ARIZONA STATE UNIVERSITY

FROM CHAOS TO HYDRO
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Aspen, CO

Relativistic-like hydrodynamics: Catching the flow

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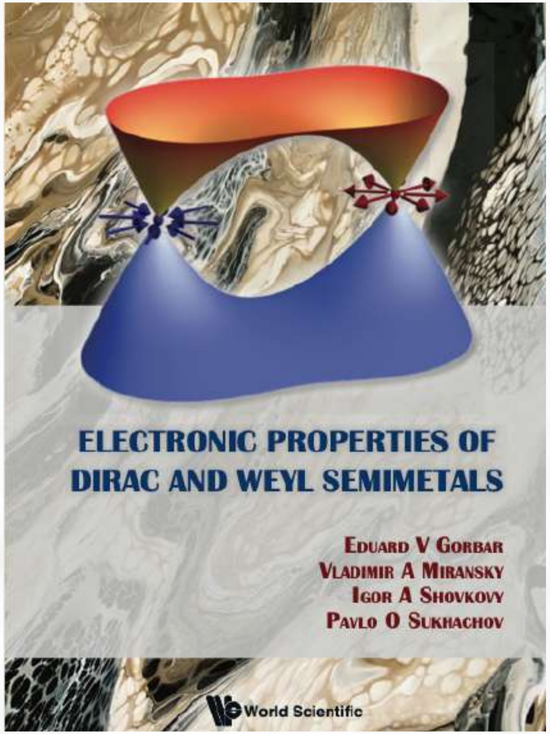
Look inside



IGOR SHOVKOVY

PAVLO SUKHACHOV

DIRAC & WEYL SEMIMETALS

Real band structures

- Electron quasiparticles with a wide range of properties are possible

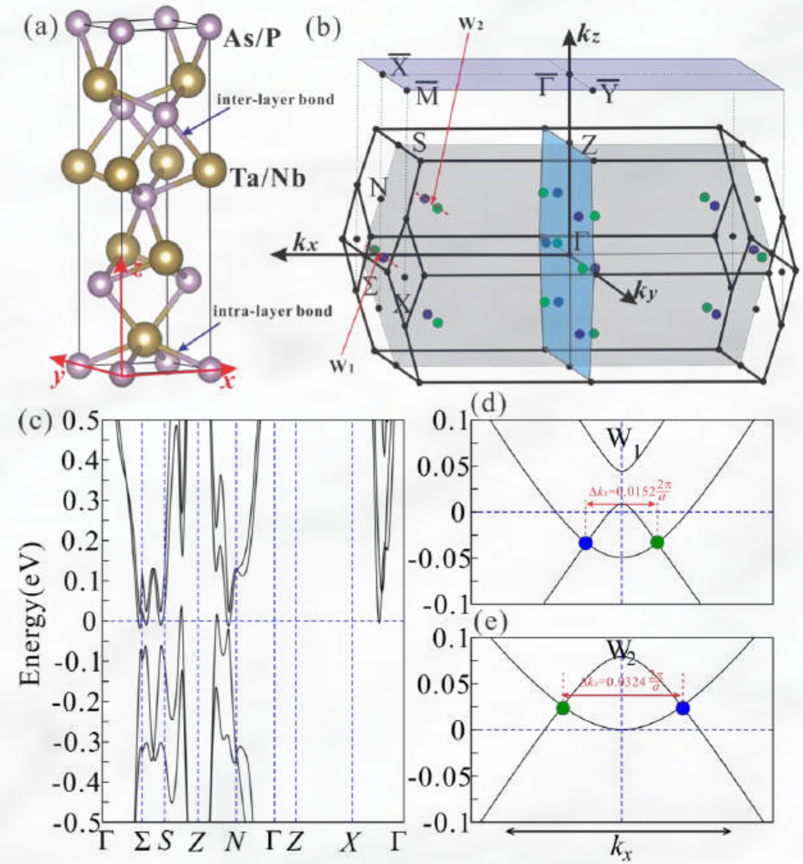
- They may even have an emergent spinor structure of Dirac/Weyl fermions, e.g.,

$$H_W \approx v_F(\vec{\boldsymbol{k}} \cdot \vec{\sigma})$$

where $\vec{\boldsymbol{k}}$ is the momentum measured from the Weyl node and v_F is the Fermi velocity

- How likely/common is this?

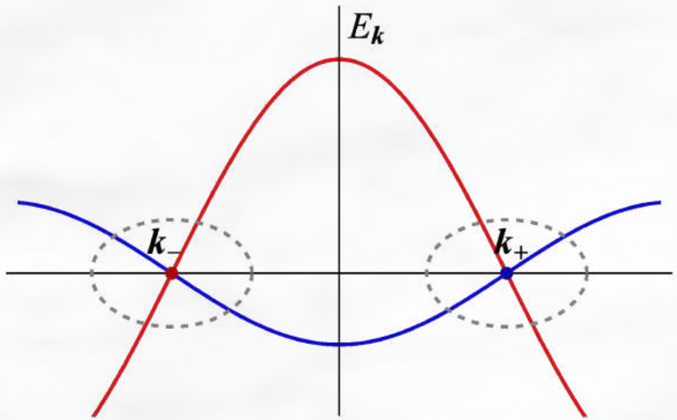
Weyl semimetals TaAs, TaP, NbAs, and NbP

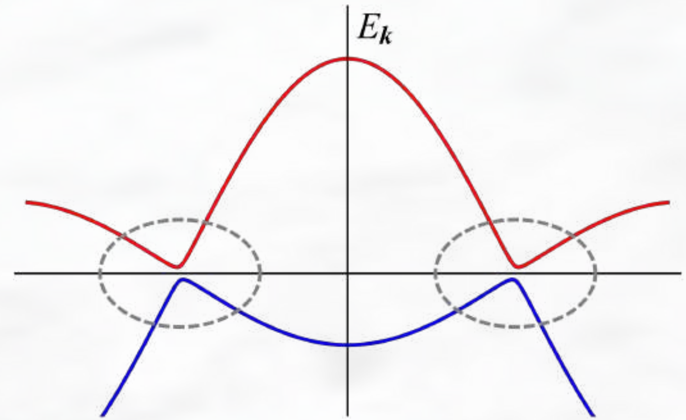


Sun, Wu & Yan, Phys. Rev. B **92**, 115428 (2015)

Relativistic-like band crossing

Do energy levels cross? Or do they repel?





A generic 2-band Hamiltonian reads

$$H_{\boldsymbol{k}} = a_{\boldsymbol{k}} + \vec{b}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{\sigma}} \quad \Longrightarrow \quad E_{\boldsymbol{k}} = a_{\boldsymbol{k}} \pm \sqrt{\left(\vec{b}_{\boldsymbol{k}}\right)^2}$$

The bands cross when [Witten, Riv. Nuovo Cimento **39**, 313 (2016)]

$$\vec{b}_{\boldsymbol{k}}=0$$

These 3 equations can be solved by adjusting $\vec{\boldsymbol{k}}$ in 3D

Emergent chirality in solids

Near a band crossing (e.g., $\vec{k} \approx \vec{k}_+$)

$$H_{\boldsymbol{k}} = a_{\boldsymbol{k}_+} + \cancel{\left(\vec{\nabla}_{\boldsymbol{k}} a_{\boldsymbol{k}} \cdot \delta\vec{\boldsymbol{k}}\right)} + \textstyle\sum_{i,j} \sigma_i b_{ij} \delta k_j$$

cone tilting

After an orthogonal transformation

$$b_{ij} \equiv \partial b_i / \partial k_j \rightarrow \hbar v_i \delta_{ij}$$

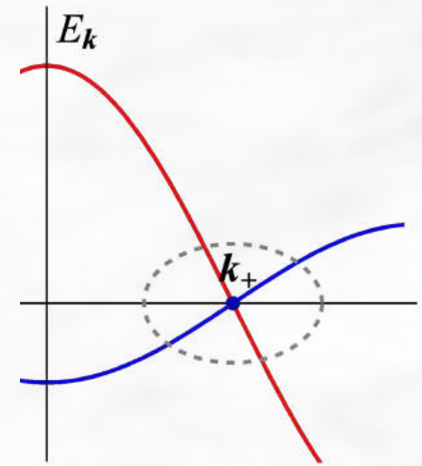
Assuming *isotropy* & a suitable *reference point*,

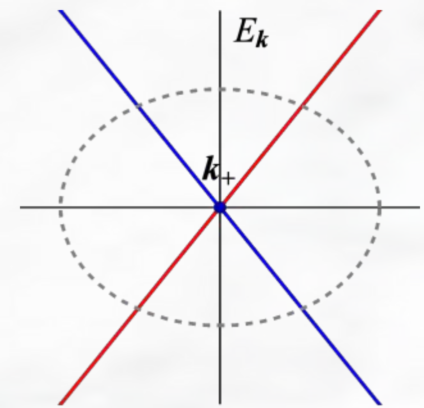
$$H_{\boldsymbol{k}} = \pm v_F \left(\vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{k}}\right)$$

which is the Weyl Hamiltonian

The *chirality* is defined by

$$\lambda = \mathrm{sign}\left[\det\left(b_{ij}\right)\right]$$





Weyl quasiparticles

- The quasiparticle eigenstates for Weyl Hamiltonian $H_\lambda = \lambda v_F(\vec{\boldsymbol{k}} \cdot \vec{\sigma})$ are

$$\psi_{\boldsymbol{k}}^{\lambda} = \frac{1}{\sqrt{2}\sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

- The quasiparticle energy $\epsilon_k = v_F\sqrt{k_x^2 + k_y^2 + k_z^2}$ is relativistic-like

- Mapping $k \to \psi_k^\lambda$ has a nontrivial topology

- Consider adiabatic evolution of the wave function from $\psi_{\boldsymbol{k}}$ to $\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}$:

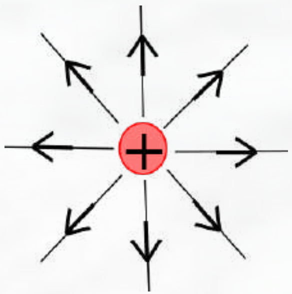
$$\langle \psi_{\boldsymbol{k}} | \psi_{\boldsymbol{k}+\delta\boldsymbol{k}} \rangle \approx 1 + \delta\boldsymbol{k} \cdot \langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle \approx e^{i\boldsymbol{a}_{\boldsymbol{k}} \cdot \delta\boldsymbol{k}}$$

where $\boldsymbol{a}_{\boldsymbol{k}} = -i\langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle$ is the Berry connection

Berry curvature & topology

- For Weyl eigenstates, the Berry curvature is

$$\boldsymbol{\Omega}_k \equiv \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}} = \lambda \frac{\vec{\boldsymbol{k}}}{2k^3}$$

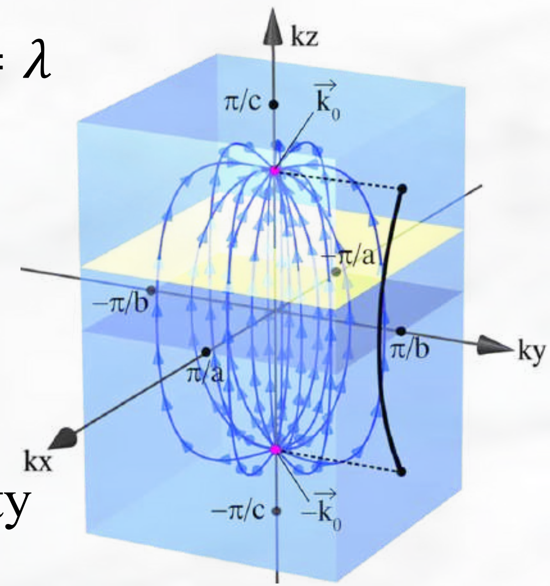


- The Chern number (topological charge)

$$C = \frac{1}{2\pi} \oint \boldsymbol{\Omega}_k \cdot d\boldsymbol{S}_{\boldsymbol{k}} = \frac{\lambda}{2\pi} \oint \frac{\vec{\boldsymbol{k}}}{2k^3} \cdot \frac{\vec{\boldsymbol{k}}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda$$

- In a solid state material, the Brillouin zone is compact
- A closed surface around a node at $\vec{\boldsymbol{k}}_{\mathbf{0}}$ is also a closed surface around the rest of the Brillouin zone
- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

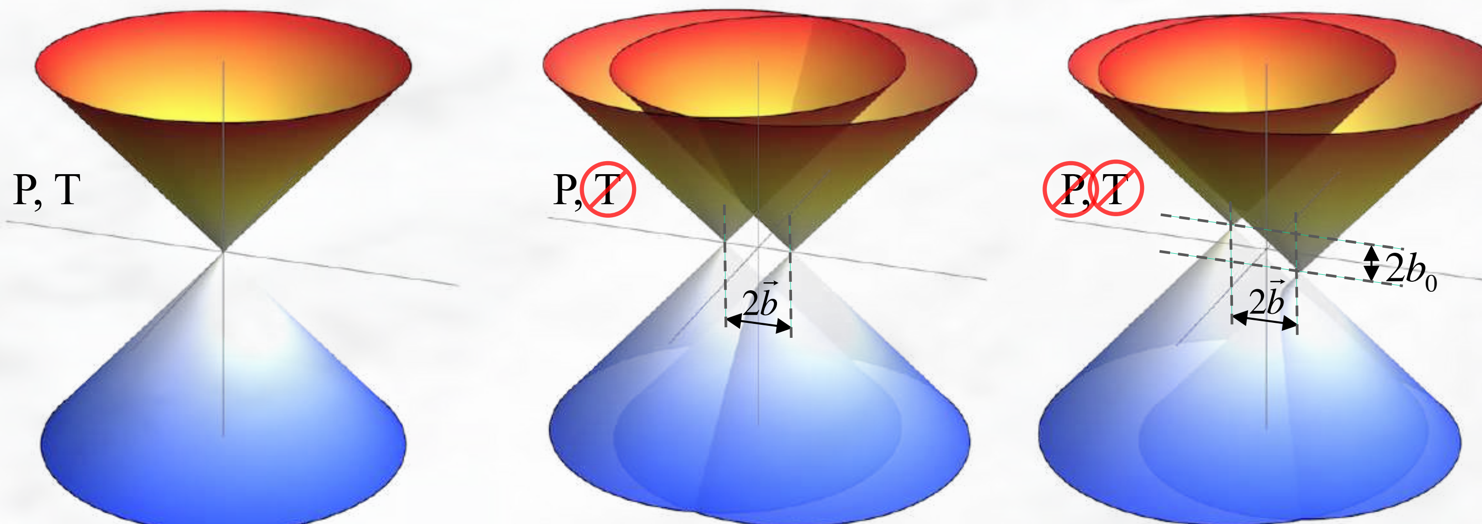
Idealized Dirac and Weyl model

- Low-energy Hamiltonians of a Dirac and Weyl materials

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5+b_0\gamma^0\gamma^5\right]\psi$$

T P

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) Weyl (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



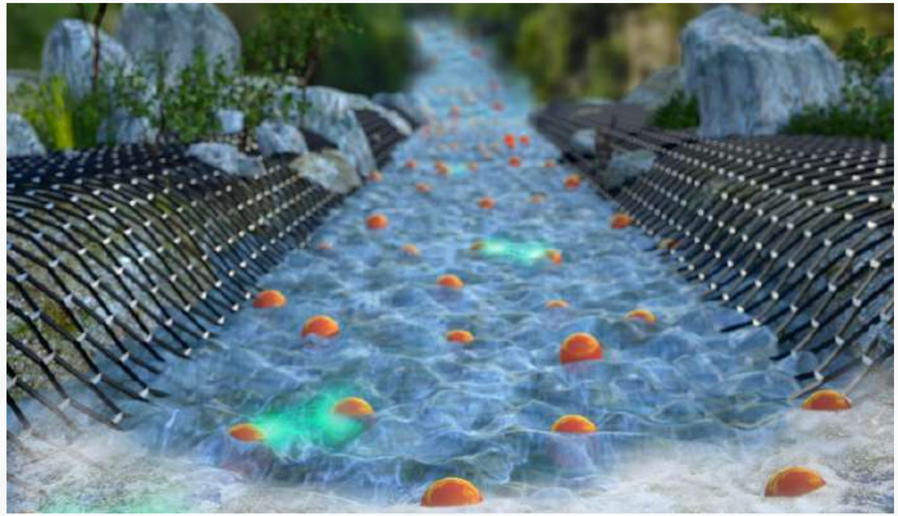
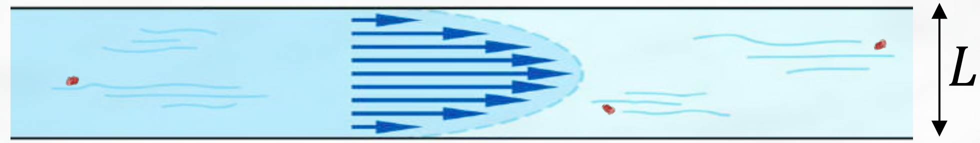


Image credit: Ryan Allen and Peter Allen, Second Bay Studios

ELECTRON HYDRODYNAMICS

Electron hydrodynamics

- First ideas date back to the 1960s [Radii Gurzhi, JETP **17**, 521 (1963)]



Ballistic:

$l_p \gg l_{ee} \gg L$

$R \sim L^{-1}$

Hydro:

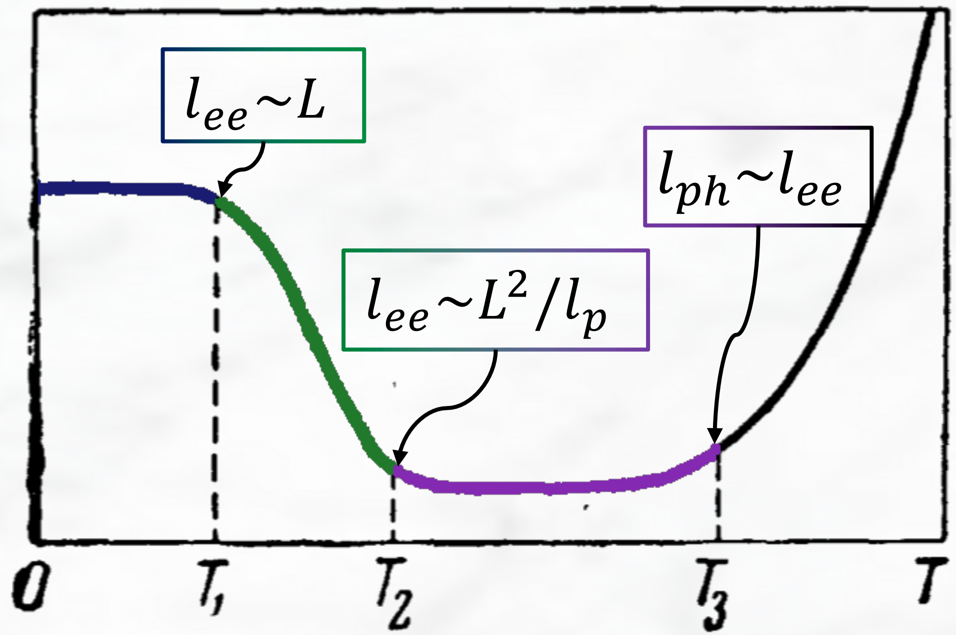
$l_{ee} \ll L \ll l_p$

$R \sim l_{ee}/\mathrm{L}^2 \sim T^{-5} L^{-2}$

Hydro + impurities:

$l_p \ll L^2/l_{ee}$

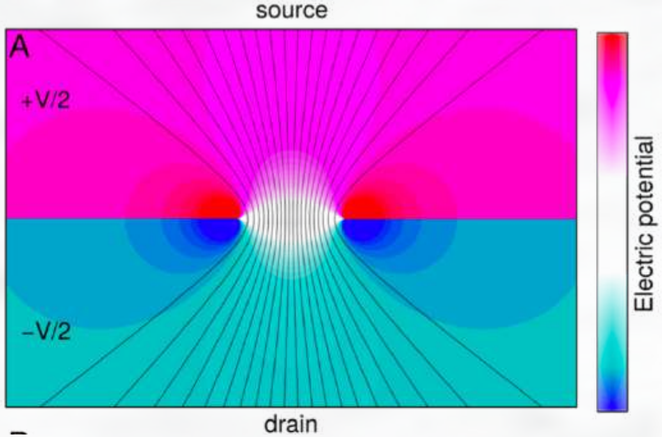
$R \sim l_p$



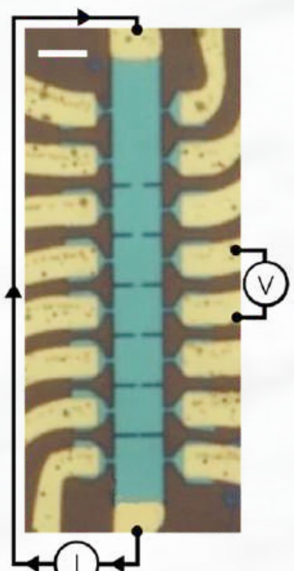
Ohmic ($l_{ph} \ll l_p$): $R \sim T^5$

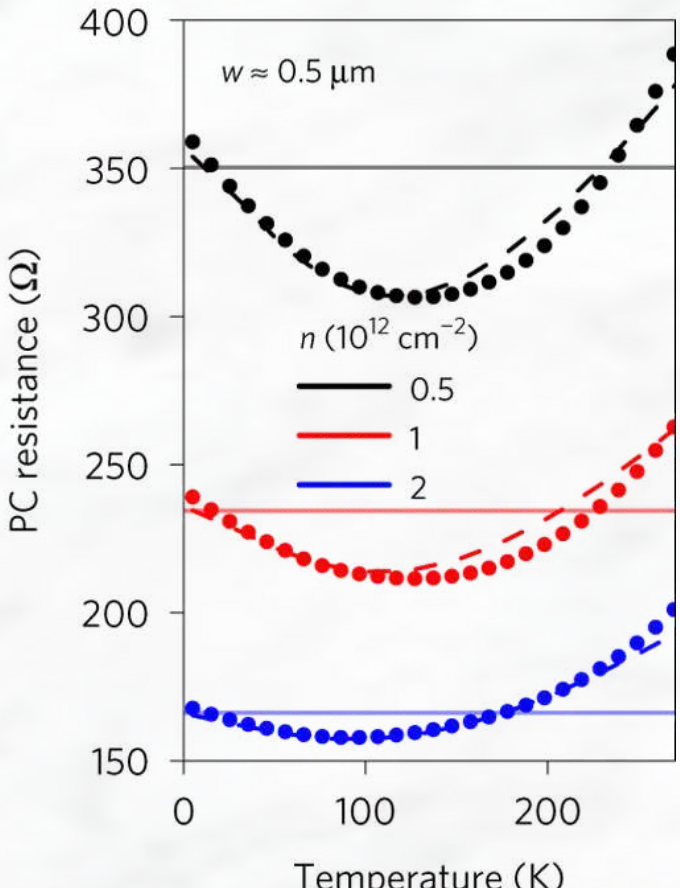
Higher than ballistic transport

[Guo et al., PNAS USA **114**, 3068 (2017)]



[Kumar et al., Nat. Phys. **13**, 1182 (2017)]





Other Signatures:

- **Negative nonlocal resistance**

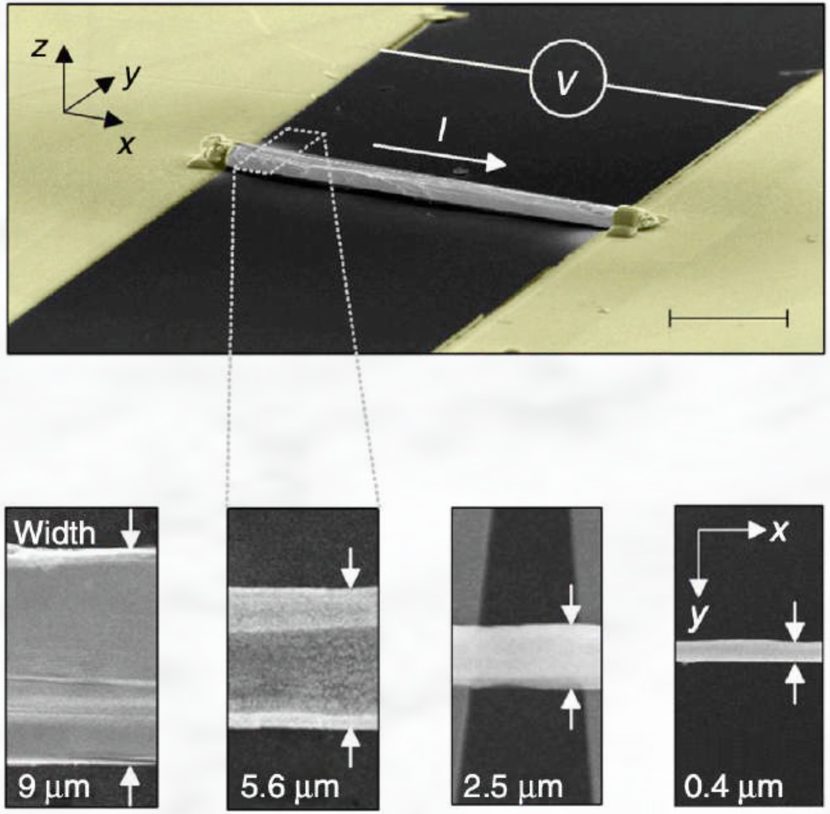
[Torre et al., Phys. Rev. B **92**, 165433 (2015)]
[Bandurin et al., Science **351**, 1055 (2016)]
[Pellegrino et al., Phys. Rev. B **94**, 155414 (2016)]
[Levitov & Falkovich, Nat. Phys. **12**, 672 (2016)]

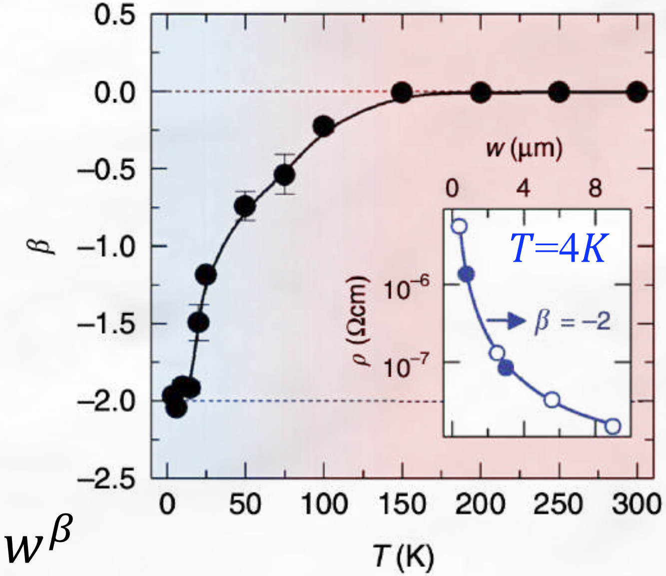
- **Visualization of the Poiseuille flow**

[Sulpizio et al., Nature **576**, 75 (2019)]

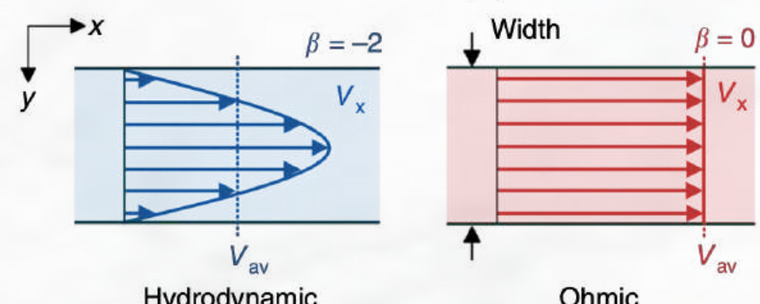
Hydrodynamics in Weyl semimetals

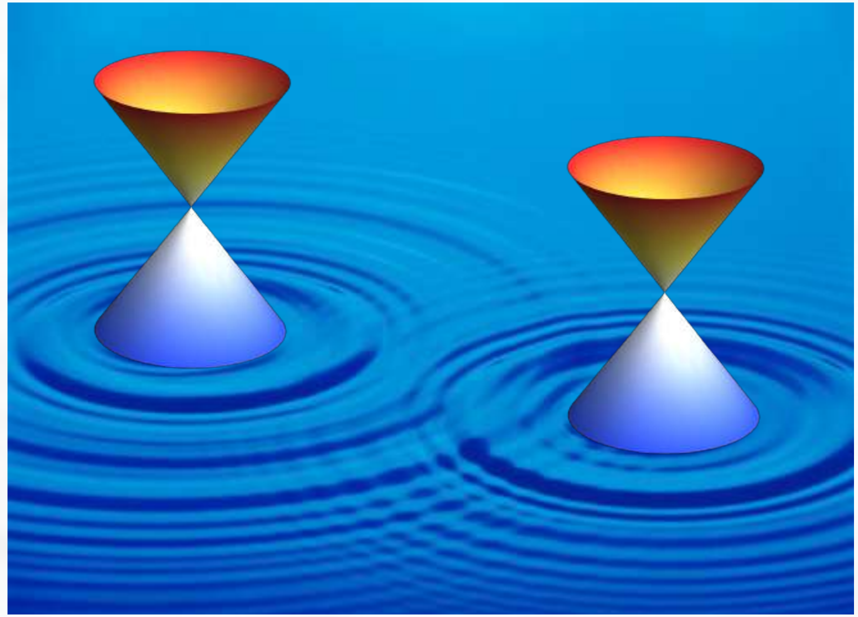
Weyl semimetals WP_2 & WTe_2 [Gooth et al., Nat. Comm. **9,** 4093 (2018); Vool, et al., arXiv:2009.04477]





$$\rho = \rho_0 + \rho_1 w^{\beta}$$





RELATIVISTIC-LIKE ELECTRON HYDRODYNAMICS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

Chiral Hydrodynamics (plasma)

- Evolution of conserved quantities:

$$\frac{\partial T^{j0}}{\partial t} + \frac{\partial T^{ji}}{\partial x^i} = -enE^j - e\left(\vec{j}\times\vec{B}\right)^j + F^j_{\text{other}}$$

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{0i}}{\partial x^i} = -e\left(\vec{E}\cdot\vec{j}\right) + W_{\text{other}}$$

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

$$\frac{\partial n_5}{\partial t} + \vec{\nabla}\cdot\vec{j}_5 = -\frac{e^2\left(\vec{B}\cdot\vec{E}\right)}{2\pi^2} - \Gamma_5 n_5$$

$\oplus$ Maxwell equations

Note:

$T^{00} = \varepsilon + \cdots$

$T^{0i} = wv^i + \cdots$

$T^{ij} = wv^i v^j - P\delta^{ij} + \cdots$

$w = \varepsilon + P$

Constitutive relations

- Expressions for currents and $T^{\mu\nu}$

$$\vec{J} = n\vec{v} + \vec{J}_a + \vec{J}_{dis}$$

$$\vec{J}_5 = n_5\vec{v} + \vec{J}_{5,a} + \vec{J}_{5,dis}$$

$$T^{\mu\nu} = \varepsilon v^\mu v^\nu - \Delta^{\mu\nu} P + h^\mu v^\nu + v^\mu h^\nu + \tau_{dis}^{\mu\nu}$$

- Anomalous terms:

$$\vec{J}_a = \vec{J}_{CS} + \sigma_\omega \vec{\omega} + \sigma_B \vec{B} \quad \& \quad \vec{J}_{5,a} = \vec{J}_{5,CS} + \sigma_\omega^5 \vec{\omega} + \sigma_B^5 \vec{B}$$

where

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma_\omega^5 = \frac{3(\mu^2+\mu_5^2)+\pi^2 T^2}{6\pi^2\hbar^2}, \qquad \sigma_B^5 = \frac{e\mu}{2\pi^2\hbar^2}$$

Hydrodynamics in Weyl metals

- The Euler equation from CKT:

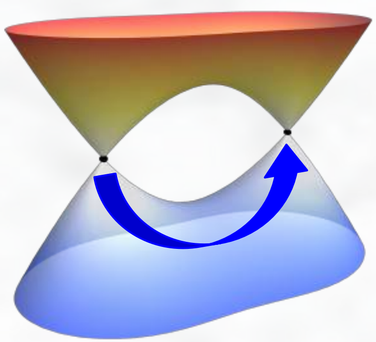
$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right)=-en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_{\mathbf{r}})$$

- The energy conservation from CKT

$$\partial_t\epsilon=-\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_{\mathbf{r}})$$

$\oplus$ Maxwell equations

- One must include topological Chern-Simons currents and densities,



$$\rho_{\mathrm{CS}}=-\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\mathrm{CS}}=-\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

ANOMALOUS HYDRO MODES

[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

Collective modes

- Local values of $\delta\mu$, $\delta\mu_5$, δT, $\delta\vec{u}$, etc. oscillate
- Seek solutions of linearized equations as plane waves,

$$\delta\mu = \delta\mu_0 \exp(-i\omega t + i\vec{k}\cdot\vec{r}), \text{ etc.}$$

- Account for all constitutive relations, e.g.,

$$\delta\rho = -e\delta n + \frac{(\mathbf{B}_0\cdot\delta\mathbf{u})\,\sigma^{(B)}}{3v_{\rm F}^2} + i\frac{5c^2\sigma^{(\epsilon,u)}(\mathbf{B}_0\cdot[\mathbf{k}\times\delta\mathbf{u}])}{2v_{\rm F}^2} - \frac{e^3(\mathbf{b}\cdot\delta\mathbf{B})}{2\pi^2\hbar^2c^2},$$

$$\delta\rho_5 = -e\delta n_5 + \frac{(\mathbf{B}_0\cdot\delta\mathbf{u})\,\sigma_5^{(B)}}{3v_{\rm F}^2},$$

$$\delta\mathbf{J} = -en_0\delta\mathbf{u} + \mathbf{B}_0\delta\sigma^{(B)} + \frac{e^3\,[\mathbf{b}\times\delta\mathbf{E}]}{2\pi^2\hbar^2c} + \frac{i}{2}\sigma^{(V)}[\mathbf{k}\times\delta\mathbf{u}] - \frac{1}{4}\sigma^{(\epsilon,V)}\,[\mathbf{k}\times[\mathbf{k}\times\delta\mathbf{u}]],$$

$$\delta\mathbf{J}_5 = -en_{5,0}\delta\mathbf{u} + \sigma_5^{(B)}\delta\mathbf{B} + \mathbf{B}_0\delta\sigma_5^{(B)} + \frac{i}{2}\sigma_5^{(V)}[\mathbf{k}\times\delta\mathbf{u}] - \frac{1}{4}\sigma_5^{(\epsilon,V)}\,[\mathbf{k}\times[\mathbf{k}\times\delta\mathbf{u}]].$$

Rich spectrum of hydro modes

- One example: longitudinal anomalous Hall wave (with $\boldsymbol{k} \parallel \boldsymbol{B}_0$ and $\boldsymbol{b} \perp \boldsymbol{B}_0$):

$$\omega_{\mathrm{lAHW},\pm} \approx \pm \frac{\hbar B_0 k_{\parallel} \sqrt{3 v_F^3 \left(\pi^3 c^4 \hbar T_0^2 + 3 e^4 \mu_m v_F^3 b_\perp^2\right)}}{c T_0 \sqrt{\pi^3 \mu_m \left(3 \varepsilon_e v_F^3 \hbar^3 B_0^2 + 4\pi e^2 T_0^2 b_\perp^2\right)}} + O(k_\parallel^3)$$

n continuity equation

$$\frac{T^2 \omega}{3 v_F^3 \hbar} \delta\mu + \frac{e B_0 k_\parallel}{2\pi^2 c} \delta\mu_5 = 0$$

n_5 continuity equation

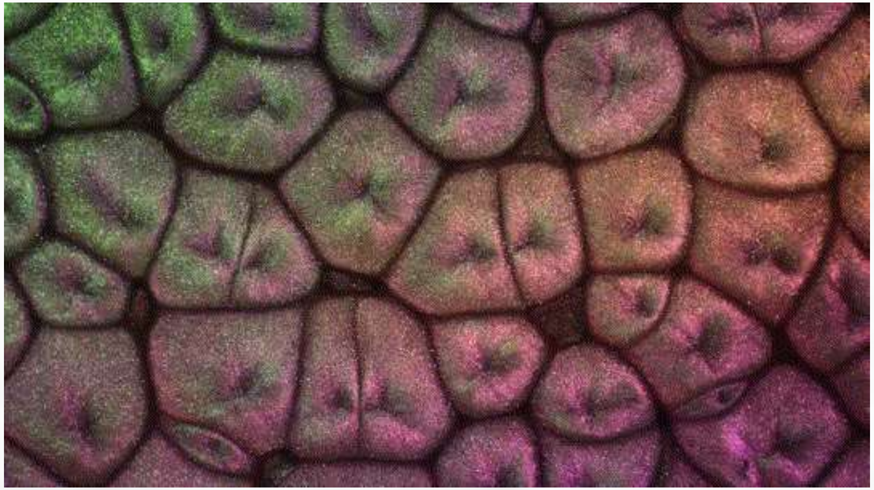
$$\frac{e B_0 k_\parallel}{2\pi^2 c} \delta\mu + \frac{T^2 \omega}{3 v_F^3 \hbar} \delta\mu_5 - i \frac{e^2 B_0}{2\pi^2 c} \delta E_\parallel = 0$$

$$\varepsilon_e \omega \delta E_\parallel + \mathrm{i} \frac{2e^2}{\pi c \hbar^2} \left(B_0 \delta\mu_5 + e b_\perp \delta\tilde{E}_\perp\right) = 0$$

$$\left(\omega^2 - \frac{c^2 k_\parallel^2}{\varepsilon_e \mu_m}\right) \delta\tilde{E}_\perp - \mathrm{i} \frac{2 e^3 \omega b_\perp}{\pi c \varepsilon_e \hbar^2} \delta E_\parallel = 0$$

Maxwell's equations

$\delta\tilde{\boldsymbol{E}}_\perp \parallel [\boldsymbol{B}_0 \times \boldsymbol{b}]$

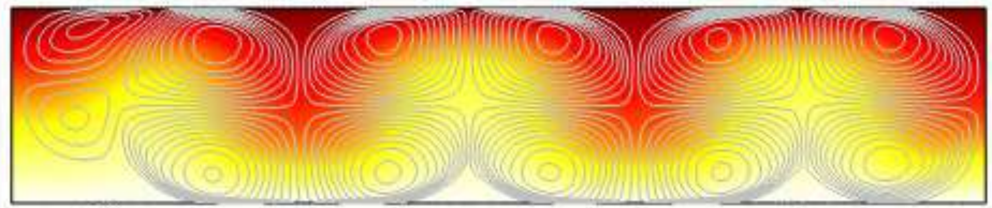

[Image credit: G. Kelemen, https://vimeo.com/233457120]

IS CONVECTION POSSIBLE?

[Sukhachov, Gorbar, Shovkovy, arXiv:2103.1583]

Motivation

- Convection is a signature effect of hydrodynamics



[Image credit: P. Sukhachov]

- Convection can have important applications
  - Efficient heat transfer, measured by Nusselt number

$$Nu = \frac{Q_{conv}}{Q_{cond}}$$

(1-10 for laminar, while 100-1000 for turbulent flow)

- Rayleigh number

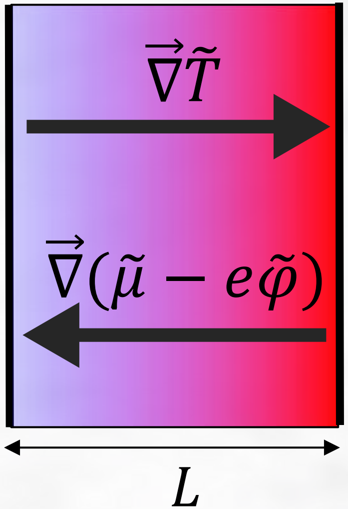
$$Ra = \frac{t_{diff}}{t_{conv}} = \frac{L^2/\kappa}{\eta/(gL\delta\varrho)}$$

Thermal conductivity helps heat transfer

Viscous drag slows convection

Pre-convection: Steady state

- Ansatz: $T \approx T_0 + \tilde{T}$, $\mu \approx \mu_0 + \tilde{\mu}$, $E \approx \tilde{E}$



$$\nabla \tilde{P} = -en_0 \tilde{\mathbf{E}},$$

$$\sigma\left(\nabla \cdot \tilde{\mathbf{E}}\right) + \frac{\sigma}{e}\Delta\left(\tilde{\mu} - \frac{\mu_0}{T_0}\tilde{T}\right) = 0,$$

$$\nabla \cdot \tilde{\mathbf{E}} = -4\pi e\tilde{n},$$

- Solution: $\tilde{E}$ and $\tilde{n}$ are very small in the bulk of the sample

$$\tilde{E} \approx \frac{4\pi e\delta T\left(n_0\partial_T n - s_0\partial_\mu n\right)}{Ln_0 q_{TF}^2} \ll \frac{\Delta\varphi}{\mathrm{L}}$$

$$\tilde{n} \approx 0$$

- Screening effects are strong already in steady state

Criterion of convective instability

- Ansatz: $T \approx T_0 + \tilde{T} + T_u$, where $T_u \simeq C_T e^{i(\vec{k}_\perp \cdot \vec{r}_\perp) + i k_x x}$, etc.

$$\nabla P_u - \eta \Delta \mathbf{u} - \left(\zeta + \frac{\eta}{d}\right) \nabla (\nabla \cdot \mathbf{u}) = -\frac{w_0 \mathbf{u}}{v_F^2 \tau} - e n_0 \mathbf{E}_u - e n_u \tilde{\mathbf{E}},$$

$$(\mathbf{u} \cdot \nabla) \tilde{w} + w_0 (\nabla \cdot \mathbf{u}) = 0,$$

$$-e n_0 (\nabla \cdot \mathbf{u}) + \sigma (\nabla \cdot \mathbf{E}_u) + \frac{\sigma}{e} \Delta \left(\mu_u - \frac{\mu_0}{T_0} T_u \right) = 0,$$

$$\nabla \cdot \mathbf{E}_u = -4\pi e n_u.$$

- Nontrivial solution exists when

$$\mathrm{Ra} = L^4 \frac{\left(k_\perp^2 + k_x^2\right)\left(k_\perp^2 + k_x^2 + \lambda_G^{-2}\right)\left(k_\perp^2 + k_x^2 + q_{\mathrm{TF}}^2\right)}{k_\perp^2}$$

where the Rayleigh number is defined by

$$\mathrm{Ra} = L^4 \frac{e^3 n_0 \tilde{E} (\partial_x \tilde{w}) T_0}{\sigma w_0^2 \eta} \left[n_0 (\partial_T n) - s_0 (\partial_\mu n) \right]$$

Phase diagram

- Note, $k_{x,\min} \sim \pi/L$
- When $\lambda_G \to \infty$ & $q_{TF} \to 0$, the minimal Ra needed is

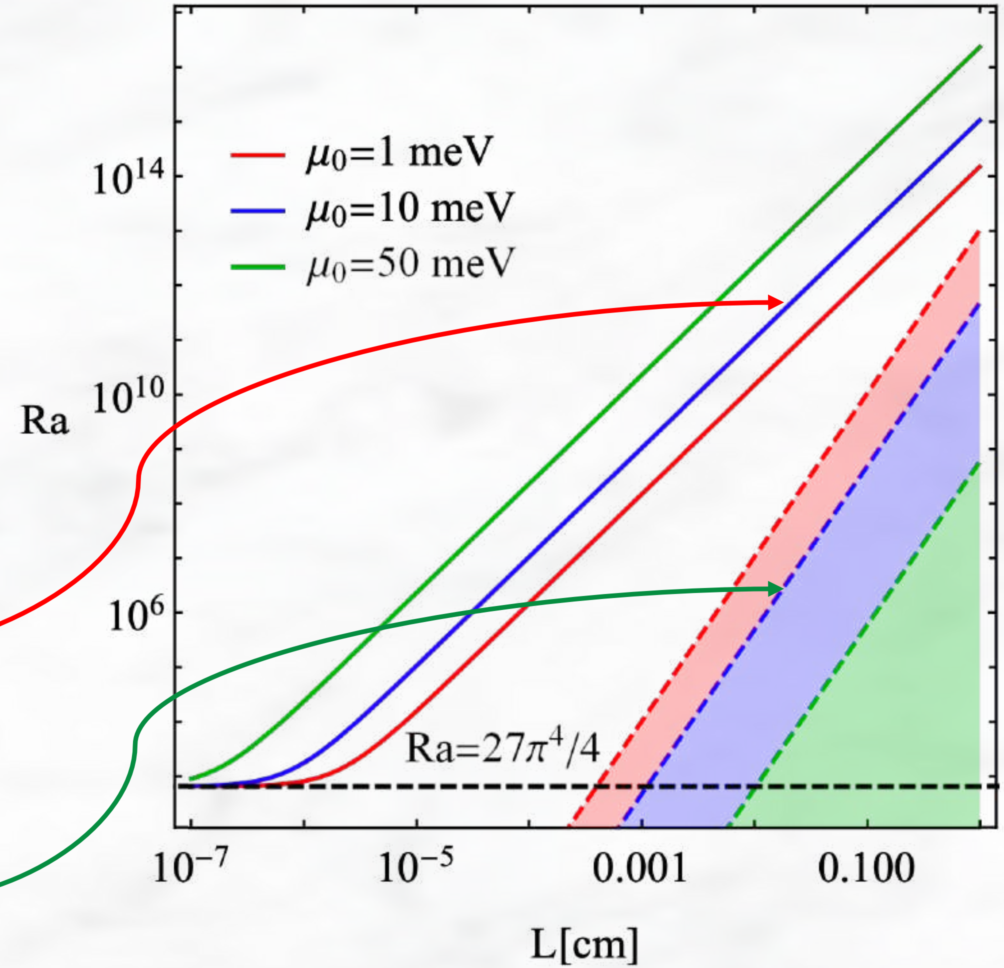
$$\mathrm{Ra}_{\min} = \frac{27\pi^4}{4} \approx 657.5$$

- For finite λ_G & nonzero q_{TF}

$$\boxed{\mathrm{Ra}_{\min} > \mathrm{Ra}_0 = \frac{L^4 q_{TF}^2}{\lambda_G^2}}$$

- Realistically achievable

$$\boxed{\mathrm{Ra} \approx 4 \times 10^9 \frac{\delta T}{T_0} \tilde{E} \left[\frac{\mathrm{V}}{\mathrm{m}}\right] L^3[\mathrm{cm}]}$$



Convection in graphene?

- Coulomb screening is much weaker
- In the "gradual channel" approximation

$$\vec{E}_u = \frac{e}{C}\vec{\nabla}n_u$$

where $C = \varepsilon/(4\pi L_g)$ is the capacitance per unit area

- Convection appears when

$$\mathrm{Ra} = L^4 \frac{\left(k_\perp^2 + k_x^2\right)^2 \left(k_\perp^2 + k_x^2 + \lambda_G^{-2}\right)\left(1 + Q^2\right)}{k_\perp^2}$$

where $Q = \sqrt{e^2(\partial_\mu n)/C}$

- For realistic parameters, convection can occur when

$$L \gtrsim 1 \text{ cm}$$

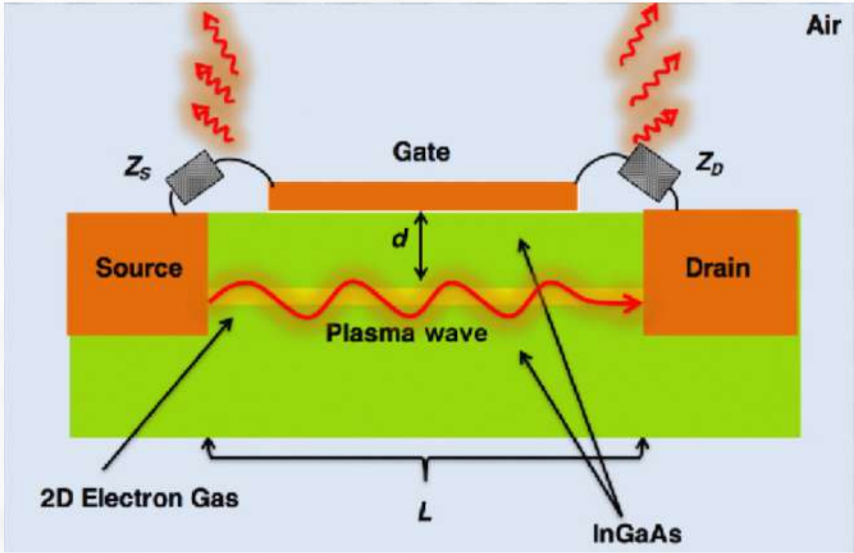


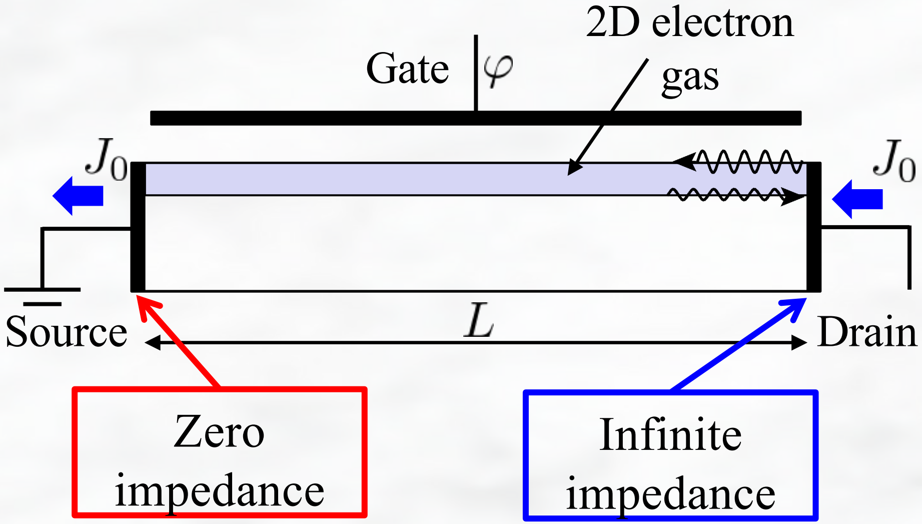
Image credit: Nafari, Aizin, Jornet, Phys. Rev. Applied **10**, 064025 (2018)

ENTROPY WAVE INSTABILITY

[Sukhachov, Gorbar, Shovkovy, arXiv:2106.11992]

Dyakonov-Shur instability

[Dyakonov, Shur, Phys. Rev. Lett. **71**, 2465 (1993)]



$$\delta\varphi(x=0)=0 \qquad \delta J(x=L)=0$$

$$\mathrm{Re}\,[\omega]=\frac{\left|v_p^2-u_0^2\right|}{v_p}\frac{\pi l}{2L}$$

$$\mathrm{Im}\,[\omega]=\frac{v_p^2-u_0^2}{2Lv_p}\ln\left|\frac{v_p+u_0}{v_p-u_0}\right|$$

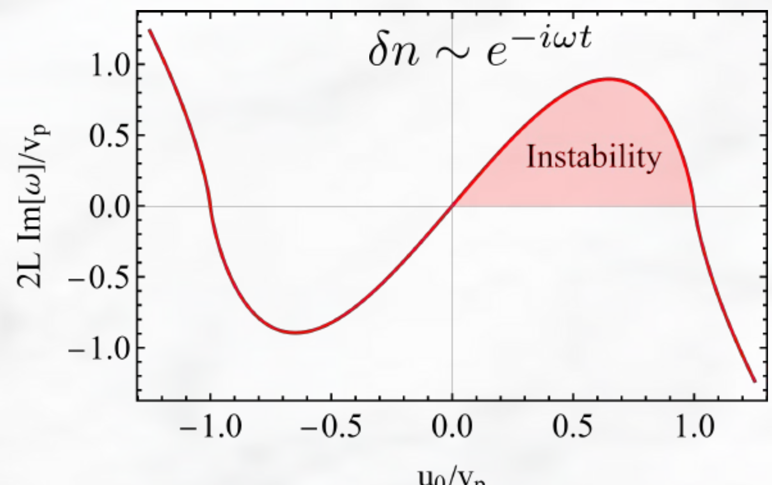
$$J_0=-enu_0$$

$$\partial_t u+u\partial_x u=-\frac{e}{m}\partial_x\varphi$$

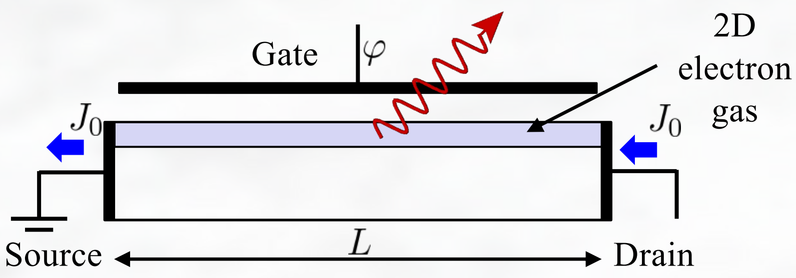
$$\partial_t\varphi+\partial_x(\varphi u)=0$$

Gradual channel approximation:

$$n\propto\varphi$$



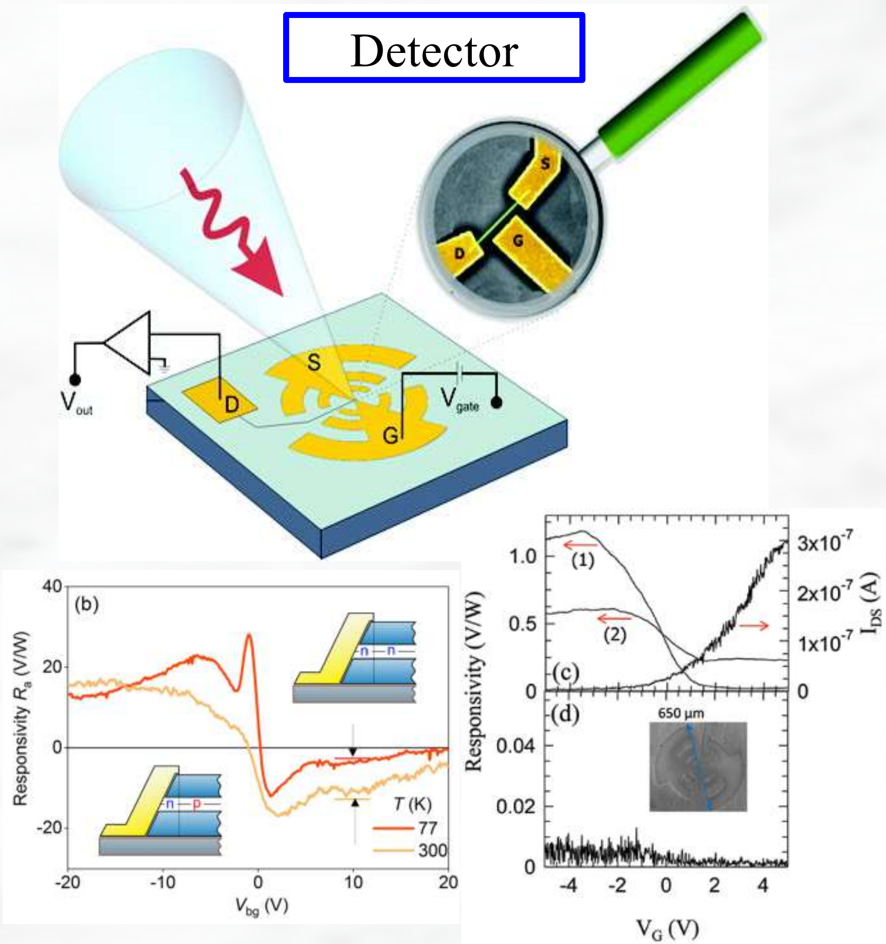
Observation & application



$\omega \sim$ THz

Experimental observation:

- ☒ Emission
- ☑ Detection



[Tauk, et al., Appl. Phys. Lett. **89**, 253511 (2006)]
[Vitiello, et al., Nano Lett. **12**, 96 (2012)]
[Vicarelli, et al., Nat. Mater. **11**, 865 (2012)]
[Giliberti, et al., Phys. Rev. B **91**, 165313 (2015)]
[Bandurin, et al., Appl. Phys. Lett. **112**, 141101 (2018)]

[Vitiello, et al., Nano Lett. **12**, 96 (2012)]

Relativistic-like case

- System of equations

$$\frac{1}{v_F^2}\left[\partial_t + (\mathbf{u}\cdot\boldsymbol{\nabla})\right](\mathbf{u}w) + \frac{1}{v_F^2} w\mathbf{u}(\boldsymbol{\nabla}\cdot\mathbf{u}) = -\boldsymbol{\nabla}P + en\boldsymbol{\nabla}\varphi + \eta\Delta\mathbf{u} + \frac{\eta}{d}\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\mathbf{u}\right) - \frac{w\mathbf{u}}{v_F^2\tau},$$

$$-e\partial_t n + (\boldsymbol{\nabla}\cdot\mathbf{J}) = 0,$$

$$\partial_t\epsilon + (\boldsymbol{\nabla}\cdot\mathbf{J}^{\epsilon}) = (\mathbf{E}\cdot\mathbf{J}),$$

$$\Delta\varphi = 4\pi e\,(n - n_0).$$

- Collective modes in an infinite system:

$$\text{3D:}\quad \omega_{\pm} \approx \pm\sqrt{\omega_p^2 + v_s^2 k_x^2} + \frac{2}{3}u_0 k_x,$$

$$\text{2D:}\quad \omega_{\pm} \approx \pm v_p k_x + \frac{1}{2}u_0 k_x,$$

$$\text{2D and 3D:}\quad \omega_e \approx u_0 k_x.$$

Plasmons

Entropy wave

where $v_s = v_F/\sqrt{d}$ and $\omega_p^2 = 4\pi e^2 n_0^2 v_F^2/w_0$

Instability in 3D: analytical results

- Boundary conditions:

$$\begin{aligned} n_1(x=0) &= 0, \\ J_x(x=L) &\equiv n_0 u_1(x=L) + u_0 n_1(x=L) = 0, \\ T_1(x=0) &= 0. \end{aligned}$$

- Frequencies of the collective modes:

$$\omega_{\pm}^{3D} \approx \pm\sqrt{\omega_p^2 + \left[v_s \frac{\pi}{L}\left(l+\frac{1}{2}\right)\right]^2} + i\frac{2u_0}{3L}\left(3-2\Lambda_p^2\right)$$

Plasmon instability

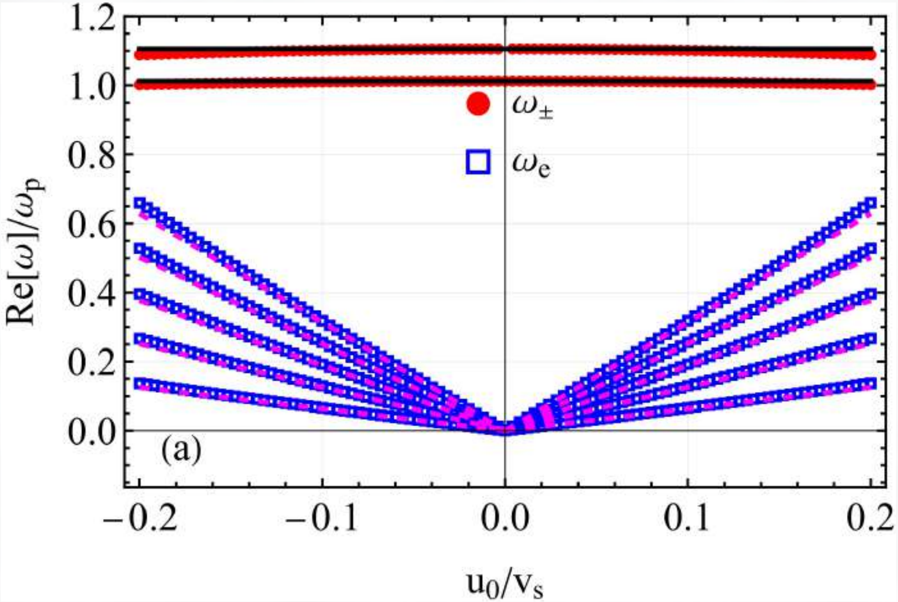
$$\omega_e^{3D} \approx \frac{2\pi l}{L}u_0 - i\frac{u_0\omega_p}{v_s} - i\frac{u_0}{L}\ln\left[\frac{3}{8}\frac{v_s^2}{u_0^2\left(1-\Lambda_p^2\right)}\right]$$

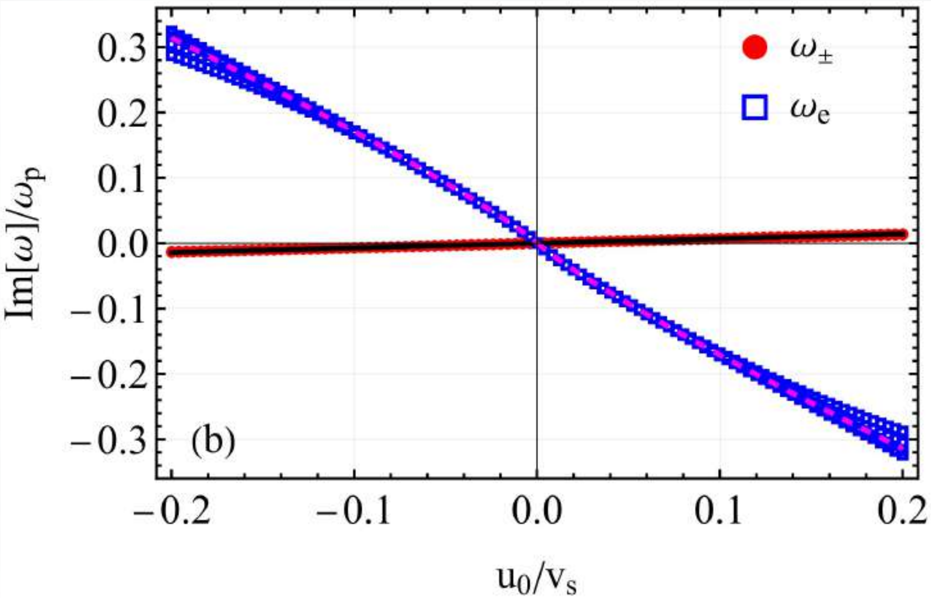
Entropy wave instability

$$l = 0, \pm 1, \pm 2, \ldots$$

$$\Lambda_p = \omega_p/(v_s q_{\mathrm{TF}}) < 1, \qquad \lim_{T\to 0}\Lambda_p = 1.$$

Instability in 3D (numerical)



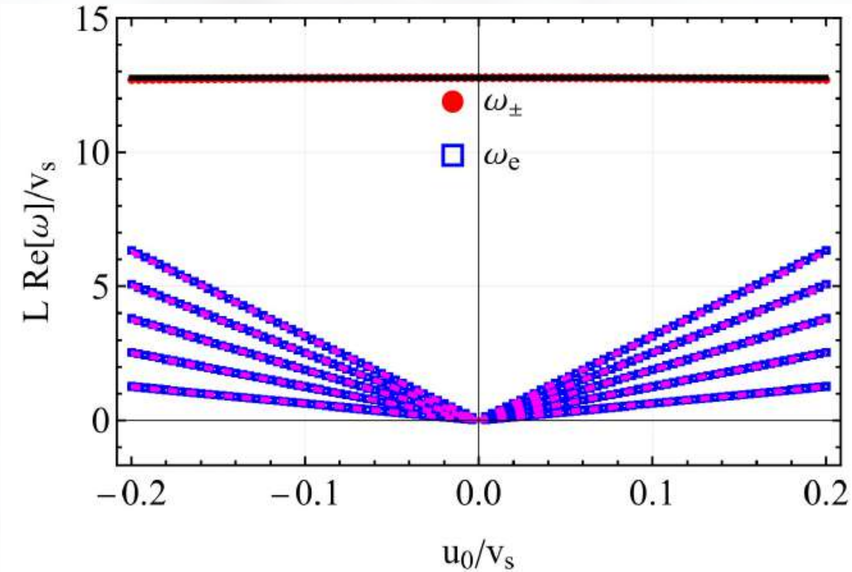


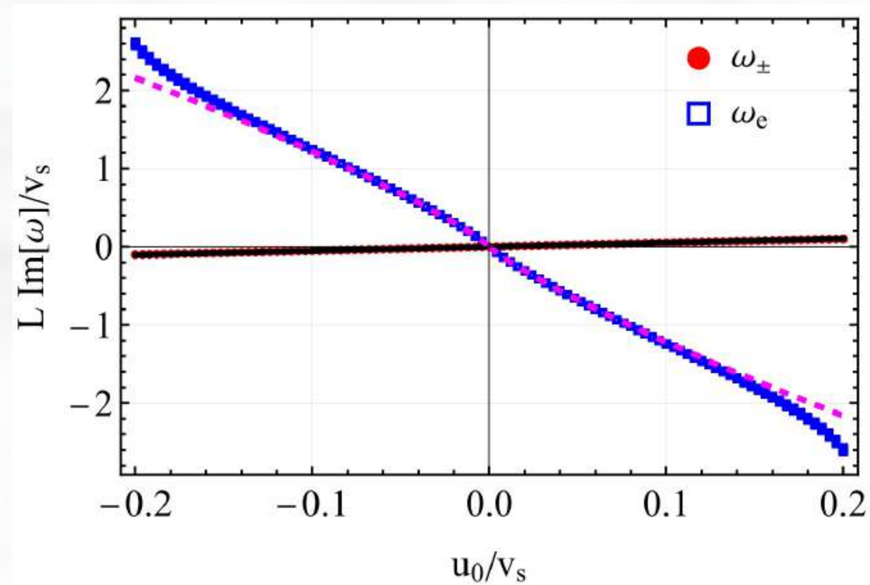
- Plasmon modes: $\mathrm{Re}(\omega_{DS}) \simeq \omega_P$
- Entropy waves: $\mathrm{Re}(\omega_{EW}) \propto u_0$
- $\mathrm{Im}(\omega_{EW}) \gg \mathrm{Im}(\omega_{DS})$
- DSI and EWI occur for opposite sign(u_0)

Instability in 2D (numerical)

[Tomadin, Polini, Phys. Rev. B **88**, 205426 (2013);
Svintsov, et al., Phys. Rev. B **88**, 245444 (2013);
Koseki, et al., Phys. Rev. B **93**, 245408 (2016)]

$$\omega_{\pm}^{2D} \approx \pm v_p \frac{\pi}{L}\left(l + \frac{1}{2}\right) + i\frac{u_0}{2L}\left(4 - 3\Lambda_p^2\right)$$





$$\omega_e^{2D} \approx \frac{2\pi l}{L} u_0 - i\frac{u_0}{L}\ln\left[\frac{2}{3}\frac{v_p^2}{u_0^2\left(1 - \Lambda_p^2\right)}\right]$$

Summary

- Electron hydrodynamics in Dirac/Weyl semimetals is chiral (if realized)

- Chern-Simon currents/densities appear and play role

  [Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

- New anomalous hydrodynamic modes are expected

  [Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

- Convection is impossible due to strong Coulomb effects (3D) and impurities (2D)

  [Sukhachov, Gorbar, Shovkovy, arXiv: 2103.1583]

- Entropy wave instability can develop (signature of relativistic-like nature)

  [Sukhachov, Gorbar, Shovkovy, arXiv:2106.11992]