



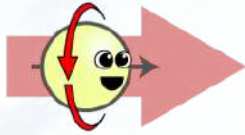
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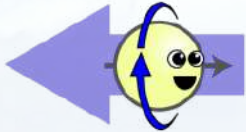
Anomalous chiral matter and all that Igor Shovkovy

Physics Colloquium, UNICAMP, Sep. 28, 2021

- Only *massless* Dirac fermions have a well-defined chirality ($\gamma^5\psi = \pm\psi$):



Right-handed (spin parallel to momentum)

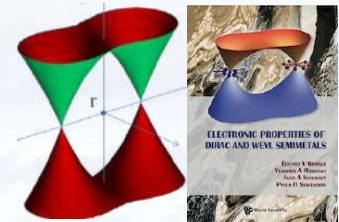
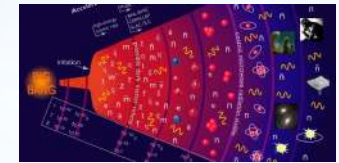
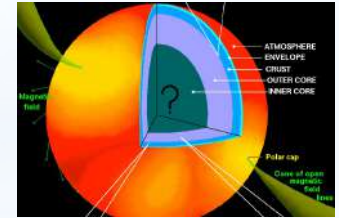
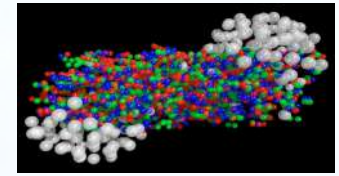


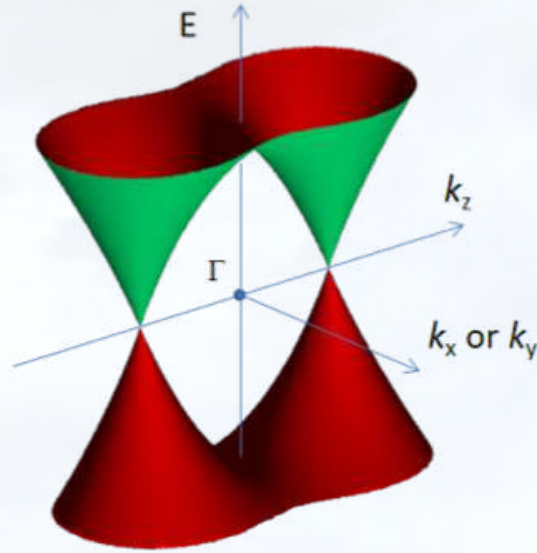
Left-handed (spin opposite to momentum)

- *Massive* Dirac fermions have an *almost* well-defined chirality in the *ultrarelativistic* regime*
 - High temperature: $T \gg m$
 - High density: $\mu \gg m$

*Chirality flip rate is nonzero: $\Gamma_{\text{flip}} \propto \alpha^2 T (m/T)^2$

- **Heavy-ion collisions (high temperature)**
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]
- **Super-dense matter in compact stars (high density)**
- **Early Universe (high temperature)**
- **Magnetospheres of magnetars**
(electron-positron plasma at moderately high temperature)
- **Electron plasma in Dirac/Weyl (semi-)metals**
[Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]
- **Other: cold atoms, superfluid $^3\text{He-A}$, etc.**
[Volovik, JETP Lett. 105, 34 (2017)]

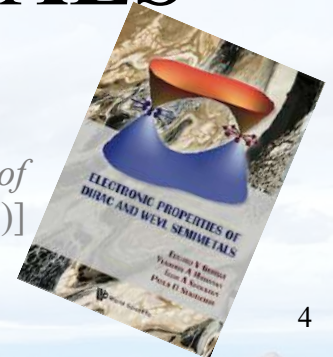




Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL SEMIMETALS

[Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]



Dirac/Weyl fermions

- Electron quasiparticles with a wide range of properties are possible
- They may even have the emergent spinor structure of *massless* Weyl fermions,

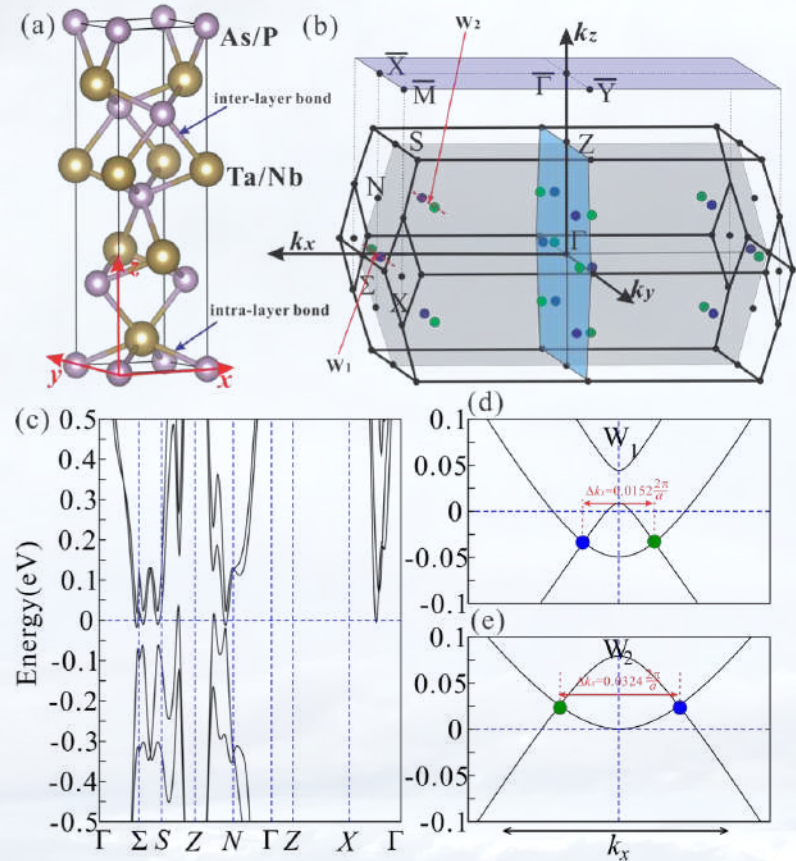
$$H_W \approx \pm v_F (\vec{\sigma} \cdot \vec{k})$$

Such nodes are not uncommon!

Na_3Bi , Cd_3As_2 , ZrTe_5 , TaAs , NbAs , ...

- [Liu et al., Science **343**, 864 (2014)]
- [Neupane et al., Nature Commun. **5**, 3786 (2014)]
- [Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
- [Li et al., Nature Physics **12**, 550 (2016)]
- [S.-Y. Xu et al., Science **349**, 613 (2015)]
- [B. Q. Lv et al., Phys. Rev. X **5**, 031013 (2015)]
- [S.-Y. Xu et al., Nature Physics **11**, 748 (2015)]
- [S.-Y. Xu et al., Science Adv. **1**, 1501092 (2015)]
- [F. Y. Bruno et al., Phys. Rev. B **94**, 121112 (2016)]

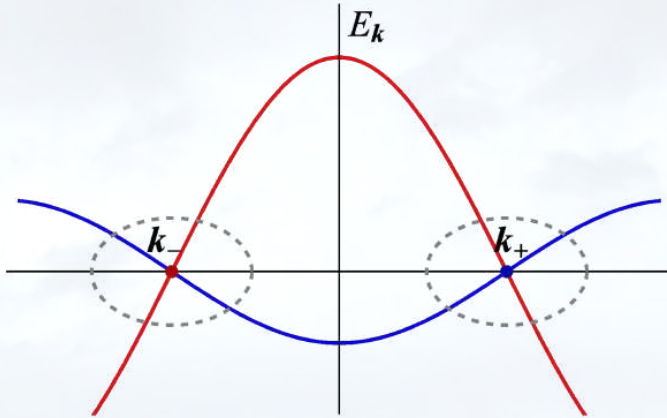
Weyl semimetals TaAs, TaP, NbAs, and NbP



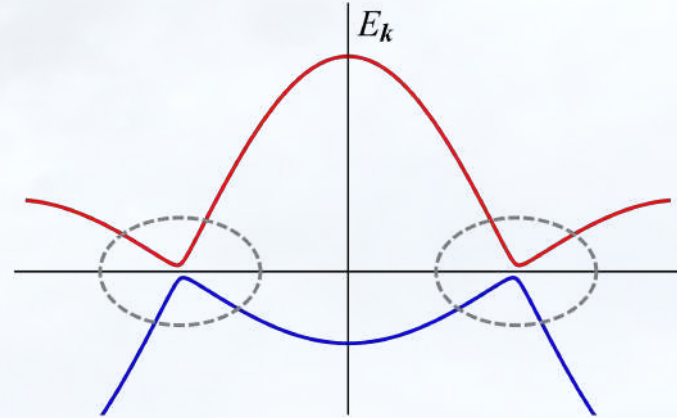
Sun, Wu & Yan, Phys. Rev. B **92**, 115428 (2015)

Relativistic-like band crossing

Do energy levels cross?



Or do they repel?



A generic 2-band Hamiltonian reads

$$H_{\mathbf{k}} = a_{\mathbf{k}} + \vec{b}_{\mathbf{k}} \cdot \vec{\sigma} \quad \Rightarrow \quad E_{\mathbf{k}} = a_{\mathbf{k}} \pm \sqrt{(\vec{b}_{\mathbf{k}})^2}$$

The bands cross when [Witten, Riv. Nuovo Cimento 39, 313 (2016)]

$$\vec{b}_{\mathbf{k}} = 0$$

These 3 equations can be solved by adjusting $\vec{\mathbf{k}}$ in 3D

Emergent chirality in solids

Near a band crossing (e.g., $\vec{k} \approx \vec{k}_+$)

$$H_{\vec{k}} = a_{\vec{k}_+} + \cancel{(\vec{\nabla}_{\vec{k}} a_{\vec{k}} \cdot \delta \vec{k})} + \sum_{i,j} \sigma_i b_{ij} \delta k_j$$

cone tilting

After an orthogonal transformation

$$b_{ij} \equiv \partial b_i / \partial k_j \rightarrow \hbar v_i \delta_{ij}$$

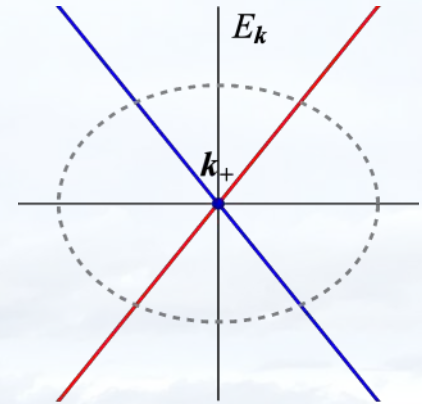
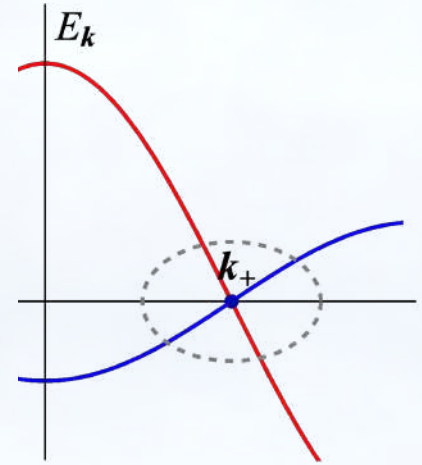
Assuming *isotropy* & a suitable *reference point*,

$$H_{\vec{k}} = \pm v_F (\vec{\sigma} \cdot \vec{k})$$

which is the Weyl Hamiltonian

The *chirality* is defined by

$$\lambda = \text{sign}[\det(b_{ij})]$$



Weyl quasiparticles

- The quasiparticle eigenstates for Weyl Hamiltonian $H_\lambda = \lambda v_F (\vec{k} \cdot \vec{\sigma})$ are

$$\psi_{\mathbf{k}}^\lambda = \frac{1}{\sqrt{2} \sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

- The quasiparticle energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$ is relativistic-like
- Mapping $k \rightarrow \psi_{\mathbf{k}}^\lambda$ has a nontrivial topology
- Consider adiabatic evolution of the wave function from $\psi_{\mathbf{k}}$ to $\psi_{\mathbf{k}+\delta\mathbf{k}}$:

$$\langle \psi_{\mathbf{k}} | \psi_{\mathbf{k}+\delta\mathbf{k}} \rangle \approx 1 + \delta\mathbf{k} \cdot \langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle \approx e^{i\mathbf{a}_{\mathbf{k}} \cdot \delta\mathbf{k}}$$

where $\mathbf{a}_{\mathbf{k}} = -i \langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle$ is the Berry connection

Berry curvature & topology

- For Weyl eigenstates, the Berry curvature is

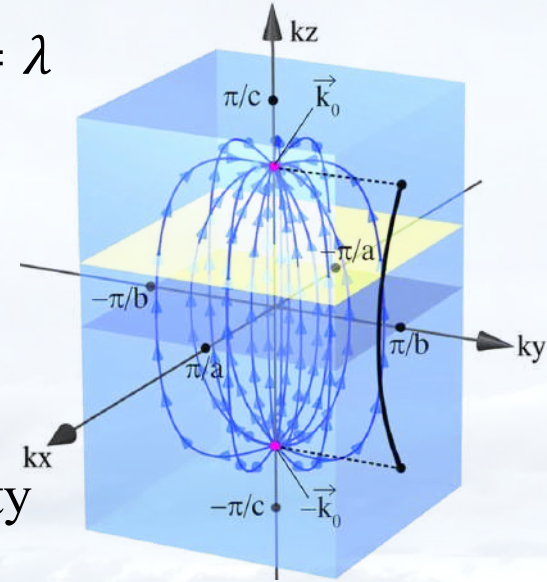
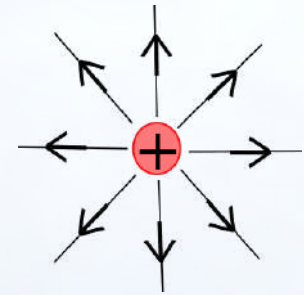
$$\mathbf{\Omega}_k \equiv \nabla_k \times \mathbf{a}_k = \lambda \frac{\vec{k}}{2k^3}$$

- The Chern number (topological charge)

$$C = \frac{1}{2\pi} \oint \mathbf{\Omega}_k \cdot d\mathbf{S}_k = \frac{\lambda}{2\pi} \oint \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin \theta d\theta d\varphi = \lambda$$

- In a solid state material, the Brillouin zone is compact
- A closed surface around a node at \vec{k}_0 is also a closed surface around the rest of the Brillouin zone
- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

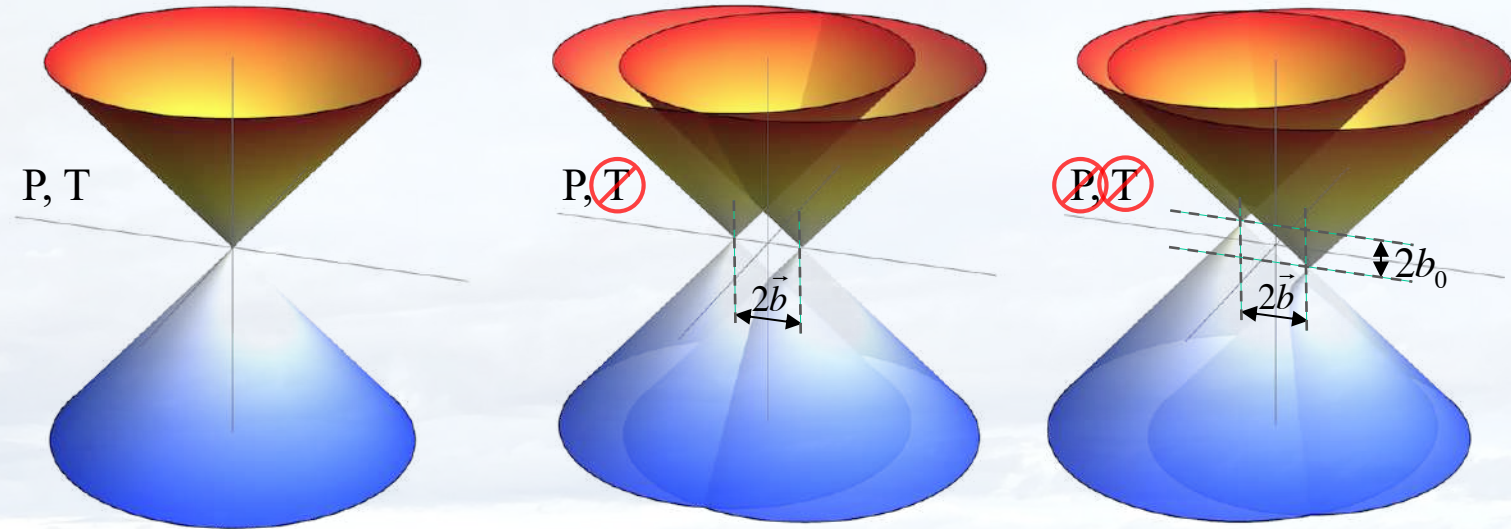
Idealized Dirac and Weyl model

- Low-energy Hamiltonians of a Dirac and Weyl materials

$$H = \int d^3\mathbf{r} \bar{\psi} \left[-iv_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - \underbrace{(\vec{b} \cdot \vec{\gamma})}_{\text{T}} \gamma^5 + \underbrace{b_0 \gamma^0 \gamma^5}_{\text{P}} \right] \psi$$

Dirac (e.g., Na₃Bi, Cd₃As₂, ZrTe₅)

Weyl (e.g., TaAs, NbAs, TaP, NbP, WTe₂)





ANOMALOUS EFFECTS

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]

Review: [Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ to persist on *macroscopic* time/distance scales
- The (collective) dynamics of $n_R + n_L$ and $n_R - n_L$ is controlled by the continuity equations

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

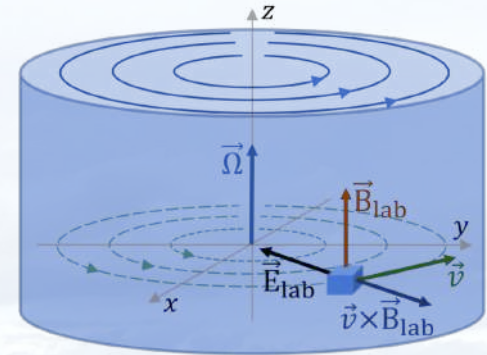
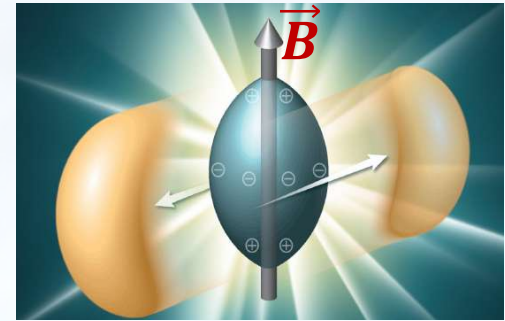
$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c} - \Gamma_{\text{flip}}(n_R - n_L)$$

Question: Can chiral anomaly produce any *macroscopic* effects in ultra-relativistic matter?

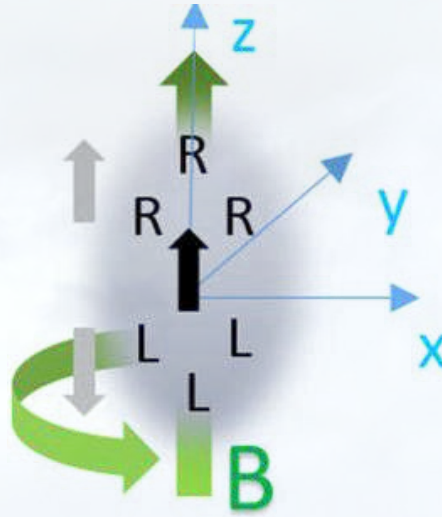
Anomalous effects

- **Theory:** Many *macroscopic* chiral anomalous effects were proposed
- Some are triggered by an external magnetic field
 - Chiral magnetic effect
 - Chiral separation effect
 - Chiral magnetic wave
 - Negative magnetoresistance
 - ...
- Others are triggered by vorticity
 - Chiral vortical effect
 - Chiral vortical wave
 - ...

} this talk



Review: [Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]



CHIRAL SEPARATION EFFECT

$$\langle \vec{J}_5 \rangle = - \frac{e \vec{B}}{2\pi^2} \mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

[Newman & Son, Phys. Rev. D 73 (2006) 045006]

Landau levels

- Landau energy levels at $m = 0$

$$E_n^\pm = \pm \sqrt{2n|eB| + p_z^2}$$

where $n = \underbrace{k + \frac{1}{2}}_{\text{orbital}} + \underbrace{\text{sgn}(eB)s_z}_{\text{spin}}$

- Lowest Landau level is spin polarized

$$E_0^\pm = \pm p_z \quad (k = 0, s_z = -\frac{1}{2})$$

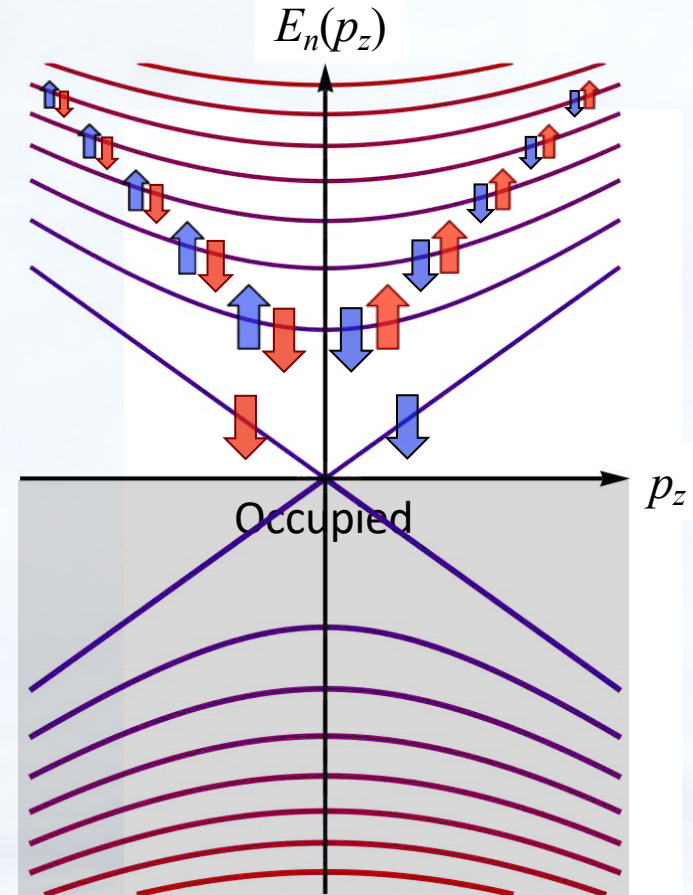
- Density of states at $E=0$:

$$\left. \frac{dn}{dE} \right|_{E=0} = \frac{|eB|}{4\pi^2}$$

- Higher Landau levels ($n \geq 1$) are twice as degenerate:

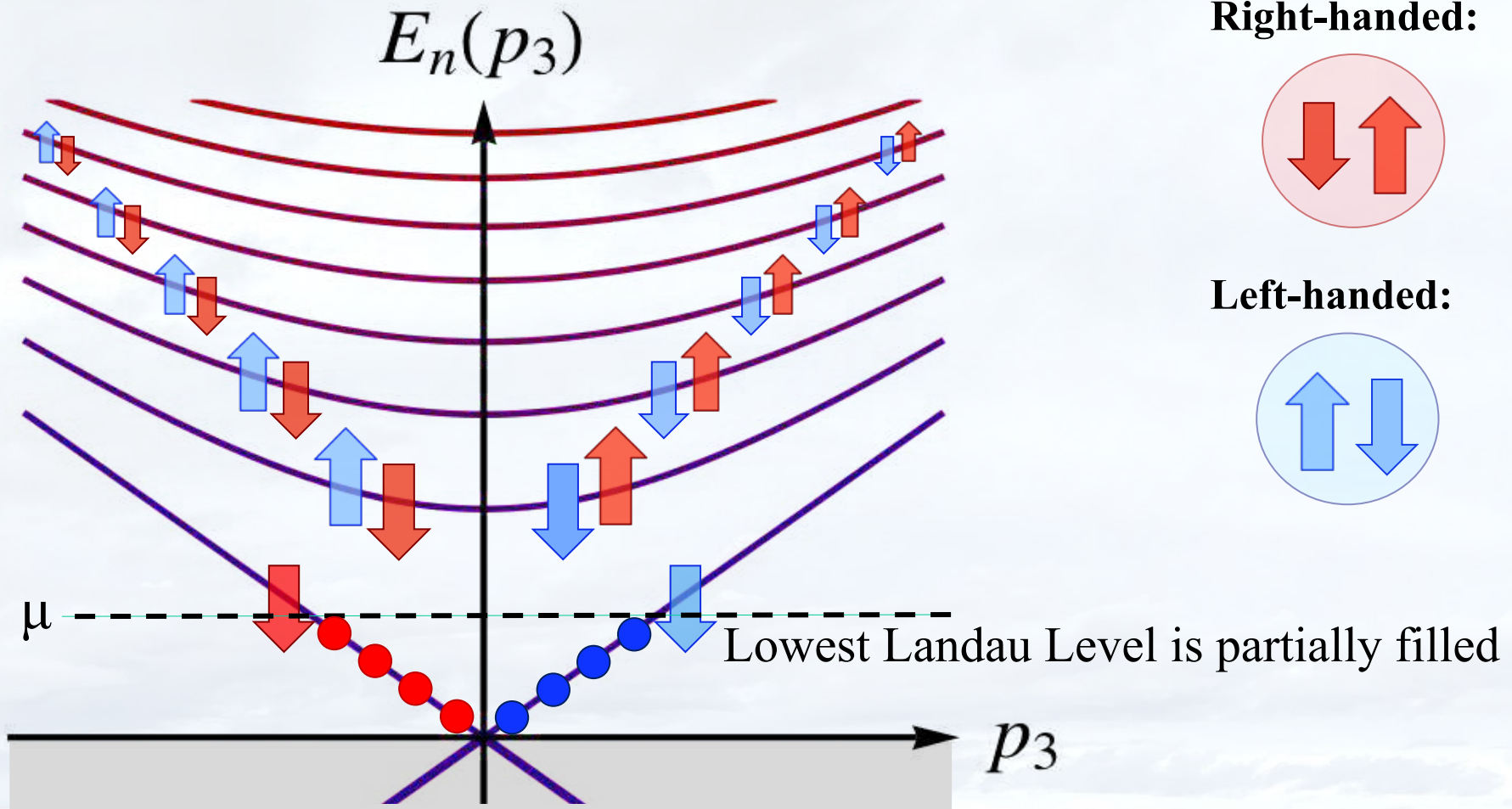
$$(i) \quad k = n \quad \& \quad s = -\frac{1}{2}$$

$$(ii) \quad k = n - 1 \quad \& \quad s = +\frac{1}{2}$$



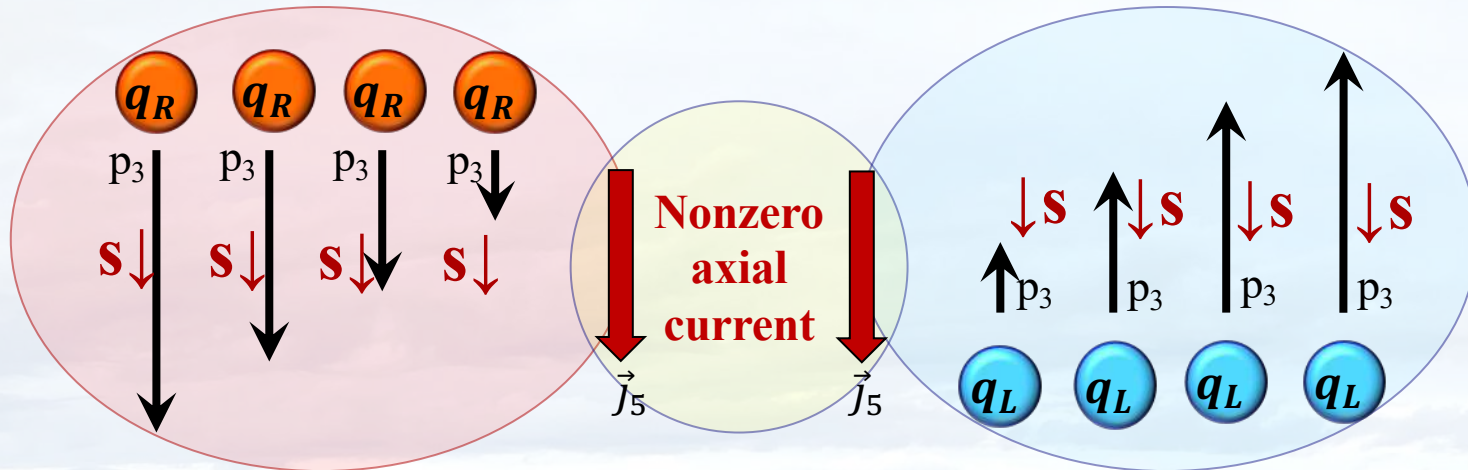
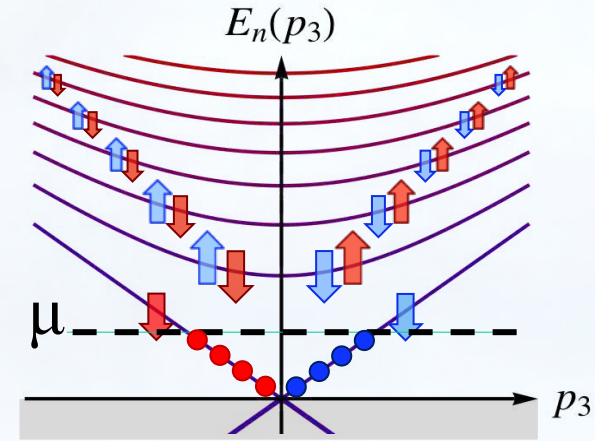
Review: [Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]

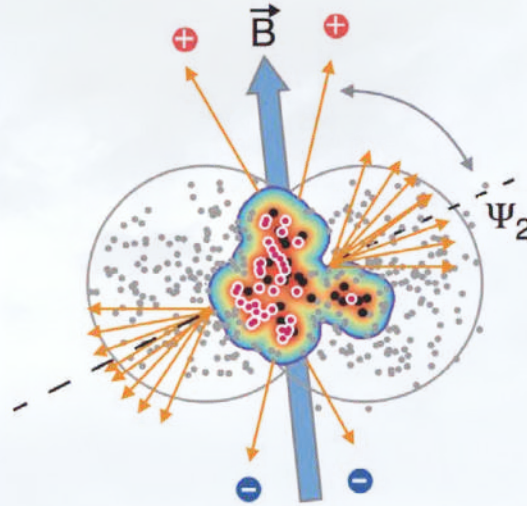
CSE: Landau spectrum & $\mu \neq 0$



- **Spin polarized** LLL is chirally asymmetric
 - states with $p_3 < 0$ (and $s = \downarrow$) are **R-handed**
 - states with $p_3 > 0$ (and $s = \downarrow$) are **L-handed**
- i.e., a nonzero **axial** current is induced

$$\langle \vec{j}_5 \rangle = -\text{tr}[\vec{\gamma} \gamma^5 S(x, x)] = -\frac{e\vec{B}}{2\pi^2} \mu$$





[Image credit: Kharzeev & Liao, Nucl. Phys. News **29**, 1 (2019)]

CHIRAL MAGNETIC EFFECT

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

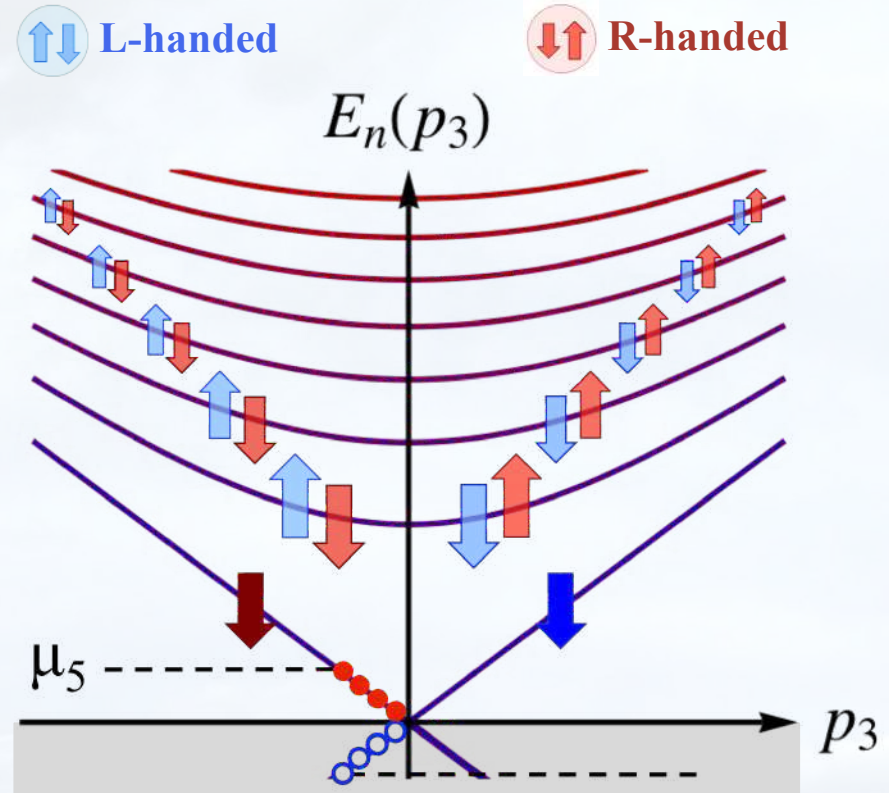
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Assume that one created a *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL ($s=\downarrow$ for particles of a *negative* charge):

- Some **R-handed** states ($p_3 < 0$ & $E < \mu_5$) are occupied
- Some **L-handed** states ($p_3 < 0$ & $|E| < \mu_5$) are empty (i.e., holes with $p_3 > 0$)



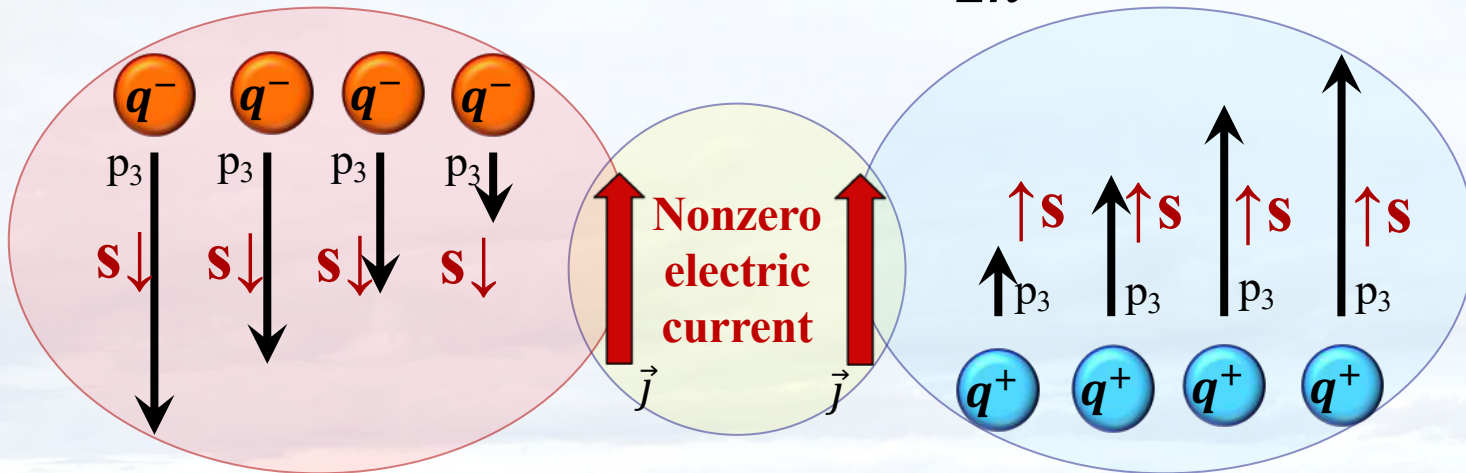
CME current: $\langle \vec{j} \rangle = -tr[\vec{\gamma}S(x, x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$

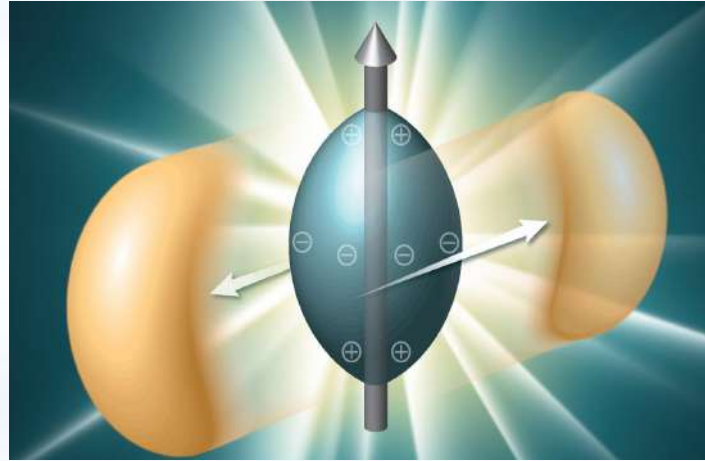
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

- **Spin polarized** LLL is chirally asymmetric
 - states with $p_3 < 0$ (and $s=\downarrow$) are **R-handed** particles
 - states with $p_3 > 0$ (and $s=\downarrow$) are **R-handed** antiparticles (**L-handed** holes)

i.e., a nonzero **electric** current is induced

$$\langle \vec{j} \rangle = -tr[\vec{\gamma}S(x, x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

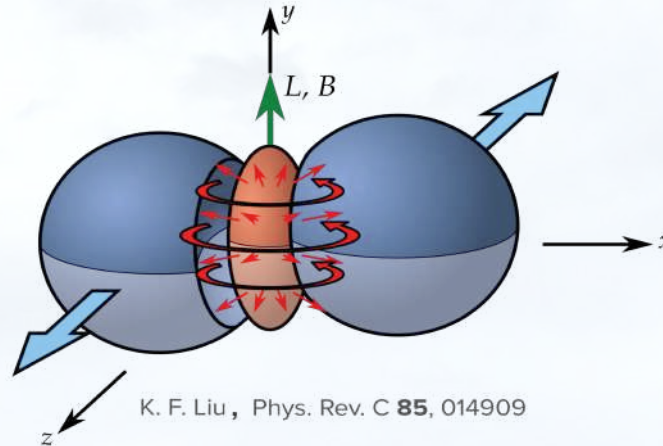




HEAVY-ION COLLISIONS

\vec{B} and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)



[Rafelski & Müller, PRL, 36, 517 (1976)],
 [Kharzeev et al., arXiv:0711.0950],
 [Skokov et al., arXiv:0907.1396],
 [Voronyuk et al., arXiv:1103.4239],
 [Bzdak & Skokov, arXiv:1111.1949],
 [Deng & Huang, arXiv:1201.5108], ...

- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19} \text{ G } (\sim 100 \text{ MeV})$$

- Vorticity estimate:

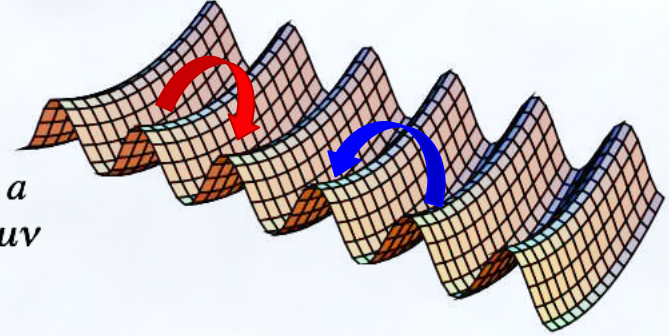
[Adamczyk et al. (STAR), Nature 548, 62 (2017)]

$$\omega \sim 9 \times 10^{21} \text{ s}^{-1} (\sim 10 \text{ MeV})$$

Source of chirality in QCD

- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$



- A random fluctuation with nonzero chirality could result in

$$N_R - N_L \neq 0 \Rightarrow \mu_5 \neq 0$$

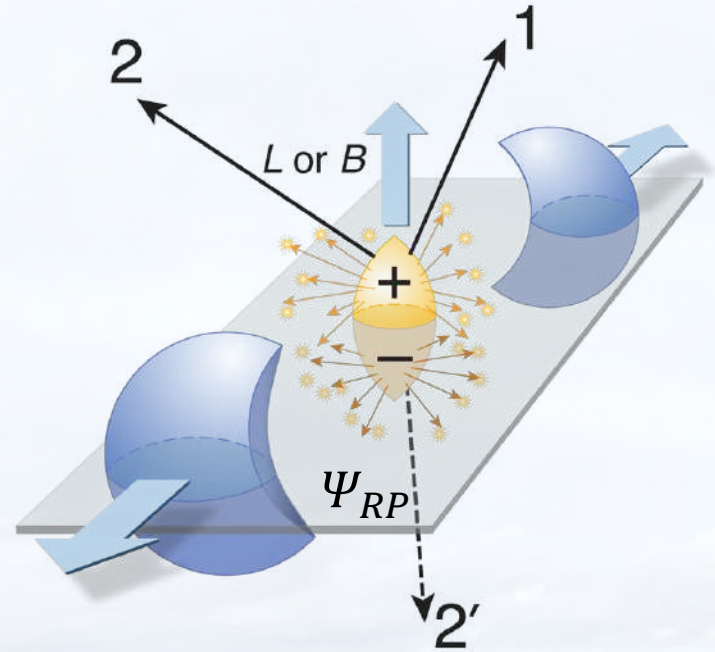
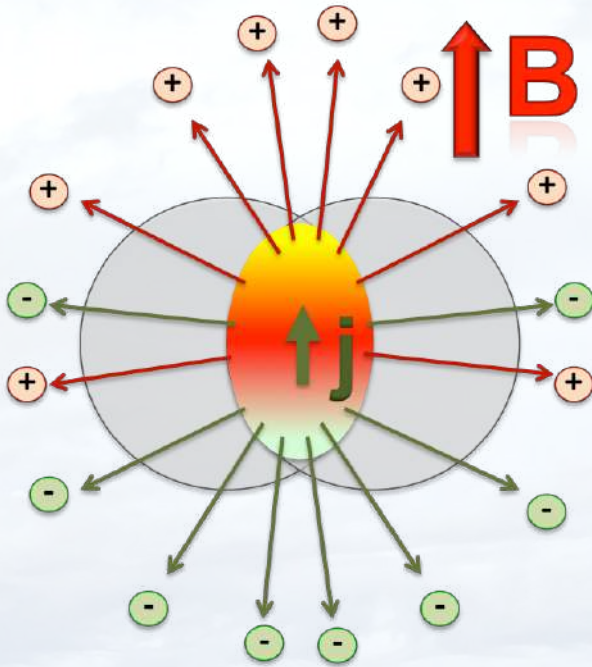
- This should lead to an electric current

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

Dipole CME

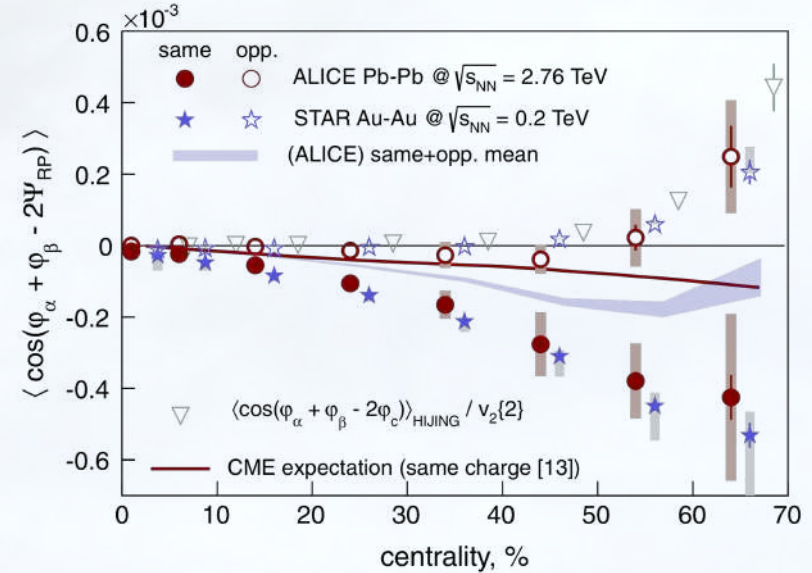
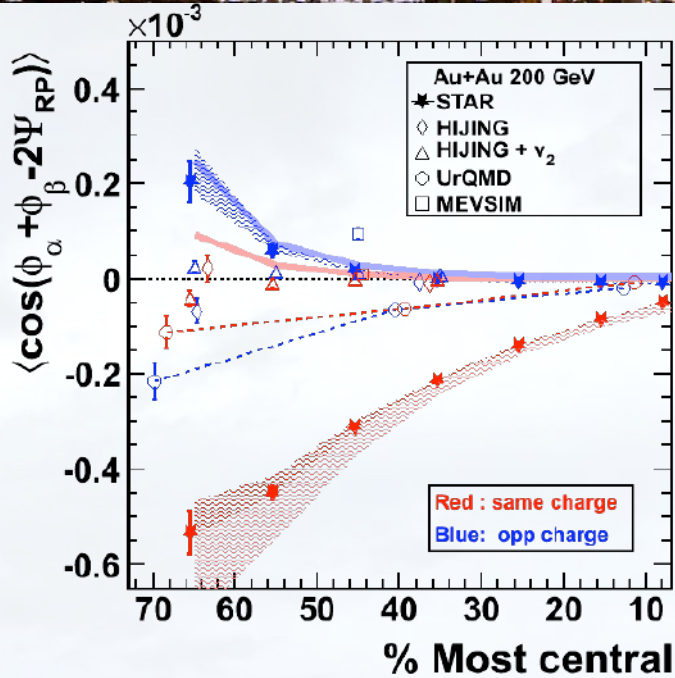
- Dipole pattern of *charged particle correlations* in heavy-ion collisions

$$\langle \cos(\varphi_{\alpha}^{\pm} + \varphi_{\beta}^{\mp} - 2\Psi_{RP}) \rangle > 0 \quad \& \quad \langle \cos(\varphi_{\alpha}^{\pm} + \varphi_{\beta}^{\pm} - 2\Psi_{RP}) \rangle < 0$$



[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
 [Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence



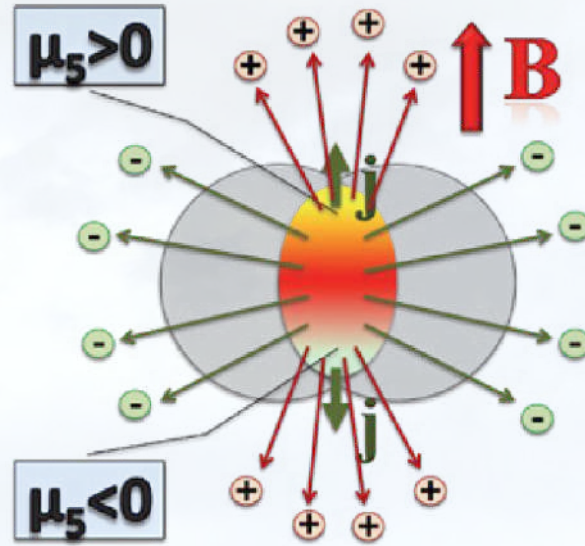
Correlations of same & opposite charge particles:

$$\left\{ \begin{array}{l} \langle \cos(\phi_\alpha^\pm + \phi_\beta^\mp - 2\Psi_{RP}) \rangle > 0 \\ \langle \cos(\phi_\alpha^\pm + \phi_\beta^\pm - 2\Psi_{RP}) \rangle < 0 \end{array} \right.$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
 [Abelev et al. (STAR), PRC **81**, 054908 (2010)]
 [Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
 [Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]
 [Adamczyk et al. (STAR), PRL **113**, 052302 (2014)]
 [Khachatryan et al. (CMS), PRL **118**, 122301 (2017)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)]
 [ALICE Collaboration, Phys. Lett. B **777**, 151 (2018)]



CHIRAL MAGNETIC WAVE

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

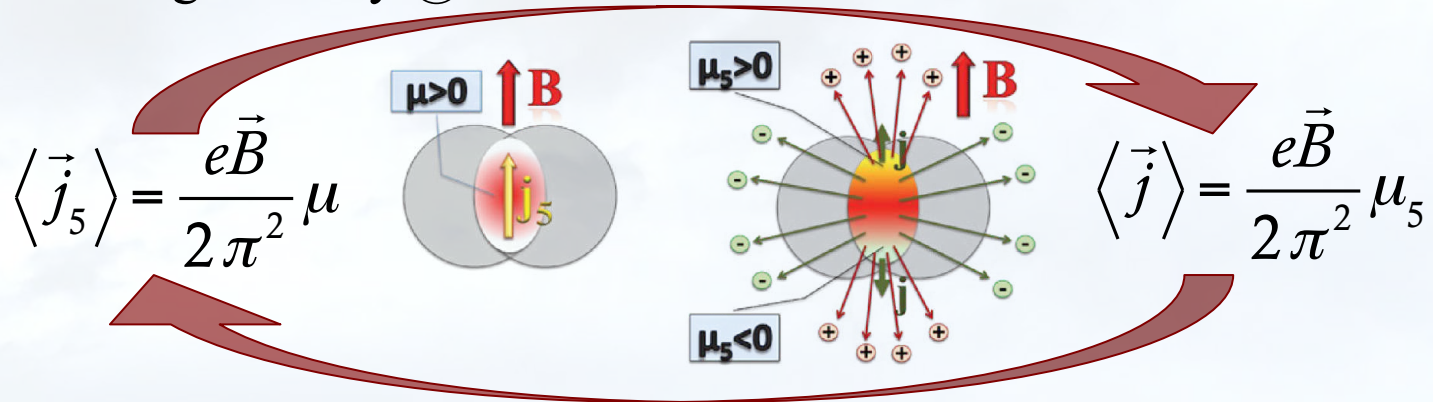
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D **99**, 016017 (2019)]

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]

Chiral Magnetic Wave

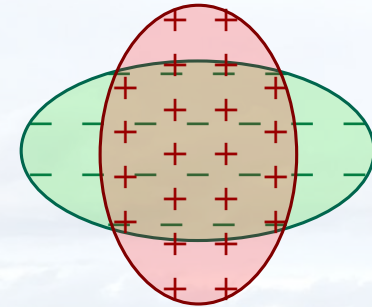
- Nonzero charge density @ $B \neq 0 \rightarrow$ CMW



[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

$$\frac{dN_{\pm}}{d\phi} \approx \bar{N}_{\pm} [1 + 2v_2 \cos(2\phi) \mp A_{\pm} r \cos(2\phi)]$$



where A_{\pm} is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

- Simple model ($\delta n, \delta n_5 \sim e^{-ik_0 t + ikz}$):

$$k_0 \delta n - \frac{eB}{2\pi^2 \chi_5} k \delta n_5 = 0$$

$$k_0 \delta n_5 - \frac{eB}{2\pi^2 \chi} k \delta n = 0$$

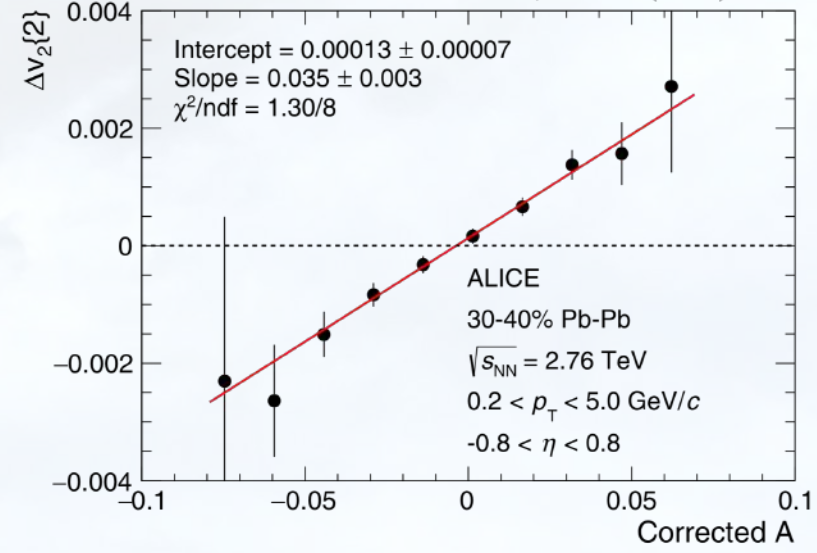
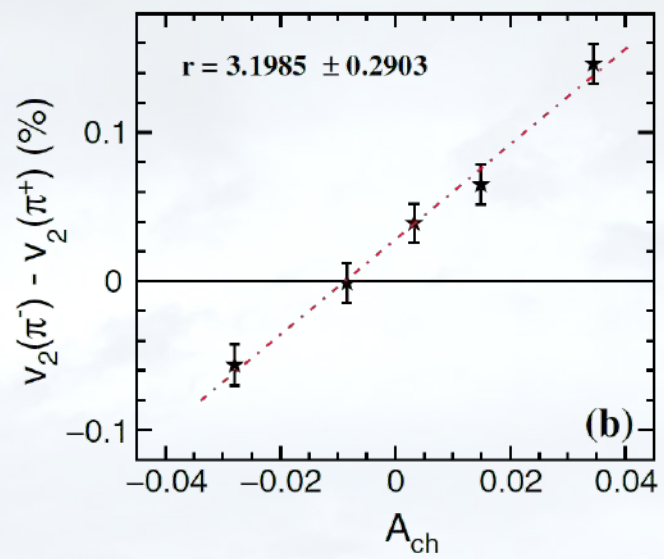
where $\chi_5 \simeq \chi = \partial n / \partial \mu \simeq T^2 / 3$

- The linear dispersion of the CMW mode:

$$k_0 \simeq \pm \frac{eB}{2\pi^2 \chi} k$$

- This is a gapless mode with speed $v \propto eB / T^2$

CMW: Experimental evidence



[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]
 [Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]
 [Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Background effects may dominate over the signal!

[CMS Collaboration, arXiv:1708.08901]

Theory: the chiral magnetic wave might be overdamped...

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608, Phys. Rev. D **99**, 016017 (2019)]
 [Shovkovy, Rybalka, Gorbar, arXiv:1811.10635, PoS (Confinement2018) 029]

CMW: careful analysis

- Simple 1-flavor model ($\mathbf{k} \parallel \mathbf{B}$):

$$k_0 \delta n - kB \delta \sigma_B + i \frac{\tau}{3} k^2 \delta n - \frac{1}{e} \sigma_E k \delta E_z = 0$$

$$k_0 \delta n_5 - kB \delta \sigma_B^5 + i \frac{\tau}{3} k^2 \delta n_5 - i \frac{e^2}{2\pi^2} B \delta E_z = 0$$

$$k \delta E_z + ie \delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i \frac{\sigma_E}{2} \pm i \frac{\sigma_E}{2} \sqrt{1 - \left(\frac{3eB}{\pi^2 T^2 \sigma_E} \right)^2 \left(k^2 + \frac{e^2 T^2}{3} \right) - i \frac{\tau}{3} k^2}$$

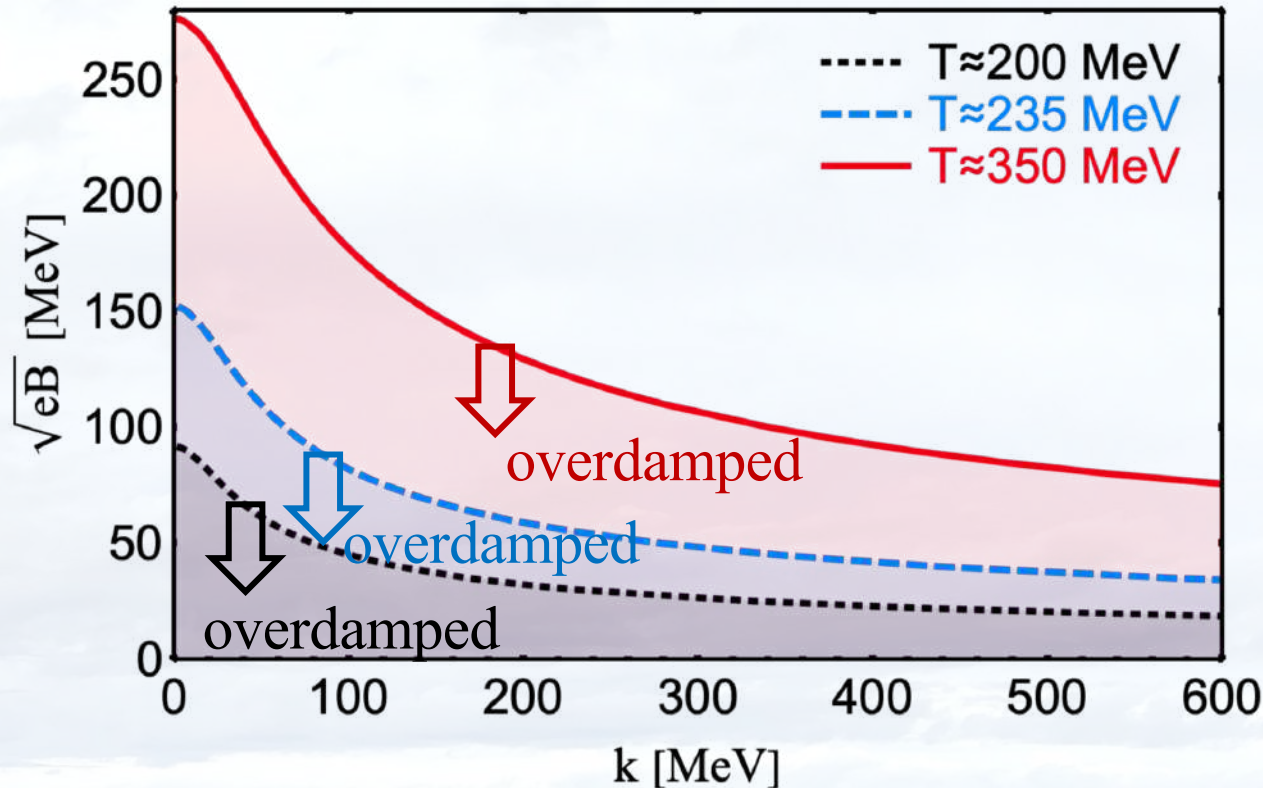
- This is a diffusive mode ($\propto e^{-ik_0 t}$) when

$$\frac{3eB}{\pi^2 T^2 \sigma_E} \sqrt{k^2 + \frac{e^2 T^2}{3}} < 1$$

CMW in QCD

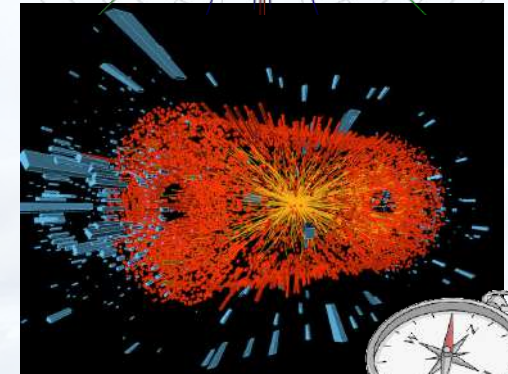
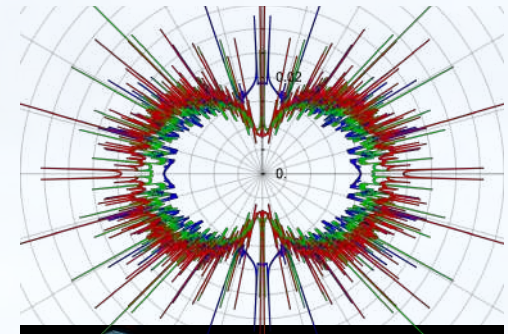
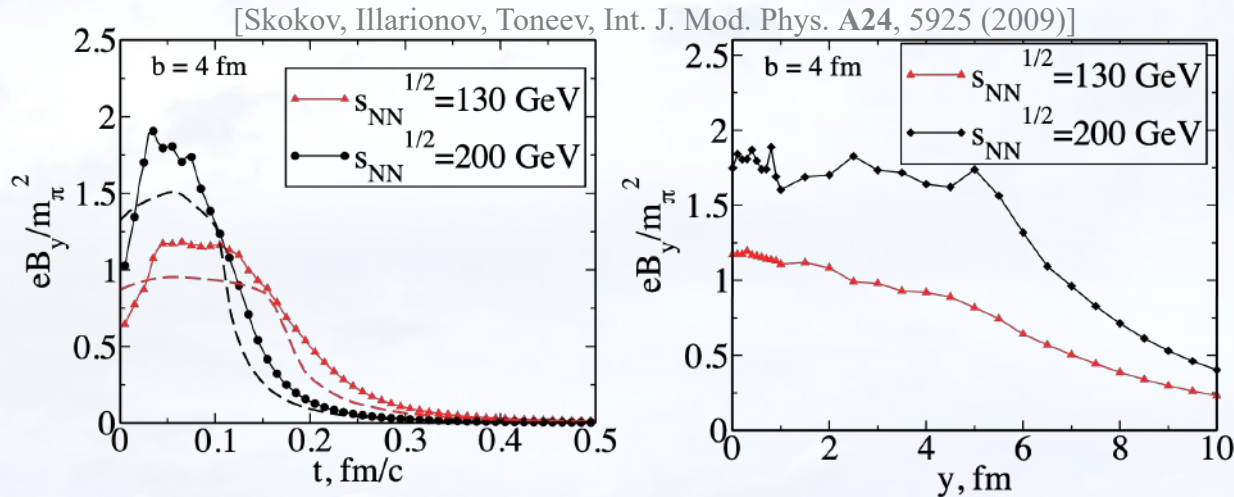
Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small eB & k :



Find “knobs” to control/measure magnetic field \vec{B} ?

- Exploit the beam-energy dependence of the field (?)



- Try to measure \vec{B} more directly (?)
 - thermal photons (?)
 - dilepton rates (?)

[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020)]

Isobar collisions

Utilize collisions of isobars, e.g.,

[Voloshin, PRL **105**, 172301 (2010)]
 [Deng et al. PRC **94**, 041901(R) (2016)]

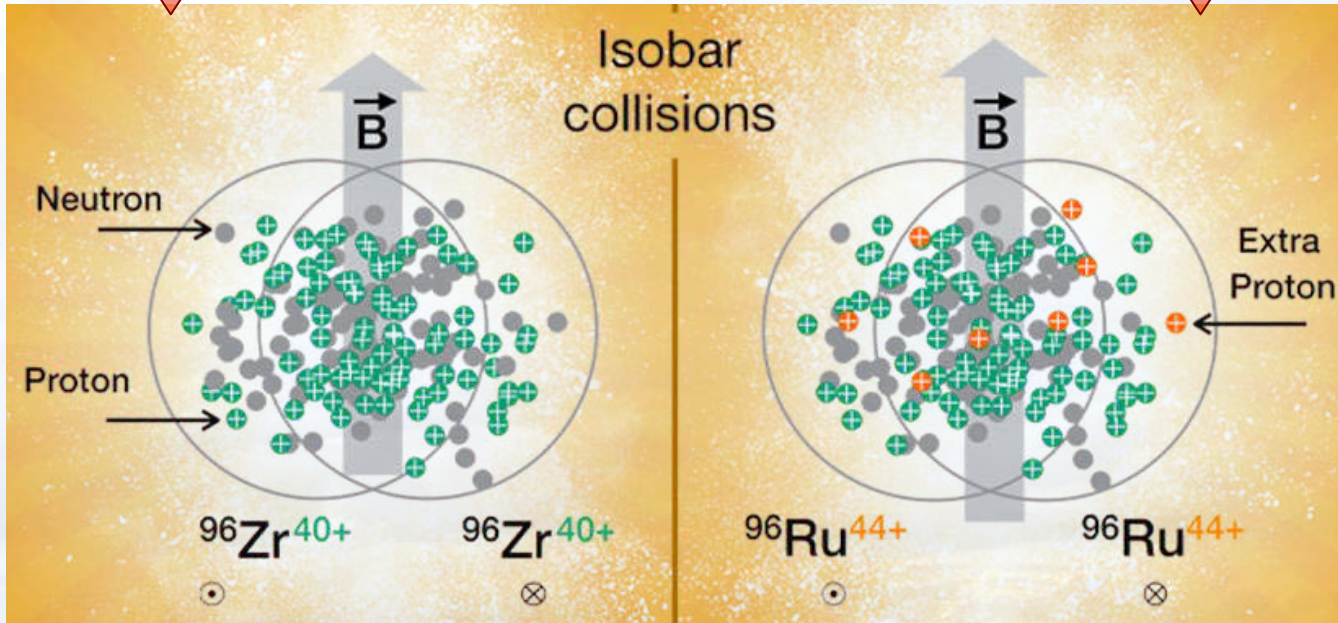
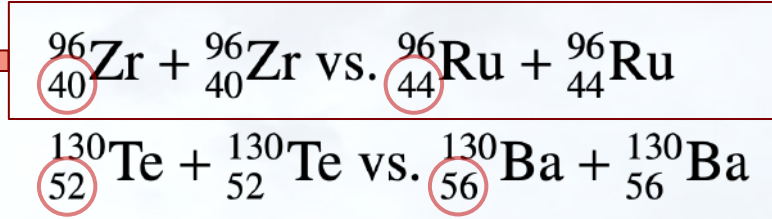
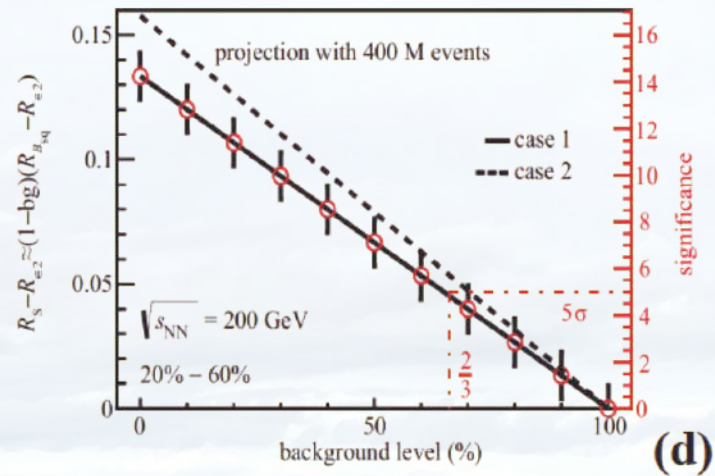
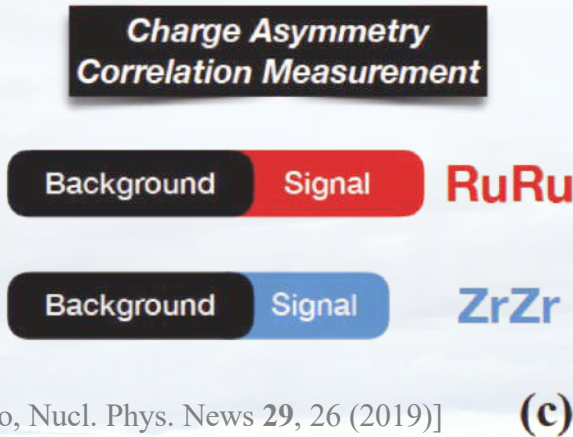
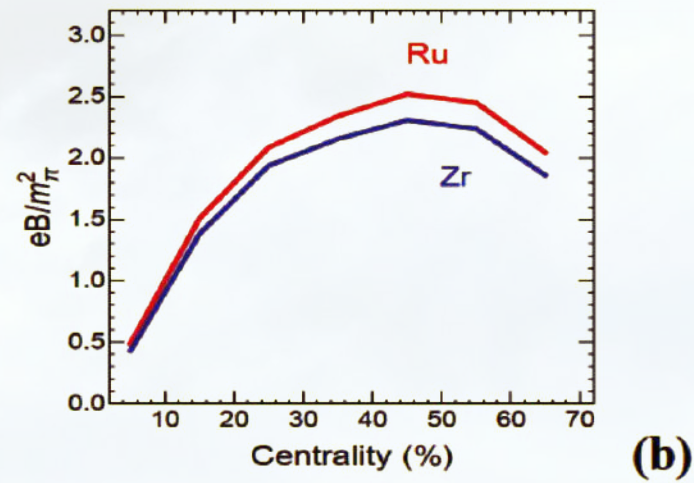
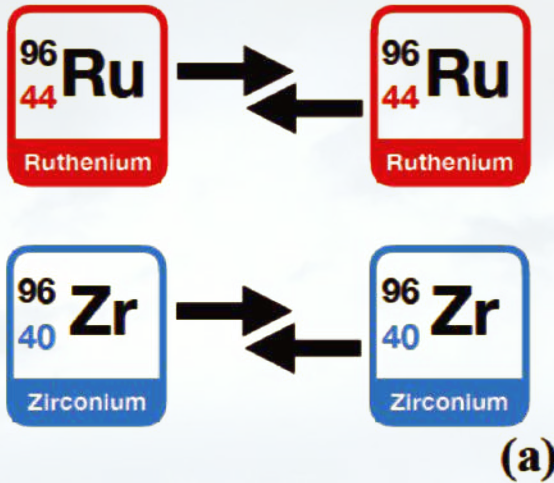


Image credit: Brookhaven National Laboratory, <https://www.bnl.gov/newsroom/news.php?a=119062>

Isobar collisions (theory)



[Kharzeev and Liao, Nucl. Phys. News 29, 26 (2019)]

Isobar collisions (experiment)

- Isobar run was completed by STAR in May 2018
- ≈ 3.8 billion collisions of $^{96}\text{Ru}+^{96}\text{Ru}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ at $\sqrt{s} = 200$ GeV
- Blind analysis by five groups of the STAR Collaboration
- Report announced on Aug. 31, 2021 (online event @ BNL)
- Preprint posted on the same day

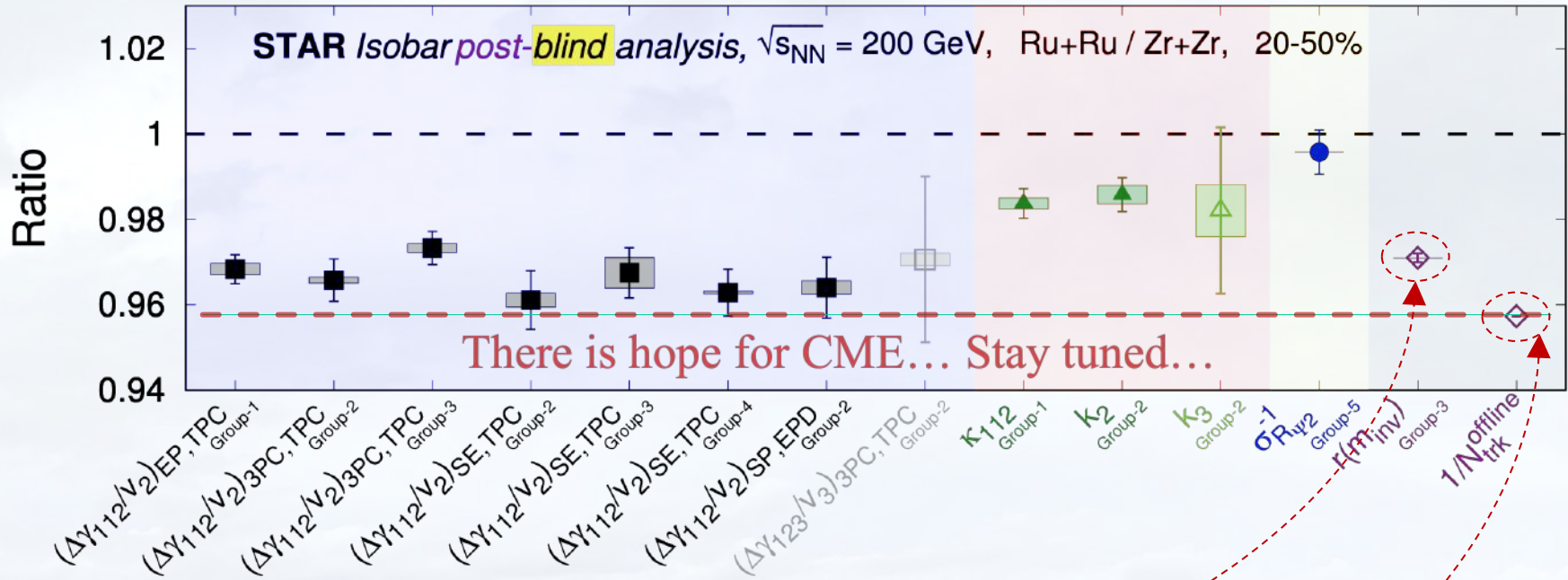
[STAR Collaboration, arXiv:2109.00131]

the STAR Collaboration performed a blind analysis of a large data sample of approximately 3.8 billion isobar collisions of $^{96}_{44}\text{Ru}+^{96}_{44}\text{Ru}$ and $^{96}_{40}\text{Zr}+^{96}_{40}\text{Zr}$ at $\sqrt{s_{NN}} = 200$ GeV. Prior to the blind analysis, the CME signatures are predefined as a significant excess of the CME-sensitive observables in Ru+Ru collisions over those in Zr+Zr collisions, owing to a larger magnetic field in the former. A precision down to 0.4% is achieved, as anticipated, in the relative magnitudes of the pertinent observables between the two isobar systems. Observed differences in the multiplicity and flow harmonics at the matching centrality indicate that the magnitude of the CME background is different between the

two species. No CME signature that satisfies the predefined criteria has been observed in isobar collisions in this blind analysis.

Isobar collisions (results)

- Compilation of post-blinding results [STAR Collaboration, arXiv:2109.00131]



- Note two extra data points (open markers):
 - the ratio of inverse multiplicities
 - the ratio of relative pair multiplicity difference

- Chiral anomaly may have macroscopic implications
- Anomaly may have observable signatures in quark-gluon plasma (and other forms of relativistic matter)
- (Dipole) chiral magnetic effect (CME) can be seen via charged particle correlations in heavy-ion collisions
- Latest isobar measurements are not conclusive yet (active studies are underway)
- Chiral magnetic wave is another phenomenon that may affect quark-gluon plasma (if not overdamped)
- Signatures of CME are tested/observed in Dirac and Weyl semimetals (and other physical systems too)