ARIZONA STATE UNIVERSITY

NSF

SWIECA SUMMER SCHOOL
Feb 8, 2021
ONLINE

Chiral anomalous effects in QGP

Igor Shovkovy

Arizona State University

XX Escola de Verão Jorge André Swieca Física Nuclear Teórica

08 a 12 de fevereiro de 2021 - Online

SBF SOCIEDADE BRASILEIRA DE FÍSICA

Quanta magazine

CHIRAL ANOMALY

Chiral symmetry

- The fermionic part of QCD Lagrangian (units with $c = 1$ & $\hbar = 1$):

$$\mathcal{L} = \bar{u}\left(i\gamma^\mu D_\mu - m\right)u + \bar{d}\left(i\gamma^\mu D_\mu - m\right)d$$

where $D_\mu = \partial_\mu + ieA_\mu + \cdots$, $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ and $g^{\mu\nu}=(1,-1,-1,-1)$

- The Lagrangian has the approximate $SU_L(2)\times SU_R(2)$ chiral symmetry:

$$\psi \to e^{\frac{i}{2}\alpha_A\tau_A}\psi \quad \text{and} \quad \psi \to e^{\frac{i}{2}\beta_A\tau_A\gamma^5}\psi$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and τ_A are the Pauli matrices (with $\tau_0 = I$)

- This chiral symmetry is spontaneously broken down to $SU_V(2)$, but plays important role in QCD
  - Light pions
  - Heavy hadron
  - Low-energy interactions
  - Etc.

Chiral symmetry

- One might also consider the following two phase transformations:

$$\psi \to e^{i\alpha}\psi \quad \text{and} \quad \psi \to e^{i\beta\gamma^5}\psi$$

which are symmetries of the *classical* QCD action

- The corresponding *classical* Noether's currents are

$$j^\mu = \bar{\psi}\gamma^\mu\psi \quad \text{and} \quad j_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

- They satisfy the *classical* continuity relations:

$$\partial_\mu j^\mu = 0 \quad \text{and} \quad \partial_\mu j_5^\mu = 2i\, m\, \bar{\psi}\gamma^5\psi$$

- Both transformations are *classical* symmetries when $m = 0$
- However, the chiral $U(1)$ symmetry is *anomalous* (even at $m = 0$)

$$\langle \partial_\mu j_5^\mu \rangle_A = \mathcal{A}(x) = -\frac{1}{16\pi^2}\varepsilon_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa}$$

i.e., symmetry of the *classical* action is not preserved in *quantum* theory when electromagnetism is accounted for

Chiral anomaly

- For a *local* transformation $\delta\psi = i\alpha(x)\gamma^5 T\psi$

$$\delta S = \int d^4x\, j_5^\mu \partial_\mu \alpha(x)$$

- The path integral integration measure [Fujikawa, Phys. Rev. Lett. 42, 1195 (1979)]

$$[d\psi][d\bar{\psi}] \to \exp\left[i \int d^4x\, \mathcal{A}(x)\alpha(x)\right][d\psi][d\bar{\psi}]$$

$$\mathcal{A}(x) = -\frac{1}{16\pi^2}\varepsilon_{\mu\nu\lambda\kappa}F_\alpha^{\mu\nu}F_\beta^{\lambda\kappa}\mathrm{Tr}\left[T^\alpha T^\beta T\right]$$

- Since a change of integration variables cannot change the result

$$\delta \int [d\psi][d\bar{\psi}]e^{iS} = i\int d^4x \int [d\psi][d\bar{\psi}]\ \left[\mathcal{A}(x)\alpha(x) + j_5^\mu \partial_\mu \alpha(x)\right]e^{iS} = 0$$

- One obtains the following anomaly relation (in QED):

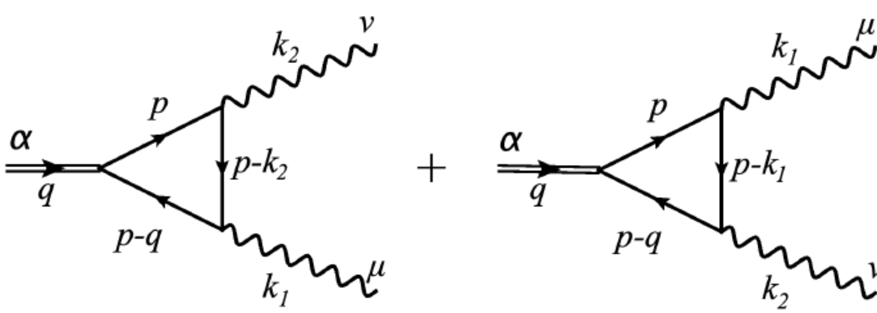
$$\left\langle \partial_\mu j_5^\mu \right\rangle_A = \mathcal{A}(x) = -\frac{1}{16\pi^2}\varepsilon_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa}$$

Chiral anomaly

- Historically, chiral anomaly was discovered by studying

$$\pi^0 \to 2\gamma$$

- The measured decay rate is $\sim 10^3$ times larger then predicted by a model with the $U(1)$ chiral symmetry enforced
- Explanation was given by Adler, Bell, and Jackiw by studying the (linearly divergent) triangle diagrams $T^{\mu\nu\alpha} \sim \langle j_5^\alpha j^\nu j^\mu \rangle$



$$\Rightarrow \begin{cases} k_1^\mu T_{\mu\nu\alpha} = \dfrac{(1+\beta)}{8\pi^2}\epsilon_{\nu\alpha\sigma\rho}k_1^\sigma k_2^\rho, \\ k_2^\nu T_{\mu\nu\alpha} = \dfrac{(1+\beta)}{8\pi^2}\epsilon_{\mu\alpha\rho\sigma}k_1^\sigma k_2^\rho, \\ q^\alpha T_{\mu\nu\alpha} = -\dfrac{1-\beta}{4\pi^2}\epsilon_{\mu\nu\sigma\rho}k_1^\sigma k_2^\rho. \end{cases}$$

where β is a regulator in the calculations

Chiral anomaly

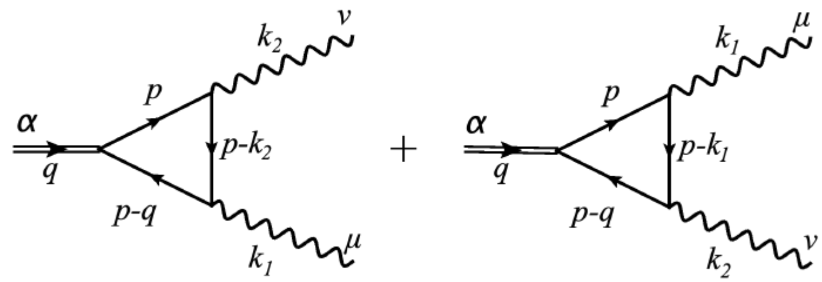
- Comparing the two types of results

$$k_1^\mu T_{\mu\nu\alpha} = \frac{(1+\beta)}{8\pi^2}\epsilon_{\nu\alpha\sigma\rho}k_1^\sigma k_2^\rho, \qquad q^\alpha T_{\mu\nu\alpha} = -\frac{1-\beta}{4\pi^2}\epsilon_{\mu\nu\sigma\rho}k_1^\sigma k_2^\rho.$$

- What is the best choice for β?

(i) β=1: $\partial_\mu j^\mu \neq 0$ and $\partial_\alpha j_5^\alpha = 0$

(ii) β=-1: $\partial_\mu j^\mu = 0$ and $\partial_\alpha j_5^\alpha \neq 0$



- Insisting that the electric charge is conserved, one finds

$$A_{\gamma\gamma} = \frac{\alpha N_c}{3\pi F_\pi}$$

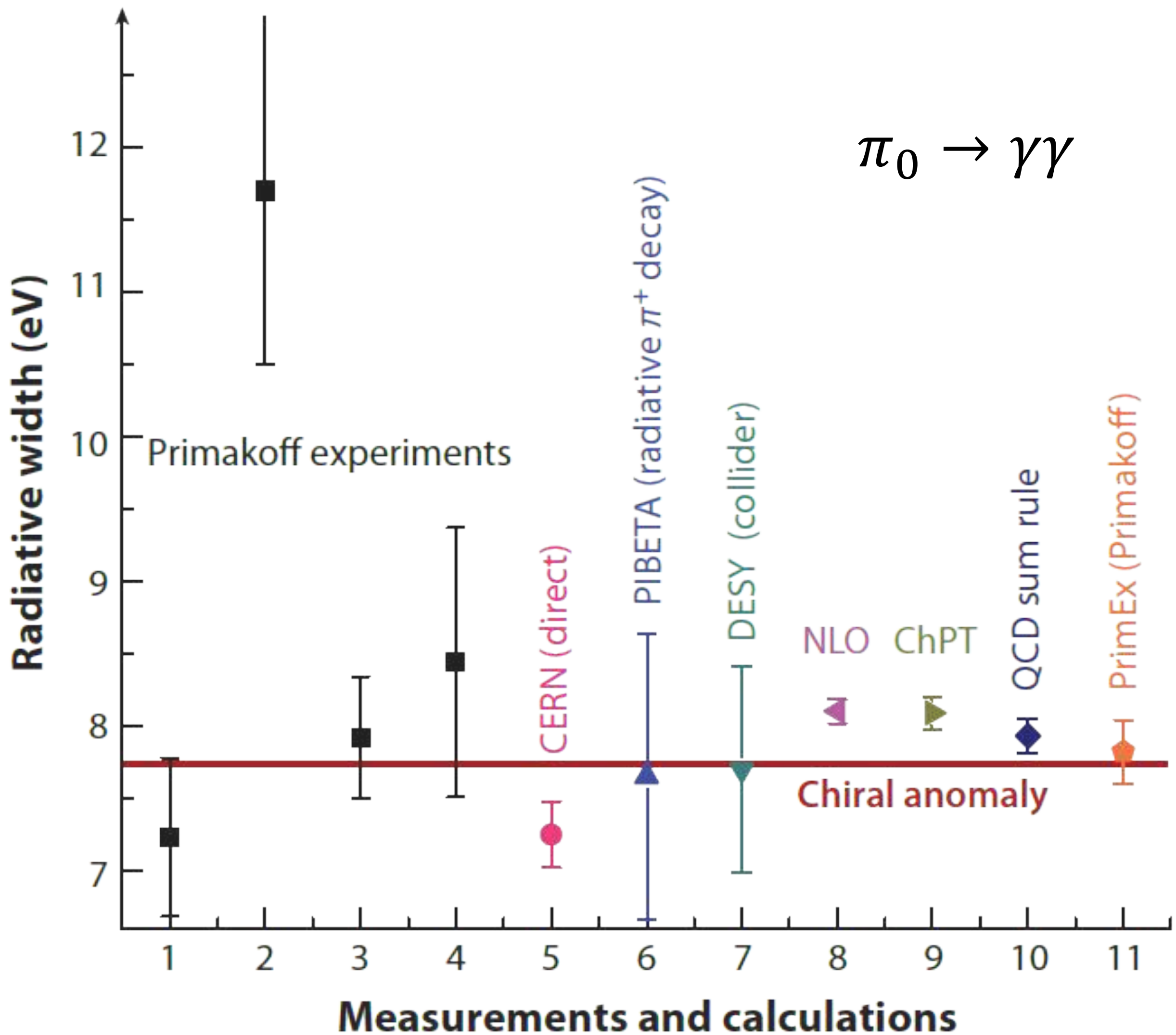
- This leads to the soft π^0 decay rate [Miskimen, Annu. Rev. Nucl. Part. Sci. **61**, 1 (2011)]

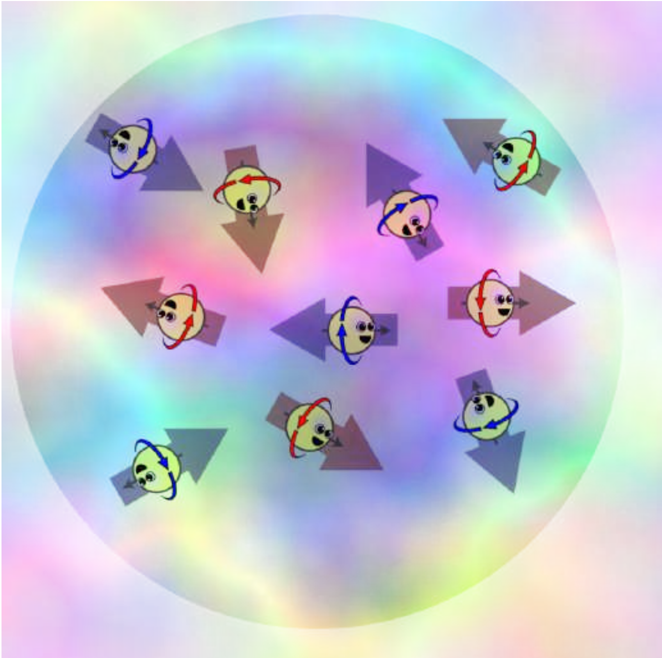
$$\Gamma(\pi^0 \to 2\gamma) = \frac{m_\pi^3}{64\pi}\left|A_{\gamma\gamma}\right|^2 \approx 7.73\text{ eV} \approx 1.17\times10^{16}\text{ s}^{-1}$$

which agrees well with the measured data!

Measurements vs. calculations

From [Miskimen, Annu. Rev. Nucl. Part. Sci. **61**, 1 (2011)]





CHIRAL MATTER

Chiral fermions

- Only *massless* Dirac fermions have a well-defined chirality ($\gamma^5\psi = \pm\psi$):

$$(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{p})\psi = 0 \quad \Longrightarrow \quad \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\psi = \mathrm{sign}(p_0)\gamma^5\psi$$

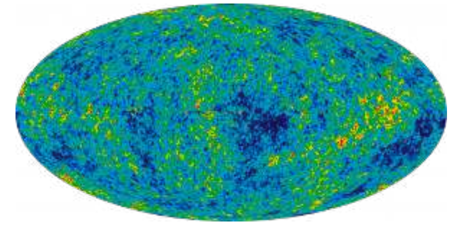
For particles ($p_0 > 0$): chirality = + helicity

For antiparticles ($p_0 < 0$): chirality = − helicity

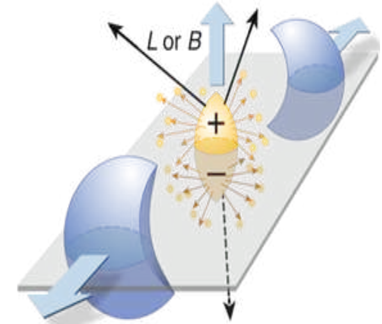
- *Massive* Dirac fermions have an almost well-defined chirality in the *ultrarelativistic* regime
  - High temperature: $T \gg m$
  - High density: $\mu \gg m$
- Chirality flip rate: $\Gamma_{\text{flip}} \propto \alpha^2 T(m/T)^2$

Chiral forms of matter

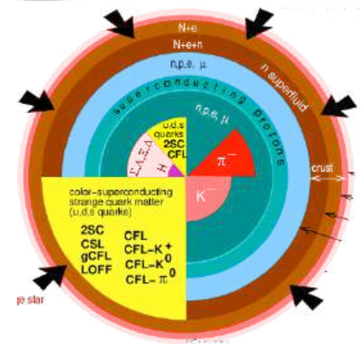
- **Early Universe**, e.g.,
  [Boyarsky, Frohlich, Ruchayskiy, Phys. Rev. Lett. 108, 031301 (2012)]



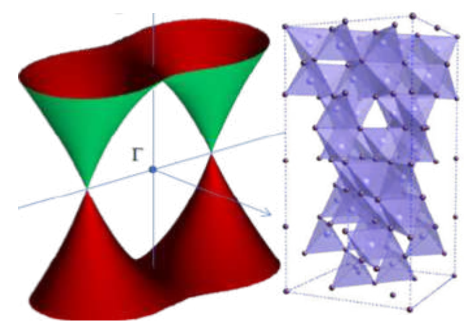
- **Heavy-ion collisions**, e.g.,
  [Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]



- **Super-dense matter in compact stars**, e.g.,
  [Yamamoto, Phys.Rev. D93, 065017 (2016)]



- **Dirac/Weyl (semi-)metals**, e.g.,
  [Li et. al. Nature Phys. 12, 550 (2016)]



- **Superfluid ^{3}He-A**, e.g.,
  [Volovik, JETP Lett. 46, 98 (1987), JETP Lett. 105, 34 (2017)]

Anomalous chiral matter

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ on *macroscopic* scales

- The (collective) dynamics of $n_R + n_L$ and $n_R - n_L$ is controlled by the continuity equations

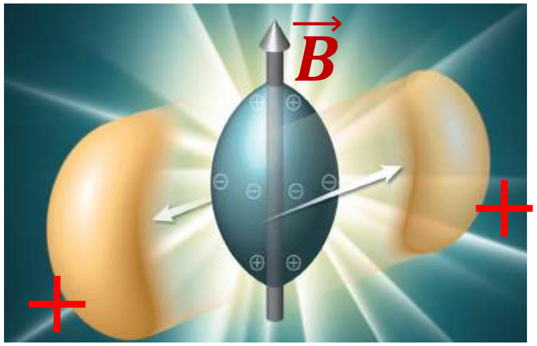
$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c} - \Gamma_{\text{flip}}(n_R - n_L)$$

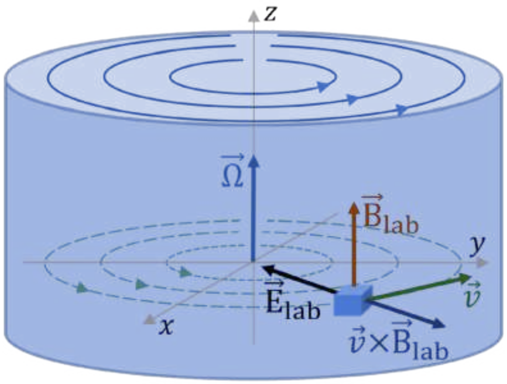
- **Question**: Could the chiral anomaly have *macroscopic* implications in the chiral forms of matter?

Anomalous effects

- **Answer**: Several *macroscopic* chiral anomalous effects were proposed
- Some are triggered by external magnetic field
  - Chiral magnetic effect
  - Chiral separation effect
  - Chiral magnetic wave
  - Negative magnetoresistance
  - etc.



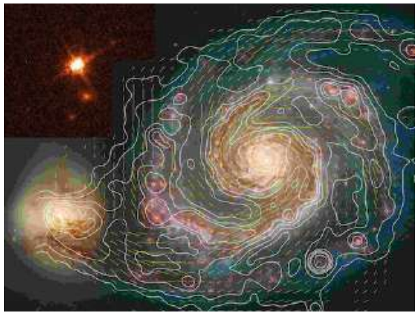
- Others are triggered by vorticity
  - Chiral vortical effect
  - Chiral vortical wave
  - etc.



Anomalous field generation

- Inverse magnetic cascade may produce seeds of helical magnetic fields in the early Universe

  [Vilenkin, Phys. Rev. D22, 3080 (1980)],
  [Joyce & Shaposhnikov, astro-ph/9703005],
  [Giovannini & Shaposhnikov, hep-ph/9710234]



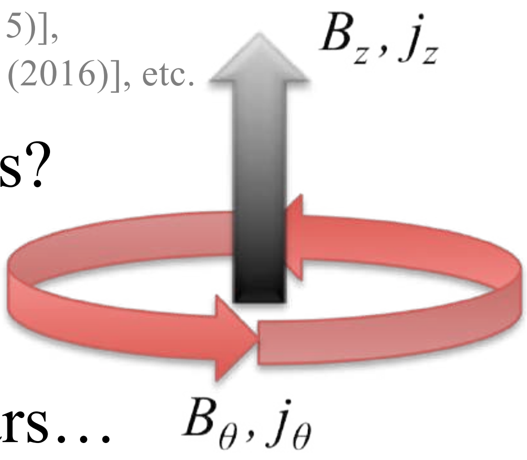
- Eigenmodes of long wavelength and fixed helicity grow:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma}\left(4\pi C_5\mu_5 - ck\right)B_k$$

  [Boyarsky et al., PRL **108**, 031301 (2012)], [Tashiro et al., PRD **86**, 105033 (2012)],
  [Manuel et al., PRD **92**, 074018 (2015)], [Hirono et al., PRD **92**, 125031 (2015)],
  [Buividovich et al., PRD **94**, 025009 (2016)], [Gorbar et al., PRD **94**, 103528 (2016)], etc.

- Strong helical magnetic field in compact stars?

  [Ohnishi, Yamamoto, arXiv:1402.4760]
  [Yamamoto, Phys. Rev. D **93**, 065017 (2016)]
  [Dvornikov, J. Exp. Theor. Phys. **123** 967 (2016)]



- Perhaps, chirality flipping is too strong in stars…

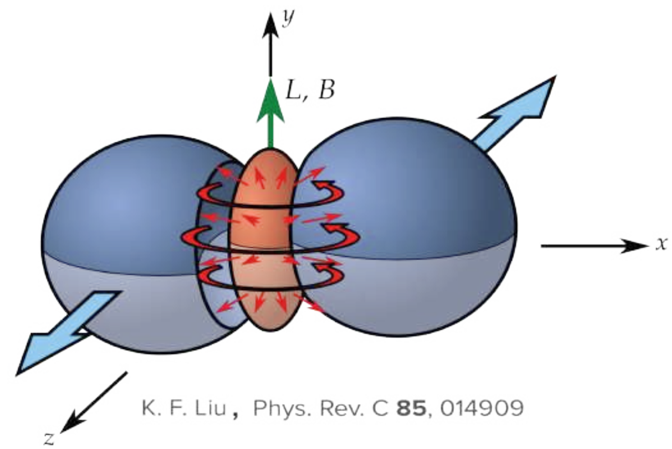
  [Grabowska, Kaplan, Reddy, Phys. Rev. D **91**, 085035 (2015)]

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



K. F. Liu , Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak &. Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108], …

- Magnetic field estimate:

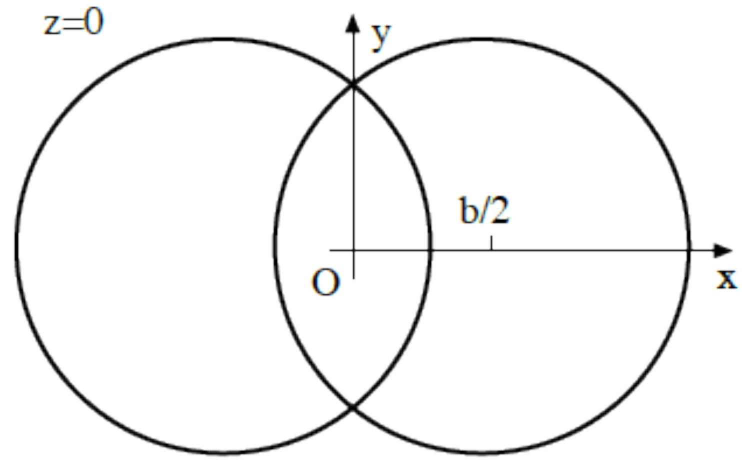
  B ~ 10^{18} to 10^{19} G (~ 100 MeV)

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

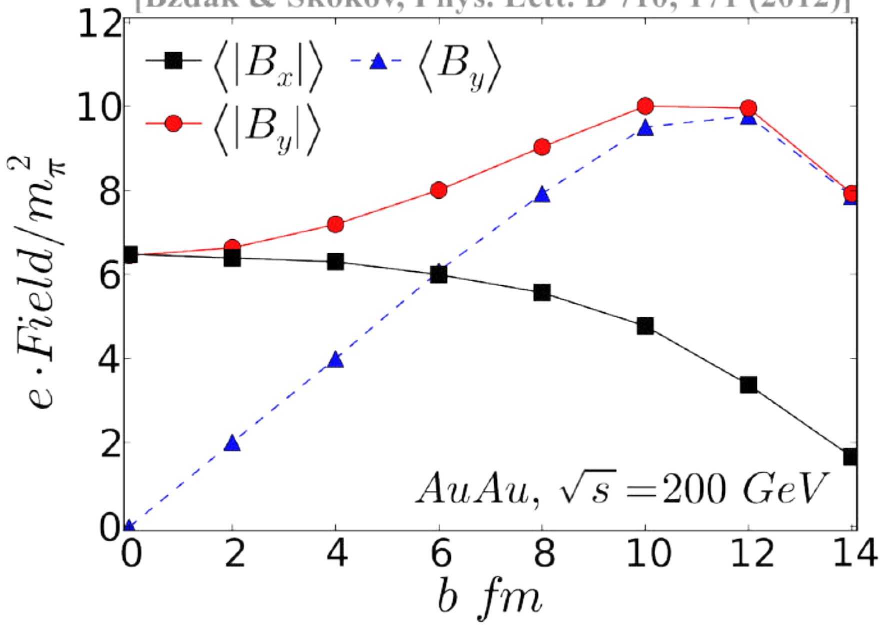
  $\omega \sim 9 \times 10^{21} s^{-1}$ (~ 10 MeV)

Magnetic field in HIC

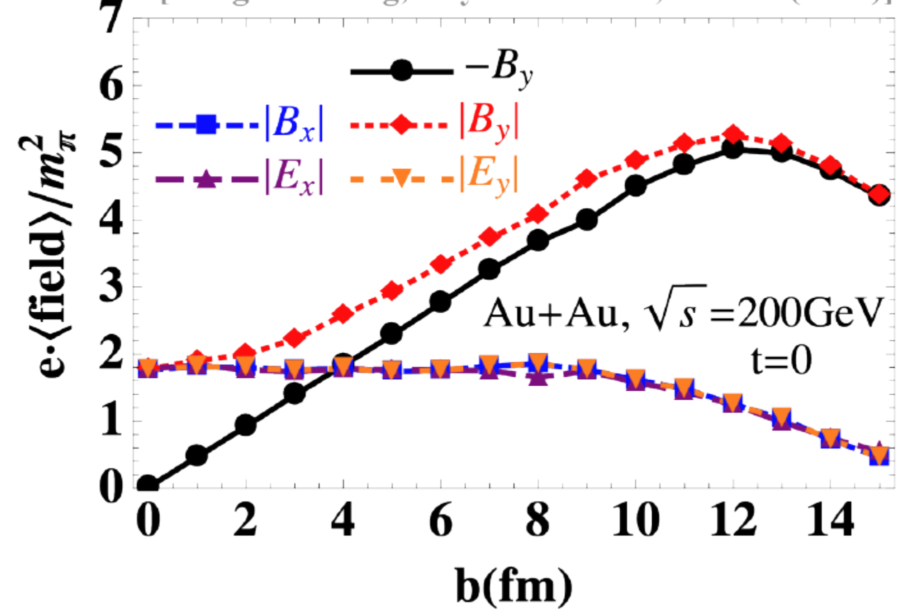
- Magnetic field
  - strong in magnitude $\sim m_\pi^2$
  - depends strongly on b
  - nonuniform
  - fluctuates from event to event

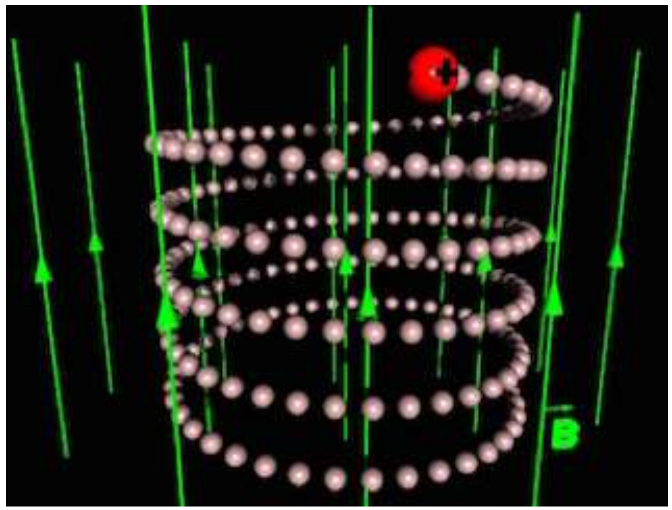


[Bzdak & Skokov, Phys. Lett. B 710, 171 (2012)]



[Deng & Huang, Phys. Rev. C 85, 044907 (2012)]





DIRAC FERMIONS IN MAGNETIC FIELD

[Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]

Dirac fermions at B≠0

- Dirac equation for charged fermions:

$$\left(i\gamma^{\mu}D_{\mu}-m\right)\psi=0$$

where $A_{\mu}=\left(A_0,-\vec{\boldsymbol{A}}\right)$ and the Landau gauge $\vec{\boldsymbol{A}}=(-By,0,0)$ is used.

- Normalized solutions for ϕ have the form

$$\phi_{k,\pm}\propto\frac{1\pm i\operatorname{sgn}(eB)\gamma^{1}\gamma^{2}}{2}\varphi_{k}(y)e^{-i\omega t+ip_{x}x+ip_{z}z}$$

where φ_k are harmonic oscillator wave functions, i.e.,

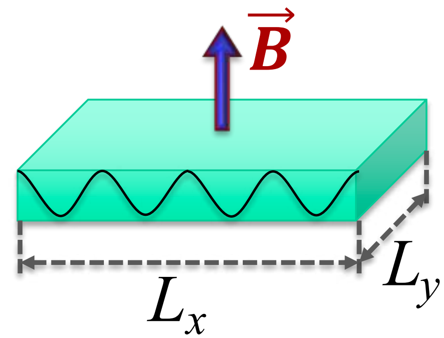
$$\varphi_k\propto H_k(\xi)e^{-\frac{\xi^2}{2}},\quad \xi=\frac{y}{l}+p_x l\operatorname{sgn}(eB)\quad\text{and}\quad l=\frac{1}{\sqrt{|eB|}}$$

- The dispersion relation is given by

$$\omega=E_n^{\pm}=\pm\sqrt{2n|eB|+p_z^2+m^2}$$

where $n=\underbrace{k+\frac{1}{2}}_{\text{orbital}}+\underbrace{\operatorname{sgn}(eB)s_z}_{\text{spin}}$ and $s_z=\pm\frac{1}{2}$ is an eigenvalue of $\frac{i}{2}\gamma^1\gamma^2$

Degeneracy of Landau levels



- The Landau level energies are independent of p_x

$$E_n^{\pm} = \pm\sqrt{2n|eB| + p_z^2 + m^2}$$

- Each level is highly degenerate
- Confine the system to a box of finite size $L_x \times L_y$

and impose periodic boundary conditions $\psi(0) = \psi(L_x)$

- The wave function is a plane wave in the x direction: $\psi(x) \propto e^{ip_x x}$

$$e^{ip_x L_x} = 1 \quad \Longrightarrow \quad p_x = \frac{2\pi n}{L_x}, \quad n = 1,2, \ldots, N_{\max}$$

- The value of p_x determines the center of orbit in y-direction:

$$y_c \approx p_x l^2 \quad \Longrightarrow \quad p_{x,\max} l^2 \lesssim L_y \quad \Longrightarrow \quad \frac{2\pi N_{\max}}{L_x}\frac{1}{|eB|} \approx L_y \quad \Longrightarrow \quad \frac{N_{\max}}{L_x L_y} \approx \frac{|eB|}{2\pi}$$

- Thus, the degeneracy is

$$N_{\max} \approx \frac{|eB|}{2\pi} L_x L_y$$

Lowest Landau level

- Landau energy levels at $m = 0$

$$E_n^{\pm} = \pm\sqrt{2n|eB| + p_z^2}$$

where $n = \underbrace{k + \frac{1}{2}}_{\text{orbital}} + \underbrace{\text{sgn}(eB)s_z}_{\text{spin}}$

- Lowest Landau level is spin polarized

$$E_0^{\pm} = \pm p_z \qquad (k = 0,\ s_z = -\tfrac{1}{2})$$

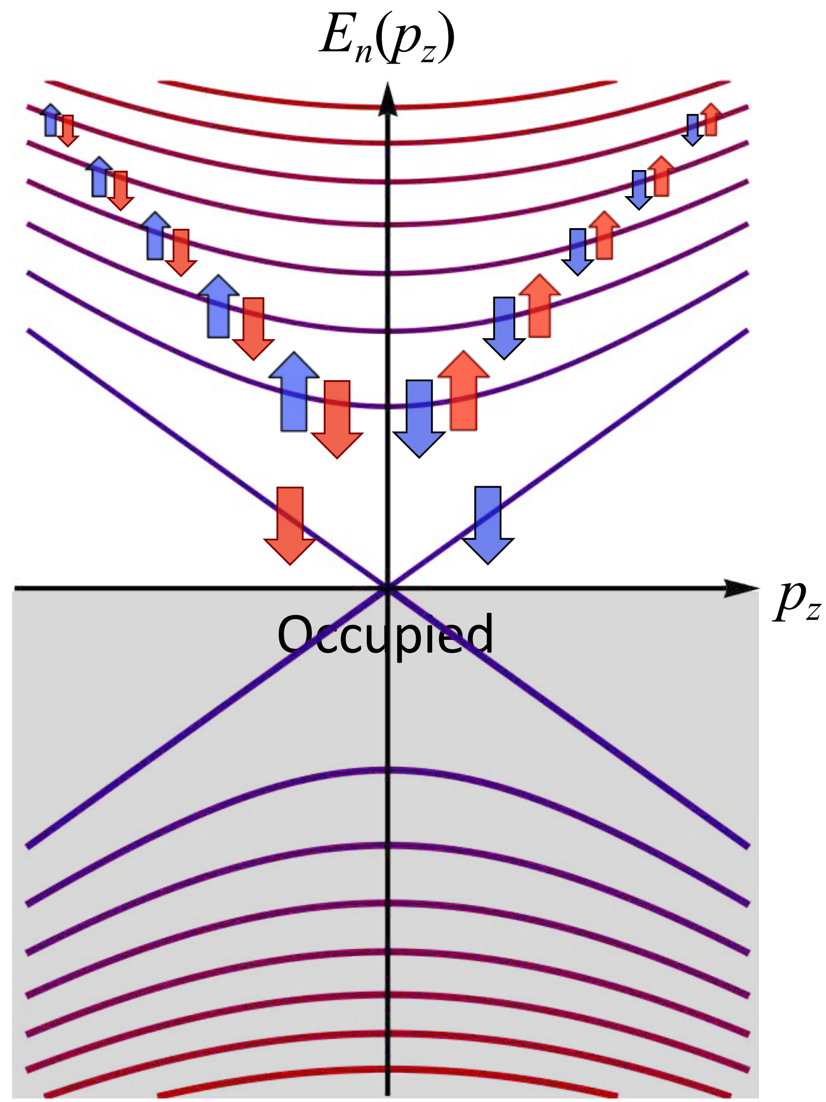
- Density of states at E=0:

$$\left.\frac{dn}{dE}\right|_{E=0} = \frac{|eB|}{2\pi}\frac{1}{2\pi} = \frac{|eB|}{4\pi^2}$$

- Higher Landau levels ($n \geq 1$) are twice as degenerate:

(i) $k = n$ & $s = -\frac{1}{2}$

(ii) $k = n - 1$ & $s = +\frac{1}{2}$



Fermion propagator

- Landau-level representation:

$$S(x,y) = \exp\left(-e\int_y^x A_\mu dz^\mu\right)\bar{S}(x-y)$$

$$\bar{S}(x) = \int \frac{d^3k}{(2\pi)^3} e^{-ikx}\bar{S}(k)$$

Schwinger phase

where

$$\bar{S}(k) = ie^{-k_\perp^2 l^2} \sum_{n=0}^{\infty} \frac{(-1)^n D_n(k)}{[k_0+\mu+i\epsilon\,\mathrm{sign}(k_0)]^2 - m^2 - k_3^2 - 2n|eB|}$$

and

$$D_n(k) = 2\left[(k_0+\mu)\gamma^0 + m - k^3\gamma^3\right]\left[\mathcal{P}_+ L_n\left(2k_\perp^2 l^2\right) - \mathcal{P}_- L_{n-1}\left(2k_\perp^2 l^2\right)\right] + 4(\boldsymbol{k}_\perp\cdot\boldsymbol{\gamma}_\perp)L_{n-1}^1\left(2k_\perp^2 l^2\right)$$

[Chodos, Everding, Owen, Phys. Rev. D42, 2881 (1990)]

ARIZONA STATE UNIVERSITY

CHIRAL SEPARATION EFFECT

$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

Chiral separation effect

- Slowly changing electric/chemical potential

$$\mu(\mathrm{z}) = e\Phi(\mathrm{z}) \quad \Longrightarrow \quad eE_z = -\partial_z(e\Phi) = -\partial_z\mu$$

- From the anomaly relation,

$$\partial_z j_5^3 = -\frac{e^2}{2\pi^2} E_z B_z = \frac{e^2}{2\pi^2} B_z\, \partial_z\mu$$

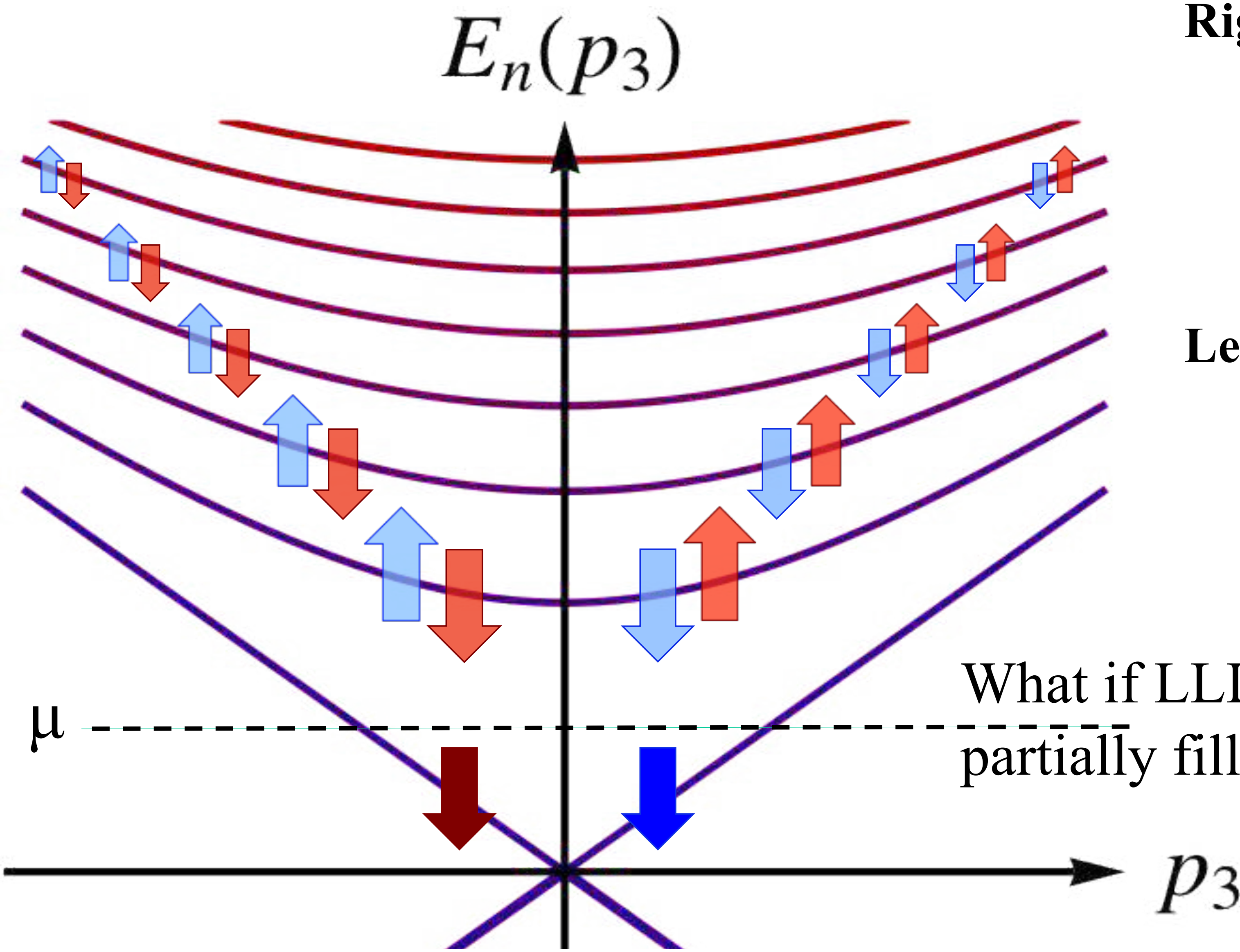
- Suggesting that for massless fermions,

$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

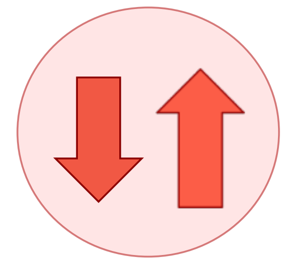
[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- This can be easily derived in free theory

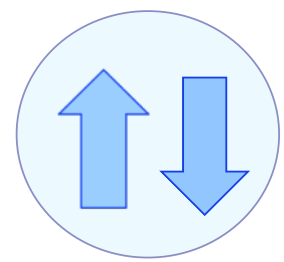
Landau spectrum & $\mu \neq 0$



Right-handed:



Left-handed:



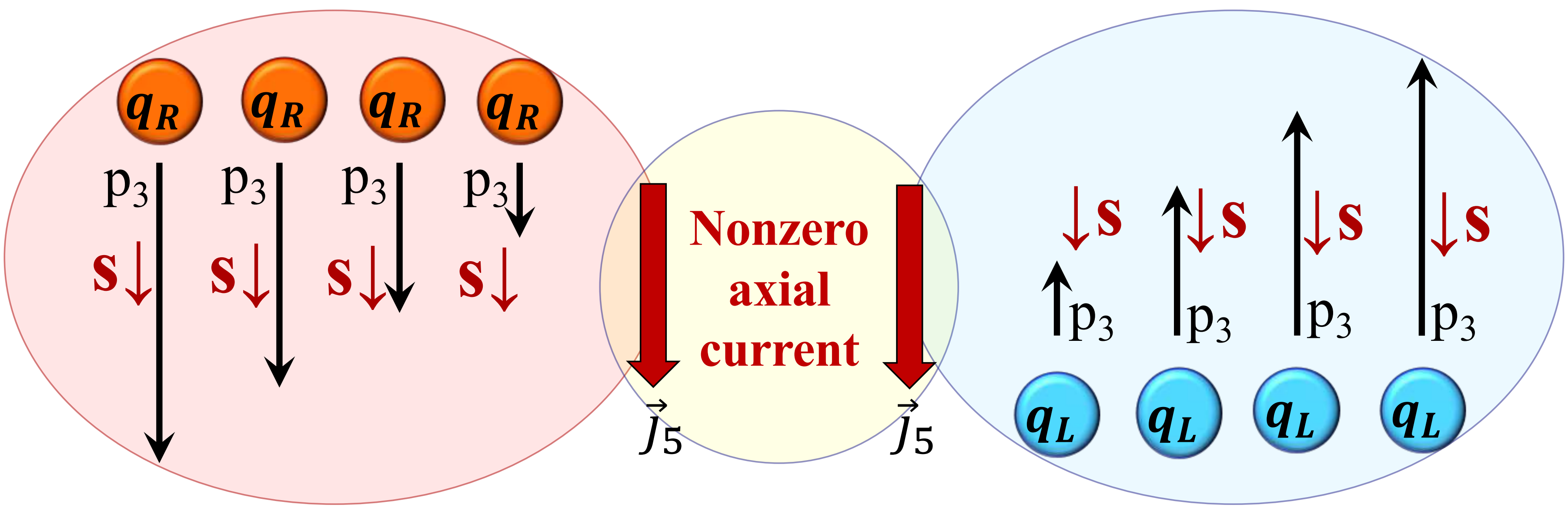
What if LLL is partially filled?

Partially filled LLL

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed

i.e., a nonzero **axial** current is induced

$$\langle \vec{j}_5 \rangle = -tr[\vec{\gamma}\gamma^5 S(x,x)] = -\frac{e\vec{B}}{2\pi^2}\mu$$



CHIRAL MAGNETIC EFFECT

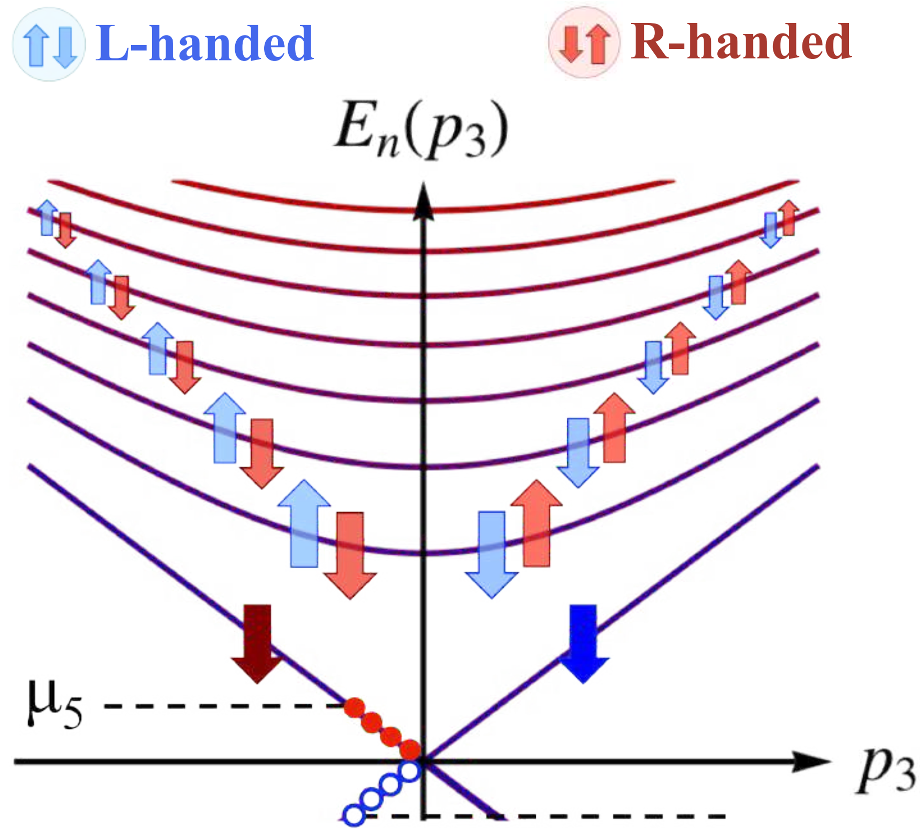
$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL ($s=\downarrow$ for particles of a *negative* charge):

- Some R-handed states ($p_3 < 0$ and $E < \mu_5$) are occupied
- Some L-handed holes ($p_3 < 0$ and $|E| < \mu_5$) are empty



CME current: $\langle \vec{j} \rangle = -tr[\vec{\gamma} S(x,x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$

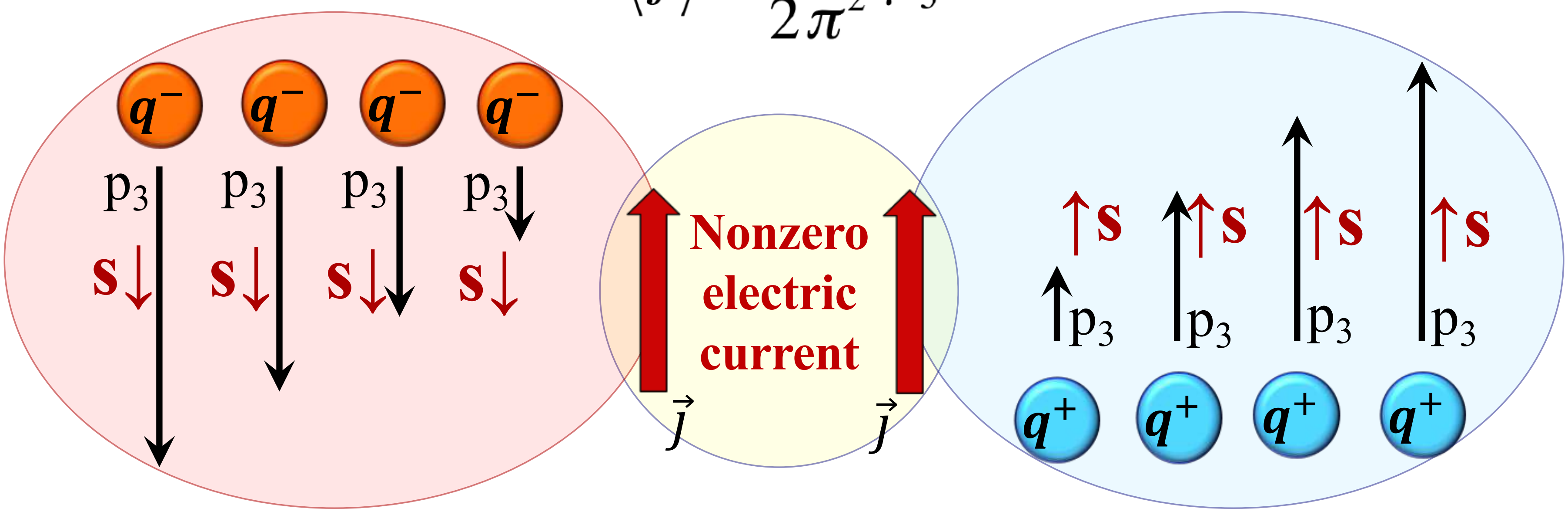
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Partially filled LLL ($\mu_5 \neq 0$)

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3 < 0$ (and s=↓) are R-handed **quarks**
  - states with $p_3 > 0$ (and s=↓) are L-handed **antiquarks**

i.e., a nonzero **electric** current is induced

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$



Radiative corrections?

- Anomaly is one-loop exact!
- **Question:** Do CME/CSE receive radiative corrections?
- CME – probably no (?)
- CSE – Yes, when dynamical gauge bosons are included!

[Gorbar, Miransky, Shovkovy, Wang, PRD **88** (2013) 025025]

- This can be shown explicitly in QED,

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(i\gamma^{\mu} D_{\mu} + \mu\gamma^{0} - m\right)\psi + \mathrm{c.t.}$$

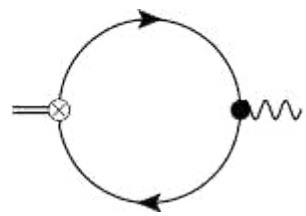
- By definition,

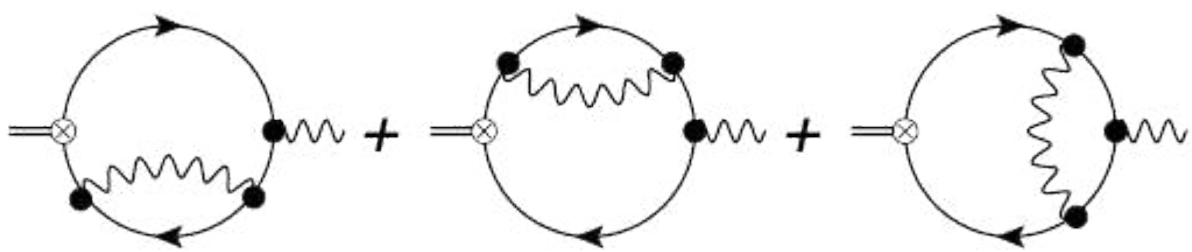
$$\langle \vec{J}_5 \rangle = -Z_2 tr\left[\vec{\gamma}\gamma^5 G(x,x)\right]$$

Radiative correction $O(\alpha)$

- To leading order in $\alpha = e^2/(4\pi)$,

$$G(x,y) = S(x,y) + i\int d^4u d\text{\textasciicircum}4v S(x,u)\Sigma(u,v)\, S(v,y)$$

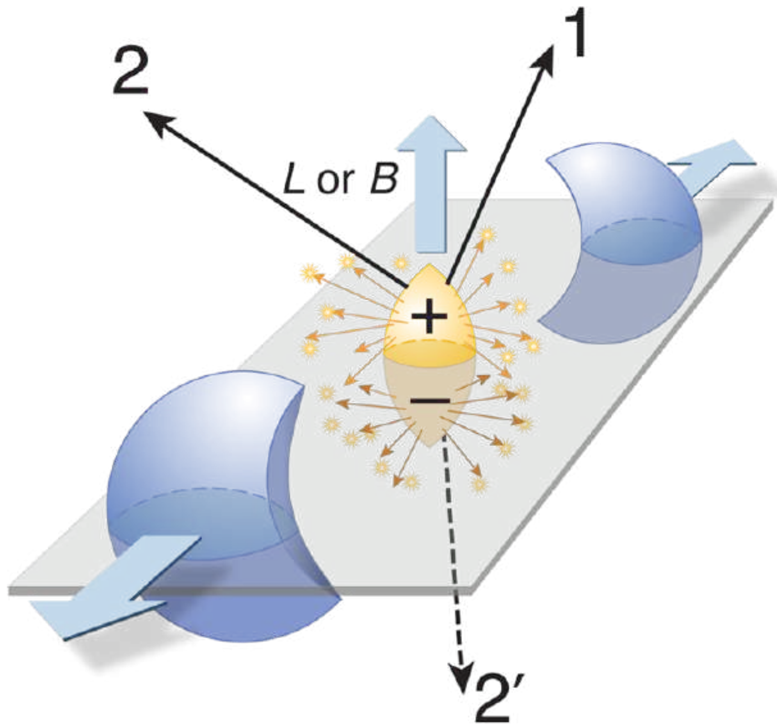
- To leading order: $\langle \vec{j}_5 \rangle_0 =$  $= -\dfrac{e\vec{B}}{2\pi^2}\mu$
- To sub-leading order,

$\langle j_5^\mu \rangle_1 =$ 

- Final result (loops + counter terms)

$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha e B m^2}{2\pi^3 \mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

[Gorbar, Miransky, Shovkovy, Wang, PRD **88** (2013) 025025]



https://physics.aps.org/articles/v2/104

HEAVY-ION COLLISIONS

CME in heavy ion collisions?

- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x\, F_a^{\mu\nu} \tilde{F}^a_{\mu\nu}$$

- A random fluctuation with nonzero chirality could result in

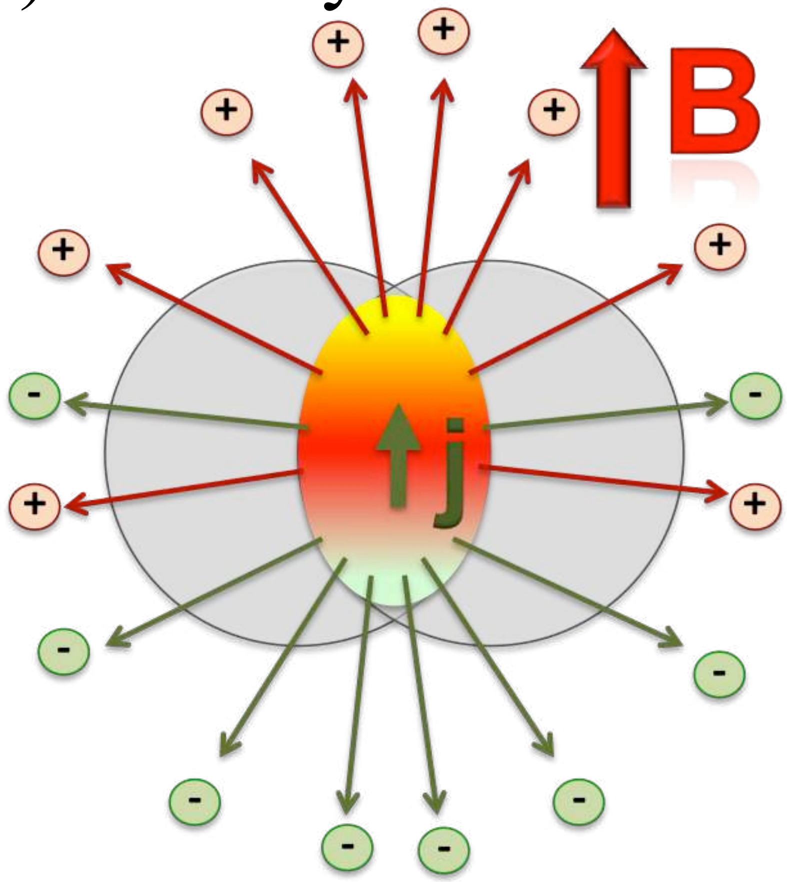
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- This should lead to an electric current

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

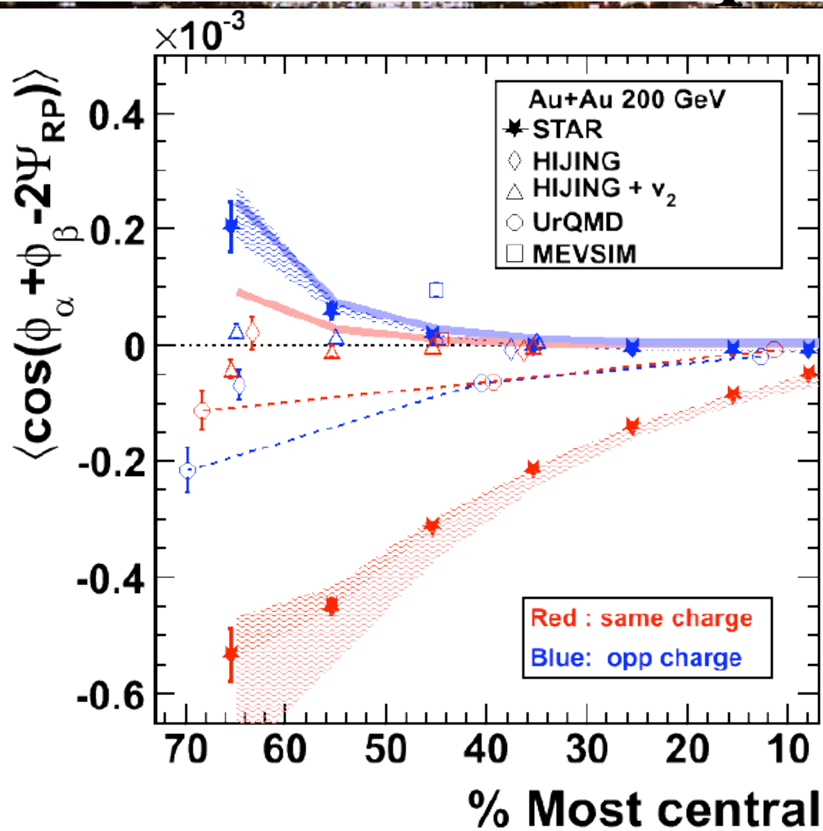
Dipole CME

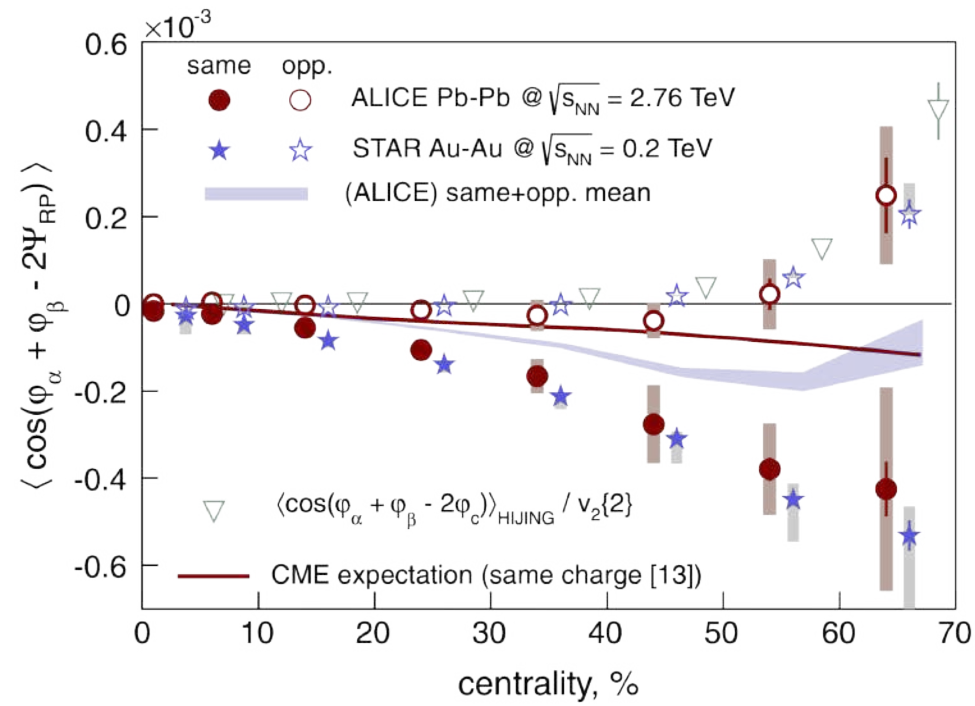
- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence





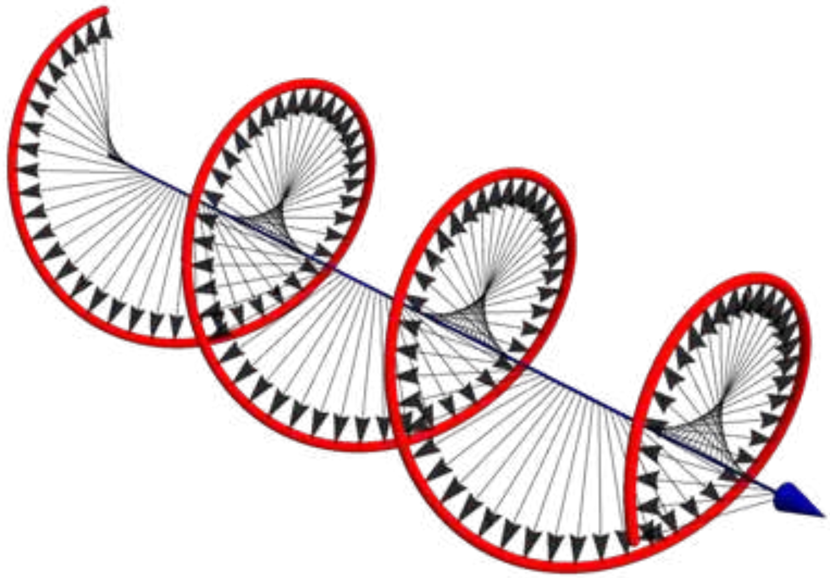
Correlations of same & opposite charge particles:

$$\begin{cases} \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\mp} - 2\Psi_{RP}) \rangle \\ \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\pm} - 2\Psi_{RP}) \rangle \end{cases}$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)], [ALICE Collaboration, Phys. Lett. B**777**, 151 (2018)]



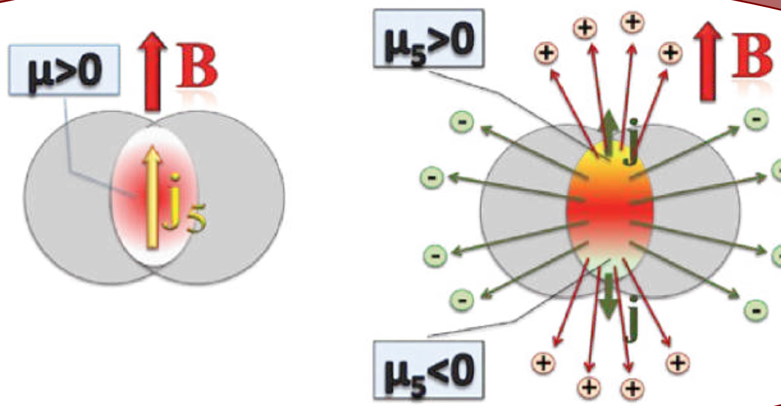
CHIRAL MAGNETIC WAVE

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

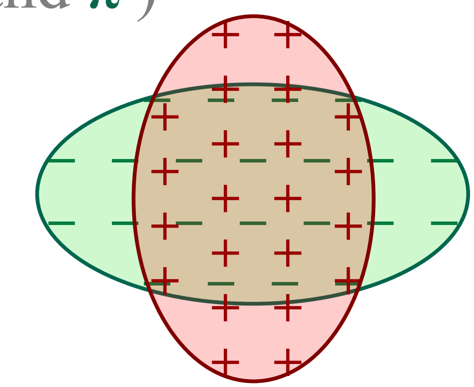


$$\left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

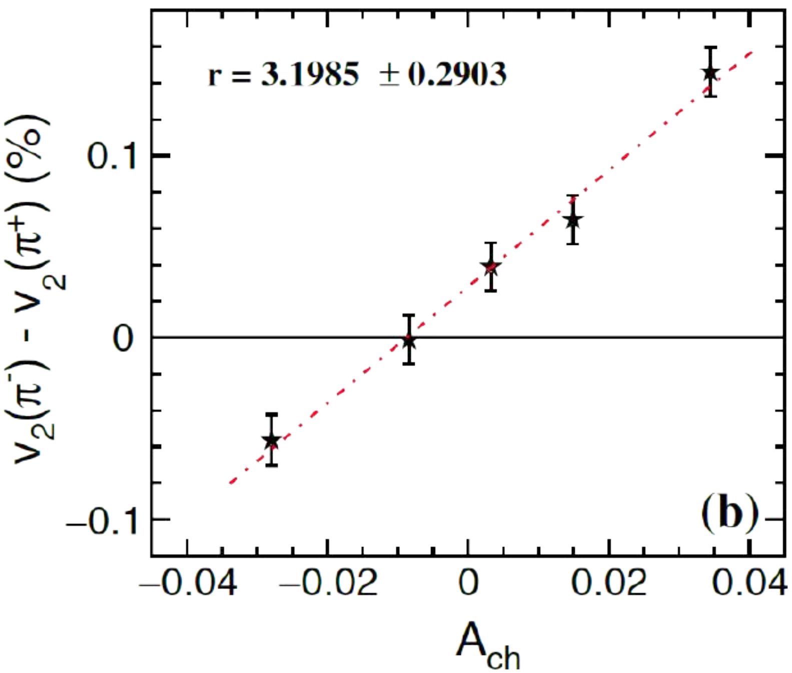
$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$

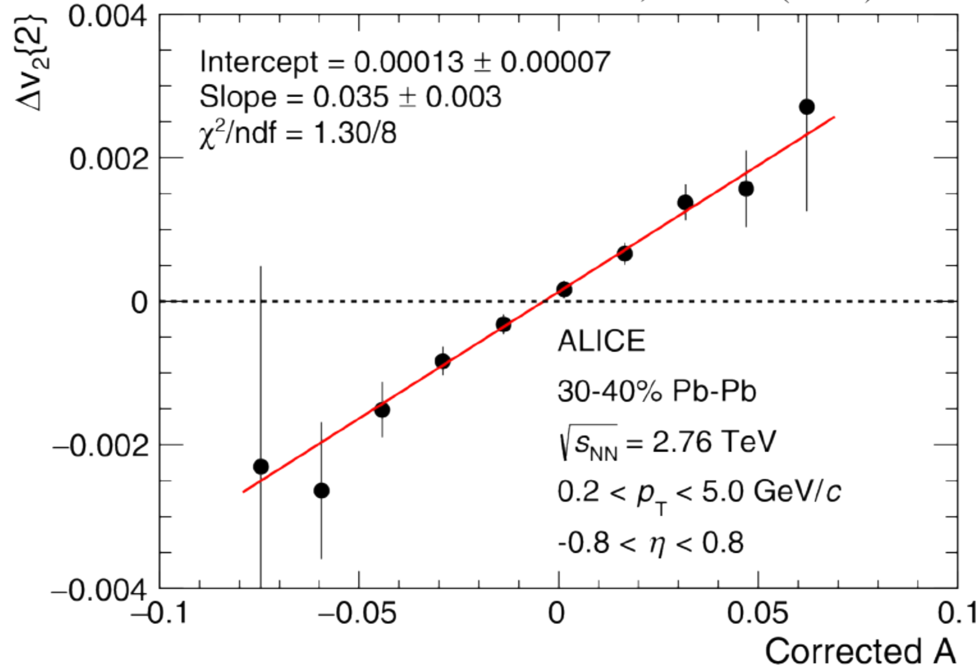


where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

CMW: Experimental evidence





[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

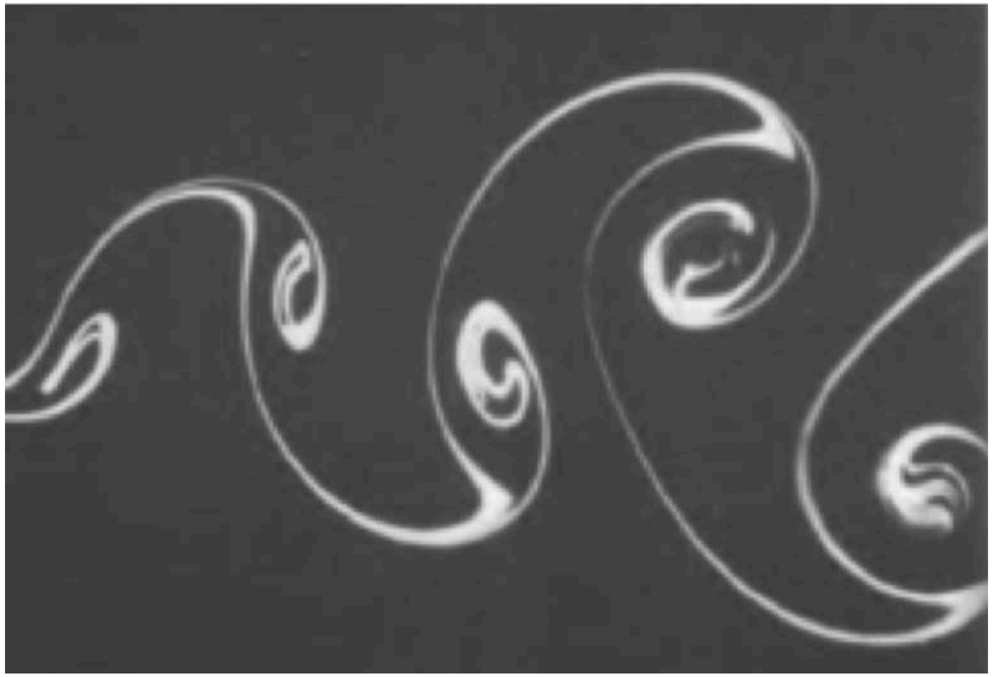
[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Higher harmonics of particle correlations are problematic…

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

In fact, the chiral magnetic wave might be overdapmed…



CHIRAL HYDRODYNAMICS

Chiral hydrodynamics

- Continuity equations: [Son, Surowka, Phys. Rev. Lett. 103, 191601 (2009)] [Neiman and Oz, JHEP 03, 023 (2011)]

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2\hbar^2} E^\mu B_\mu$$

$$\partial_\nu T^{\mu\nu} = eF^{\mu\nu} j_\nu$$

together with the constitutive relations:

$$j^\mu = nu^\mu + \nu^\mu$$

$$j_5^\mu = n_5 u^\mu + \nu_5^\mu$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu} P + (h^\mu u^\nu + u^\mu h^\nu) + \pi^{\mu\nu}$$

Anomalous contributions

- Currents included new non-dissipative terms:

$$j^\mu = nu^\mu + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$j_5^\mu = n_5 u^\mu + \sigma_\omega^5 \omega^\mu + \sigma_B^5 B^\mu$$

where the anomalous coefficients are

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2 T^2}{6\pi^2\hbar^2}, \qquad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$

CMW: $eB \ll T^2$

- Simple 1-flavor model (**k** ∥ **B**):

$$k_0 \delta n - kB\delta\sigma_B + i\frac{\tau}{3}k^2\delta n - \frac{1}{e}\sigma_E k\delta E_z = 0$$

$$k_0 \delta n_5 - kB\delta\sigma_B^5 + i\frac{\tau}{3}k^2\delta n_5 - i\frac{e^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i\frac{\sigma_E}{2} \pm i\frac{\sigma_E}{2}\sqrt{1-\left(\frac{3eB}{\pi^2 T^2\sigma_E}\right)^2\left(k^2+\frac{e^2T^2}{3}\right)} - i\frac{\tau}{3}k^2$$

- This is a completely diffusive mode when

$$\frac{3eB}{\pi^2 T^2\sigma_E}\sqrt{k^2+\frac{e^2T^2}{3}} < 1$$

Naïve CMW ($\sigma_E \to 0$)

- In contrast, if Gauss's law is ignored, the mode is non-diffusive, i.e.,

$$k_0^{(\pm)} = \pm \frac{3eB}{2\pi^2 T^2} \sqrt{k^2 + \frac{e^2 T^2}{3}} - i\frac{\tau}{3}k^2, \quad \text{when} \quad \sigma_E \to 0.$$

- There is only a small dissipation due to the charge diffusion when $k \to 0$
- Notably, the CMW is gapped!
- The gap comes from the anomaly due to δE_z

$$-i\frac{e^2}{2\pi^2}B\delta E_z = -i\frac{e^2 B}{2\pi^2}\left(\frac{-ie\delta n}{k}\right)$$

CMW in HIC

- Model with two light u- and d-quarks:

$$k_0\delta n_f - \frac{eq_f Bk}{2\pi^2\chi_{f,5}}\delta n_{f,5} + iD_f k^2\delta n_f - \frac{1}{eq_f}\sigma_{E,f}k\delta E_z = 0$$

$$k_0\delta n_{f,5} - \frac{eq_f Bk}{2\pi^2\chi_f}\delta n_f + iD_f k^2\delta n_{f,5} - i\frac{e^2q_f^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\sum_f q_f\delta n_f = 0$$

where $f = u, d$, and $q_u = {}^2/_3$, $q_d = -{}^1/_3$

χ_f, D_f, and $\sigma_{E,f}$ are susceptibilities, diffusion coefficients and electrical conductivities for each quark flavor, respectively

Non-perturbative regime

Near-critical strongly coupled quark-gluon plasma

$$\sigma_E = \sum_f \sigma_{E,f} = c_\sigma C_{\rm em}^{\ell} T$$

$$\chi_f = c_\chi \chi_f^{(SB)}$$

$$D_f = \frac{c_D}{2\pi T}$$

$$C_{\rm em}^{\ell} = \left({}^5/_9\right) 4\pi\alpha_{\rm em} \approx 0.051$$

Lattice data

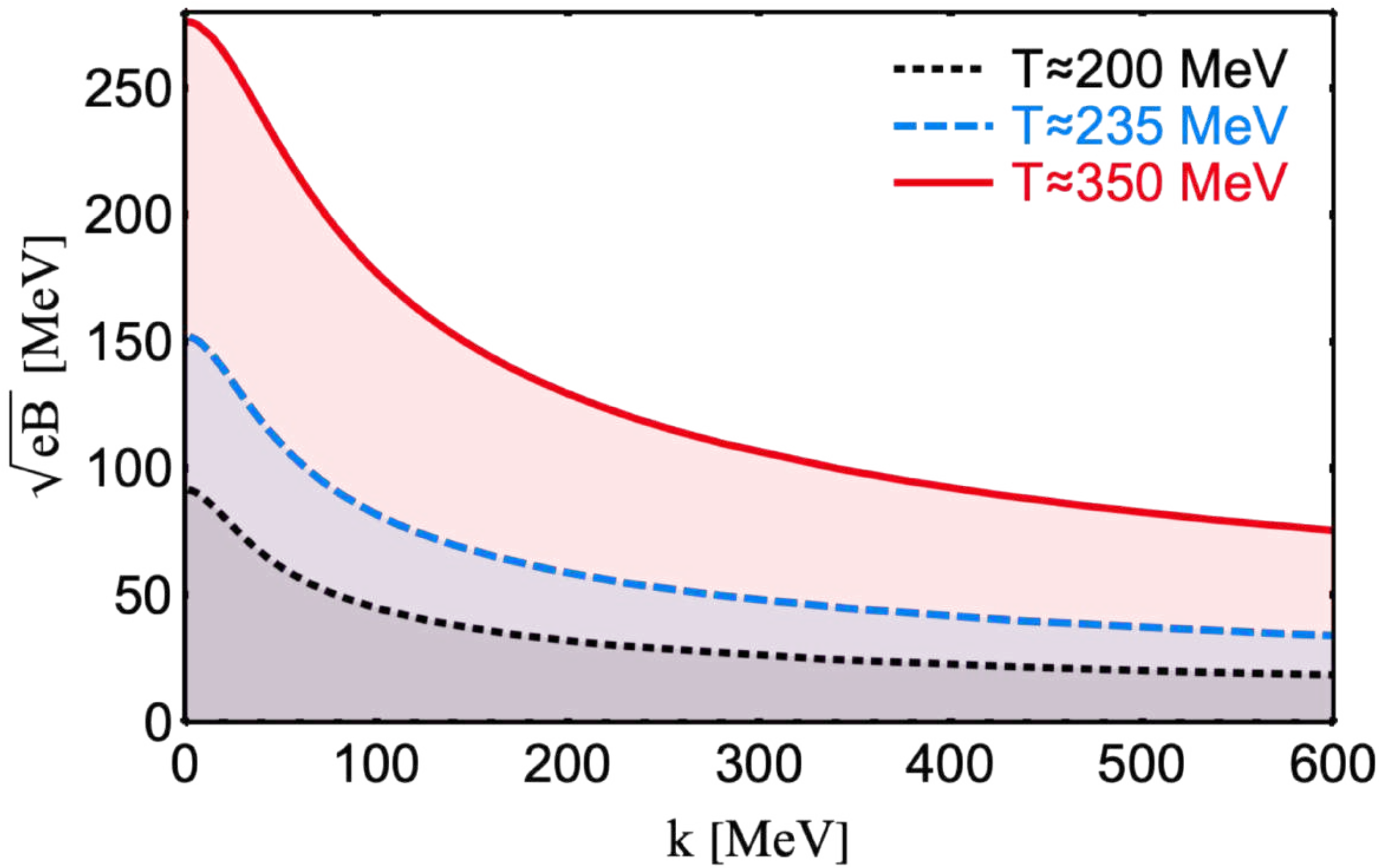
[Aarts, et. al. JHEP 1502, 186 (2015)]

	c_σ	c_χ	c_D
T=200 MeV	0.111	0.804	0.758
T=235 MeV	0.214	0.885	1.394
T=350 MeV	0.316	0.871	1.826

Results

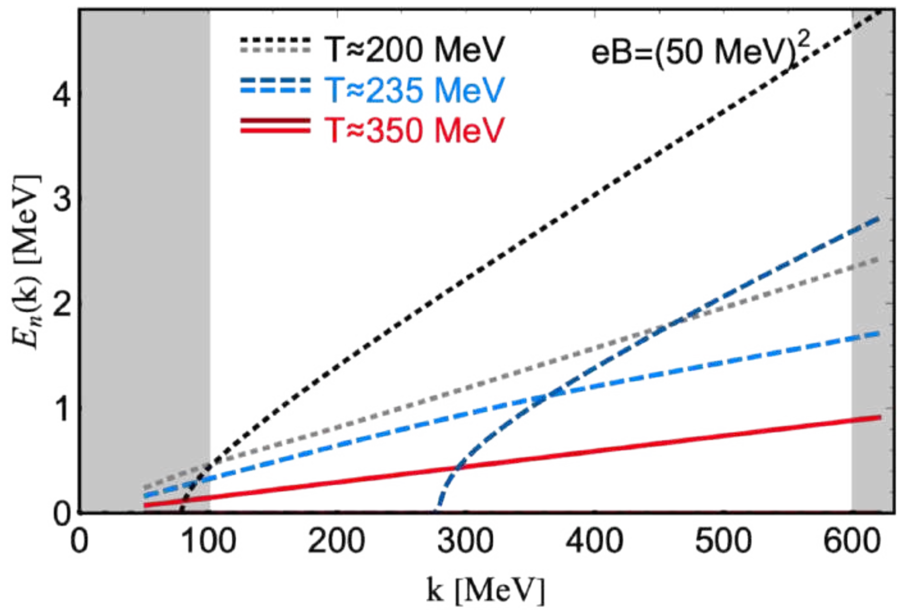
Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

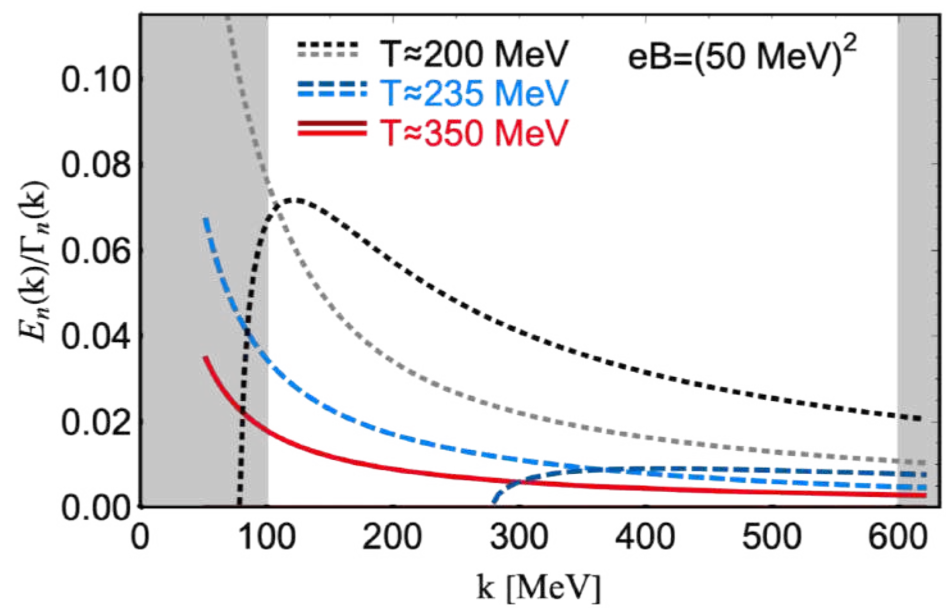
CMW is completely diffusive at small eB & k:



Moderately strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$





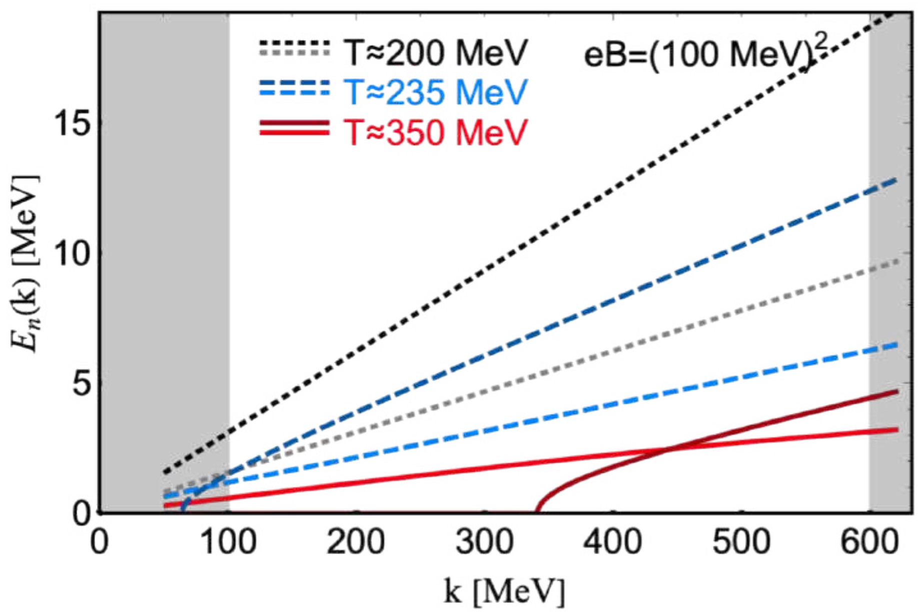
Allowed range of wave vectors:

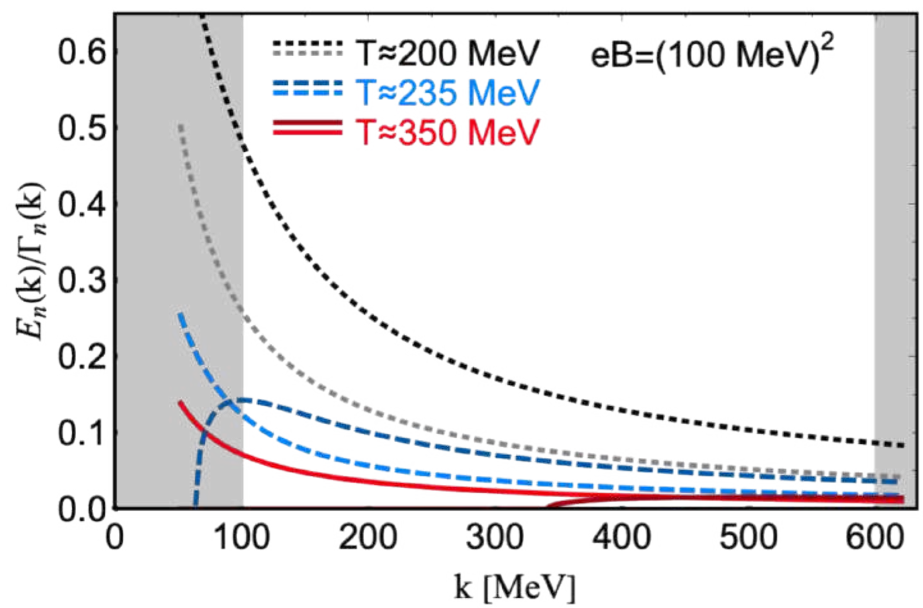
(50 MeV) $100\ \text{MeV} \lesssim k \lesssim 600\ \text{MeV}$

Wavelengths: $2\ \text{fm} \lesssim \lambda_k \lesssim 12\ \text{fm}$ (24 fm)

Strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



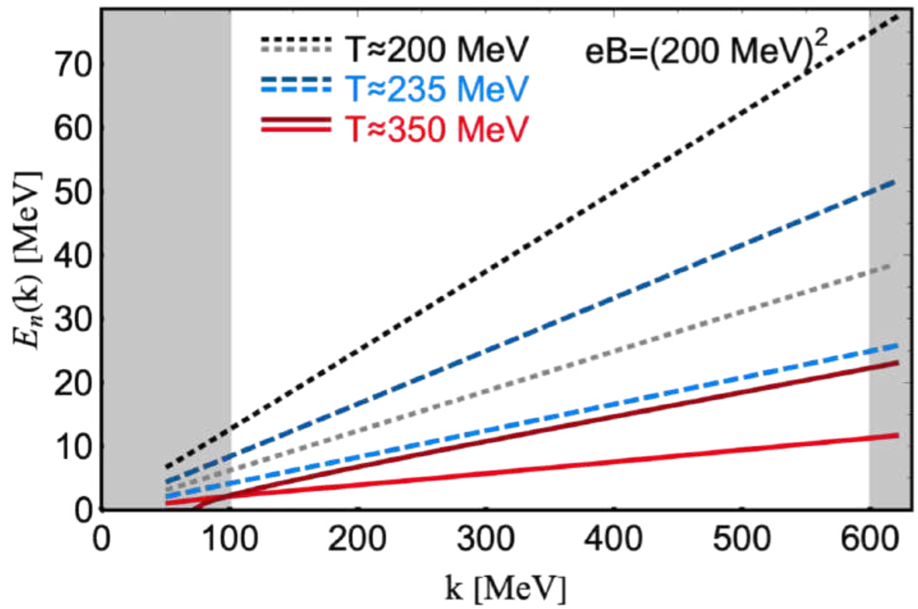


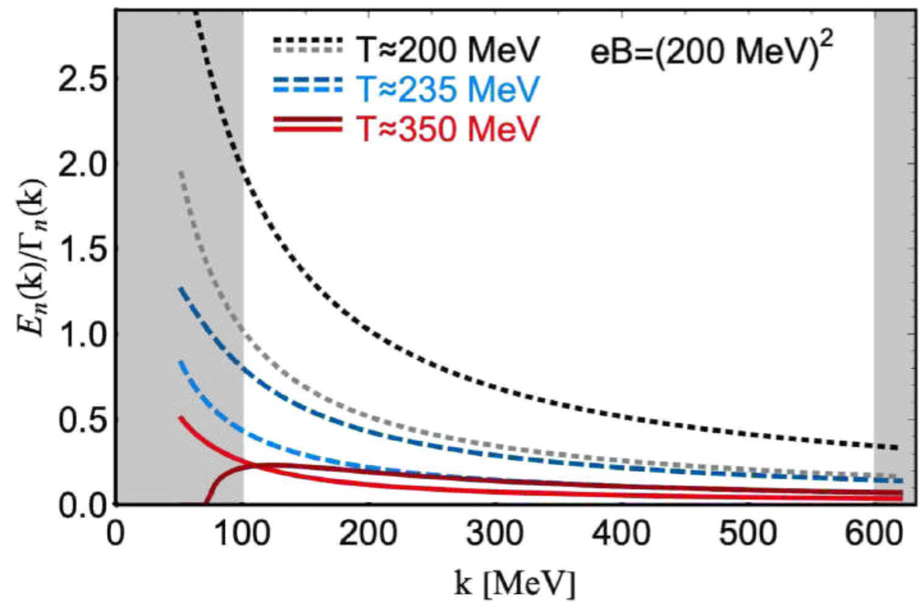
Even for such strong B-field, the CMW is strongly overdampled

Charge diffusion $iD_f k^2$ plays a big role $(k \gtrsim {}^{2\pi}/_{R})$

Very strong B-field

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$





The CMW may become a propagating mode only in extremely strong B-fields, $eB \gtrsim (200\ \mathrm{MeV})^2$

Otherwise, it is overdampled

Summary

- Anomalous physics can be seen in a chiral plasma in hydrodynamic regime
- Chiral magnetic/separation effects can modify charged particle correlators
- Anomaly can modify some features of hydro modes in chiral plasmas
- Dynamical electromagnetism plays a crucial role
  - Electrical conductivity screens charge fluctuations
  - Charge diffusion is not negligible in finite-size systems
- Chiral magnetic wave in HIC is likely overdamped