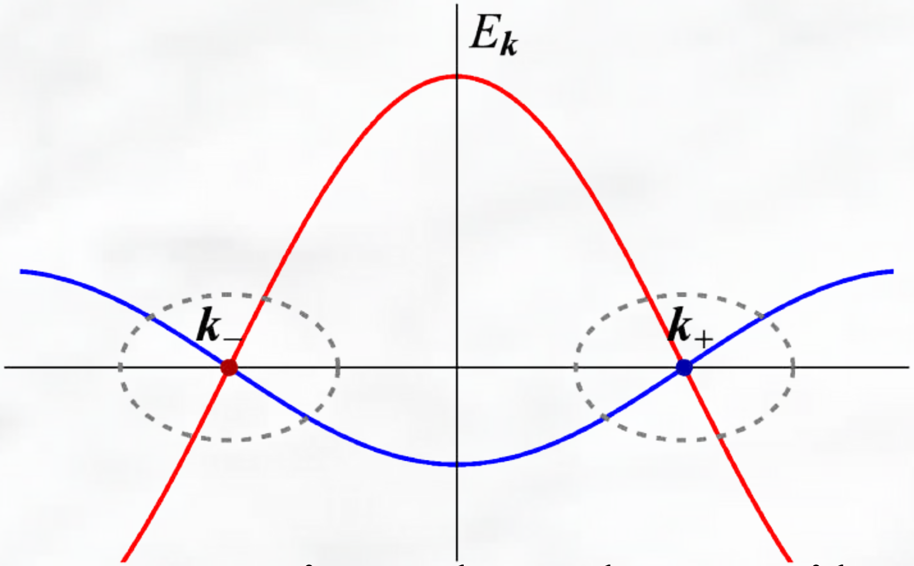




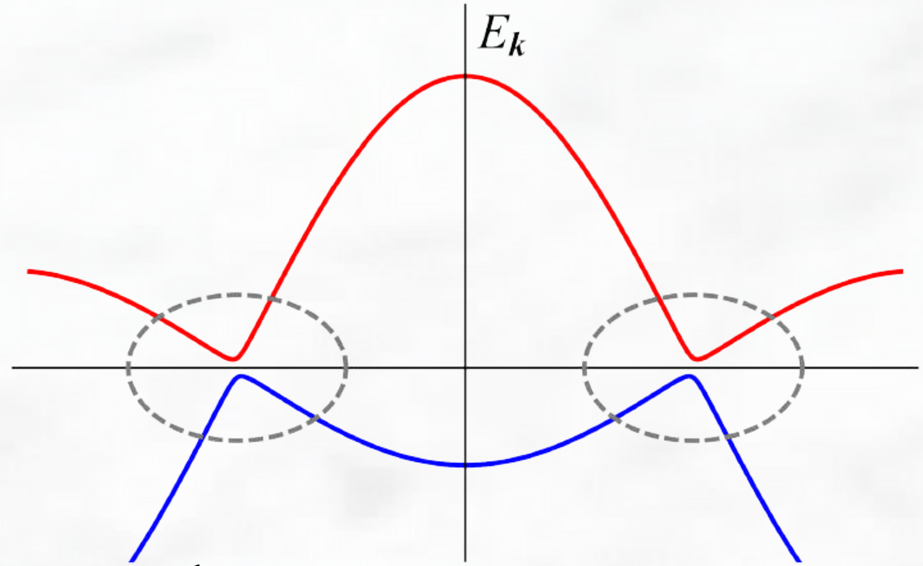


Relativistic-like band crossing

Do energy levels cross?



Or do they repel?



A generic 2-band Hamiltonian reads

$$H_{\boldsymbol{k}} = a_{\boldsymbol{k}} + \vec{b}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{\sigma}} \quad \Longrightarrow \quad E_{\boldsymbol{k}} = a_{\boldsymbol{k}} \pm \sqrt{\left(\vec{b}_{\boldsymbol{k}}\right)^2}$$

The bands cross when

[Witten, Riv. Nuovo Cimento 39, 313 (2016)]

$$\vec{b}_{\boldsymbol{k}}=0$$

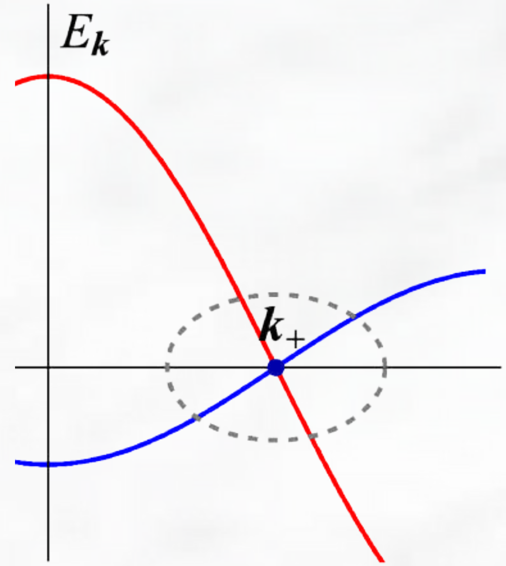
These 3 equations can be solved by adjusting $\vec{\boldsymbol{k}}$ in 3D

Emergent chirality in solids

Near a band crossing (e.g., $\vec{\boldsymbol{k}} \approx \vec{\boldsymbol{k}}_+$)

$$H_{\boldsymbol{k}} = a_+ + \cancel{\left(\vec{\nabla}_{\boldsymbol{k}} a_{\boldsymbol{k}} \cdot \delta\vec{\boldsymbol{k}}\right)} + \textstyle\sum_{i,j} \sigma_i b_{ij} \delta k_j$$

cone tilting



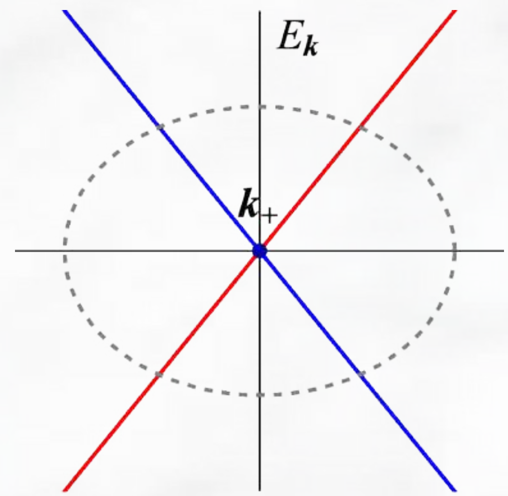
Using an orthogonal transformation

$$b_{ij} \equiv \partial b_i / \partial k_j \to \hbar v_i \delta_{ij}$$

Assuming *isotropy* & choosing a suitable *reference point*,

$$H_{\boldsymbol{k}} = \pm v_F (\vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{k}})$$

which is the Hamiltonian for Weyl fermions



Note that the *chirality* is defined by

$$\lambda = \mathrm{sign}\left[\det\left(b_{ij}\right)\right]$$

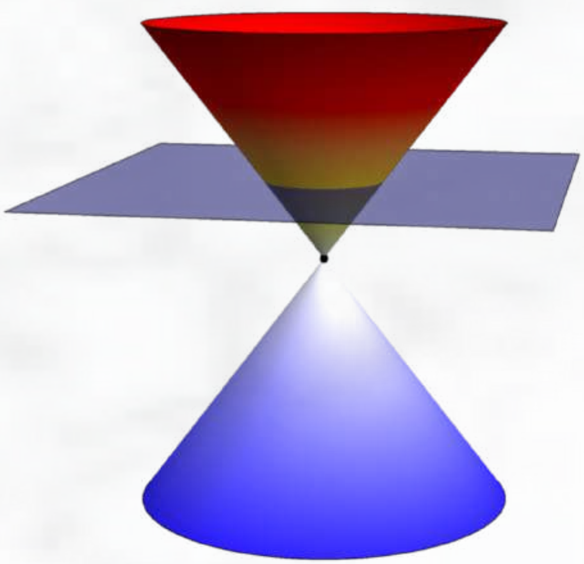
Type I & II Weyl materials

Tilting of the Weyl cone

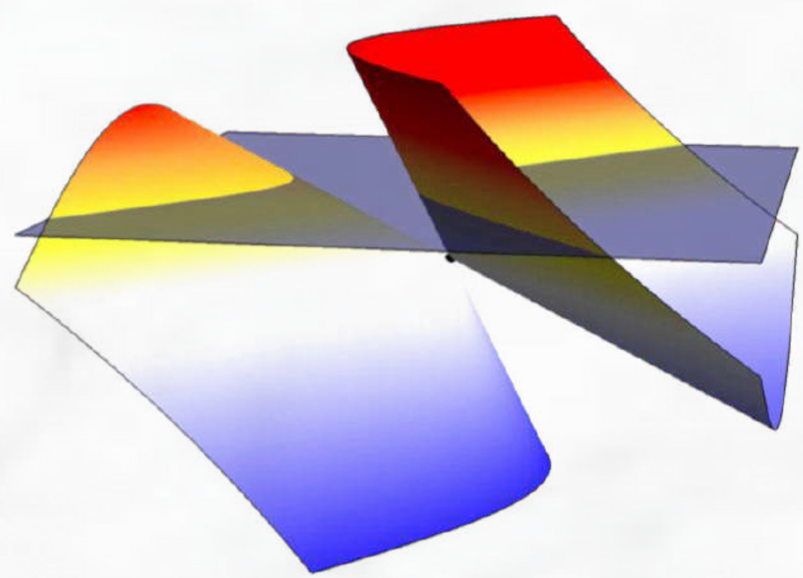
$$H_{\boldsymbol{k}} = \underbrace{\vec{\boldsymbol{t}} \cdot \vec{\boldsymbol{k}}}_{\text{cone tilting}} \pm v_F (\vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{k}})$$

The energy spectrum: $E_{\boldsymbol{k}} = \vec{\boldsymbol{t}} \cdot \vec{\boldsymbol{k}} \pm v_F |\vec{\boldsymbol{k}}|$

Type-I Weyl material



Type-II Weyl material

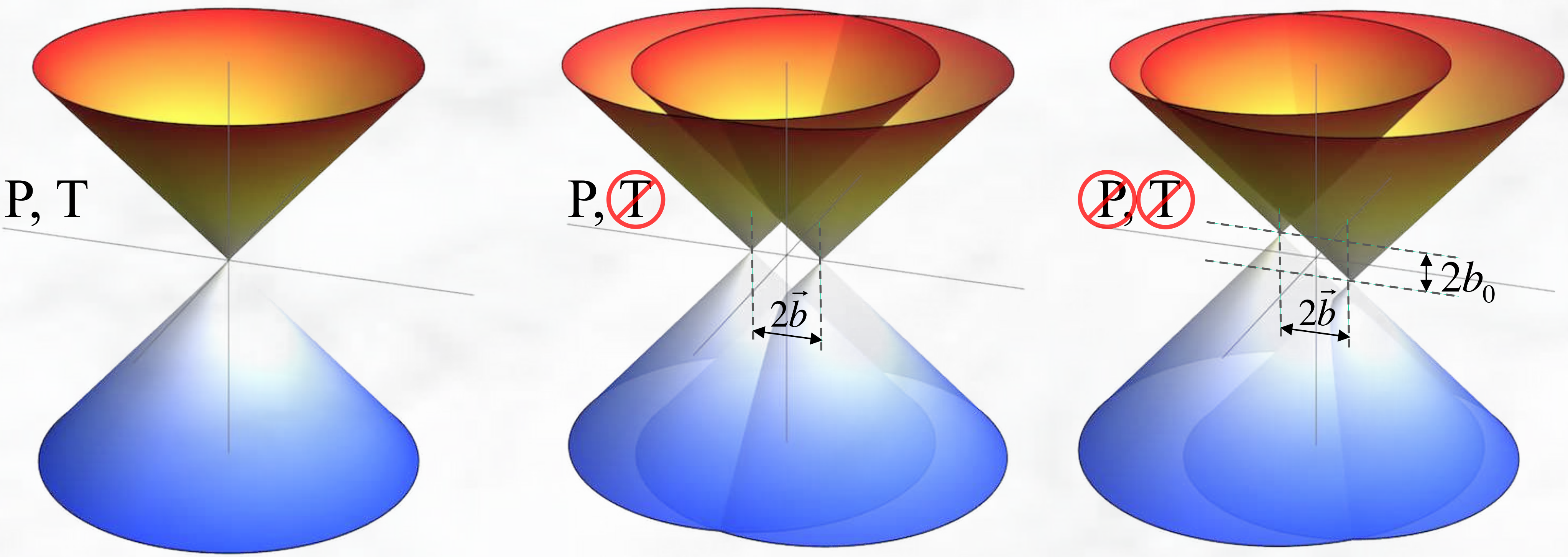


Idealized models

- Low-energy Hamiltonians of a Dirac and Weyl materials

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \overset{\cancel{\mathrm{T}}}{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5} + \overset{\cancel{\mathrm{P}}}{b_0\gamma^0\gamma^5}\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) **Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Low-energy Hamiltonian

- The Hamiltonian for massless Dirac fermions is given by

$$H_D = \begin{pmatrix} v_F(\vec{\boldsymbol{k}} \cdot \vec{\sigma}) & 0 \\ 0 & -v_F(\vec{\boldsymbol{k}} \cdot \vec{\sigma}) \end{pmatrix}$$

- This can we viewed as a combination of two Weyl fermions

$$H_\lambda = \lambda v_F(\vec{\boldsymbol{k}} \cdot \vec{\sigma})$$

where $\lambda = \pm 1$ is a chirality

The Weyl energy eigenstates are given by

$$\psi_{\boldsymbol{k}}^{\lambda} = \frac{1}{\sqrt{2}\sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

They described particles of energy $\epsilon_k = v_F\sqrt{k_x^2 + k_y^2 + k_z^2}$

The mapping $k \rightarrow \psi_k^{\lambda}$ has a nontrivial topology

Berry connection & curvature

- Consider evolution from $\psi_{\boldsymbol{k}}$ to $\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}$:

$$\langle\psi_{\boldsymbol{k}}|\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}\rangle \approx 1 + \delta\boldsymbol{k}\cdot\langle\psi_{\boldsymbol{k}}|\boldsymbol{\nabla}_{\boldsymbol{k}}|\psi_{\boldsymbol{k}}\rangle \approx e^{i\boldsymbol{a}_{\boldsymbol{k}}\cdot\delta\boldsymbol{k}}$$

where $\boldsymbol{a}_{\boldsymbol{k}} = -i\langle\psi_{\boldsymbol{k}}|\boldsymbol{\nabla}_{\boldsymbol{k}}|\psi_{\boldsymbol{k}}\rangle$ is the Berry connection

- The Berry curvature is defined as follows:

$$\boldsymbol{\Omega}_k = \boldsymbol{\nabla}_k\times\boldsymbol{a}_{\boldsymbol{k}}$$

- Note the similarity with gauge fields, but $\boldsymbol{a}_{\boldsymbol{k}}$ and $\boldsymbol{\Omega}_k$ are defined in the momentum space
- It is convenient to define the Chern number (flux of $\boldsymbol{\Omega}_k$)

$$C = \frac{1}{2\pi}\oint \boldsymbol{\Omega}_k\cdot d\boldsymbol{S}_{\boldsymbol{k}}$$

- A nonzero (integer) Chern number indicates a nonzero (topological) charge inside the k-volume surrounded by the closed surface (Gauss's law)

Berry curvature for Weyl fermions

- In the case of Weyl fermions ($v_F = 1$),

$$\psi_{\boldsymbol{k}}^{\lambda} = \frac{1}{\sqrt{2}\sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

- This leads to the Berry connection

$$a_{\boldsymbol{k},x} \equiv -i\langle \psi_{\boldsymbol{k}}^{\lambda} | \partial_{k_x} | \psi_{\boldsymbol{k}}^{\lambda} \rangle = -\frac{v_F^2 k_y}{2(\epsilon_k^2 + \lambda v_F \epsilon_k k_z)}$$

$$a_{\boldsymbol{k},y} \equiv -i \left\langle \psi_{\boldsymbol{k}}^{\lambda} \middle| \partial_{k_y} \middle| \psi_{\boldsymbol{k}}^{\lambda} \right\rangle = \frac{v_F^2 k_x}{2(\epsilon_k^2 + \lambda v_F \epsilon_k k_z)}$$

$$a_{\boldsymbol{k},z} \equiv -i\langle \psi_{\boldsymbol{k}}^{\lambda} | \partial_{k_z} | \psi_{\boldsymbol{k}}^{\lambda} \rangle = 0$$

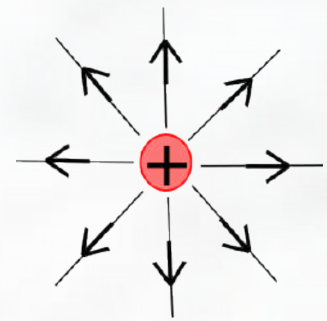
- The Berry curvature is

$$\boldsymbol{\Omega}_k \equiv \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}} = \lambda \frac{\vec{\boldsymbol{k}}}{2k^3}$$

Berry curvature for Weyl fermions

- Note that the Berry curvature (in momentum space)

$$\boldsymbol{\Omega}_k \equiv \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}} = \lambda \frac{\vec{\boldsymbol{k}}}{2k^3}$$



has the shape of a *monopole* @ $\vec{\boldsymbol{k}} = 0$

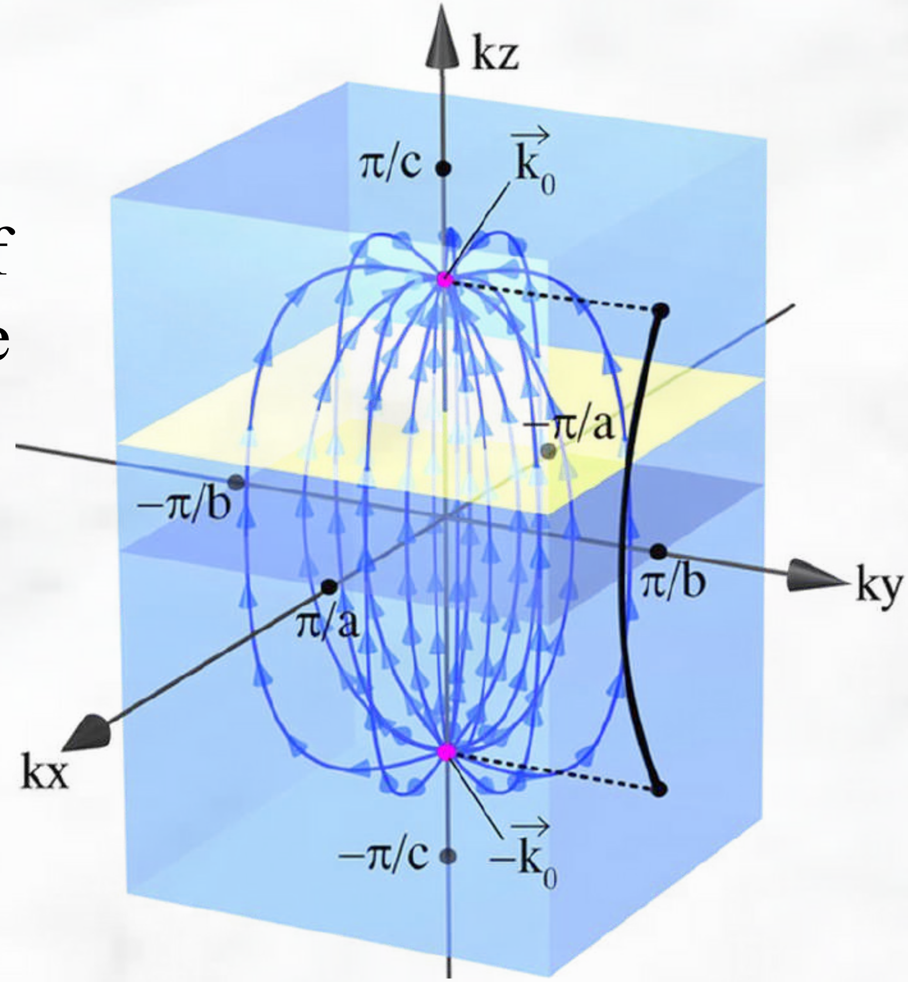
- The total flux of $\boldsymbol{\Omega}_k$-field through the spherical surface of radius K with the center at $\vec{k} = 0$

$$C = \frac{1}{2\pi} \oint \lambda \frac{\vec{\boldsymbol{k}}}{2k^3} \cdot \frac{\vec{\boldsymbol{k}}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda = \pm 1$$

- Thus, the electronic structure of massless Weyl fermions is characterized by a topological monopole at $\vec{\boldsymbol{k}} = 0$
- Is the Berry monopole just a mathematical curiosity?
- Are there any observable consequences?

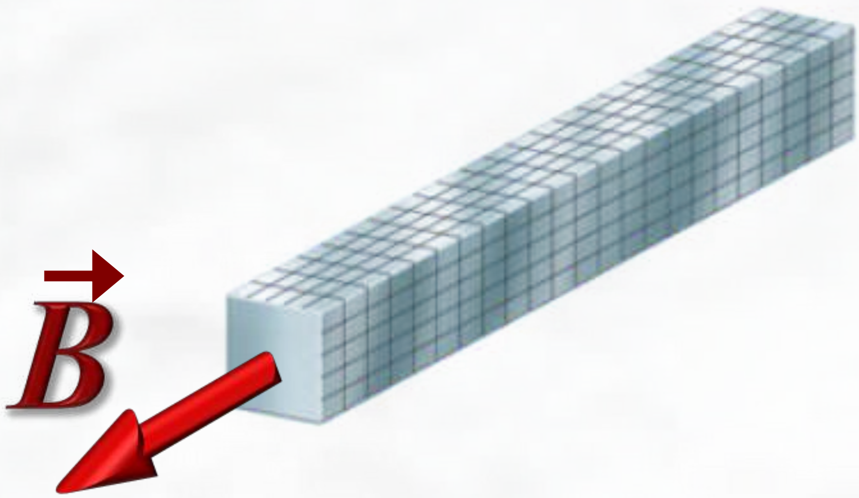
Weyl fermions on a lattice

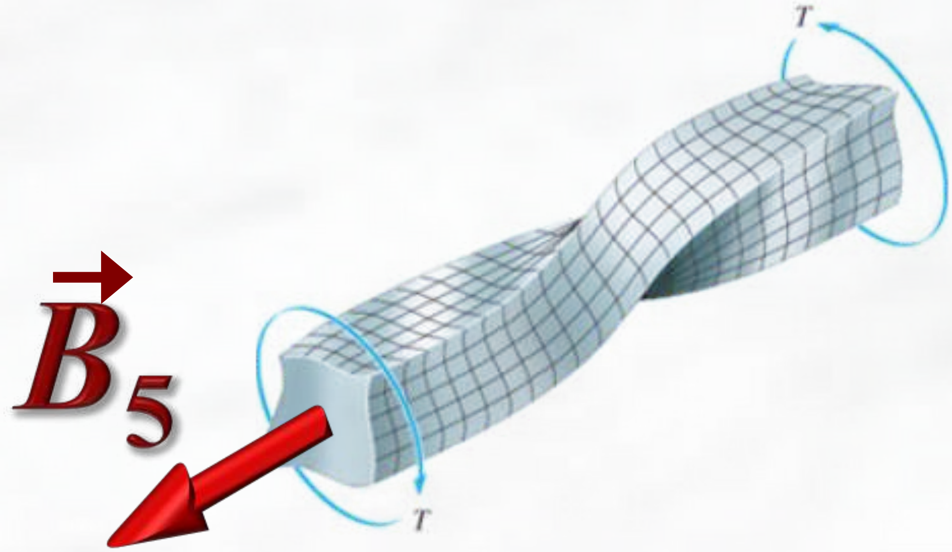
- In solid state physics, the momentum space (Brillouin zone) is compact
- A closed surface around a Weyl node is also a closed surface (of opposite orientation) around the rest of the Brillouin zone
- Flipping surface orientation changes the sign of the flux
- There must be an opposite charge in the rest of the zone



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

- Thus, Weyl fermions come in pairs of opposite chirality
[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]





PSEUDO-ELECTROMAGNETIC FIELDS

$$\mathbf{E}_\lambda = \mathbf{E} + \lambda \mathbf{E}_5 \text{ and } \mathbf{B}_\lambda = \mathbf{B} + \lambda \mathbf{B}_5$$

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]
[Grushin, Venderbos,Vishwanath, Ilan, Phys. Rev. X **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B **94**, 241405 (2016)]
[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Strain in Weyl materials

- Strains affect low-energy quasiparticles in Weyl materials

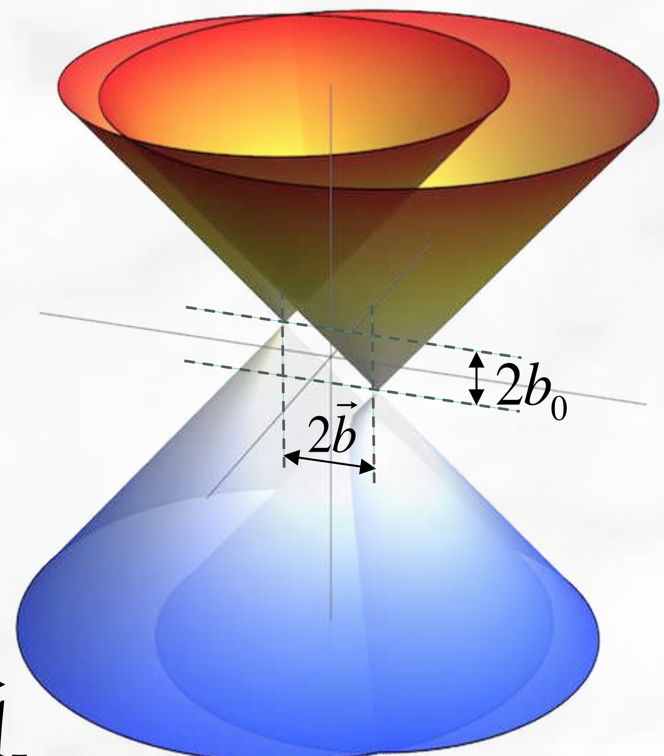
$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\,\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

where the components of the chiral gauge fields are

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$



The associated pseudo-EM fields are

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$

Chiral effects in Weyl materials

- Any qualitative properties of Weyl materials directly sensitive to b_0 and $\vec{b}$?
- Some proposals:
  - Anomalous Hall effect
  - Anomalous Alfven waves
  - Strain/torsion induced CME
  - Strain/torsion induced quantum oscillations
  - Strain/torsion dependent resistance
  - etc.
- How about collective modes?

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

CONSISTENT CHIRAL KINETIC THEORY

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral kinetic theory (1)

- The transition amplitude: [Stephanov & Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\langle f|e^{iH(t_f-t_i)}|i\rangle = \left[\int \mathcal{D}x\mathcal{D}p\mathcal{P}\exp\left\{i\int_{t_i}^{t_f}(\boldsymbol{p}\cdot\dot{\boldsymbol{x}}-\boldsymbol{\sigma}\cdot\boldsymbol{p})dt\right\}\right]_{fi}$$

- The Hamiltonian can be diagonalized

$$V_{\boldsymbol{p}}^{\dagger}\boldsymbol{\sigma}\cdot\boldsymbol{p}V_{\boldsymbol{p}} = |\boldsymbol{p}|\sigma_3$$

- Discretizing the path integral and inserting unit matrices

$$\ldots V_{\boldsymbol{p}_2}V_{\boldsymbol{p}_2}^{\dagger}\exp\{-i\boldsymbol{\sigma}\cdot\boldsymbol{p}_2\Delta t\}V_{\boldsymbol{p}_2}V_{\boldsymbol{p}_2}^{\dagger}V_{\boldsymbol{p}_1}V_{\boldsymbol{p}_1}^{\dagger}\exp\{-i\boldsymbol{\sigma}\cdot\boldsymbol{p}_1\Delta t\}V_{\boldsymbol{p}_1}V_{\boldsymbol{p}_1}^{\dagger}\ldots$$

one derives

$$\langle f|e^{iH(t_f-t_i)}|i\rangle = \left[V_{\boldsymbol{p}_f}\int \mathcal{D}x\mathcal{D}p\mathcal{P}\exp\left\{i\int_{t_i}^{t_f}(\boldsymbol{p}\cdot\dot{\boldsymbol{x}}-|\boldsymbol{p}|\sigma_3-\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}})dt\right\}V_{\boldsymbol{p}_i}^{\dagger}\right]_{fi}.$$

Note that we used

$$V_{\boldsymbol{p}_2}^{\dagger}V_{\boldsymbol{p}_1}\approx\exp(-i\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\Delta\boldsymbol{p}),\quad \text{where } \hat{\boldsymbol{a}}_{\boldsymbol{p}}=iV_{\boldsymbol{p}}^{\dagger}\boldsymbol{\nabla}_{\boldsymbol{p}}V_{\boldsymbol{p}}$$

Chiral kinetic theory (2)

- Explicit expressions:

$$V_{\boldsymbol{p}} = \begin{pmatrix} \sqrt{\dfrac{|\boldsymbol{p}| + p_z}{2|\boldsymbol{p}|}} & \sqrt{\dfrac{|\boldsymbol{p}| - p_z}{2|\boldsymbol{p}|}} \exp[-i\phi_p] \\ \sqrt{\dfrac{|\boldsymbol{p}| - p_z}{2|\boldsymbol{p}|}} \exp[i\phi_p] & -\sqrt{\dfrac{|\boldsymbol{p}| + p_z}{2|\boldsymbol{p}|}} \end{pmatrix}$$

where $\phi_p = \arctan(p_y/p_x)$

$$\widehat{\boldsymbol{\Omega}}_{\boldsymbol{p}} = \boldsymbol{\nabla}\times\widehat{\boldsymbol{a}}_{\boldsymbol{p}} = -\frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}\begin{pmatrix} 1 & \dfrac{|\boldsymbol{p}| - p_z}{p_x + ip_y} \\ \dfrac{|\boldsymbol{p}| - p_z}{p_x - ip_y} & -1 \end{pmatrix}$$

- Classical approximation ⇒ ignore off-diagonal terms

Chiral kinetic theory

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

- Kinetic equation:

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_\mathbf{p} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$

$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_\mathbf{r} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_\mathbf{r}\epsilon_\mathbf{p}$, $\mathbf{v} = \boldsymbol{\nabla}_\mathbf{p}\epsilon_\mathbf{p}$,

$$\epsilon_\mathbf{p} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Current and chiral anomaly

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right]f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda)\right]f_\lambda$$

$$+e\mathbf{\nabla}\times\int\frac{d^3p}{(2\pi\hbar)^3}f_\lambda\epsilon_{\mathbf{p}}\mathbf{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t}+\mathbf{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B})+(\mathbf{E}_5\cdot\mathbf{B}_5)\Big] \quad ✔$$

$$\frac{\partial\rho}{\partial t}+\mathbf{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B}_5)+(\mathbf{E}_5\cdot\mathbf{B})\Big] \quad ✘$$

Consistent definition of current

- Additional Bardeen-Zumino term is needed,

$$\delta j^{\mu} = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda}A^5_{\nu}F_{\rho\lambda}$$

- In components,

$$\delta\rho = \frac{e^3}{2\pi^2\hbar^2 c^2}\left(\mathbf{A}^5\cdot\mathbf{B}\right)$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}A^5_0\mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c}\left(\mathbf{A}^5\times\mathbf{E}\right)$$

- Its role and implications:
  - Electric charge is conserved locally ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)

Maxwell equations

Faraday's law:

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

or in momentum space:

$$\frac{c}{\omega}\,\vec{k}\times\vec{E} = \vec{B}$$

Ampere-Maxwell's law:

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

or in momentum space:

$$\vec{P} = \frac{i}{\omega}\vec{J}$$

$$\frac{c}{\omega}\vec{k}\times\left(\frac{c}{\omega}\vec{k}\times\vec{E}\right) = -\left(4\pi\frac{i}{\omega}\vec{J} + \vec{E}\right)$$

Gauss's law (not independent): $i\vec{k}\cdot\vec{E} = 4\pi\rho$

Collective modes

We search for plane-wave solutions with

$$\mathbf{E}' = \mathbf{E}e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}, \quad \mathbf{B}' = \mathbf{B}e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

and the distribution function $f_\lambda = f_\lambda^{(\mathrm{eq})} + \delta f_\lambda$,

where

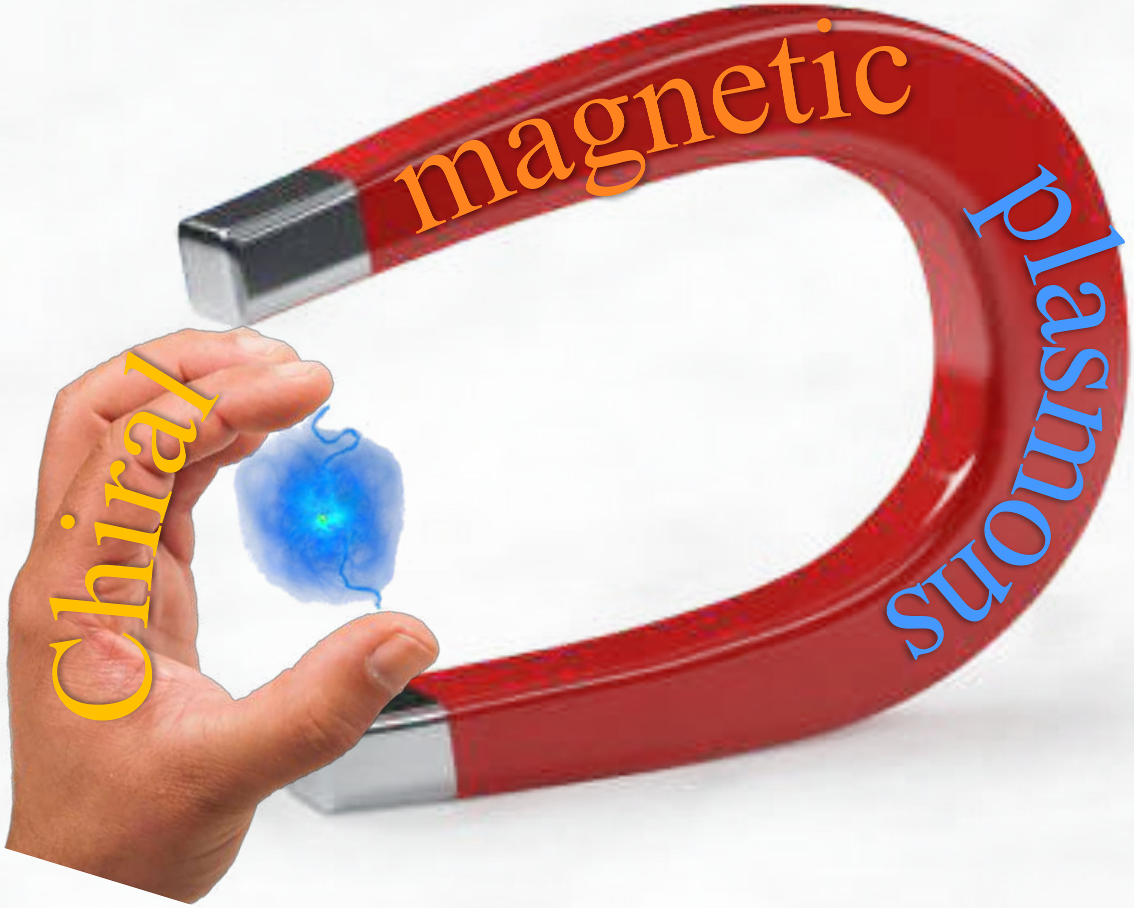
$$\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

The polarization vector & susceptibility tensor:

$$P^m = i\frac{J'^m}{\omega} = \chi^{mn} E'^n$$

The plasmon dispersion relations follow from

$$\det\left[\left(\omega^2 - c^2 k^2\right)\delta^{mn} + c^2 k^m k^n + 4\pi\omega^2\chi^{mn}\right] = 0$$



SOME RESULTS:

CHIRAL MAGNETIC PLASMONS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

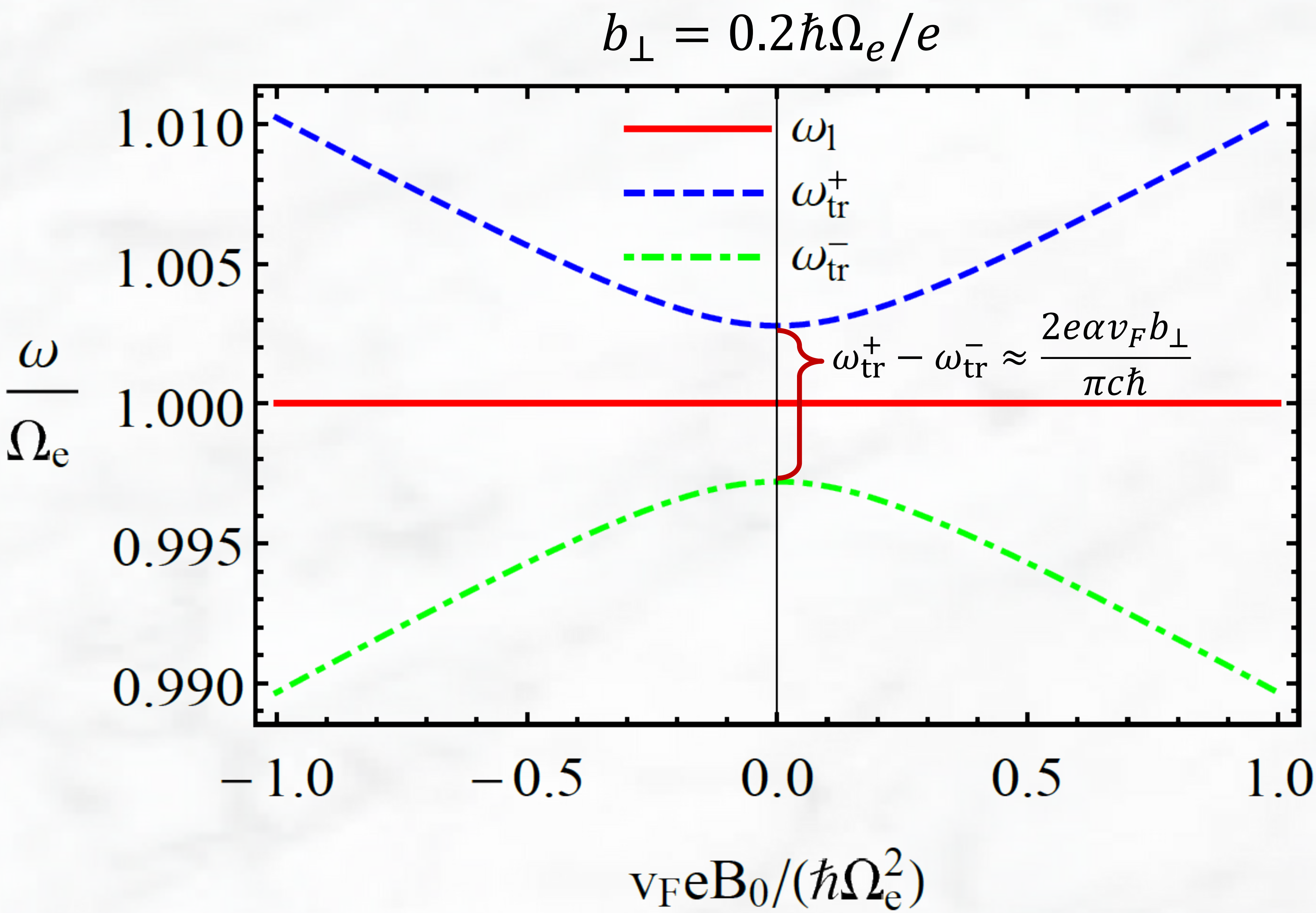
$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

and

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\left\{9\hbar^2 b_\perp^2 + \left[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_\parallel - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\right]^2\right\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

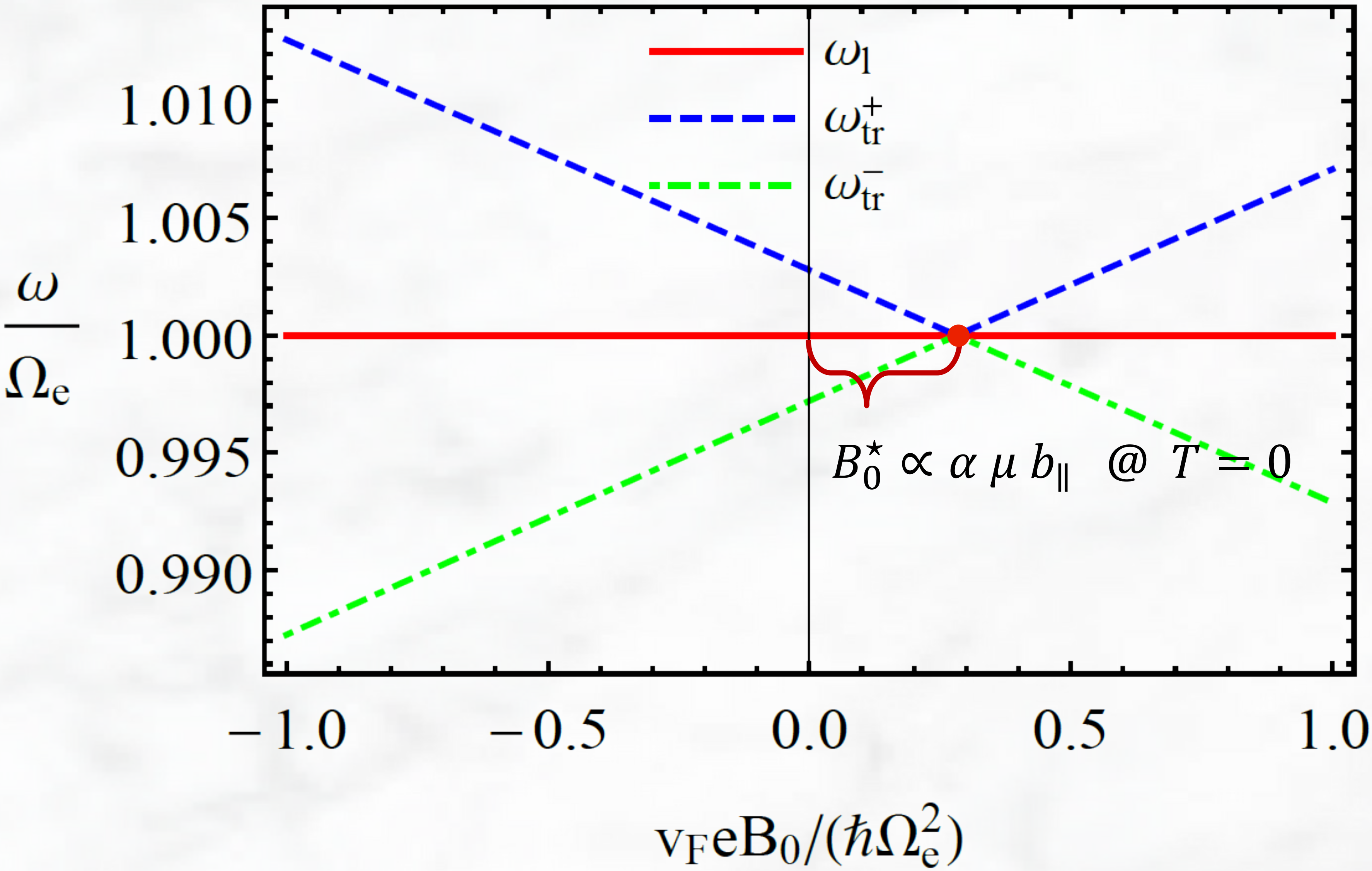
Plasmon frequencies, $\vec{B} \perp \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmon frequencies, $\vec{B} \parallel \vec{b}$

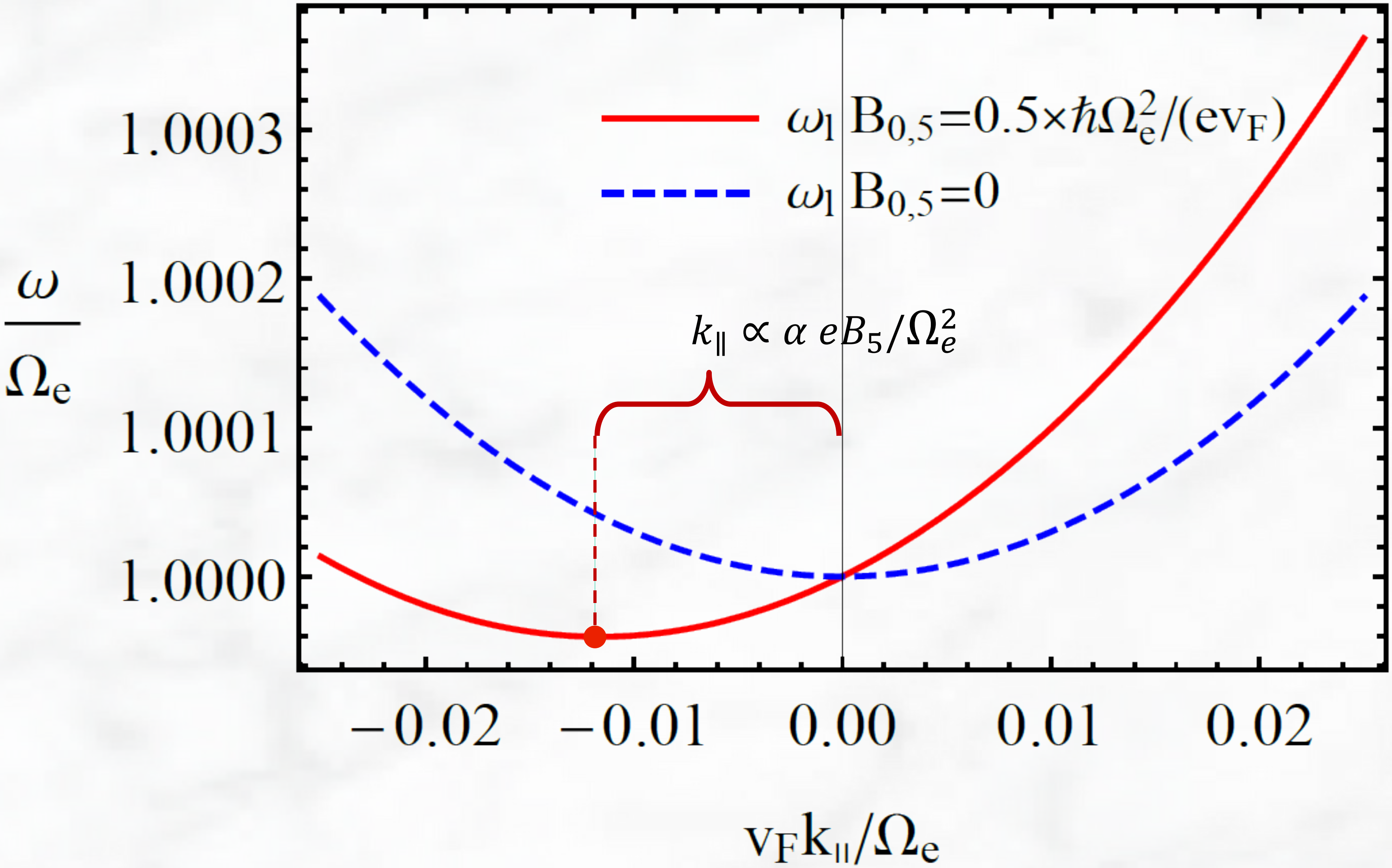
$b_{\parallel} = 0.2\hbar\Omega_e/e$



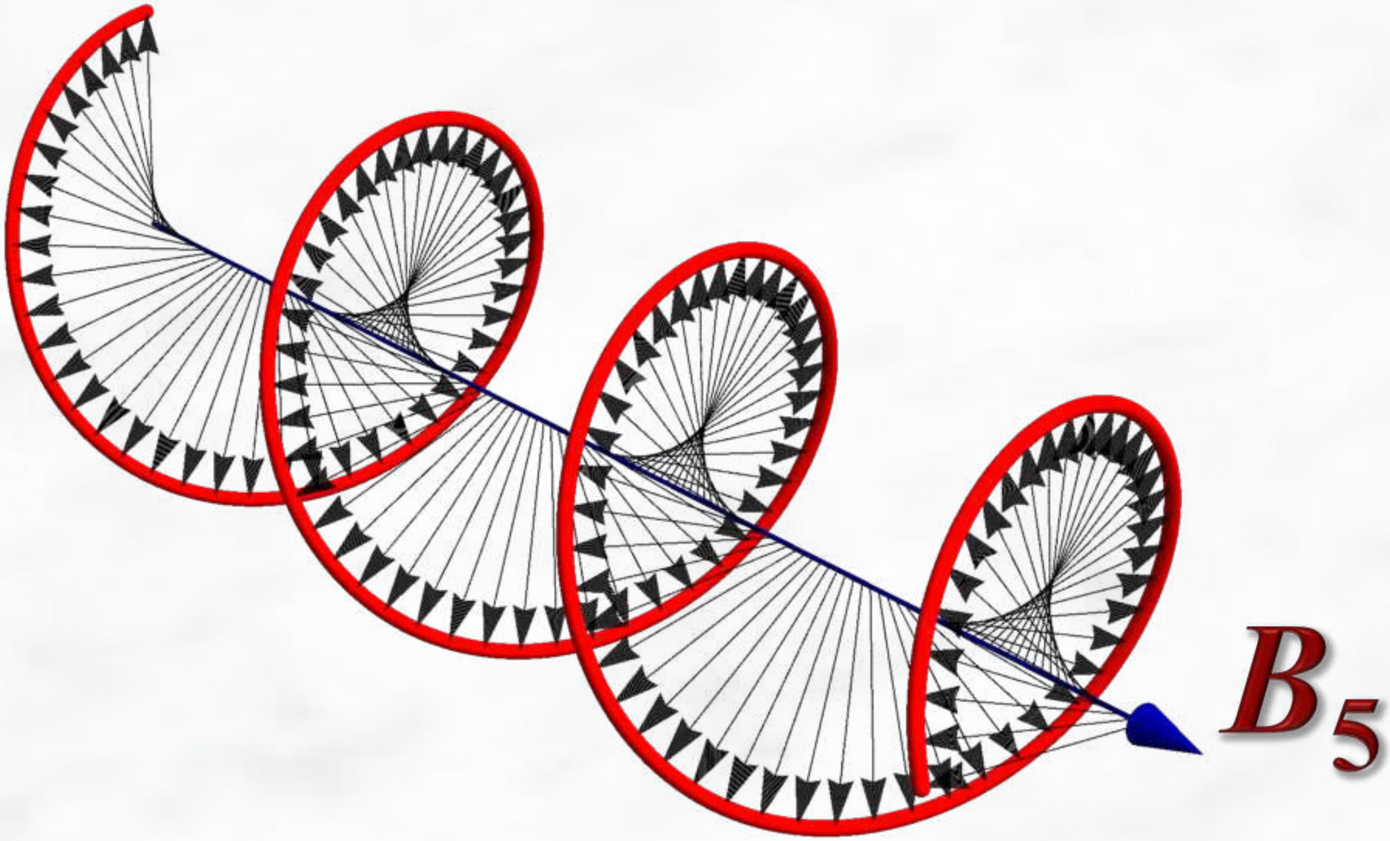
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmons with $\vec{k} \neq 0$, $\vec{k} \parallel \vec{B}_5$

- The longitudinal mode is sensitive to $\vec{B}_5$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]



RESULTS:

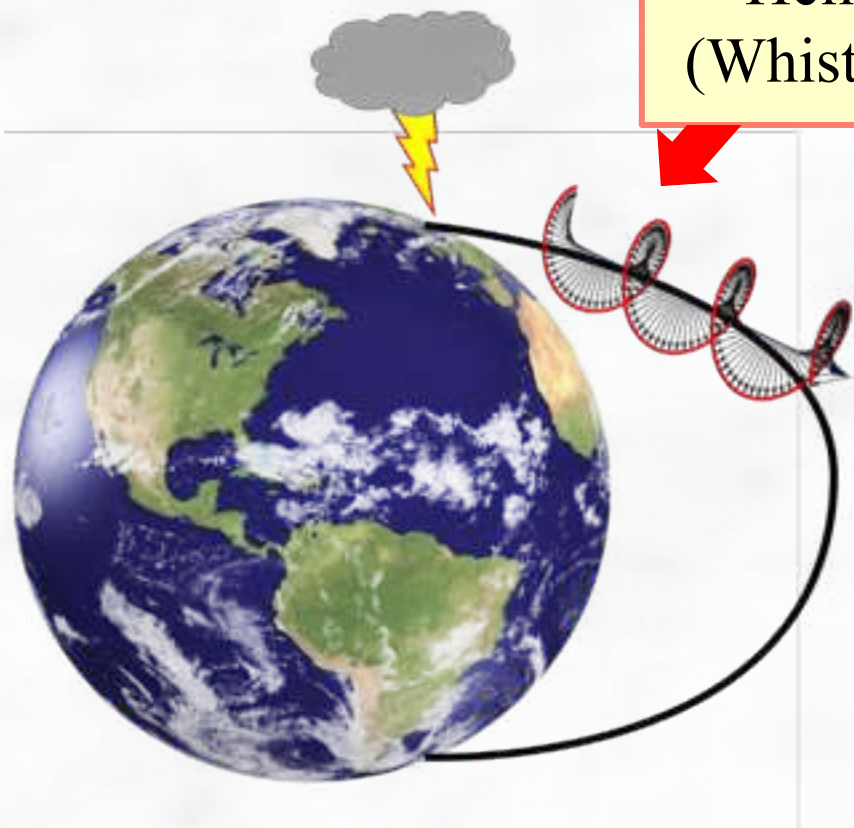
PSEUDOMAGNETIC HELICONS

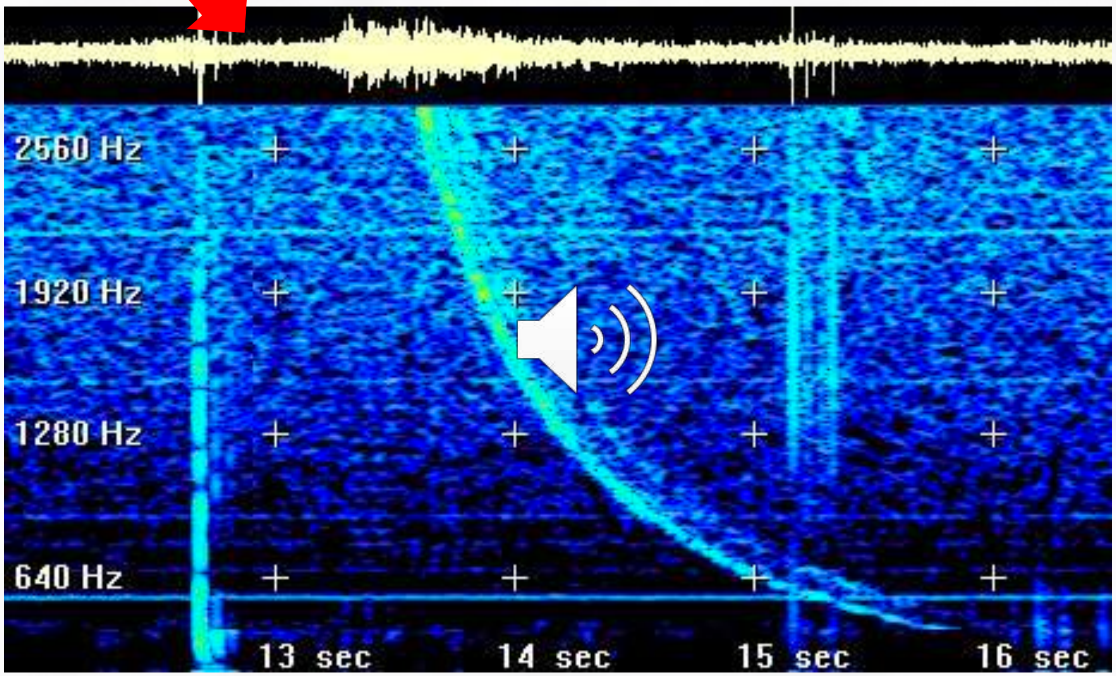
[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

Pseudo-magnetic helicons

- Usual helicons are transverse low-energy gapless excitations propagating along the background magnetic field $\vec{\boldsymbol{B}}_0$ in uncompensated media (e.g., metals with different electron and hole densities)

Helicon in the ionosphere (Whistler) and its spectrogram





(Pseudo-)magnetic helicon

- Helicon dispersion law at $T \to 0$:

$$\omega_h|_{B_{0,5}\to 0, \mu_5 \to 0} \overset{b_0 \to 0}{=} \frac{eB_0 c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu + 2B_0 e^4 v_F^2 b_\parallel} + O(k^3)$$

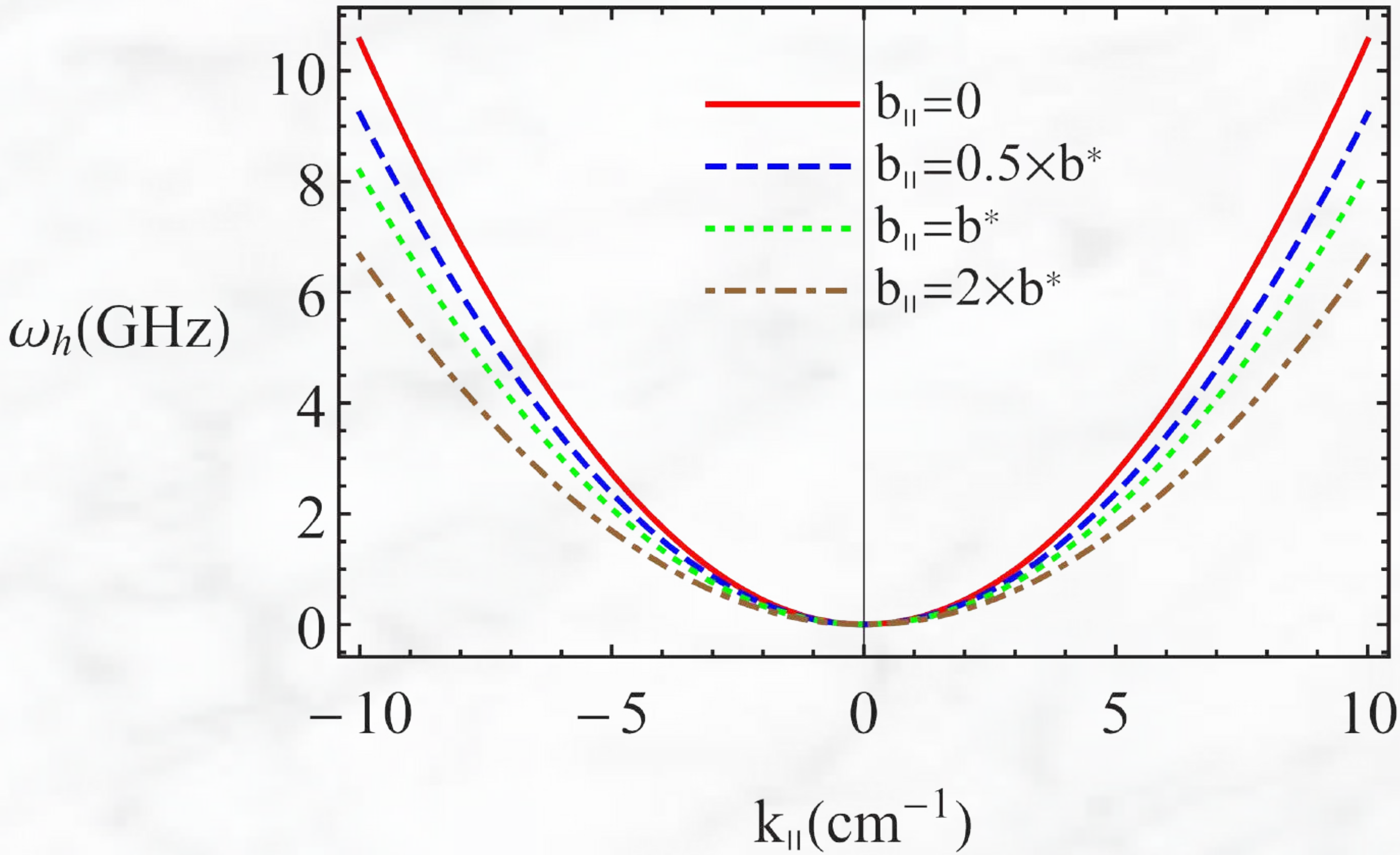
$$\omega_h|_{B_0 \to 0, \mu \to 0} \overset{b_0 \to -\mu_5/e}{=} \frac{eB_{0,5} c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu_5 + 2B_{0,5} e^4 v_F^2 b_\parallel} + O(k^3)$$

- Properties:
  - Gapless electromagnetic wave propagates in metals **without magnetic field**!
  - Chiral shift modifies effective helicon dispersion
  - In equilibrium, i.e., $\mu_5 = -eb_0$, the term linear in the wave vector is **absent**

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

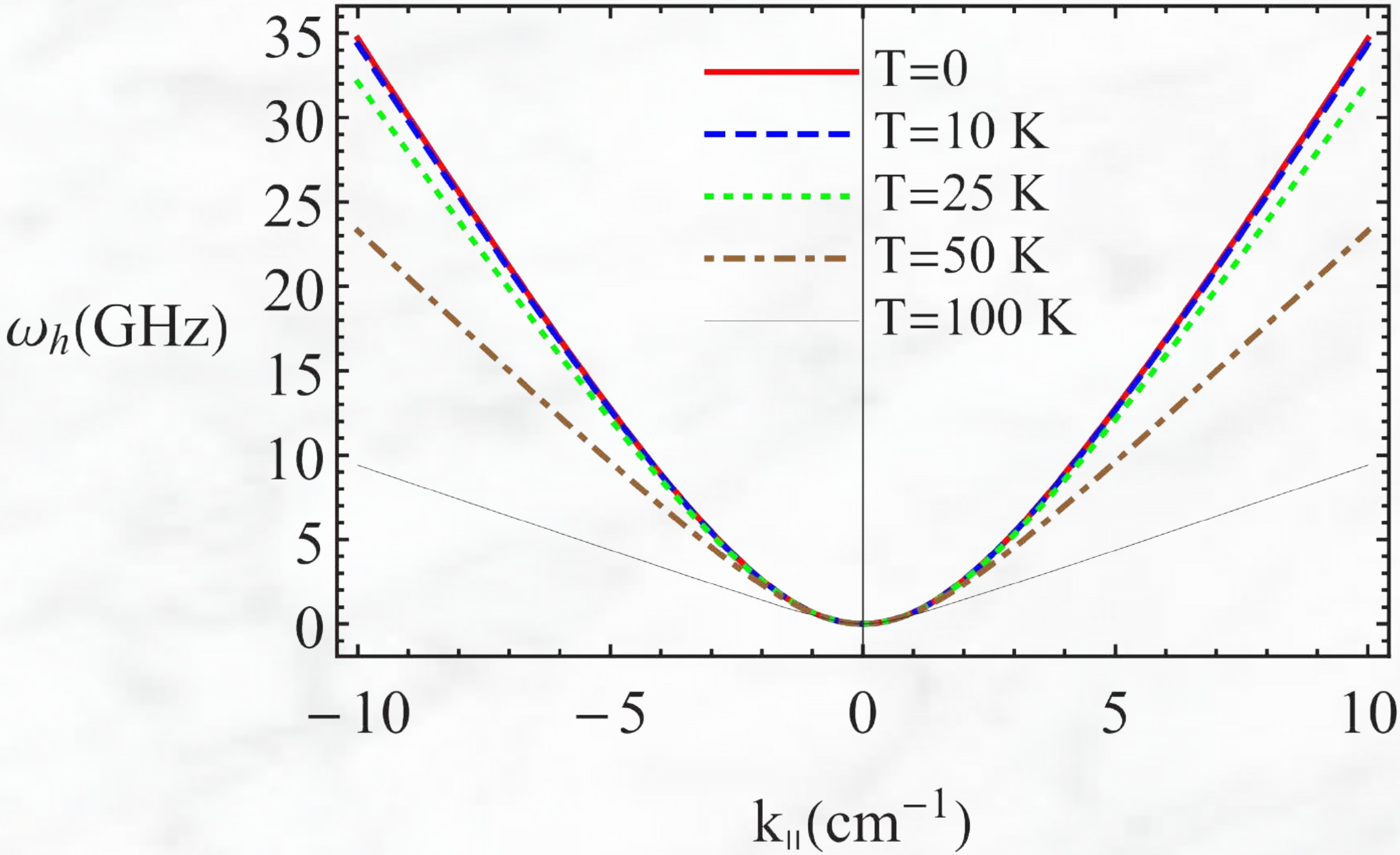
Helicons at different $b_{\parallel}$

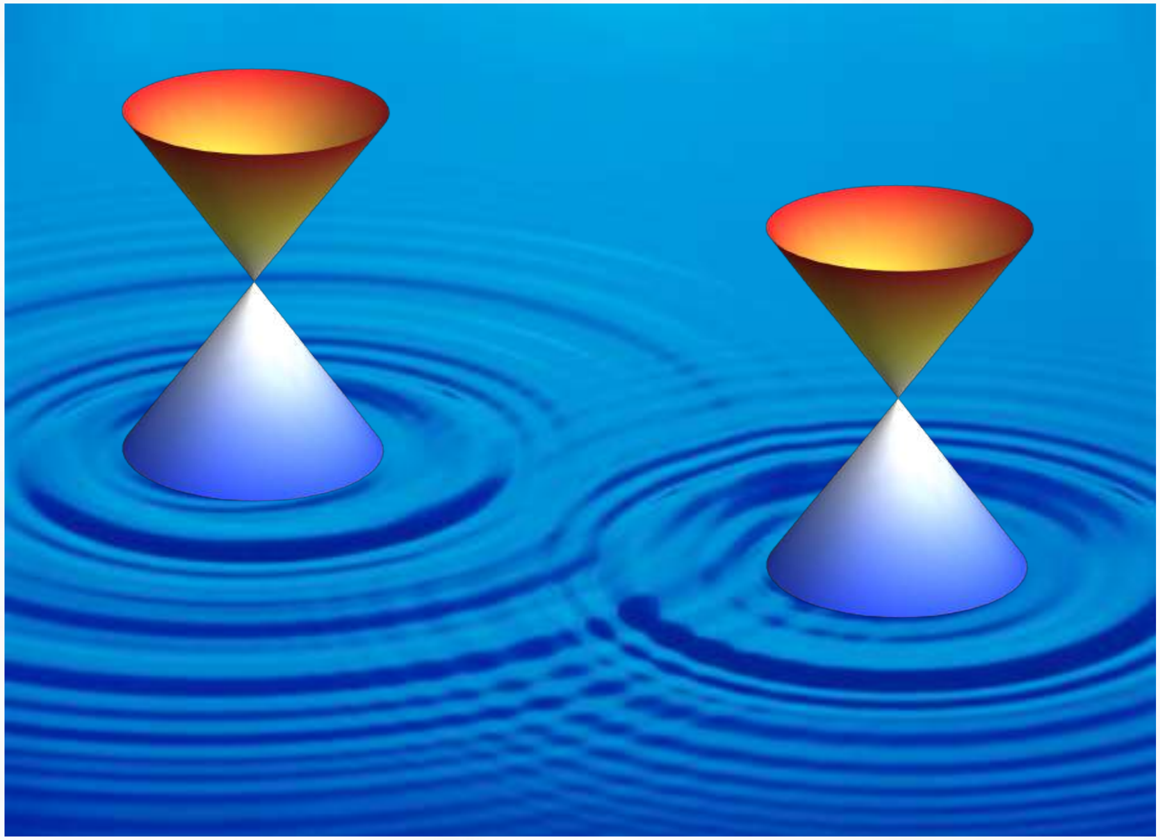
$eb_0 = -\mu_5$, $B_{0,5} = 10^{-2}$T, $\mu_5 = 5$ meV, $\mu = 0$, $eb^* = 0.3\pi\hbar v_F/c_3$



Helicons at different T

$eb_0 = -\mu_5$, $B_{0,5} = 10^{-2}$T, $b_\parallel = 0.5b^*$, $\mu_5 = 5$ meV, $\mu = 0$





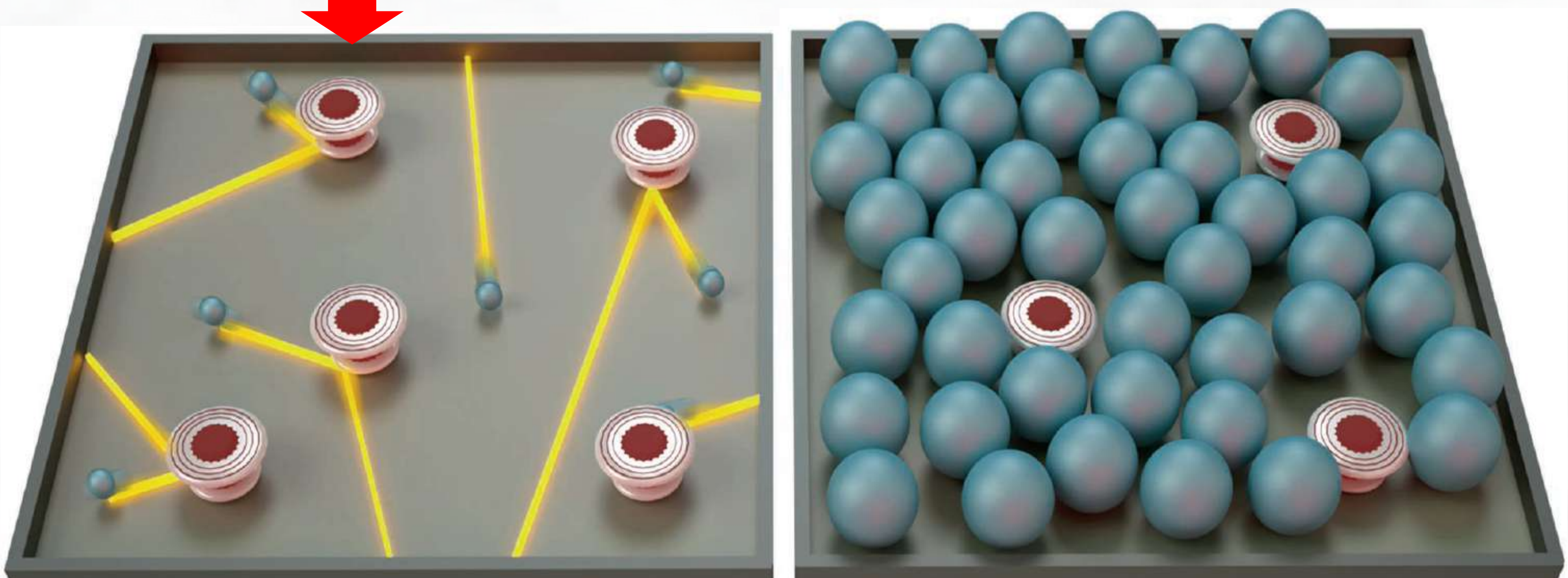
CONSISTENT HYDRODYNAMICS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 97, 121105(R) (2018)]

Two regimes

- Momentum-relaxing (l_{MR}) vs. momentum-conserving (l_{MC}) collisions [Gurzhi, J. Exp. Theor. Phys. **17**, 521 (1963); Sov. Phys. Usp. **11**, 255 (1968).]

Ohmic regime: $l_{MR} \ll l_{MC}, L$, where L is the sample size



[Zaanen, Science **351**, 1058 (2016)]

Hydrodynamic regime: $l_{MC} \ll L \ll l_{MR}$

Hydrodynamics in Weyl metals

The Euler equation from CKT:

$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right)=-en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_{\mathbf{r}})$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

The energy conservation from CKT

$$\partial_t\epsilon=-\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_{\mathbf{r}})$$

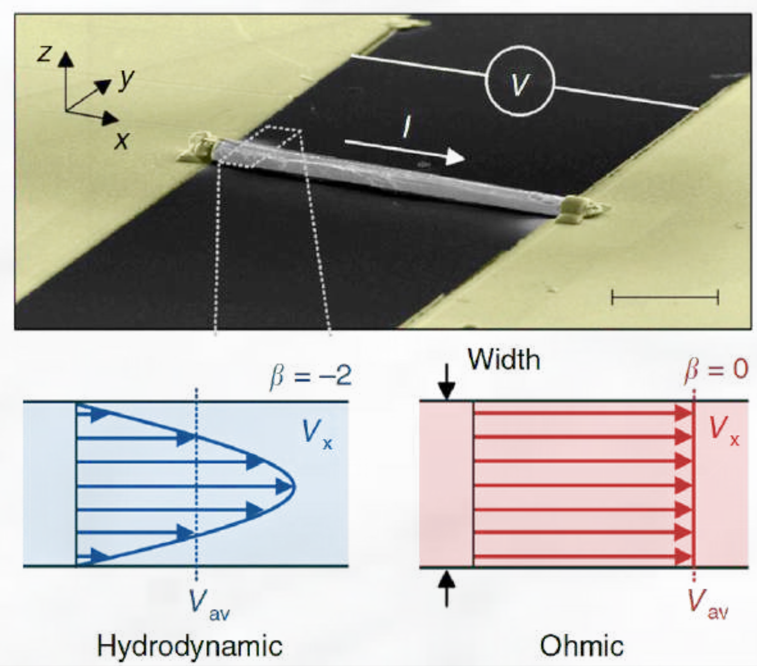
+ Maxwell equations with the Chern-Simons currents

$$\rho_{\rm CS}=-\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\rm CS}=-\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$

Experimental evidence in tungsten diphosphide (WP_2): $\rho=\rho_0+\rho_1 w^{\beta}$

[Gooth et al., Nature Commun. **9**, 4093 (2018)]



Rich spectrum of hydro modes

- Magneto-acoustic wave ($\rho = 0$):

$$\omega_{\mathrm{s},\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau}\sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)}\left[2|\mathbf{k}|^2 B_0^2 - (\mathbf{k}\cdot\mathbf{B}_0)\right]}{3w_0}}$$

- *Gapped* chiral magnetic wave ($\rho = 0$):

$$\omega_{\mathrm{gCMW},\pm} = \pm\frac{eB_0\sqrt{3v_F^3\left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_\parallel^2\right)}}{2\pi^2 T_0^2 c\sqrt{\varepsilon_e \hbar}}$$

- Helicons ($\rho \neq 0$):

$$\omega_{\mathrm{h},\pm} \approx \mp\frac{ck_\parallel^2 B_0}{4\pi\mu_m\rho_0} - \frac{i}{\tau}\frac{c^2 k_\parallel^2 w_0}{4\pi\mu_m v_F^2 \rho_0^2} + O(k_\parallel^3)$$

- New anomalous Hall waves at $\vec{b} \neq 0$, etc.

[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

Anomalous Hall Waves (AHW)

- Longitudinal AHW (lAHW) ($\boldsymbol{k} \parallel \boldsymbol{B}_0$ and $\boldsymbol{b} \perp \boldsymbol{B}_0$)

$$\omega_{\mathrm{lAHW},\pm} \approx \pm \frac{\hbar B_0 k_{\parallel} \sqrt{3 v_F^3 \left(\pi^3 c^4 \hbar T_0^2 + 3 e^4 \mu_m v_F^3 b_\perp^2\right)}}{c T_0 \sqrt{\pi^3 \mu_m \left(3 \varepsilon_e v_F^3 \hbar^3 B_0^2 + 4\pi e^2 T_0^2 b_\perp^2\right)}} + O(k_\parallel^3)$$

ρ continuity equation

ρ_5 continuity equation

$$\frac{T^2 \omega}{3 v_F^3 \hbar} \delta\mu + \frac{e B_0 k_\parallel}{2\pi^2 c} \delta\mu_5 = 0$$

$$\frac{e B_0 k_\parallel}{2\pi^2 c} \delta\mu + \frac{T^2 \omega}{3 v_F^3 \hbar} \delta\mu_5 - i \frac{e^2 B_0}{2\pi^2 c} \delta E_\parallel = 0$$

$$\varepsilon_e \omega \delta E_\parallel + \mathrm{i} \frac{2 e^2}{\pi c \hbar^2} \left(B_0 \delta\mu_5 + e b_\perp \delta\tilde{E}_\perp\right) = 0$$

$$\left(\omega^2 - \frac{c^2 k_\parallel^2}{\varepsilon_e \mu_m}\right) \delta\tilde{E}_\perp - \mathrm{i} \frac{2 e^3 \omega b_\perp}{\pi c \varepsilon_e \hbar^2} \delta E_\parallel = 0$$

Maxwell's equations

$\delta\tilde{\boldsymbol{E}}_\perp \parallel [\boldsymbol{B}_0 \times \boldsymbol{b}]$

Summary

- Currents in consistent chiral kinetic theory must include topological Chern-Simons terms
- Collective modes are affected
  - local chiral charge non-conservation (anomaly)
  - separation between Weyl nodes (b_0 and $\vec{b}$)
  - oscillation of both *electric* and *chiral* charge
- New types of collective modes, pseudomagnetic helicons, may exist in Weyl materials
- Chiral kinetic theory can be used to obtain consistent chiral hydrodynamics