ARIZONA STATE UNIVERSITY

UNIVERSITY OF ARIZONA
Mar 19, 2021
TUCSON

Anomalous physics of magnetized quark-gluon plasma

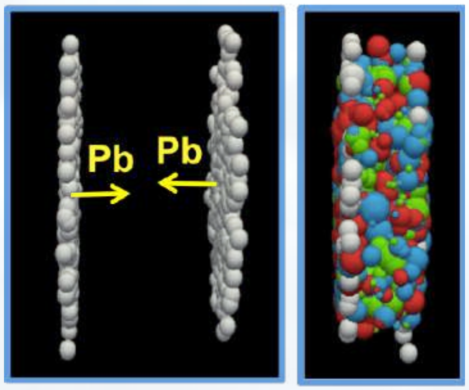
Igor Shovkovy

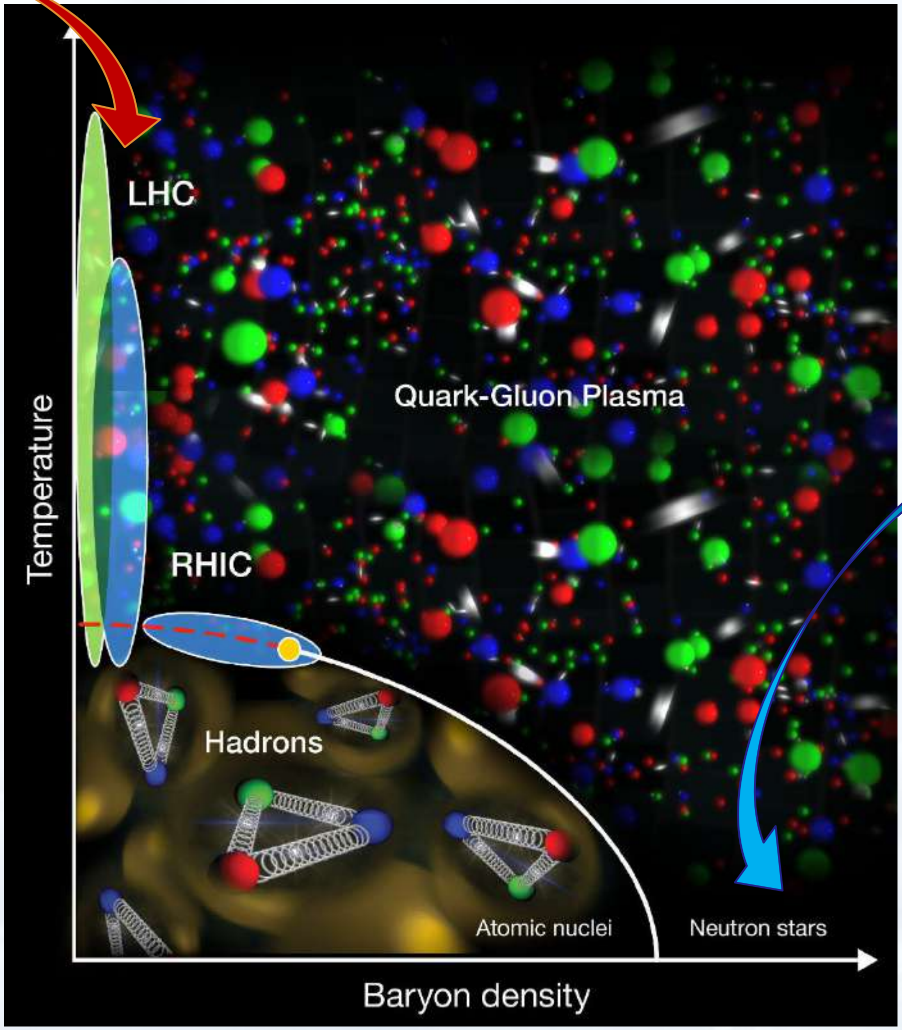
Arizona State University

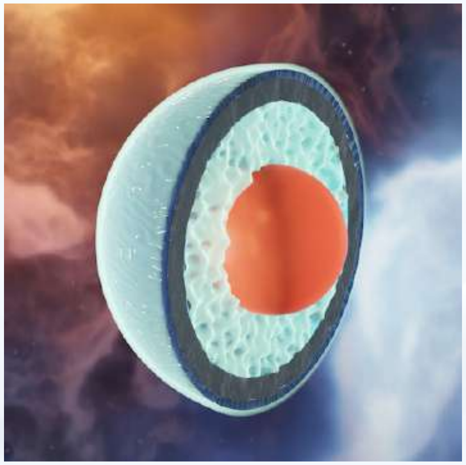
[Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]

Quark-gluon plasma

- **Quark–gluon plasma** (QGP) is a state of matter at high energy density (made of deconfined quarks and gluons)







Relativistic Matter

- **Non-relativistic**
  - Particles move much slower than the speed of light
  - Kinetic energies are much smaller than the rest energy

$$E_{\text{kin}} \ll E_{\text{rest}}: \; E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}$$

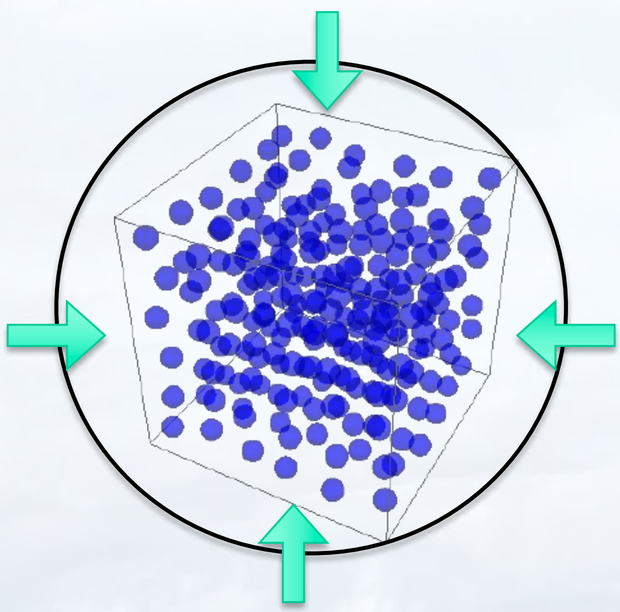
- **Relativistic**
  - Particle velocities approach the speed of light
  - Kinetic energies are comparable to, or larger than E_{rest}

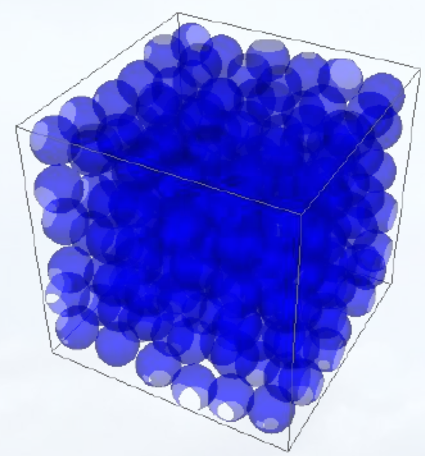
$$E_{\text{kin}} \gtrsim E_{\text{rest}}: \; E = c\sqrt{p^2 + m^2c^2} \approx cp$$

Super-dense Matter

- **What happens when you squeeze matter to very high density?**

Pauli exclusion principle: fermions cannot occupy same quantum states (they end up filling out all states from $p_{\min} \approx 0$ to $p_{\max} \propto \hbar n^{1/3}$)

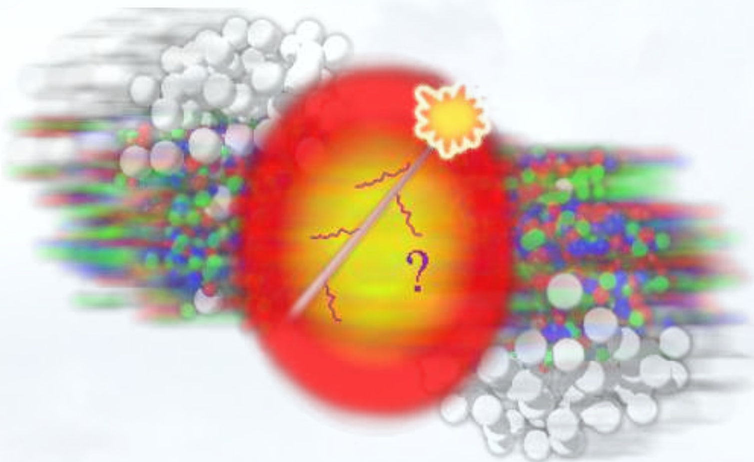


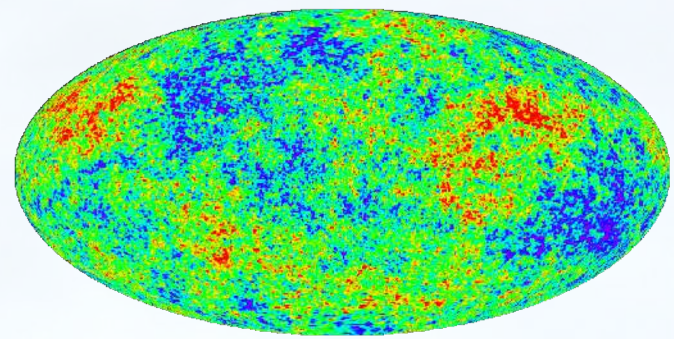


$$p_{\max} \approx 200 \left(\frac{n}{1\ \mathrm{fm}^3}\right)^{\frac{1}{3}} \mathrm{MeV/c}$$

Super-hot Matter

- **What happens when you heat matter to very high temperature?** (e.g., matter in heavy ion collisions)



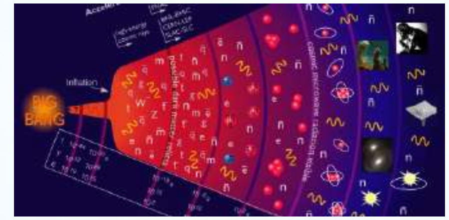


Heat is equivalent to **kinetic energy**: average kinetic energy of particles is proportional to temperature:

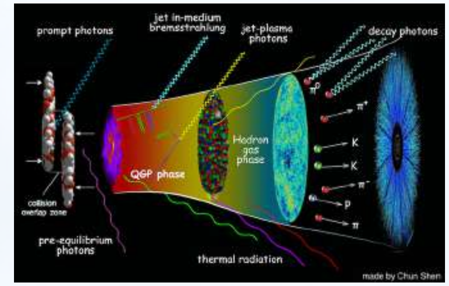
$$p \approx k_B T/c \propto 200 \left(\frac{k_B T}{200\ \text{MeV}}\right) \text{MeV/c (assuming } p \gg mc\text{)}$$

Relativistic matter

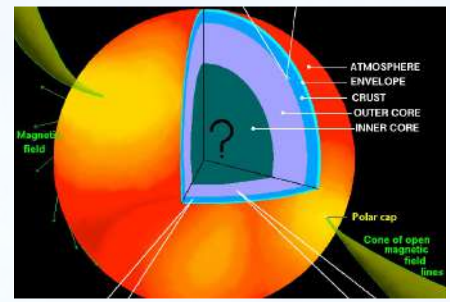
- **Early Universe** **(high temperature)**



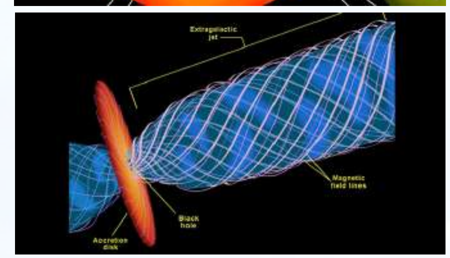
- **Heavy-ion collisions** **(high temperature)**
  [Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]



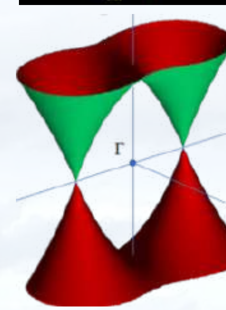
- **Super-dense matter in compact stars (high density)**

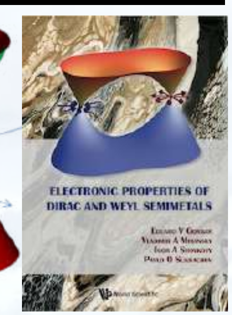


- **Ultra-relativistic jets from black holes (moderately high temperature and density)**



- **Dirac/Weyl (semi-)metals**
  [Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]

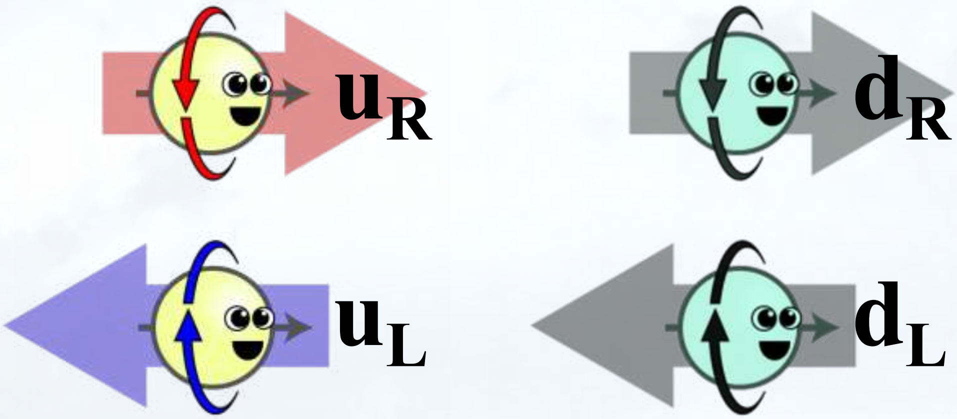




- **Other: cold atoms, superfluid ^{3}He-A, etc.**
  [Volovik, JETP Lett. 105, 34 (2017)]

Chiral fermions

- Only *massless* Dirac fermions have a well-defined chirality ($\gamma^5\psi = \pm\psi$):



Righ-handed

Left-handed

- *Massive* Dirac fermions have an *almost* well-defined chirality in the *ultrarelativistic* regime
  - High temperature: $T \gg m$
  - High density: $\mu \gg m$
- Chirality flip rate: $\Gamma_{\text{flip}} \propto \alpha^2 T (m/T)^2$

Anomalous chiral matter

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ on *macroscopic* scales

- The (collective) dynamics of $n_R + n_L$ and $n_R - n_L$ is controlled by the continuity equations

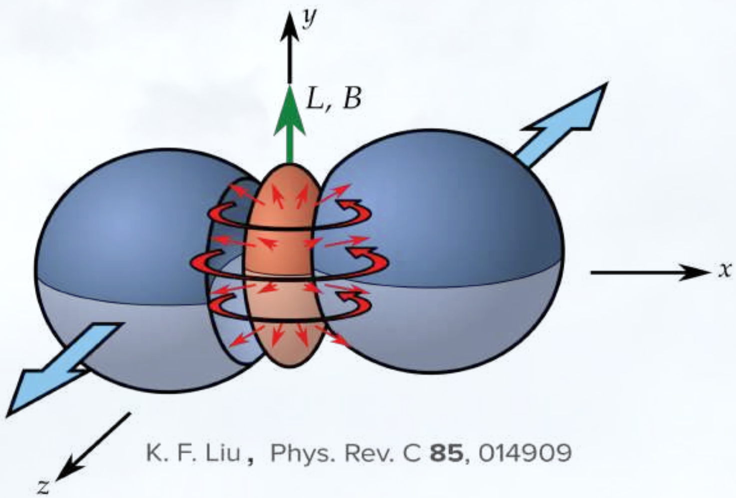
$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c} - \Gamma_{\text{flip}}(n_R - n_L)$$

Question: Can chiral anomaly produce any *macroscopic* effects in ultra-relativistic matter?

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)



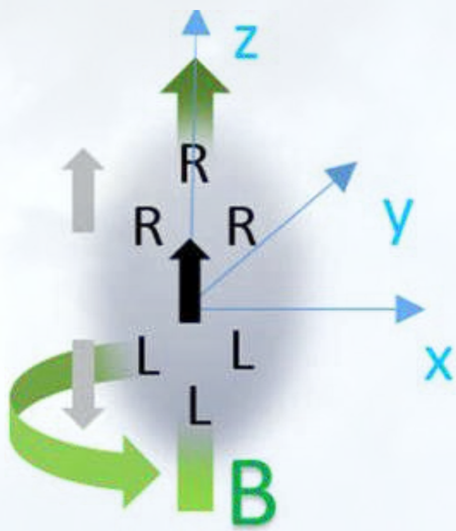
[Rafelski & Müller, PRL, 36, 517 (1976)], [Kharzeev et al., arXiv:0711.0950], [Skokov et al., arXiv:0907.1396], [Voronyuk et al., arXiv:1103.4239], [Bzdak &. Skokov, arXiv:1111.1949], [Deng & Huang, arXiv:1201.5108], …

- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19} \text{ G } (\sim 100 \text{ MeV})$$

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

$$\omega \sim 9 \times 10^{21} s^{-1} \ (\sim 10 \text{ MeV})$$



CHIRAL SEPARATION EFFECT

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Landau levels

- Landau energy levels at $m = 0$

$$E_n^{\pm} = \pm\sqrt{2n|eB| + p_z^2}$$

where $n = \underbrace{k + \frac{1}{2}}_{\text{orbital}} + \underbrace{\text{sgn}(eB)s_z}_{\text{spin}}$

- Lowest Landau level is spin polarized

$$E_0^{\pm} = \pm p_z \qquad (k = 0,\ s_z = -\frac{1}{2})$$

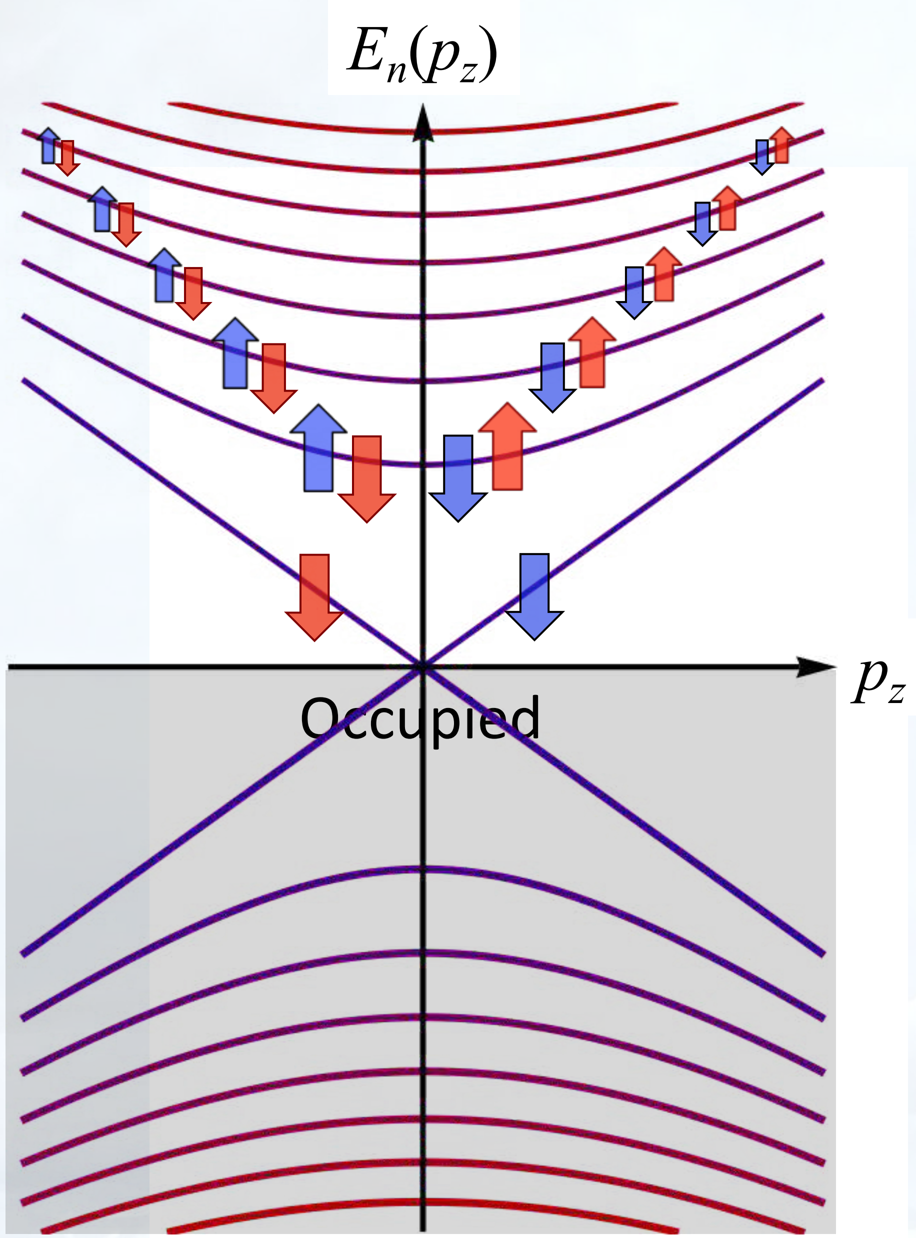
- Density of states at $E=0$:

$$\left.\frac{dn}{dE}\right|_{E=0} = \frac{|eB|}{2\pi}\frac{1}{2\pi} = \frac{|eB|}{4\pi^2}$$

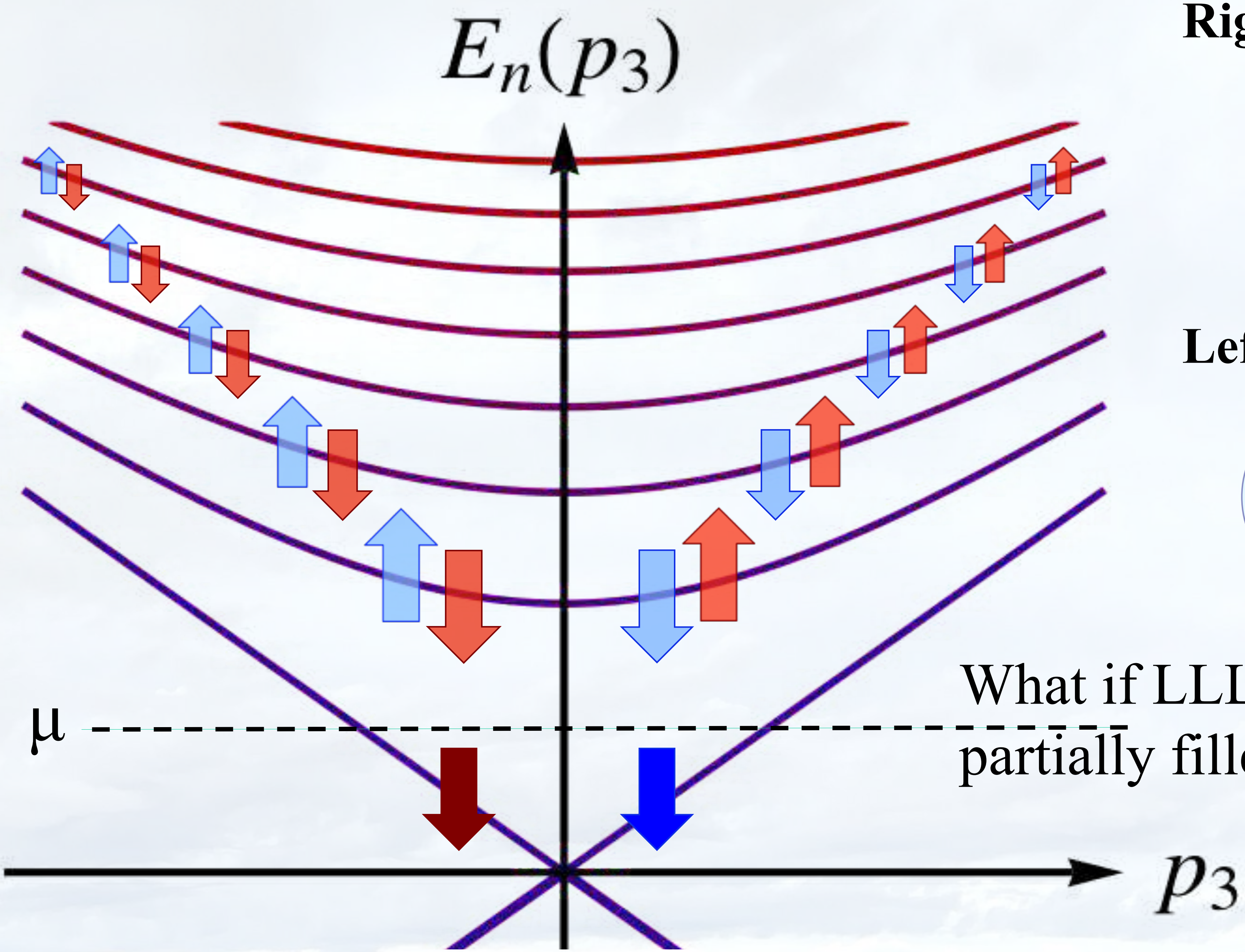
- Higher Landau levels ($n \geq 1$) are twice as degenerate:

(i) $k = n$ & $s = -\frac{1}{2}$

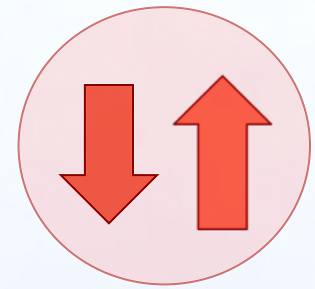
(ii) $k = n - 1$ & $s = +\frac{1}{2}$



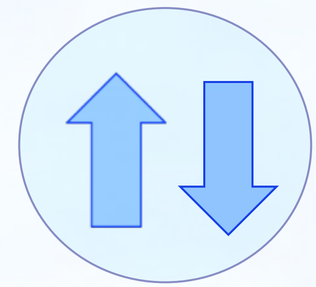
Landau spectrum & $\mu \neq 0$



Right-handed:



Left-handed:



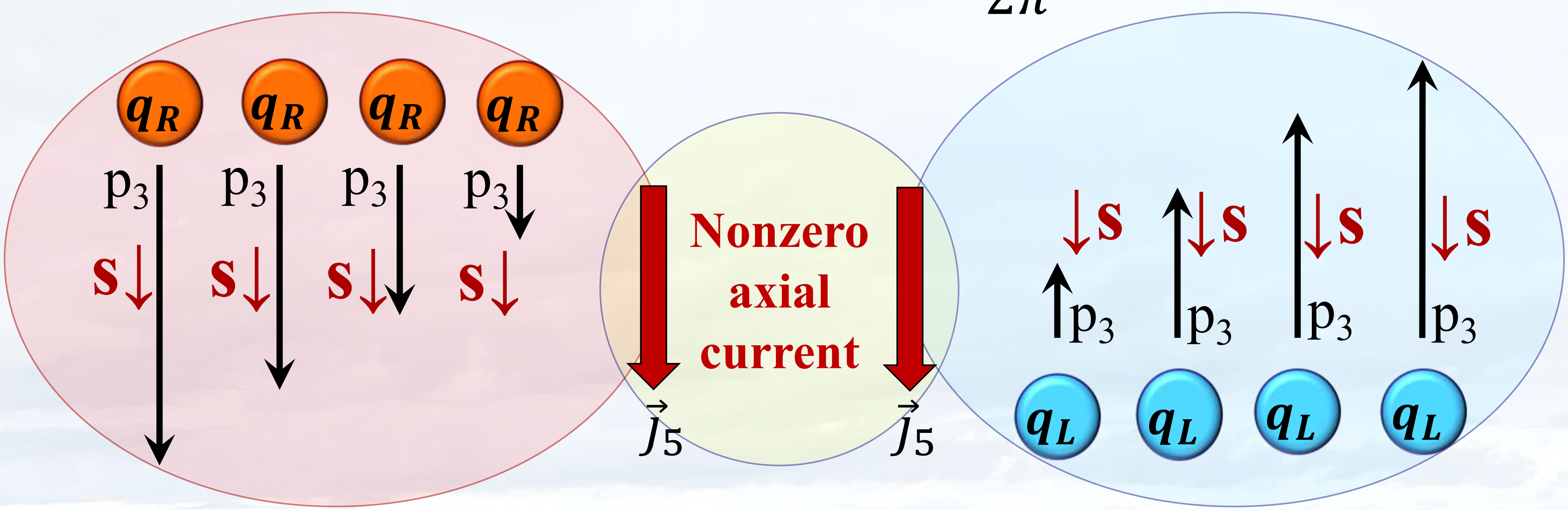
What if LLL is partially filled?

Partially filled LLL

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and s=↓) are R-handed
  - states with $p_3>0$ (and s=↓) are L-handed

i.e., a nonzero **axial** current is induced

$$\langle \vec{j}_5 \rangle = -tr[\vec{\gamma}\gamma^5 S(x,x)] = -\frac{e\vec{B}}{2\pi^2}\mu$$



ARIZONA STATE UNIVERSITY

CHIRAL MAGNETIC EFFECT

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

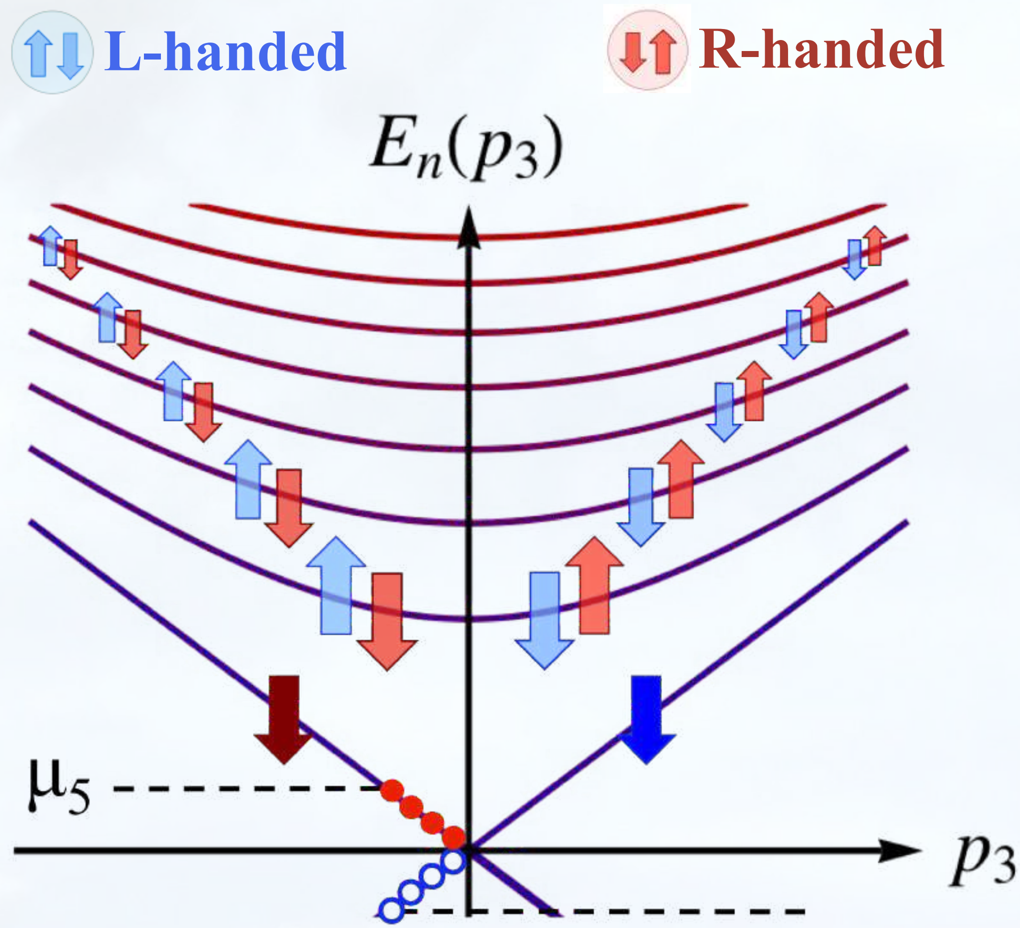
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Topological fluctuations could induce *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL (s=↓ for particles of a *negative* charge):

- Some R-handed states ($p_3 < 0$ and $E < \mu_5$) are occupied
- Some L-handed holes ($p_3 < 0$ and $|E| < \mu_5$) are empty



CME current: $$\langle \vec{j} \rangle = -tr[\vec{\gamma} S(x,x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

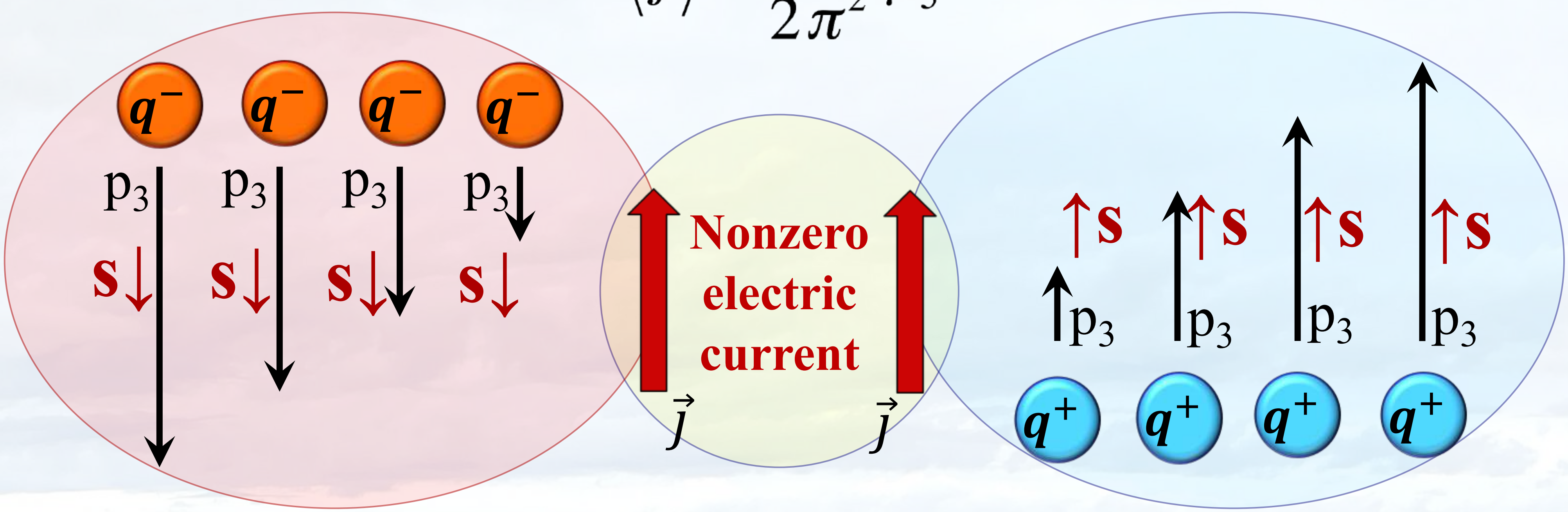
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

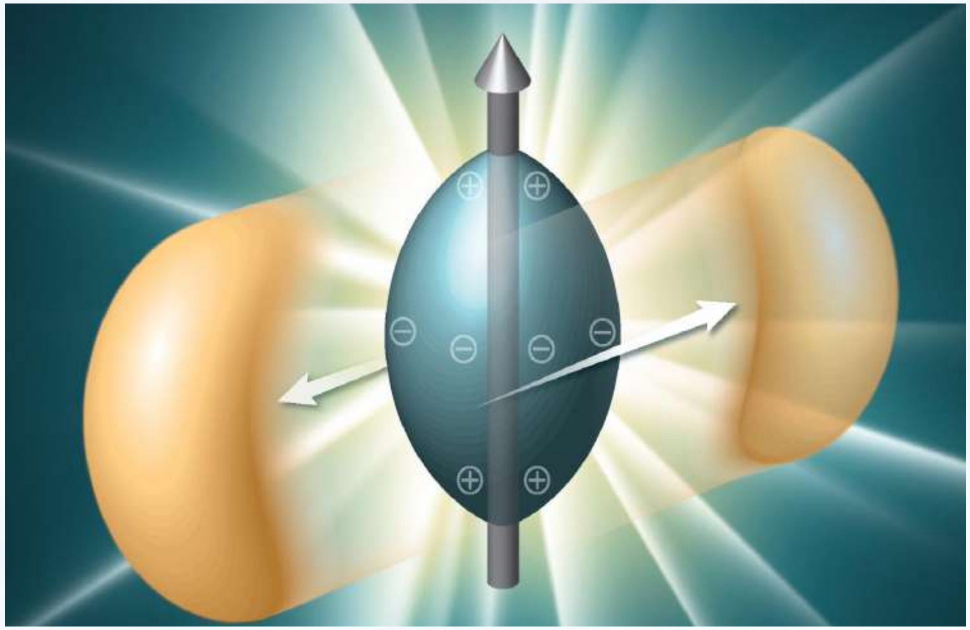
Partially filled LLL ($\mu_5 \neq 0$)

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3 < 0$ (and s=↓) are R-handed **quarks**
  - states with $p_3 > 0$ (and s=↓) are L-handed **antiquarks**

i.e., a nonzero **electric** current is induced

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$





HEAVY-ION COLLISIONS

CME in heavy ion collisions?

- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_a^{\mu\nu} \tilde{F}^a_{\mu\nu}$$

- A random fluctuation with nonzero chirality could result in

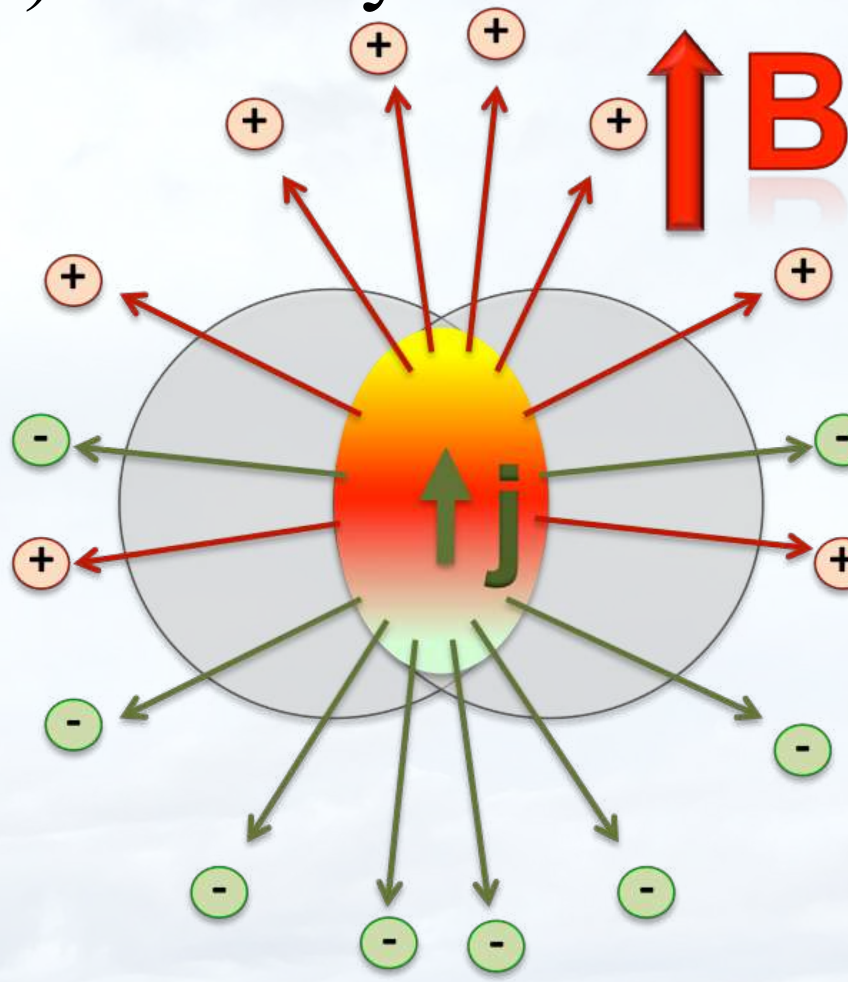
$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- This should lead to an electric current

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

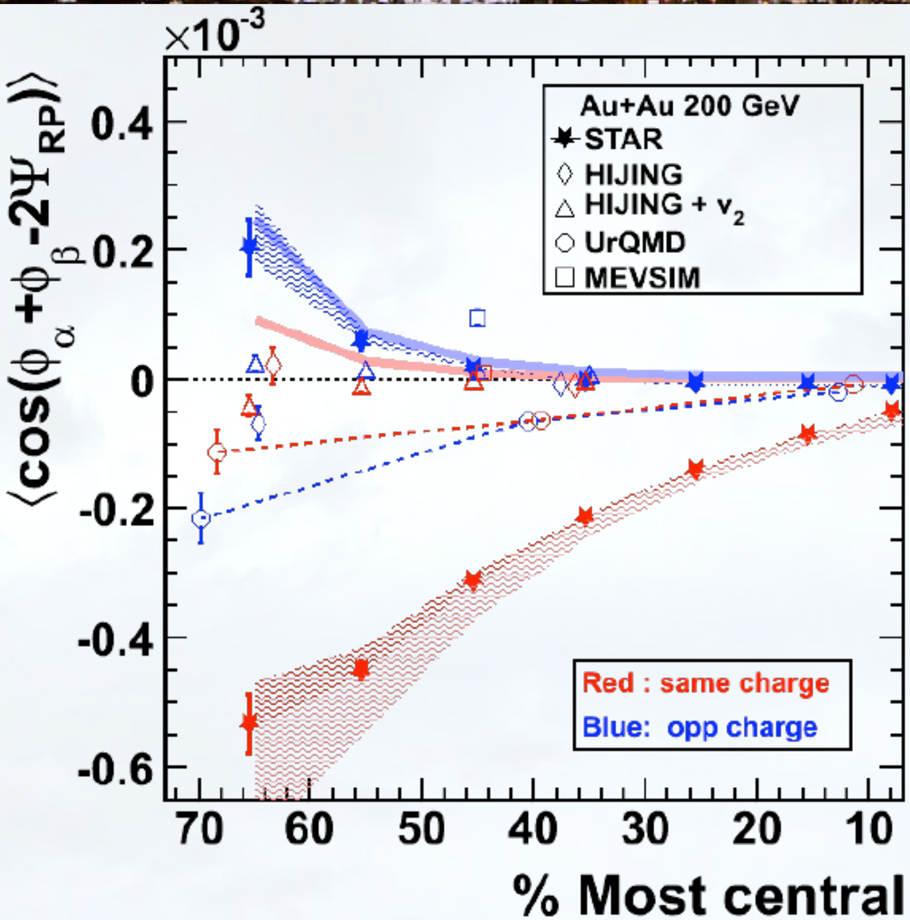
Dipole CME

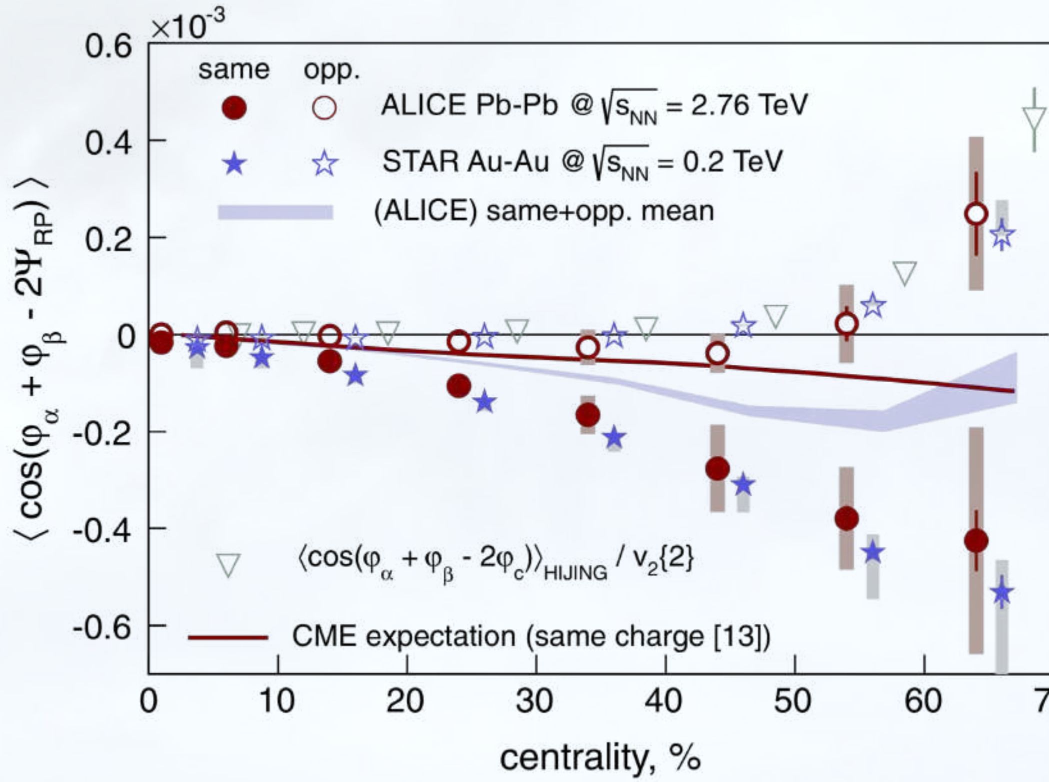
- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence





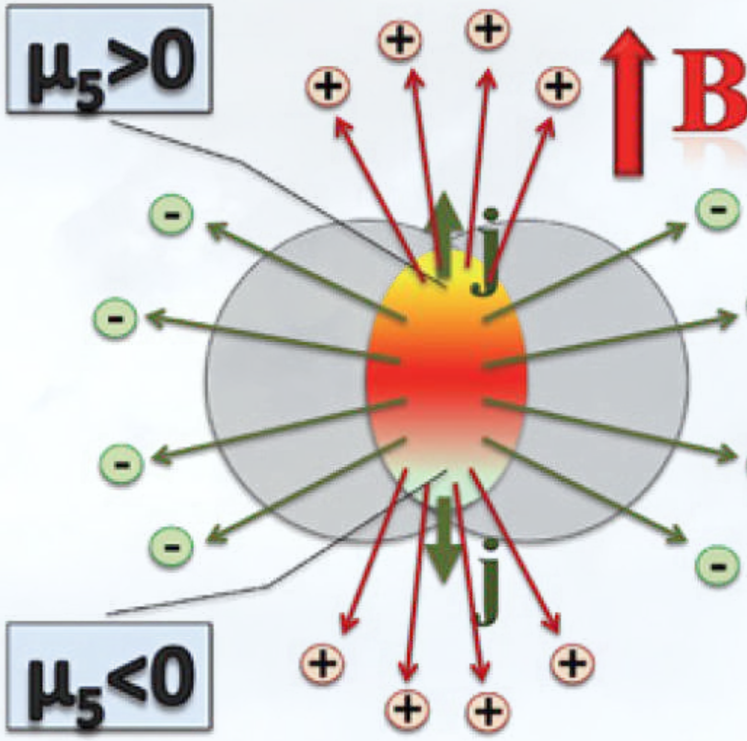
Correlations of same & opposite charge particles:

$$\begin{cases} \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\mp} - 2\Psi_{RP}) \rangle \\ \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\pm} - 2\Psi_{RP}) \rangle \end{cases}$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)], [ALICE Collaboration, Phys. Lett. B**777**, 151 (2018)]



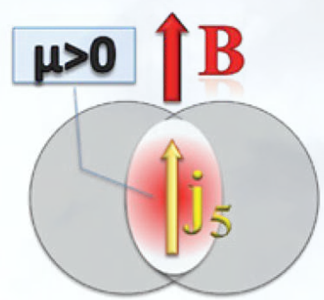
CHIRAL MAGNETIC WAVE: NAÏVE APPROACH

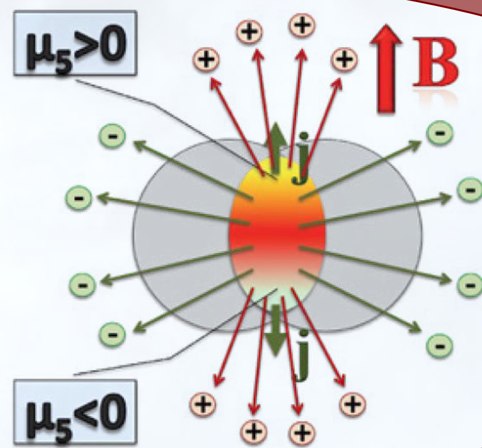
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW

$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2}\mu$$



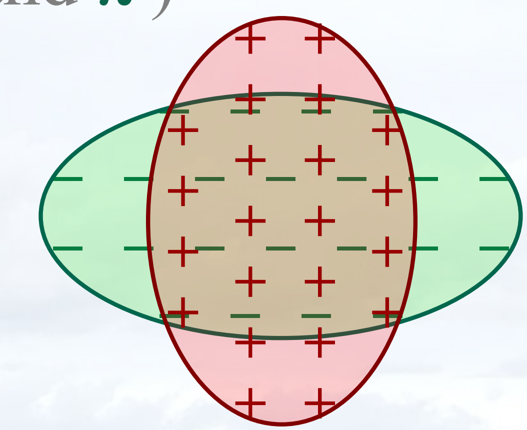


$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$



where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Dispersion of CMW ($\mathbf{k} \parallel \mathbf{B}$)

- Simple model ($\delta n, \delta n_5 \sim e^{-i\omega t + ikz}$):

$$k_0 \delta n - \frac{eB}{2\pi^2 \chi_5} k \delta n_5 = 0$$

$$k_0 \delta n_5 - \frac{eB}{2\pi^2 \chi} k \delta n = 0$$

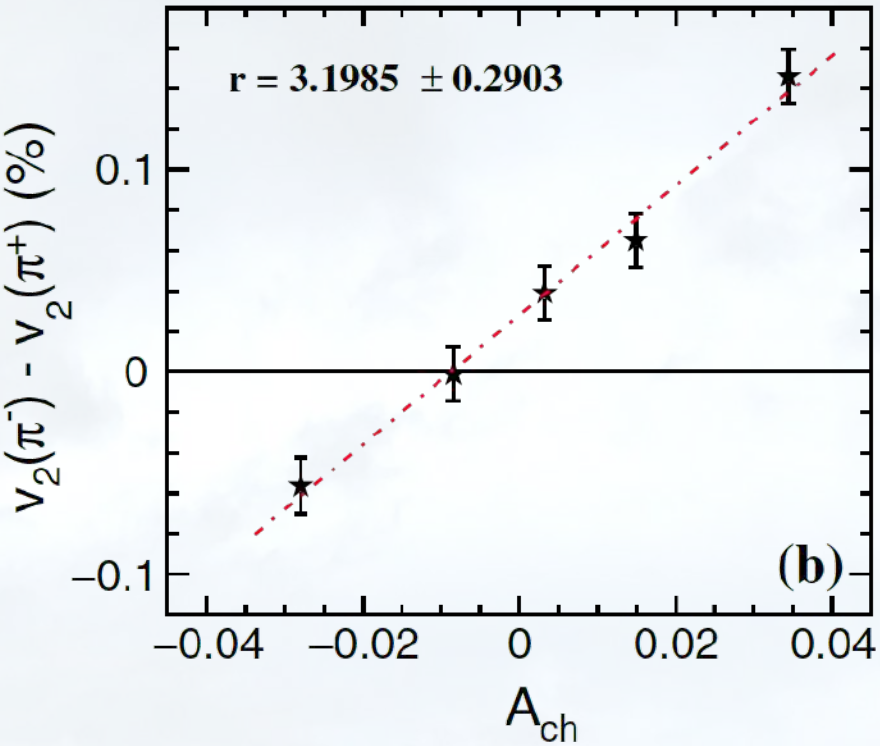
where $\chi_5 \simeq \chi = \partial n / \partial \mu \simeq T^2/3$

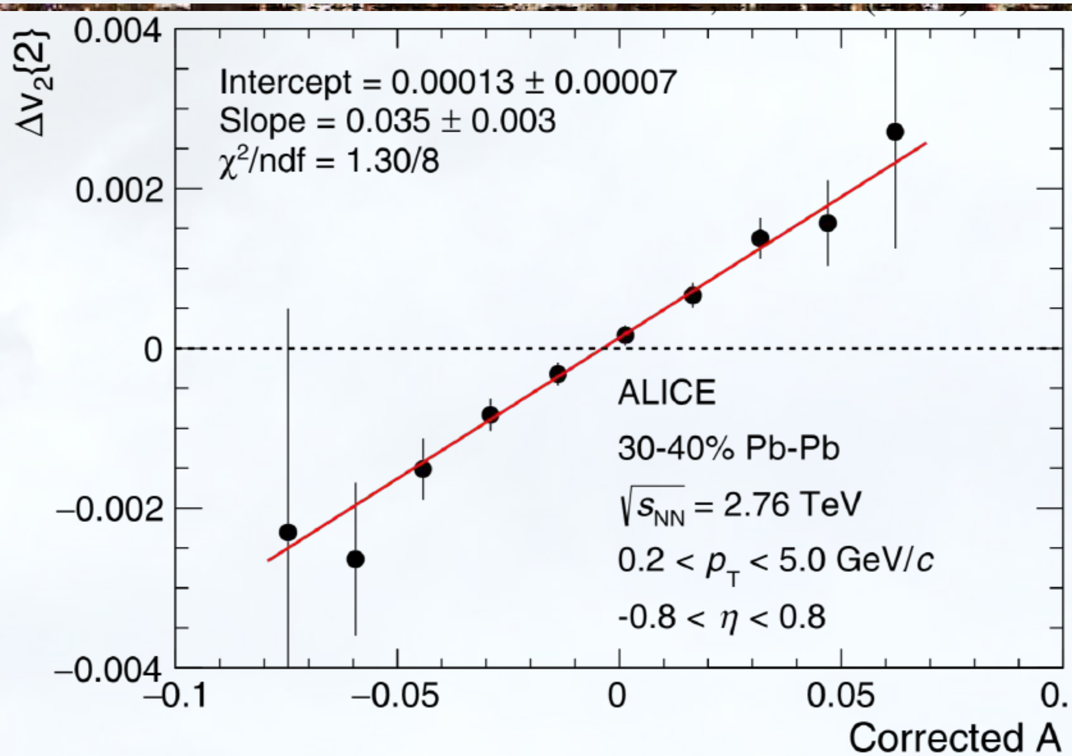
- The linear dispersion of the CMW mode:

$$k_0 \simeq \pm \frac{eB}{2\pi^2 \chi} k$$

- This is a gapless mode with speed $v \propto eB/T^2$

CMW: Experimental evidence





[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

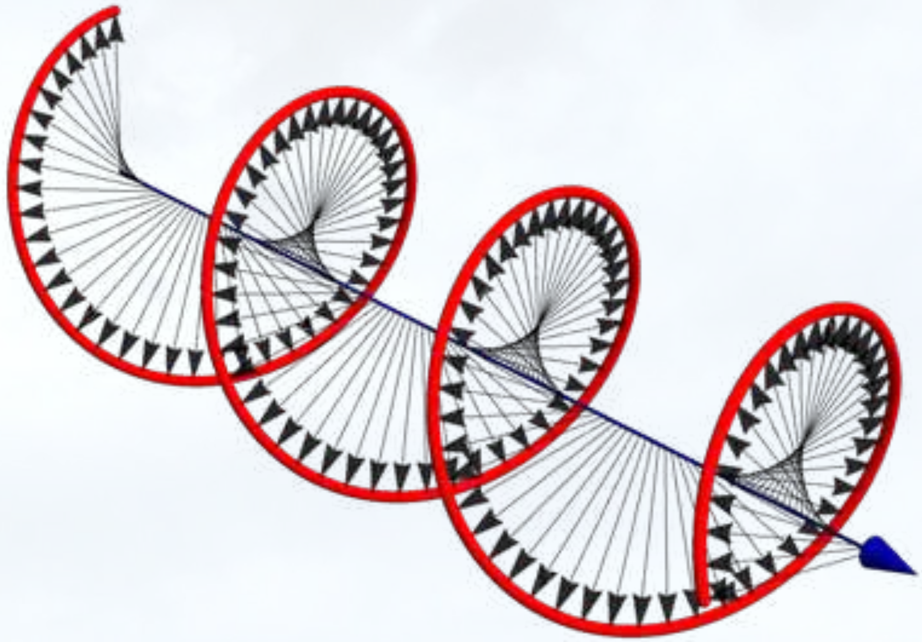
[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Higher harmonics of particle correlations are problematic…

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

In fact, the chiral magnetic wave might be overdapmed…



CHIRAL MAGNETIC WAVE: MORE RIGOROUS

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D 99, 016017 (2019)]
[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]

$eB \ll T^2$: Diffusive CMW

- Simple 1-flavor model ($\mathbf{k} \parallel \mathbf{B}$):

$$k_0 \delta n - kB\delta\sigma_B + i\frac{\tau}{3}k^2\delta n - \frac{1}{e}\sigma_E k\delta E_z = 0$$

$$k_0 \delta n_5 - kB\delta\sigma_B^5 + i\frac{\tau}{3}k^2\delta n_5 - i\frac{e^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i\frac{\sigma_E}{2} \pm i\frac{\sigma_E}{2}\sqrt{1 - \left(\frac{3eB}{\pi^2 T^2 \sigma_E}\right)^2 \left(k^2 + \frac{e^2 T^2}{3}\right)} - i\frac{\tau}{3}k^2$$

- This is a completely diffusive mode when

$$\frac{3eB}{\pi^2 T^2 \sigma_E}\sqrt{k^2 + \frac{e^2 T^2}{3}} < 1$$

CMW in QGP

- Model with two light u- and d-quarks:

$$k_0\delta n_f - \frac{eq_f Bk}{2\pi^2\chi_{f,5}}\delta n_{f,5} + iD_f k^2\delta n_f - \frac{1}{eq_f}\sigma_{E,f}k\delta E_z = 0$$

$$k_0\delta n_{f,5} - \frac{eq_f Bk}{2\pi^2\chi_f}\delta n_f + iD_f k^2\delta n_{f,5} - i\frac{e^2 q_f^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\sum_f q_f\delta n_f = 0$$

where $f = u, d$, and $q_u = 2/3$, $q_d = -1/3$

χ_f, D_f, and $\sigma_{E,f}$ are susceptibilities, diffusion coefficients and electrical conductivities for each quark flavor, respectively

Non-perturbative regime

Near-critical strongly coupled quark-gluon plasma

$$\sigma_E = \sum_f \sigma_{E,f} = c_\sigma C^\ell_{\text{em}} T$$

$$C^\ell_{\text{em}} = \left(5/9\right) 4\pi\alpha_{\text{em}} \approx 0.051$$

$$\chi_f = c_\chi \chi_f^{(SB)}$$

$$D_f = \frac{c_D}{2\pi T}$$

Lattice data

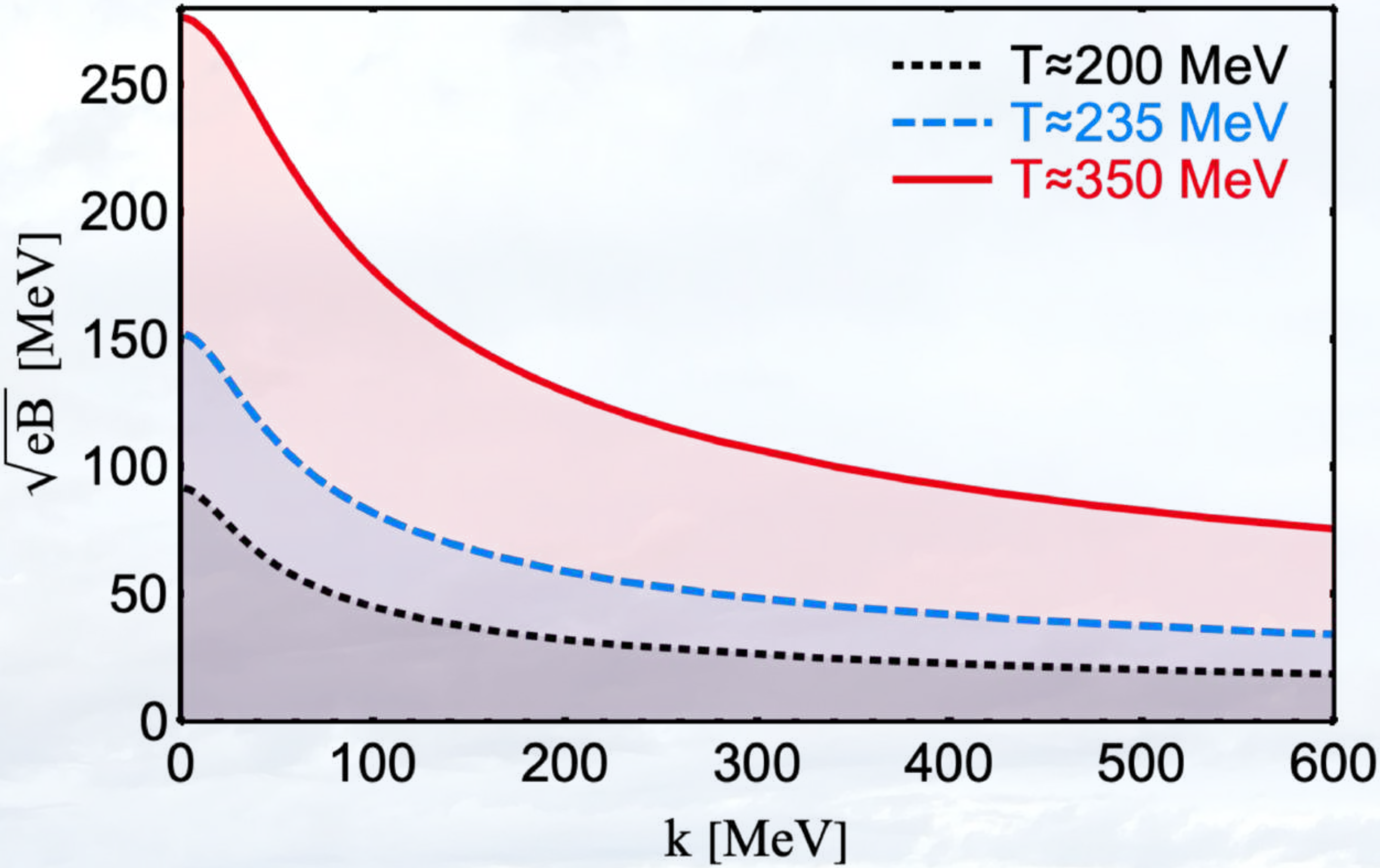
[Aarts, et. al. JHEP 1502, 186 (2015)]

	c_σ	c_χ	c_D
T=200 MeV	0.111	0.804	0.758
T=235 MeV	0.214	0.885	1.394
T=350 MeV	0.316	0.871	1.826

Results

Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small eB & k:



[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]

Summary

- Chiral anomalous effects can have observable signatures in (quasi-)relativistic matter
- Dynamical electromagnetism is important for CMW
  – Electrical conductivity screens charge fluctuations
  – Anomaly makes wave gapped
  – Charge diffusion is not negligible in finite-size systems
- Chiral magnetic wave is likely to be badly overdamped
- Empirical data will have to resolve the controversy
- Similar effects can be tested/observed in Dirac/Weyl semimetals, trapped cold atoms, superfluid helium, etc.