Arizona State University

Chirality, Vorticity, etc.
Nov 3, 2021
Stony Brook U.

Anomalous phenomena in Dirac and Weyl semimetals

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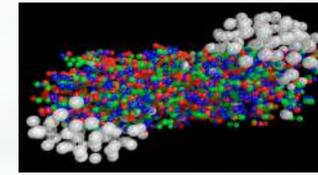
The 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

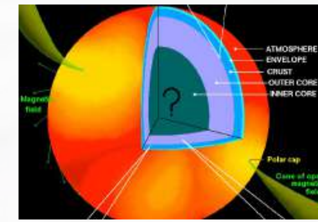
Stony Brook University, November 1-5, 2021

Chiral plasma

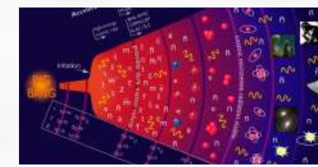
- **Heavy-ion collisions** **(high temperature)**
  [Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]



- **Super-dense matter in compact stars** **(high density)**
  [Effects seem negligible due slow chirality production and a large chirality flip rate]

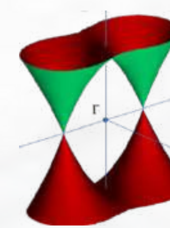


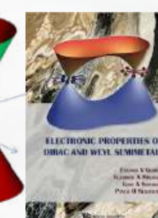
- **Early Universe** **(high temperature)** } talk by Kahniashvili
  [Joyce & Shaposhnikov, PRL **79**, 1193 (1997)]



- **Magnetospheres of magnetars**
  (electron-positron plasma at moderately high temperature)
  [Gorbar & Shovkovy, arXiv:2110.11380]

- **Electron plasma in Dirac/Weyl (semi-)metals** } this talk
  [Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]

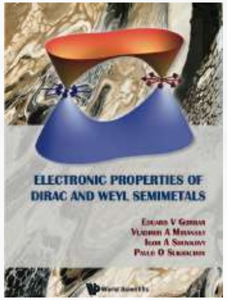




- **Other: cold atoms, superfluid ^{3}He-A, etc.**
  [Bevan, Manninen, Cook, Hook, Hall, Vachaspati, Volovik, Nature 386, 689 (1997)]

Reviews on Dirac & Weyl semimetals

- Gorbar, Miransky, Shovkovy & Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals*, (World Scientific, Singapore, 2021)



- Chernodub, Ferreiros, Grushin, Landsteiner & Vozmediano, *Thermal transport, geometry, and anomalies*, arXiv:2110.05471
- Ong & Liang, *Experimental signatures of the chiral anomaly in Dirac–Weyl semimetals*, Nature Rev. Phys. **3**, 394 (2021); arXiv:2010.08564
- Armitage, Mele & Vishwanath, *Weyl and Dirac semimetals in three-dimensional solids*, Rev. Mod. Phys. **90**, 015001 (2018); arXiv:1705.01111
- Gorbar, Miransky, Shovkovy & Sukhachov, *Anomalous transport properties of Dirac and Weyl semimetals,* Low Tem. Phys. **44**, 487 (2018); arXiv:1712.08947
- Landsteiner, Acta Phys. Polon. B **47**, 2617 (2016); arXiv:1610.04413

Dirac/Weyl quasiparticles

- Electron quasiparticles with a wide range of properties are possible
- They may even have the emergent spinor structure of *massless* Weyl fermions,

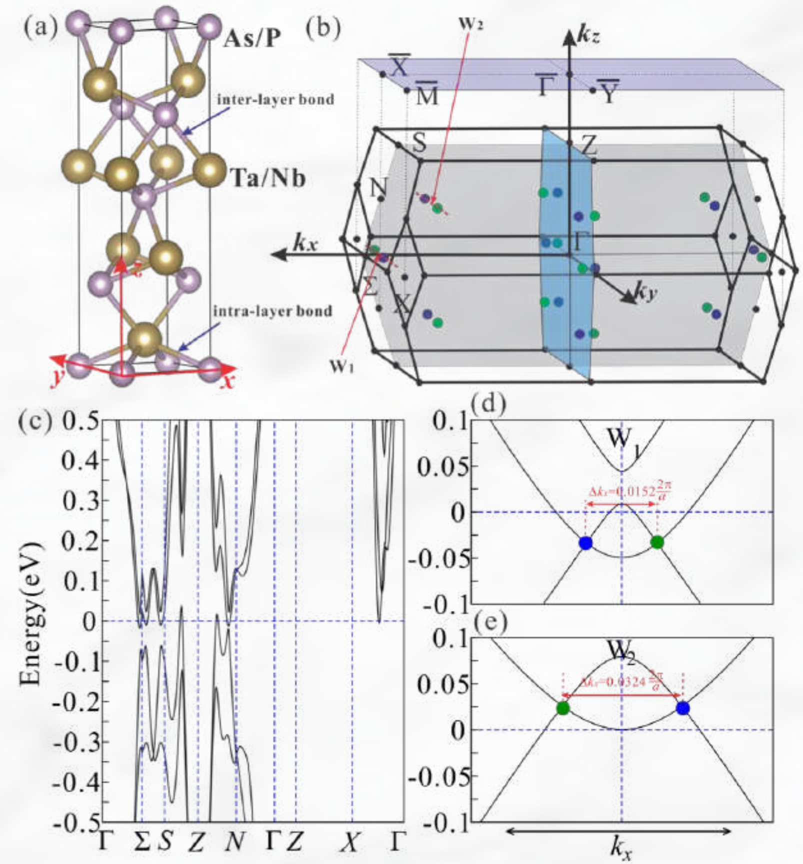
$$H_W \approx \pm v_F (\vec{\sigma} \cdot \vec{\boldsymbol{k}})$$

In fact, such nodes are common!

Na_3Bi, Cd_3As_2, $ZrTe_5$, TaAs, NbAs, …

[Liu et al., Science **343**, 864 (2014)]
[Neupane et al., Nature Commun. **5**, 3786 (2014)]
[Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
[Li et al., Nature Physics **12**, 550 (2016)]
[S.-Y. Xu et al., Science **349**, 613 (2015)]
[B. Q. Lv et al., Phys. Rev. X **5**, 031013 (2015)]
[S.-Y. Xu et al., Nature Physics **11**, 748 (2015)]
[S.-Y. Xu et al., Science Adv. **1**, 1501092 (2015)]
[F. Y. Bruno et al., Phys. Rev. B **94**, 121112 (2016)]

Weyl semimetals TaAs, TaP, NbAs, and NbP

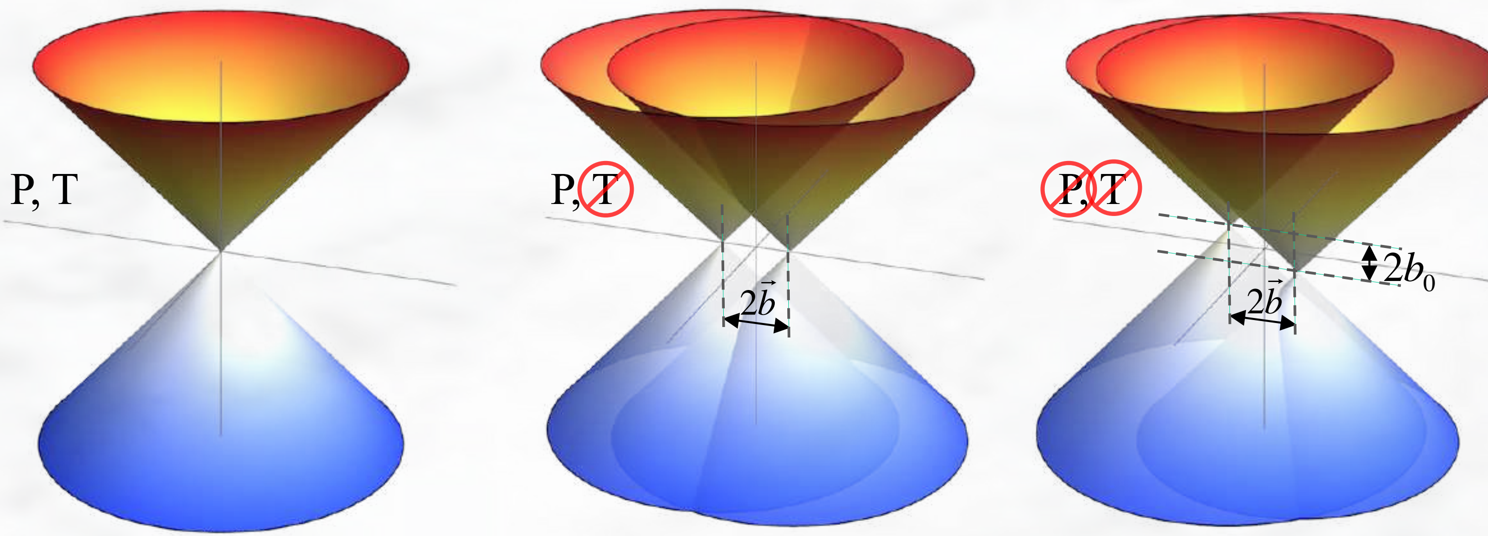


Sun, Wu & Yan, Phys. Rev. B **92**, 115428 (2015)

- Idealized Hamiltonians of Dirac and Weyl materials

$$H = \int d^3\mathbf{r}\, \bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5 + b_0\gamma^0\gamma^5\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$)   Weyl (e.g., TaAs, NbAs, TaP, NbP, $EuCd_2As_2$)



Weyl quasiparticles

- The quasiparticle eigenstates for Weyl Hamiltonian $H_\lambda = \lambda v_F(\vec{\boldsymbol{k}} \cdot \vec{\sigma})$ are

$$\psi_{\boldsymbol{k}}^{\lambda} = \frac{1}{\sqrt{2}\sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

- The quasiparticle energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$ is relativistic-like

- Mapping $k \rightarrow \psi_k^\lambda$ has a nontrivial topology

- Consider adiabatic evolution of the wave function from $\psi_{\boldsymbol{k}}$ to $\psi_{\boldsymbol{k}+\delta \boldsymbol{k}}$:

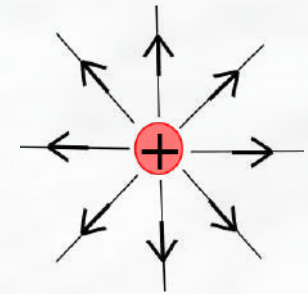
$$\langle \psi_{\boldsymbol{k}} | \psi_{\boldsymbol{k}+\delta \boldsymbol{k}} \rangle \approx 1 + \delta \boldsymbol{k} \cdot \langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle \approx e^{i \boldsymbol{a}_{\boldsymbol{k}} \cdot \delta \boldsymbol{k}}$$

where $\boldsymbol{a}_{\boldsymbol{k}} = -i \langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle$ is the Berry connection

Berry curvature & topology

- For Weyl eigenstates, the Berry curvature is

$$\boldsymbol{\Omega}_k \equiv \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}} = \lambda \frac{\vec{\boldsymbol{k}}}{2k^3}$$



- The Chern number (topological charge)

$$C = \frac{1}{2\pi} \oint \boldsymbol{\Omega}_k \cdot d\boldsymbol{S}_{\boldsymbol{k}} = \frac{\lambda}{2\pi} \oint \frac{\vec{\boldsymbol{k}}}{2k^3} \cdot \frac{\vec{\boldsymbol{k}}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda$$

- In a solid state material, the Brillouin zone is compact
- A closed surface around a node at $\vec{\boldsymbol{k}}_{\boldsymbol{0}}$ is also a closed surface around the rest of the Brillouin zone
- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]

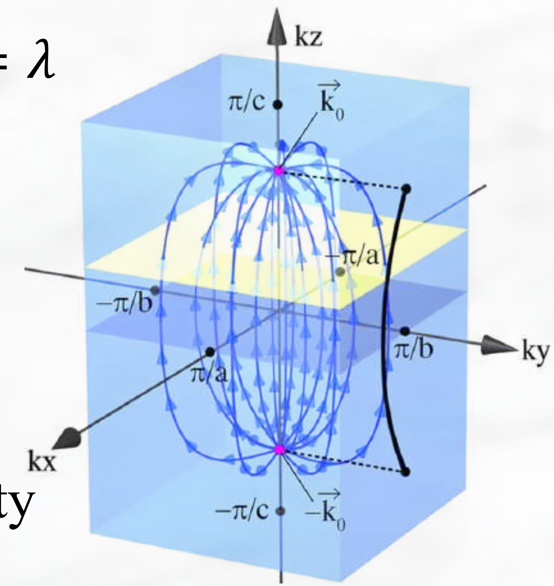


Image credit [Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

Anomalous effects in semimetals

- Observable properties of Dirac/Weyl semimetals are sensitive to (i) the chiral anomaly, (ii) the values of b_0 and $\vec{b}$, and (iii) nontrivial topology
- Partial list of potential anomalous effects:
  - Negative magnetoresistance ($\rho_{\parallel}$ decreasing with B)
  - New types of collective modes (anomalous Hall waves, pseudo-magnetic helicons, chiral zero sound, etc.)
  - Anomalous thermoelectric effects (e.g., $\vec{J}_Q \propto \vec{b}\times\vec{E}$ and $\vec{J}_Q \propto \vec{b}\times\vec{\nabla}T$)
  - Strain/torsion induced CME ($\vec{J} \propto u_{33}\vec{B}$ and $\vec{J} \propto \mu\vec{B}_5$)
  - Strain/torsion dependent conductivity/resistance
  - Quantum oscillations in thin films [$T \propto v_F/(\mu b)$]
  - Strain/torsion induced quantum oscillations (pseudo-Landau levels)
  - Nonlocal anomalous transport

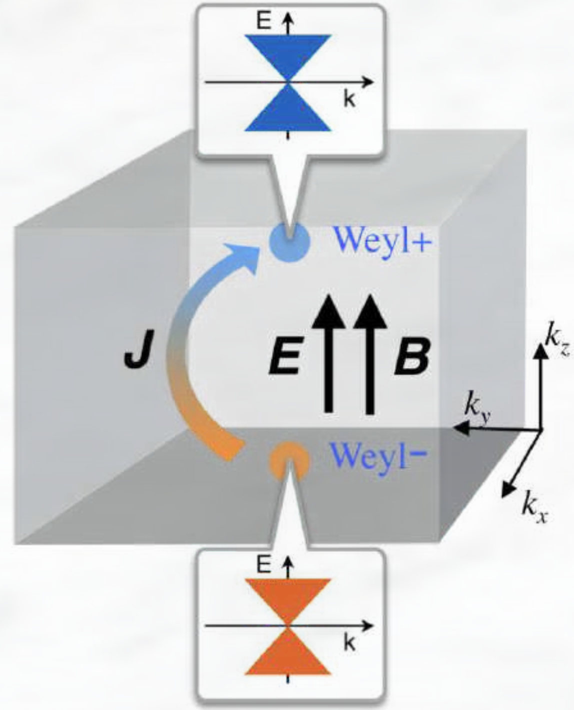


Image credit [Zhang et al., Nat. Commun. 7, 10735 (2016)]

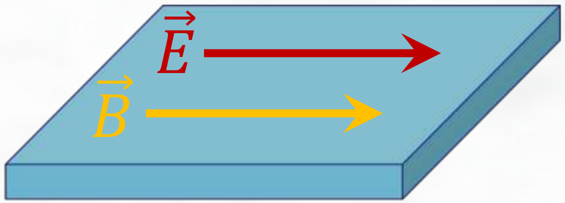
NEGATIVE MAGNETORESISTANCE

Steady CME current

- Homogeneous chiral plasma:

[Nielsen & Ninomiya, Phys. Lett. B **130**, 390 (1983)]
[Son & Spivak, Phys. Rev. B **88**, 104412 (2013)]

$$\frac{\partial n_5}{\partial t} + \vec{\nabla}\cdot\vec{j}_5 = \frac{e^2\vec{E}\cdot\vec{B}}{2\pi^2} - \frac{n_5}{\tau_{\text{ch}}}$$



- Steady state $(\tau_{\text{ch}} \sim 1 \text{ ps to } 1 \text{ ns})$

$$n_5 = \frac{e^2\vec{E}\cdot\vec{B}}{2\pi^2}\tau_{\text{ch}} \quad \rightarrow \quad \mu_5 = \frac{n_5}{\chi_5} \approx \frac{3v^3 n_5}{T^2+\mu^2/\pi^2}$$

- The CME current

$$J_i = \frac{e^2}{2\pi^2}\mu_5 B_i = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{\text{ch}} \frac{B_i B_k}{\chi_5} E_k \quad \rightarrow \quad \sigma_{CME}^{\parallel} = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{\text{ch}} \frac{B^2}{\chi_5}$$

i.e.,

$$\rho_{\text{total}}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}$$

Negative Magnetoresistance

- Experimental confirmation

$$\rho^{\parallel}_{\text{total}} = \frac{1}{\sigma_0 + a(T)B^2}$$

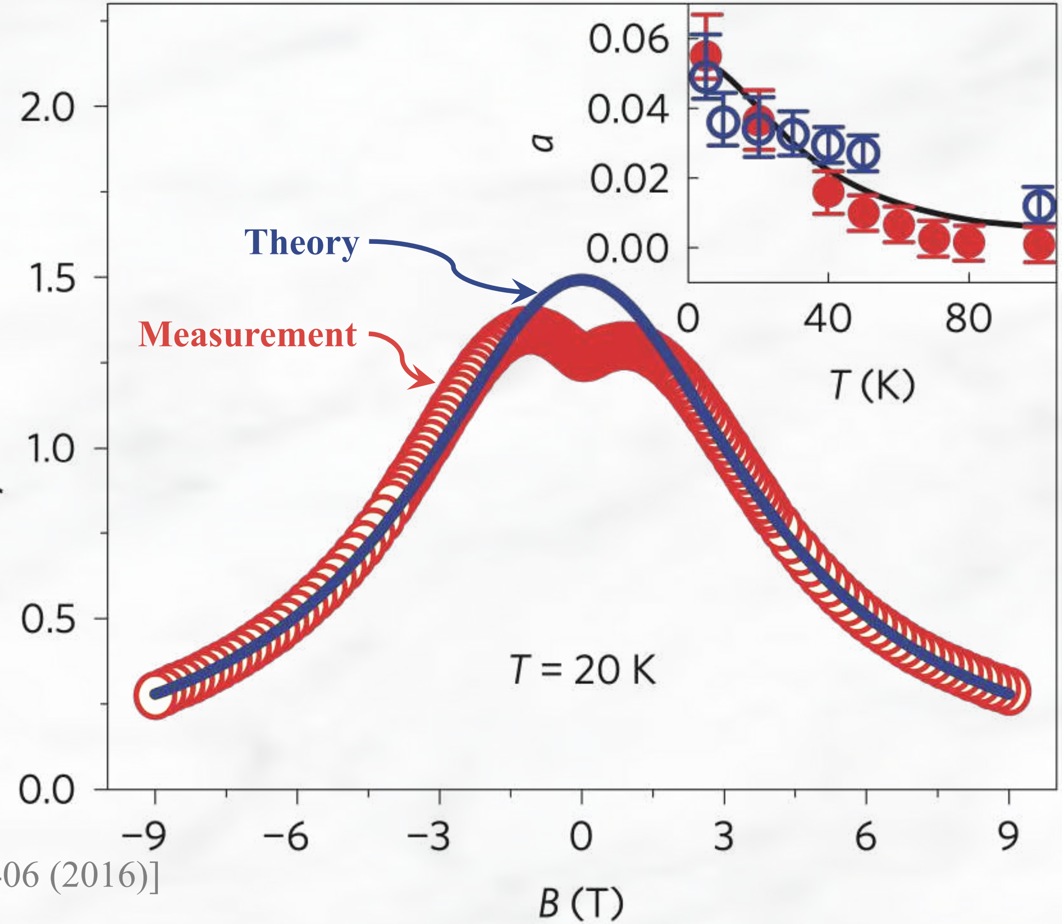
Dirac semimetals:

[Kim et al, Phys. Rev. Lett. **111**, 246603 (2013)]
[Li et al., Nat. Mater. **12**, 550 (2016)]
[Xiong et al., Science **350**, 413 (2015)]
[Feng, et al., Phys. Rev. B **92**, 081306 (2015)]
[Li et al., Nat. Commun. **6**, 10137 (2015)]
[Li et al., Nat. Commun. **7**, 10301 (2016)]

Weyl semimetals:

[Huang et al., Phys. Rev. X **5**, 031023 (2015)]
[Zhang et al., Nat. Commun. **7**, 10735 (2016)]
[Hirschberger et al., Nat. Mater. **15**, 1161 (2016)]
[Wang et al., Phys. Rev. B **93**, 121112 (2016)]
[Du et al., Sci. China Phys. Mech. Astron. **59**, 657406 (2016)]
[Li et al., Front. Phys. **12**, 127205 (2017)]

[Q. Li et al, Nature Physics **12**, 550 (2016)]



Chiral charge pumping (theory)

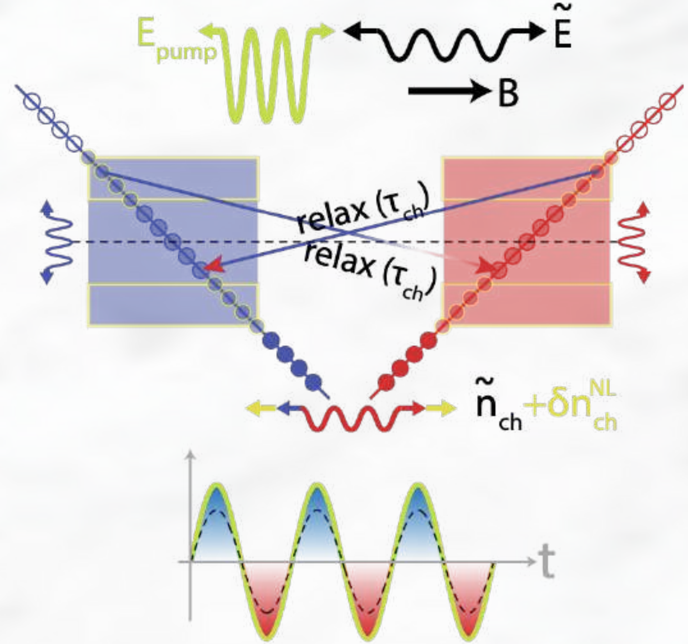
[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

- Weyl semimetal TaAs
  - $\vec{B} \neq 0$ & oscillating $\vec{E} \parallel \vec{B}$
- The nonlinear contribution to chiral charge-pumping conductivity

$$\delta\sigma_{\rm ch}^{\rm NL} = i\frac{9\alpha^2 e^5 v^3}{8h^2\omega^3}\left(\frac{\tilde{\mathbf{E}}_{\rm pump}\cdot\mathbf{B}}{B}\right)^2 B$$

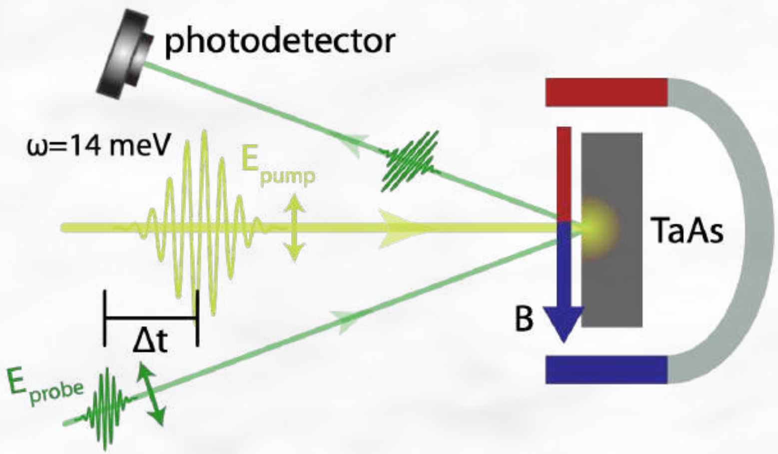
- The reflection coefficient

$$R(T) = \left|\frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}}\right|^2 \quad \text{where} \quad \epsilon = \epsilon_\infty + i\frac{\sigma}{\omega\epsilon_0}$$

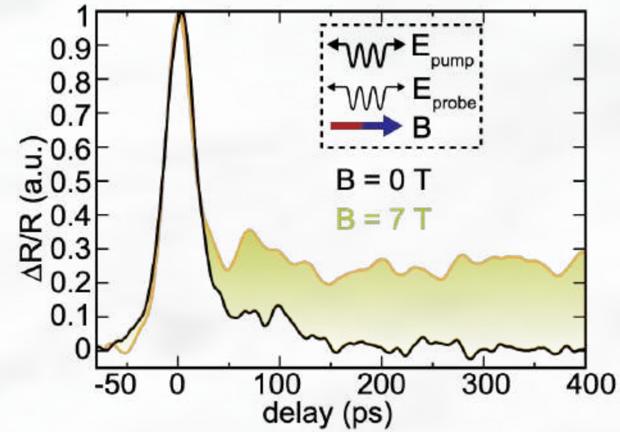


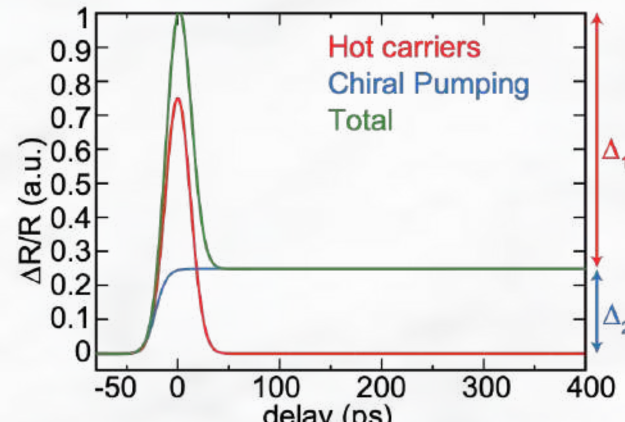
Chiral charge pumping (data)

- Experimental setup



- Measurements:





- Chiral charge relaxation time

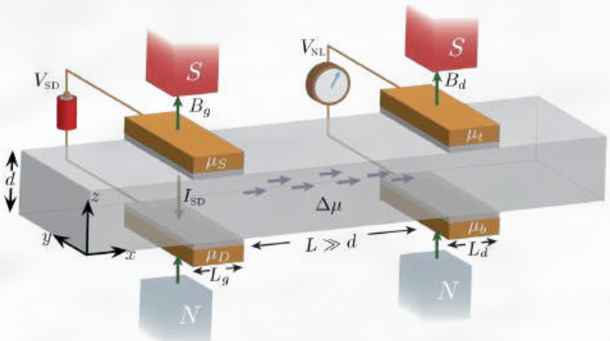
$$1\ \text{ns} \ll \tau_{ch} < 77\ \text{ns}$$

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

Nonlocal anomalous transport

- Theory

[Parameswaran, Grover, Abanin, Pesin, Vishwanath, PRX 4, 031035 (2014)]



$$\alpha_{NL} = \frac{R_{NL}}{R_L} \propto e^{-L/L_V}$$

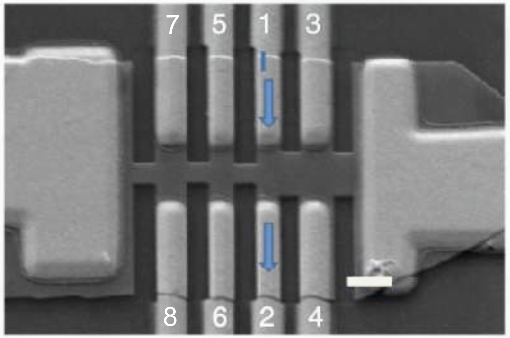
- Measurements:

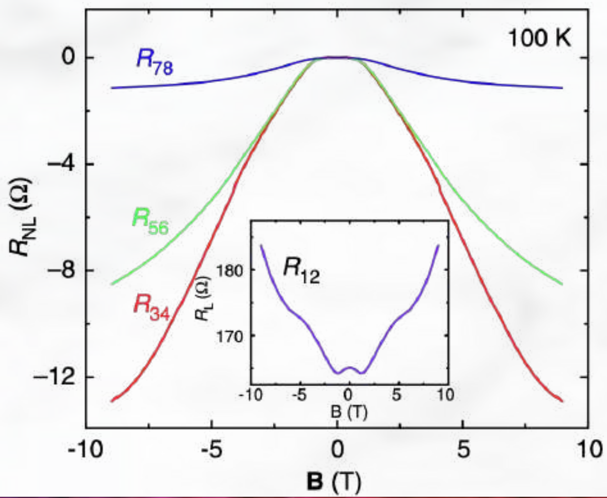
$L_V \sim 2$ μm

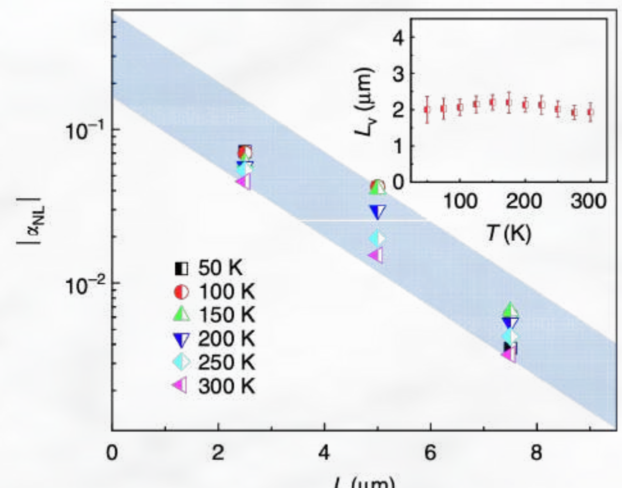
- Experiment (challenge: Ohmic diffusion)

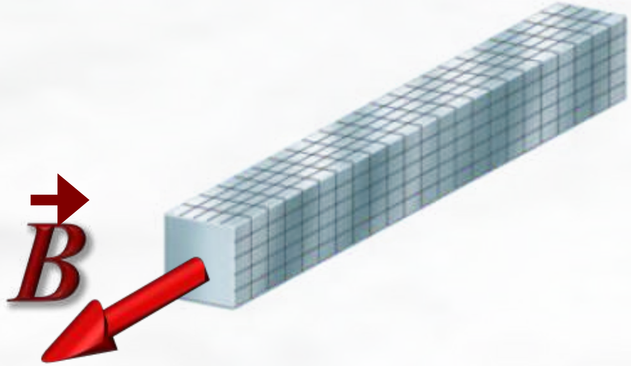
[Zhang et al., Nat. Commun. 8, 13741 (2017)]

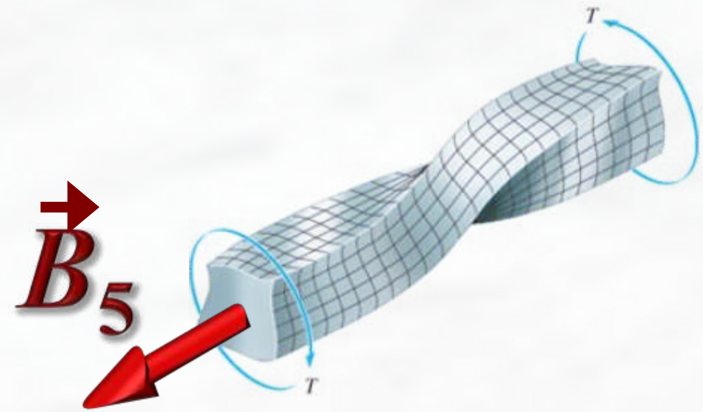
Cd_3As_2











CHIRAL STRAINTRONICS

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]
[Grushin, Venderbos,Vishwanath, Ilan, Phys. Rev. X **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B **94**, 241405 (2016)]
[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Pseudo-electromagnetic fields

- **Strains** modify the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

via the emergent **chiral gauge fields** are

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

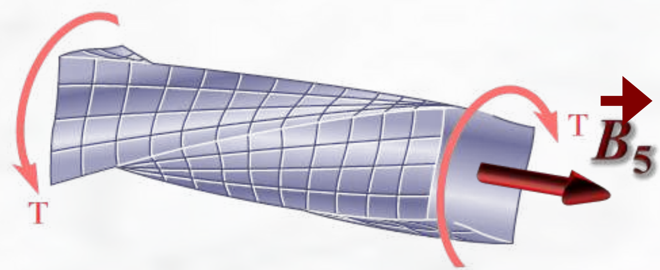
$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
[Grushin, Venderbos,Vishwanath, Ilan, PRX **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

leading to the **pseudo-EM** fields

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$



Continuity relations (with $\vec{B}_5$ & $\vec{E}_5$)

- Naïve continuity relations from chiral kinetic theory:

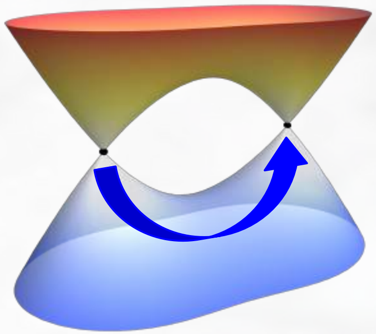
$$\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B}) + (\mathbf{E}_5\cdot\mathbf{B}_5)\right] \quad ✔$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B}_5) + (\mathbf{E}_5\cdot\mathbf{B})\right] \quad ✘$$

- Extra Bardeen-Zumino (Chern-Simons) terms are needed, i.e.

$$\delta\rho = -\frac{e^3}{2\pi^2\hbar^2 c^2}\left((\vec{b}+\vec{A}_5)\cdot\vec{B}\right)$$

$$\delta\vec{j} = -\frac{e^3}{2\pi^2\hbar^2 c^2}\left(b_0 + A_{5,0}\right)\vec{B} + \frac{e^3}{2\pi^2\hbar^2 c^2}\left[(\vec{b}+\vec{A}_5)\times\vec{E}\right]$$



- Electric charge is conserved ($\partial_\mu J^\mu = 0$)
- Anomalous Hall effect is reproduced
- No CME in equilibrium ($\mu_5 = -eb_0$)

[Landsteiner, PRB **89**, 075124 (2014)]
[Landsteiner, Acta Phys. Polon. B **47**, 2617 (2016)]
[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **96**, 085130 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

INSTRUCTIVE EXAMPLE: COLLECTIVE MODES

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

Maxwell equations

Faraday's law:

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

or in momentum space:

$$\frac{c}{\omega}\vec{k}\times\vec{E} = \vec{B}$$

Ampere-Maxwell's law:

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

or in momentum space:

$$\frac{c}{\omega}\vec{k}\times\left(\frac{c}{\omega}\vec{k}\times\vec{E}\right) = -\left(4\pi\frac{i}{\omega}\vec{J} + \vec{E}\right)$$

$$\vec{P} = \frac{i}{\omega}\vec{J}$$

Gauss's law constrains $\vec{E}$ as follows: $i\vec{k}\cdot\vec{E} = 4\pi\rho$

Anomalous contributions enter here

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

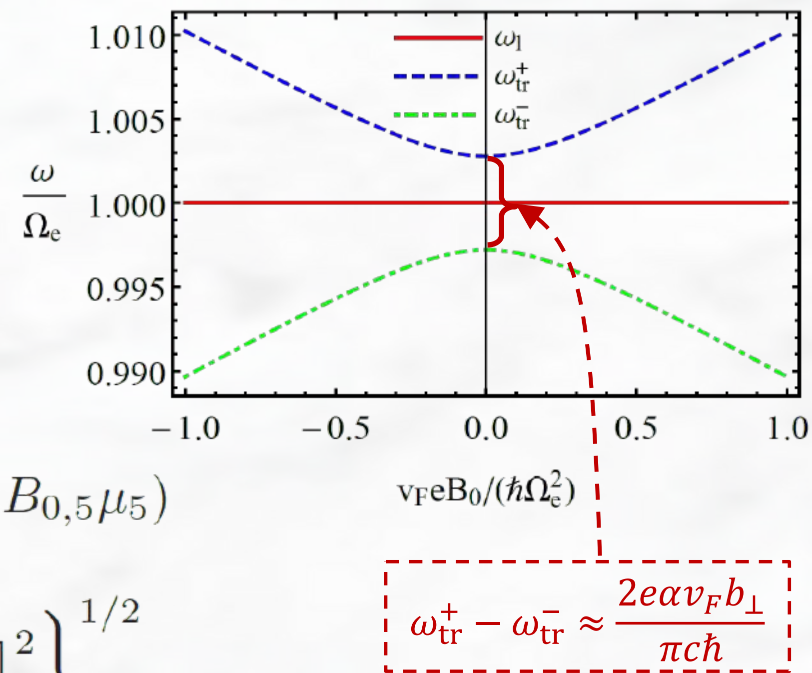
where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

and $\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\Bigg\{9\hbar^2 b_\perp^2 + \Big[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5)$

$$- 3\hbar b_\parallel - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\Big]^2\Bigg\}^{1/2}$$

$b_\perp = 0.2\hbar\Omega_e/e$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Strain-induced helicons ($B = 0$)

- Strain-induced pseudo-magnetic field $B_{0,5}$ leads to

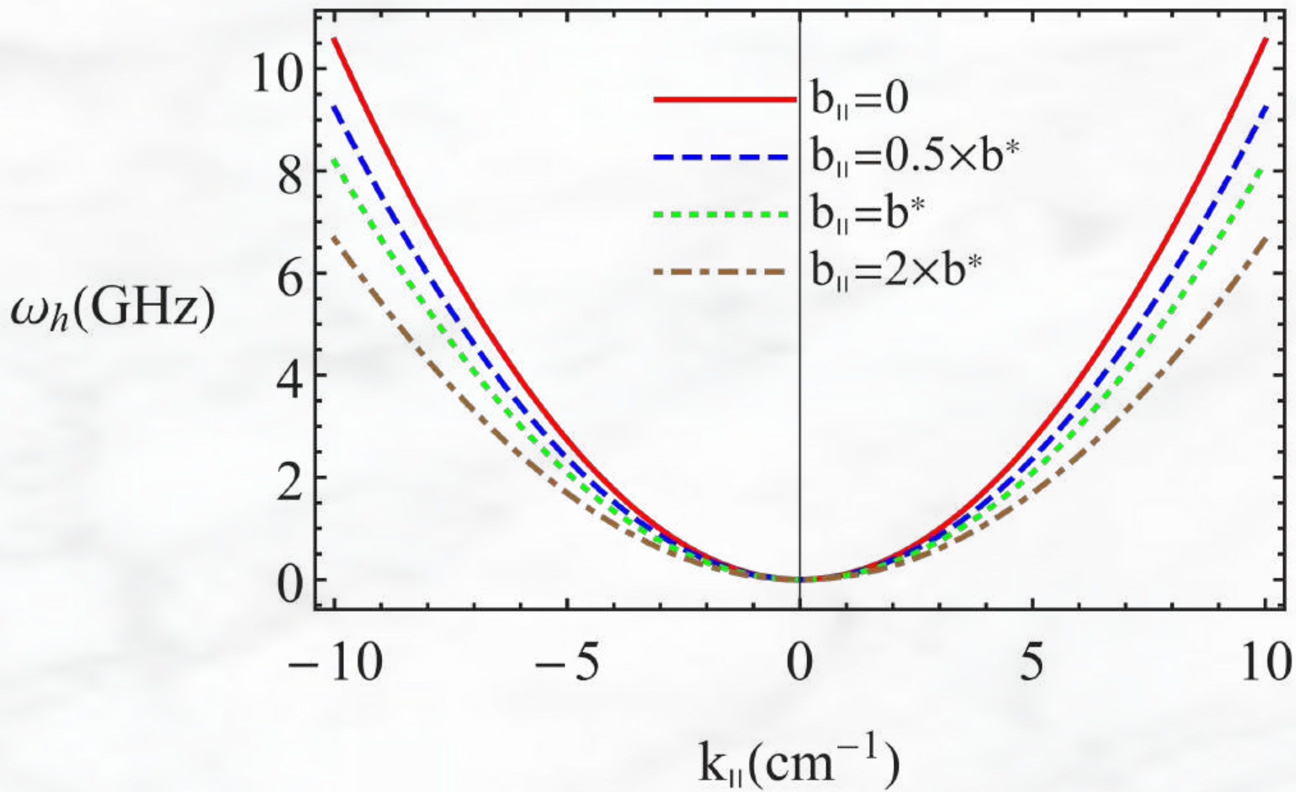
$$\omega_h|_{B_0\to 0,\mu\to 0} \stackrel{b_0\to -\mu_5/e}{=} \frac{eB_{0,5}c^3\hbar^2\pi v_F^2 k^2}{\pi\hbar^2 c^2\Omega_e^2\mu_5 + 2B_{0,5}e^4 v_F^2 b_{\parallel}} + O(k^3)$$

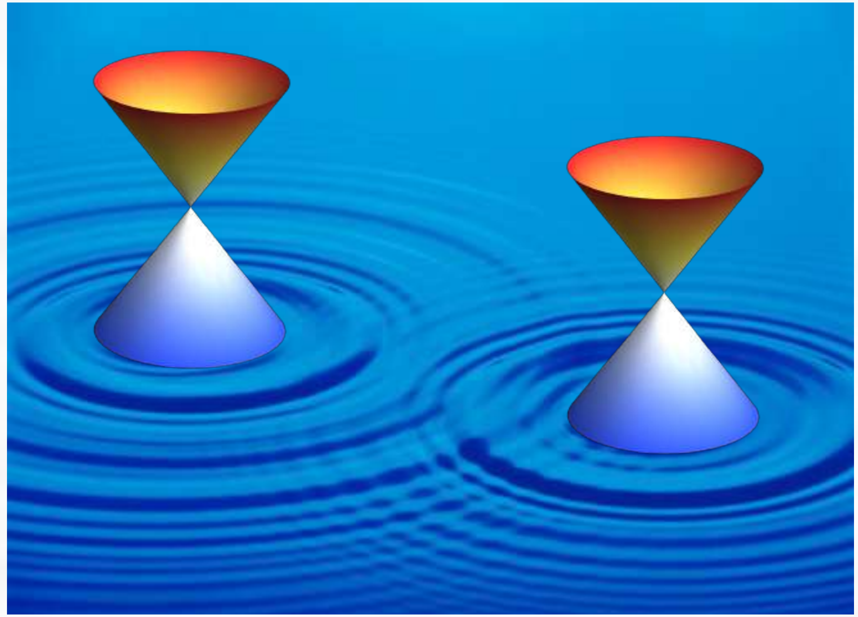
[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

- Properties:
  – Gapless electromagnetic wave propagates in metals **without magnetic field**!
  – Chiral shift modifies effective helicon dispersion
  – In equilibrium, i.e., $\mu_5 = -eb_0$, the term linear in the wave vector is **absent**

Helicons at different $b_{\parallel}$

$$eb_0 = -\mu_5,\ B_{0,5} = 10^{-2}\mathrm{T},\ \mu_5 = 5\ \mathrm{meV},\ \mu = 0,\ eb^* = 0.3\pi\hbar v_F/c_3$$





ANOMALOUS CHIRAL HYDRODYNAMICS

[Son, Surowka, Phys. Rev. Lett. **103**, 191601 (2009)]
[Neiman and Oz, JHEP **03**, 023 (2011)]

Electron hydrodynamics

- First ideas date back to the 1960s [Radii Gurzhi, JETP **17**, 521 (1963)]

Ballistic:

$l_p \gg l_{ee} \gg L$

$R \sim L^{-1}$

Hydro:

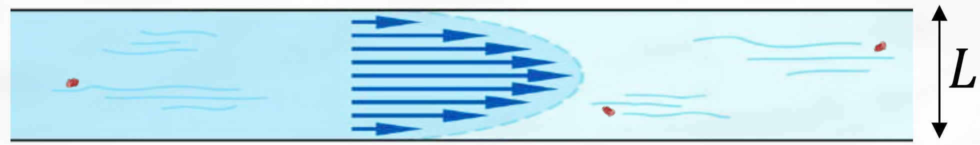
$l_{ee} \ll L \ll l_p$

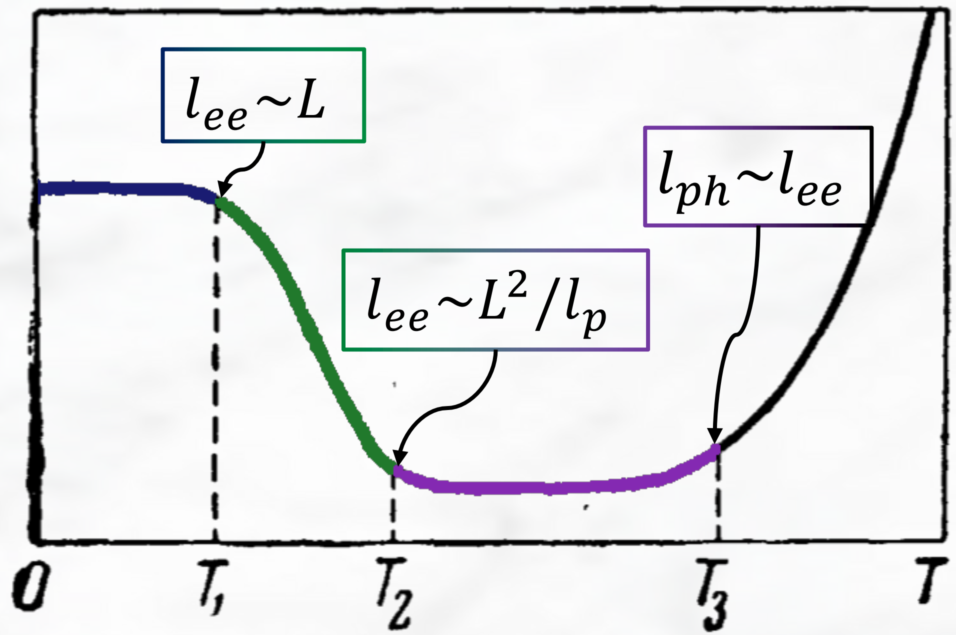
$R \sim l_{ee}/L^2 \sim T^{-5} L^{-2}$

Hydro + impurities:

$l_p \ll L^2/l_{ee}$

$R \sim l_p$

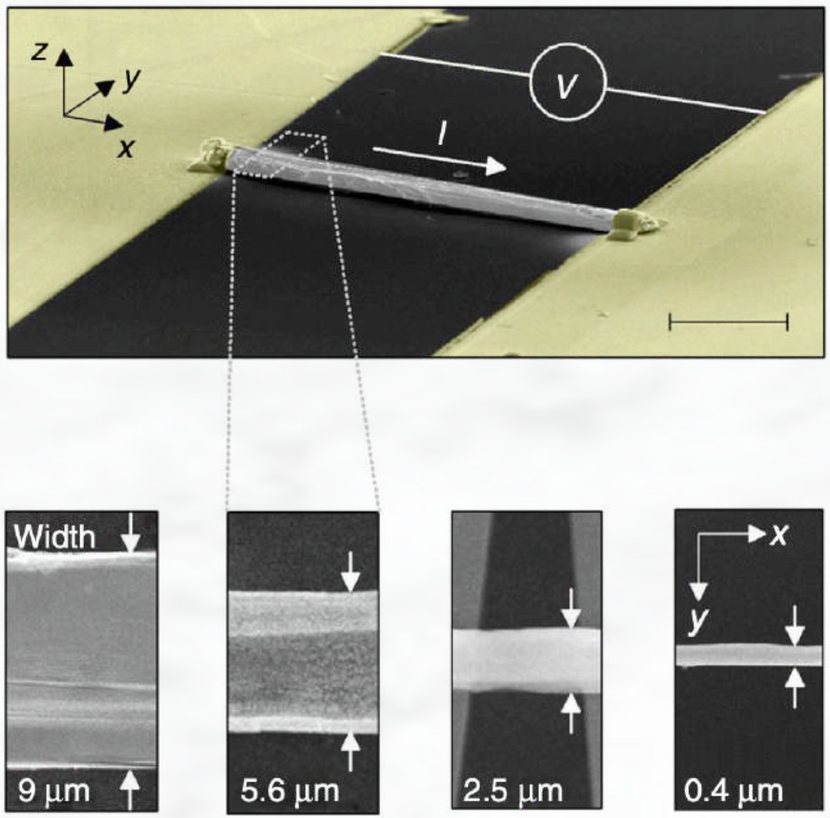


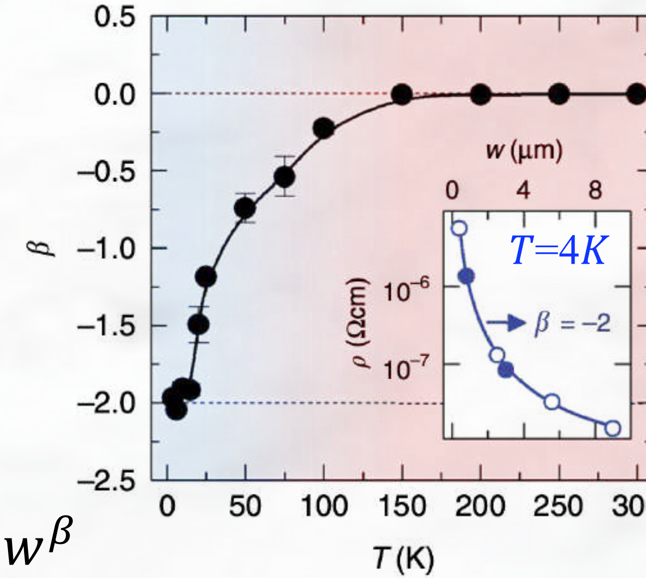


Ohmic ($l_{ph} \ll l_p$): $R \sim T^5$

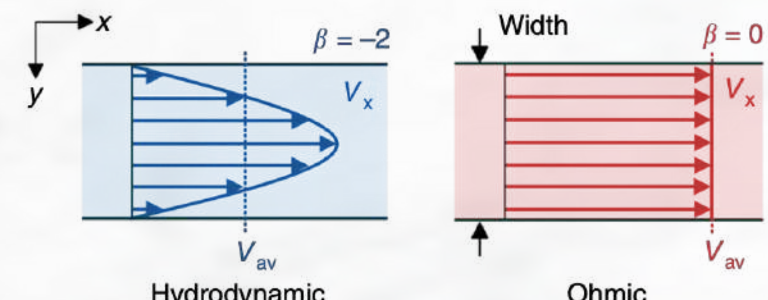
Hydrodynamics in Weyl semimetals

Weyl semimetals WP_2 & WTe_2 [Gooth et al., Nat. Comm. **9,** 4093 (2018); Vool, et al., arXiv:2009.04477]





$$\rho = \rho_0 + \rho_1 w^{\beta}$$



Chiral Hydrodynamics (plasma)

- Evolution of conserved quantities:

[Son, Surowka, Phys. Rev. Lett. **103**, 191601 (2009)]
[Neiman and Oz, JHEP **03**, 023 (2011)]

$$\frac{\partial T^{j0}}{\partial t} + \frac{\partial T^{ji}}{\partial x^i} = -enE^j - e\left(\vec{j}\times\vec{B}\right)^j + F^j_{\mathrm{other}}$$

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{0i}}{\partial x^i} = -e\left(\vec{E}\cdot\vec{j}\right) + W_{\mathrm{other}}$$

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

$$\frac{\partial n_5}{\partial t} + \vec{\nabla}\cdot\vec{j}_5 = -\frac{e^2(\vec{B}\cdot\vec{E})}{2\pi^2} - \Gamma_5 n_5$$

$\oplus$ Maxwell equations

Note:
$T^{00} = \varepsilon + \cdots$
$T^{0i} = wv^i + \cdots$
$T^{ij} = wv^i v^j - P\delta^{ij} + \cdots$
$w = \varepsilon + P$

Constitutive relations

- Expressions for currents and $T^{\mu\nu}$

[Son, Surowka, Phys. Rev. Lett. **103**, 191601 (2009)]
[Neiman and Oz, JHEP **03**, 023 (2011)]

$$\vec{J} = n\vec{v} + \vec{J}_a + \vec{J}_{dis}$$

$$\vec{J}_5 = n_5\vec{v} + \vec{J}_{5,a} + \vec{J}_{5,dis}$$

$$T^{\mu\nu} = \varepsilon v^\mu v^\nu - \Delta^{\mu\nu} P + h^\mu v^\nu + v^\mu h^\nu + \tau^{\mu\nu}_{dis}$$

- Anomalous terms:

$$\vec{J}_a = \vec{J}_{CS} + \sigma_\omega \vec{\omega} + \sigma_B \vec{B} \qquad \& \qquad \vec{J}_{5,a} = \vec{J}_{5,CS} + \sigma^5_\omega \vec{\omega} + \sigma^5_B \vec{B}$$

where

$$\sigma_\omega = \frac{\mu\mu_5}{\pi^2\hbar^2}, \qquad \sigma_B = \frac{e\mu_5}{2\pi^2\hbar^2}$$

$$\sigma^5_\omega = \frac{3(\mu^2+\mu_5^2)+\pi^2T^2}{6\pi^2\hbar^2}, \qquad \sigma^5_B = \frac{e\mu}{2\pi^2\hbar^2}$$

Rich spectrum of hydro modes

- One example: longitudinal anomalous Hall wave (with $\boldsymbol{k} \parallel \boldsymbol{B}_0$ and $\boldsymbol{b} \perp \boldsymbol{B}_0$):

$$\omega_{\mathrm{lAHW},\pm} \approx \pm \frac{\hbar B_0 k_{\parallel} \sqrt{3 v_{\mathrm{F}}^3 \left(\pi^3 c^4 \hbar T_0^2 + 3 e^4 \mu_m v_{\mathrm{F}}^3 b_{\perp}^2\right)}}{c T_0 \sqrt{\pi^3 \mu_m \left(3 \varepsilon_e v_{\mathrm{F}}^3 \hbar^3 B_0^2 + 4\pi e^2 T_0^2 b_{\perp}^2\right)}} + O(k_{\parallel}^3)$$

n continuity equation

$$\frac{T^2 \omega}{3 v_{\mathrm{F}}^3 \hbar} \delta\mu + \frac{e B_0 k_{\parallel}}{2\pi^2 c} \delta\mu_5 = 0$$

n_5 continuity equation

$$\frac{e B_0 k_{\parallel}}{2\pi^2 c} \delta\mu + \frac{T^2 \omega}{3 v_{\mathrm{F}}^3 \hbar} \delta\mu_5 - i \frac{e^2 B_0}{2\pi^2 c} \delta E_{\parallel} = 0$$

$$\varepsilon_e \omega \delta E_{\parallel} + \mathrm{i} \frac{2e^2}{\pi c \hbar^2} \left(B_0 \delta\mu_5 + e b_{\perp} \delta\tilde{E}_{\perp}\right) = 0$$

$$\left(\omega^2 - \frac{c^2 k_{\parallel}^2}{\varepsilon_e \mu_m}\right) \delta\tilde{E}_{\perp} - \mathrm{i} \frac{2 e^3 \omega b_{\perp}}{\pi c \varepsilon_e \hbar^2} \delta E_{\parallel} = 0$$

Maxwell's equations

$\delta\tilde{\boldsymbol{E}}_{\perp} \parallel [\boldsymbol{B}_0 \times \boldsymbol{b}]$

Summary

- **Chiral anomaly** can be realized and tested in Dirac/Weyl semimetals
- **Chiral magnetic effect** is strongly supported by experimental data
- **Chiral charge**, which is relatively long-lived, can be optically pumped and manipulated
- Many other **anomalous effects** are proposed theoretically
- Plethora of **collective modes** with anomalous features may exist
- Some anomalous properties can also appear in the regime of **electron hydrodynamics**