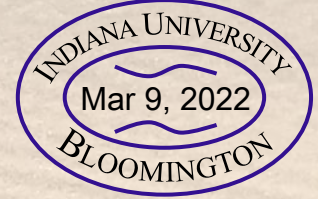




ASU College of Integrative
Sciences and Arts
ARIZONA STATE UNIVERSITY

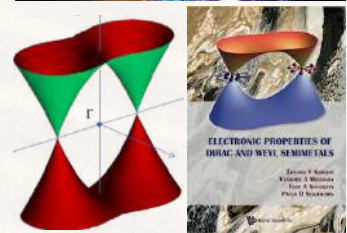
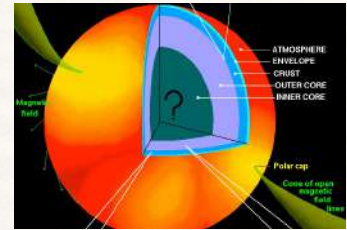
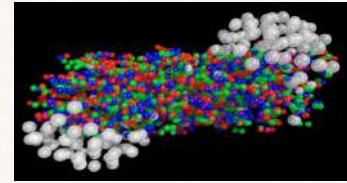
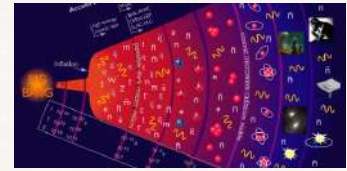


CHIRAL MATTER: from Quark Gluon Plasma to Topological Semimetals

Igor Shovkovy

Physics Colloquium, Indiana University Bloomington

- **Early Universe**
(extremely high temperature $> 10^{15}$ K)
- **Heavy-ion collisions**
(high temperature $\lesssim 4 \times 10^{12}$ K)
- **Super-dense matter in compact stars**
(high densities $\lesssim 10^{17}$ kg/m³)
- **Magnetospheres of magnetars**
(electron-positron plasma at temperatures $\lesssim 10^{11}$ K)
- **Electron plasma in Dirac/Weyl (semi-)metals**
(chiral quasiparticle plasma at temperatures $\lesssim 10^2$ K)
- **Other: cold atoms, superfluid ³He-A, etc.**
(chiral quasiparticles at temperatures $\sim 10^{-3}$ K)





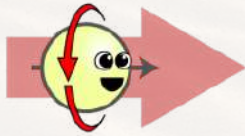
ANOMALOUS MATTER

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]

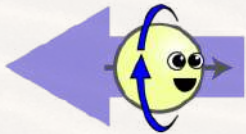
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]

[Becattini, Liao, Lisa, Lect. Notes Phys. **987**, 1 (2021)]

- Only *massless* Dirac/Weyl fermions have a well-defined chirality ($\gamma^5 \psi = \pm \psi$)*:



Right-handed (spin parallel to momentum)



Left-handed (spin opposite to momentum)

- The chirality of *massive* Dirac fermions is *almost* well-defined in the *ultra-relativistic* regime*
 - High temperature: $T \gg m$
 - High density: $\mu \gg m$

*Note: like the particle spin, chirality is a quantum property

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ existing on *macroscopic* time/distance scales
- The spacetime dynamics of $n = n_R + n_L$ and $n_5 = n_R - n_L$ is governed by continuity equations

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial n_5}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_m n_5$$

chiral anomaly

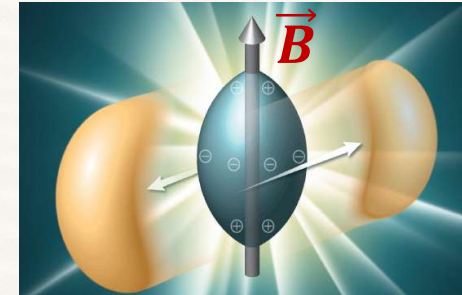
where the chirality flip rate: $\Gamma_m \propto \alpha^2 T (m/T)^2$

- Chiral anomaly can produce *macroscopic* effects in plasmas

Anomalous effects

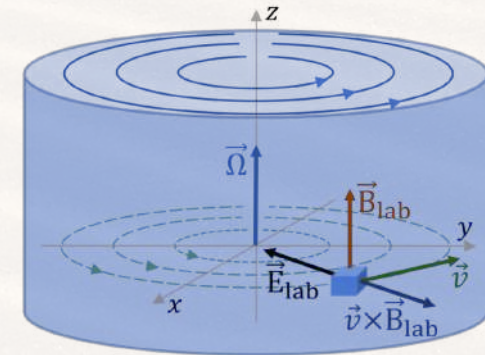
- **Theory:** Many *macroscopic* chiral anomalous effects were proposed
- Some are triggered by an external magnetic field
 - Chiral magnetic effect
 - Chiral separation effect
 - Chiral magnetic wave
 - Negative magnetoresistance
 - ...

} this talk

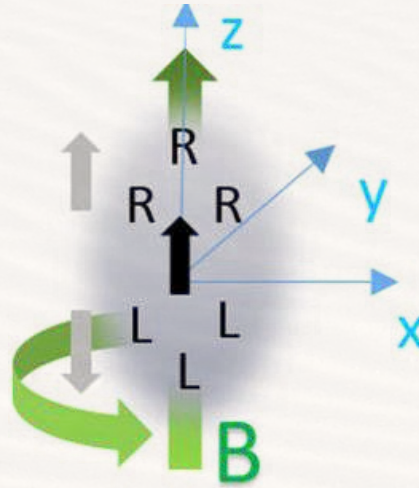


Review: [Shovkovy, arXiv:2111.11416]

- Others are triggered by vorticity
 - Chiral vortical effect
 - Chiral vortical wave
 - ...



Review: [Becattini, Liao, Lisa, Lect. Notes Phys. **987**, 1 (2021)]



CHIRAL ANOMALOUS EFFECT

$$\langle \vec{J}_5 \rangle = -\frac{e\vec{B}}{2\pi^2} \mu \quad \& \quad \langle \vec{j} \rangle = \frac{e^2\vec{B}}{2\pi^2} \mu_5$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

[Newman & Son, Phys. Rev. D 73 (2006) 045006]

- Landau energy levels at $m = 0$

$$E_n^\pm = \pm \sqrt{2n|eB| + p_z^2}$$

where $n = \underbrace{k + \frac{1}{2}}_{\text{orbital}} + \underbrace{\text{sgn}(eB)s_z}_{\text{spin}}$

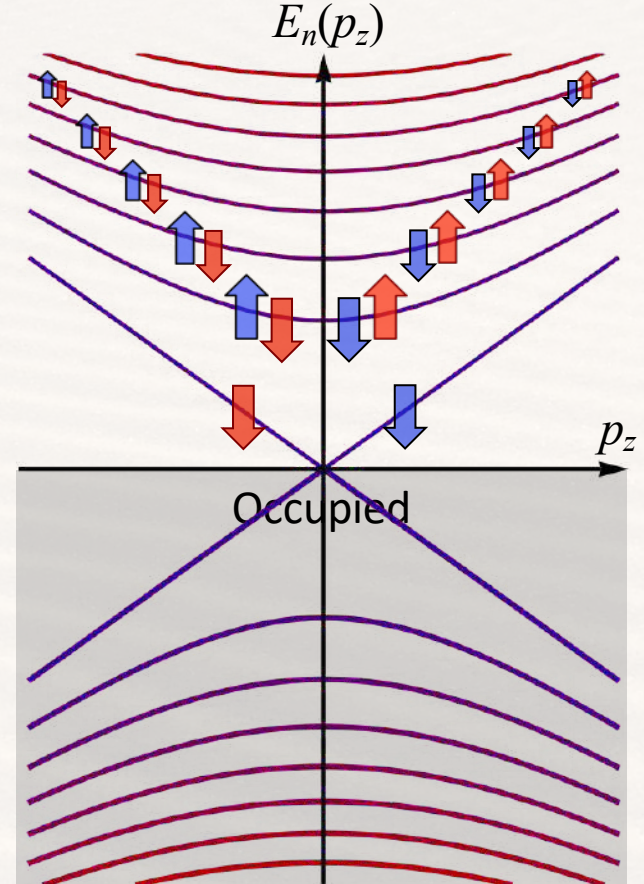
- Lowest Landau level is spin polarized

$$E_0^\pm = \pm p_z \quad (k = 0, s_z = -\frac{1}{2})$$

- Higher Landau levels ($n \geq 1$) are twice as degenerate and non-polarized:

(i) $k = n$ & $s = -\frac{1}{2}$

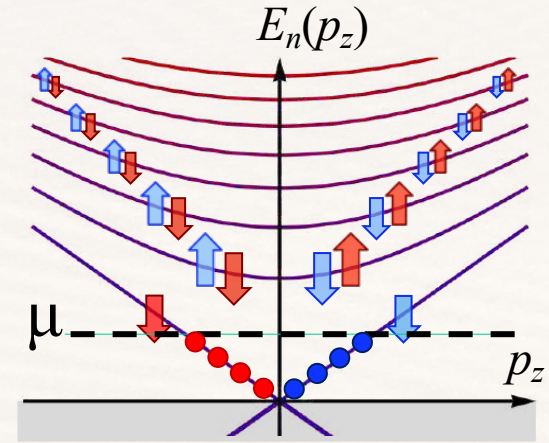
(ii) $k = n - 1$ & $s = +\frac{1}{2}$



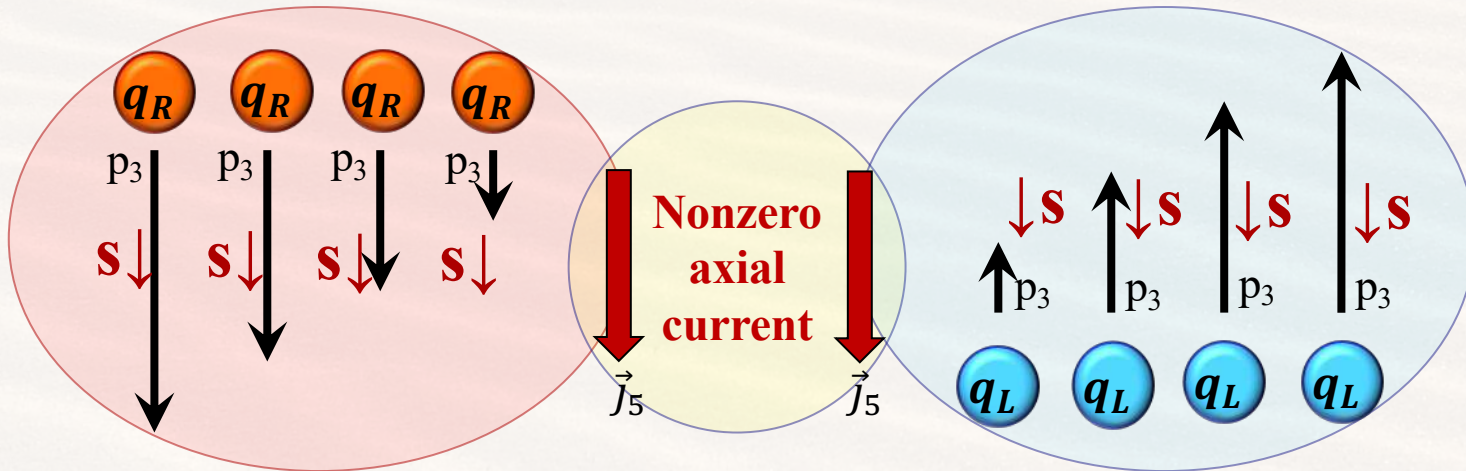
[Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]

Chiral Separation Effect ($\mu \neq 0$)

- **Spin polarized LLL** is chirally asymmetric
 - states with $p_3 < 0$ (and $s = \downarrow$) are **R-handed**
 - states with $p_3 > 0$ (and $s = \downarrow$) are **L-handed**
- i.e., a nonzero **axial** current is induced



$$\langle \vec{j}_5 \rangle = -tr[\vec{\gamma}\gamma^5 S(x, x)] = -\frac{e\vec{B}}{2\pi^2} \mu$$

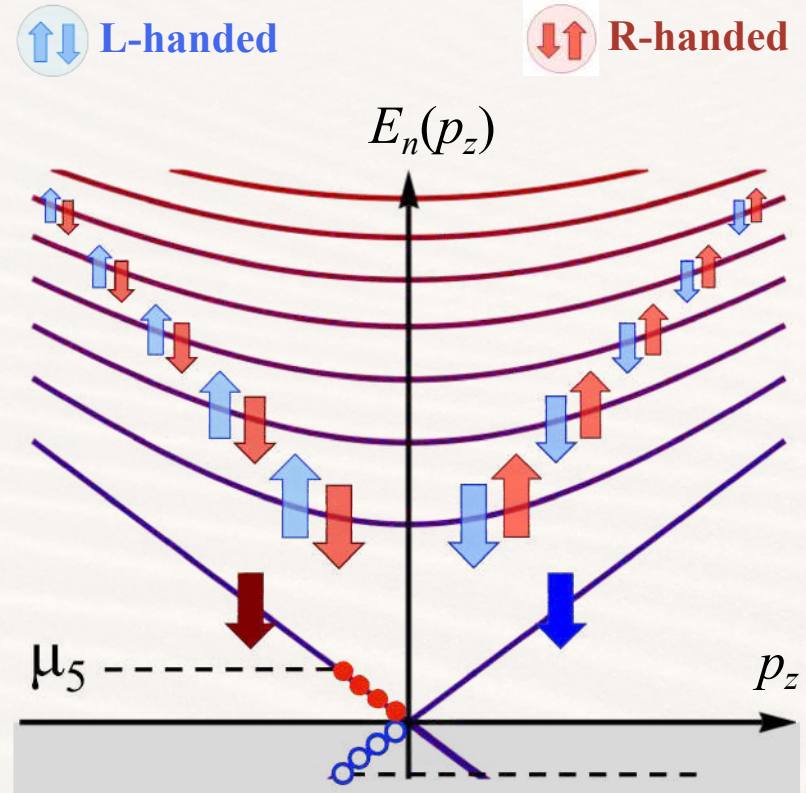


Chiral Magnetic Effect ($\mu_5 \neq 0$)

Assume a *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

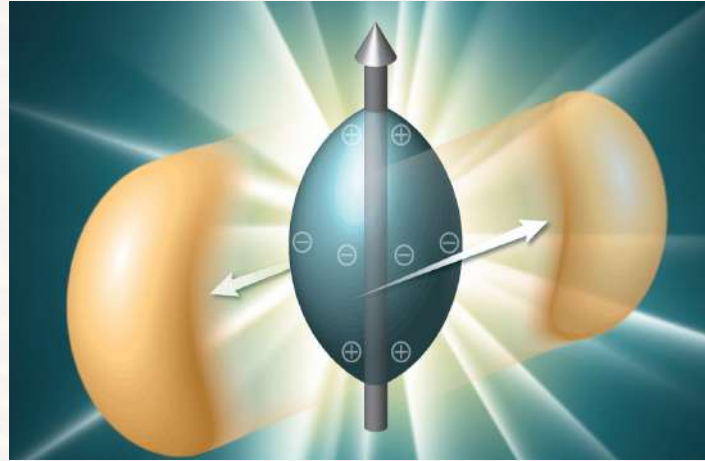
Spin polarized LLL ($s=\downarrow$ for particles of a *negative* charge):

- Some **R-handed** states ($p_3 < 0$ & $E < \mu_5$) are occupied
- Some **L-handed** states ($p_3 < 0$ & $|E| < \mu_5$) are empty (i.e., holes with $p_3 > 0$)



CME current: $\langle \vec{j} \rangle = -tr[\vec{\gamma}S(x, x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$

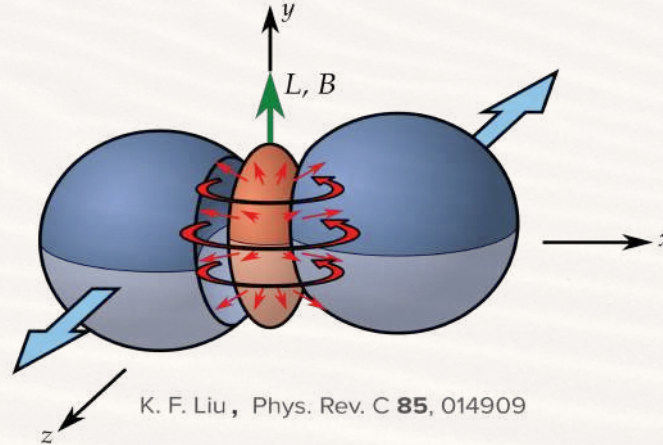
[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]



HEAVY-ION COLLISIONS

\vec{B} and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields are induced by the currents of passing charged ions



[Rafelski & Müller, PRL, 36, 517 (1976)],
 [Kharzeev et al., arXiv:0711.0950],
 [Skokov et al., arXiv:0907.1396],
 [Voronyuk et al., arXiv:1103.4239],
 [Bzdak & Skokov, arXiv:1111.1949],
 [Deng & Huang, arXiv:1201.5108], ...

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

$$\omega \sim 9 \times 10^{21} \text{ s}^{-1} (\sim 10 \text{ MeV})$$

- Magnetic field estimate:

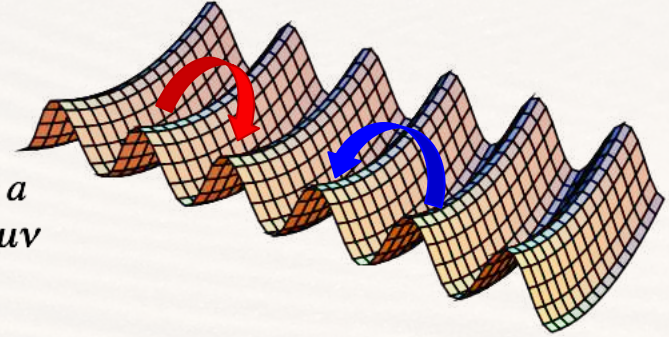
$$B \sim 10^{18} \text{ to } 10^{19} \text{ G } (\sim 100 \text{ MeV})$$



Source of chirality in QCD

- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$

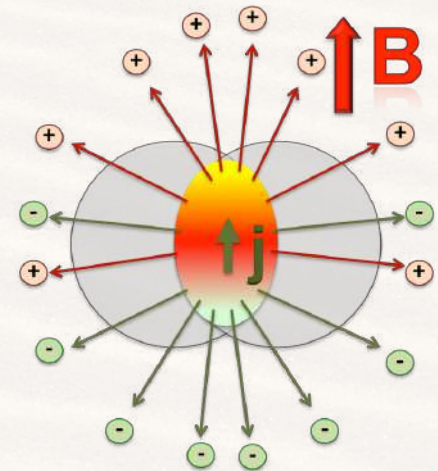


- A random fluctuation with nonzero chirality should produce

$$N_R - N_L \neq 0 \Rightarrow \mu_5 \neq 0$$

- The latter leads to an electric CME current

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$



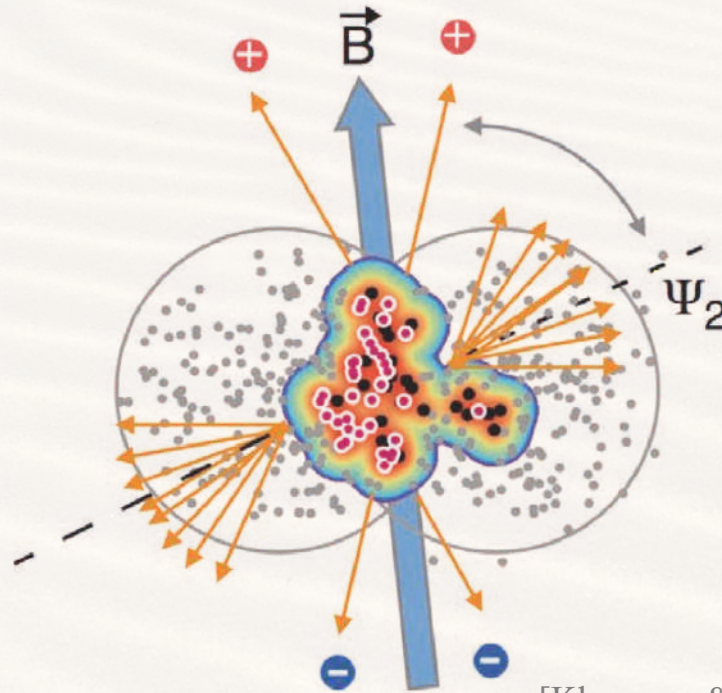
Dipole CME

- Dipole pattern of *charged particle correlations* in heavy-ion collisions

$$\langle \cos(\varphi_{\alpha}^{\pm} + \varphi_{\beta}^{\mp} - 2\Psi_{RP}) \rangle > 0 \quad \& \quad \langle \cos(\varphi_{\alpha}^{\pm} + \varphi_{\beta}^{\pm} - 2\Psi_{RP}) \rangle < 0$$

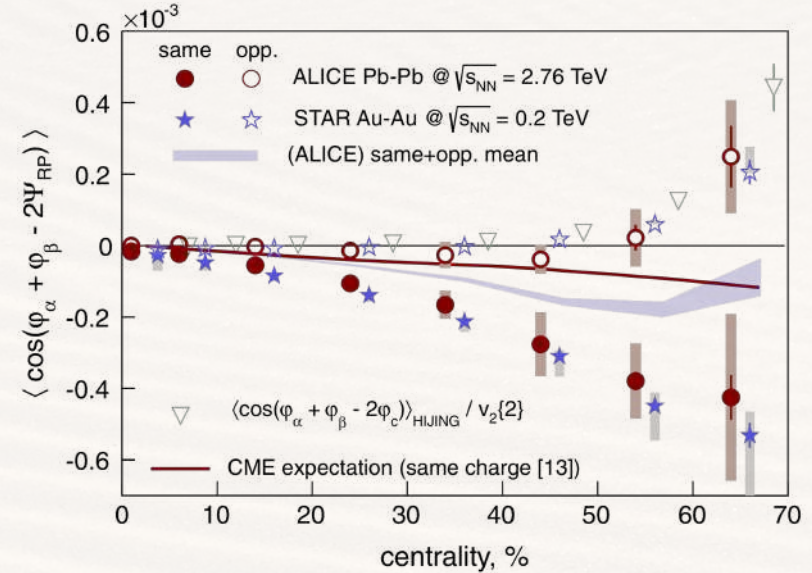
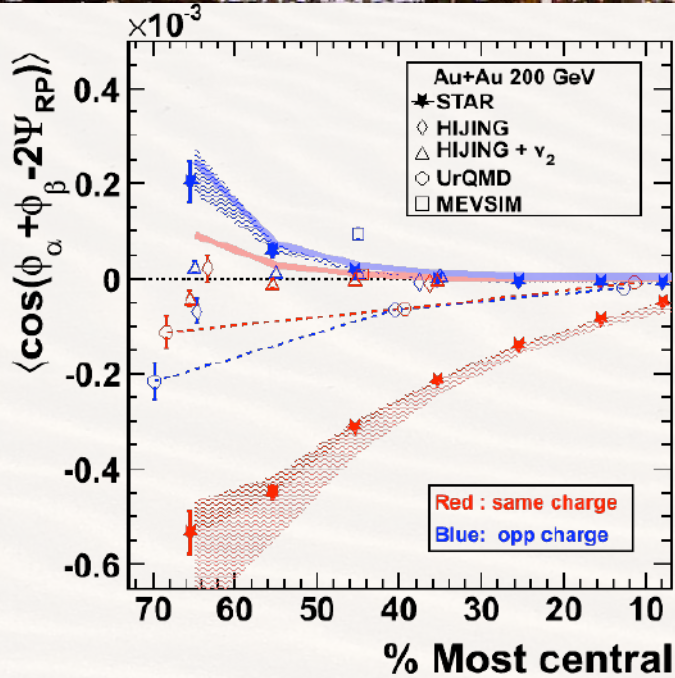
[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]



[Kharzeev & Liao, Nucl. Phys. News **29**, 1 (2019)]

CME: Experimental evidence



Correlations of same & opposite charge particles: $\left\{ \begin{array}{l} \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\mp - 2\Psi_{RP}) \rangle > 0 \\ \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}) \rangle < 0 \end{array} \right.$

- [Abelev et al. (STAR), PRL **103**, 251601 (2009)]
- [Abelev et al. (STAR), PRC **81**, 054908 (2010)]
- [Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
- [Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]
- [Adamczyk et al. (STAR), PRL **113**, 052302 (2014)]
- [Khachatryan et al. (CMS), PRL **118**, 122301 (2017)]

Large background effects!

- [Belmont & Nagle, PRC **96**, 024901 (2017)]
- [ALICE Collaboration, Phys. Lett. B **777**, 151 (2018)]

Isobar collisions

Utilize collisions of isobars, e.g.,

[Voloshin, PRL **105**, 172301 (2010)]
 [Deng et al. PRC **94**, 041901(R) (2016)]

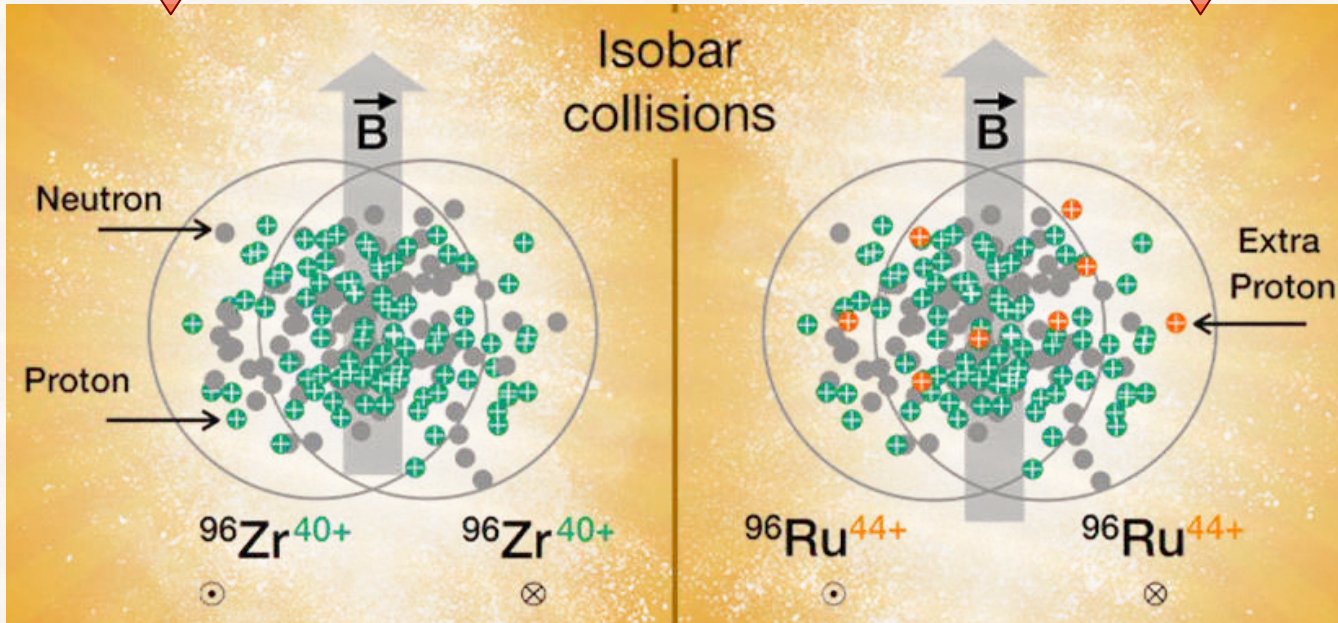
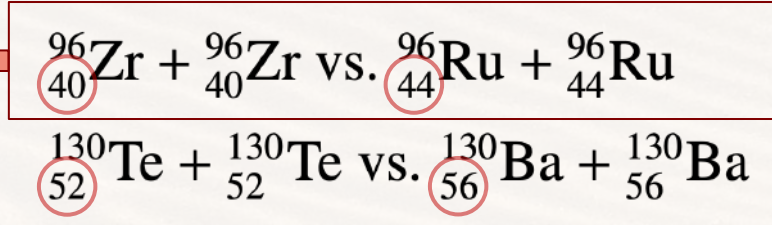
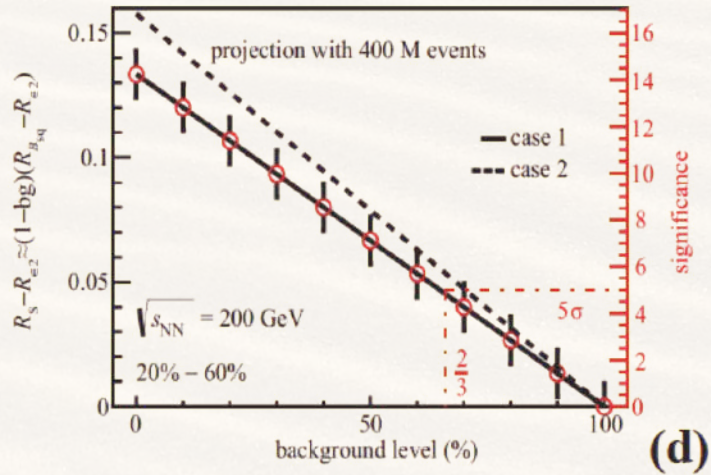
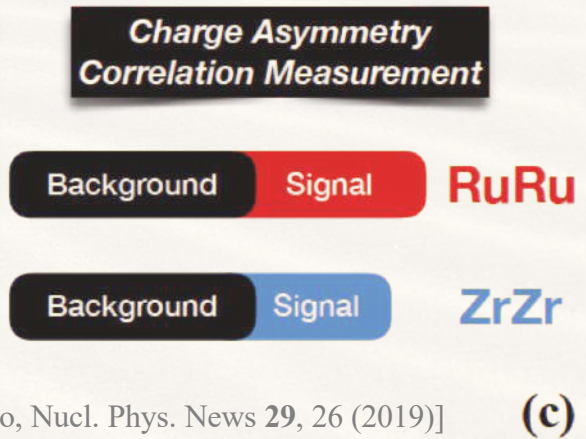
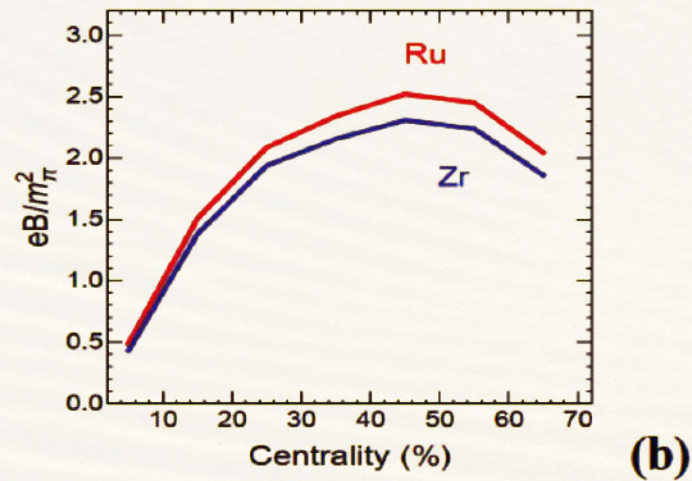
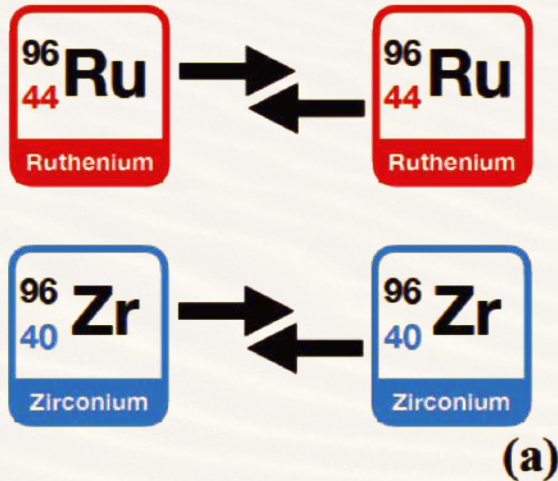


Image credit: Brookhaven National Laboratory, <https://www.bnl.gov/newsroom/news.php?a=119062>

Isobar collisions (theory)



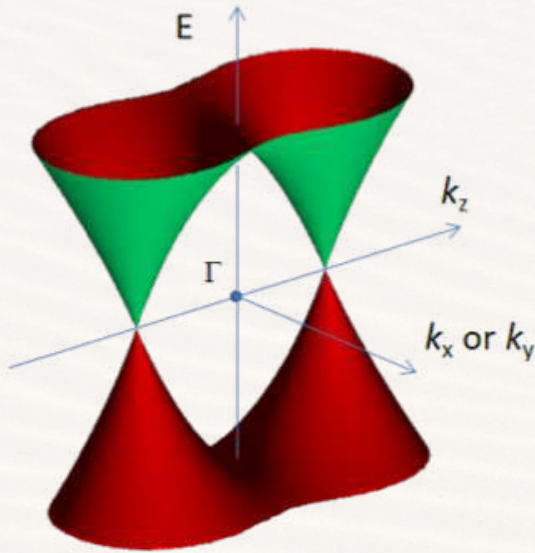
[Kharzeev and Liao, Nucl. Phys. News 29, 26 (2019)]

Isobar collisions (experiment)

- Isobar run was completed by STAR in May 2018
- ≈ 3.8 billion collisions of $^{96}\text{Ru} + ^{96}\text{Ru}$ and $^{96}\text{Zr} + ^{96}\text{Zr}$ at $\sqrt{s} = 200$ GeV
- Blind analysis by five groups of the STAR Collaboration
- Report announced on Aug. 31, 2021 (online event @ BNL)
- Paper published on Jan. 3, 2022

[STAR Collaboration, Phys.Rev.C **105**, 014901 (2022); arXiv:2109.00131]

influence of signal and backgrounds, the STAR Collaboration performed a blind analysis of a large data sample of approximately 3.8 billion isobar collisions of $^{96}_{44}\text{Ru} + ^{96}_{44}\text{Ru}$ and $^{96}_{40}\text{Zr} + ^{96}_{40}\text{Zr}$ at $\sqrt{s_{NN}} = 200$ GeV. Prior to the blind analysis, the CME signatures are predefined as a significant excess of the CME-sensitive observables in Ru + Ru collisions over those in Zr + Zr collisions, owing to a larger magnetic field in the former. A precision down to 0.4% is achieved, as anticipated, in the relative magnitudes of the pertinent observables between the two isobar systems. Observed differences in the multiplicity and flow harmonics at the matching centrality indicate that the magnitude of the CME background is different between the two species. No CME signature that satisfies the predefined criteria has been observed in isobar collisions in this blind analysis.



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL SEMIMETALS

[Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]



Dirac/Weyl fermions

- Electron quasiparticles with a wide range of properties are possible
- They may even have the emergent spinor structure of *massless* Weyl fermions,

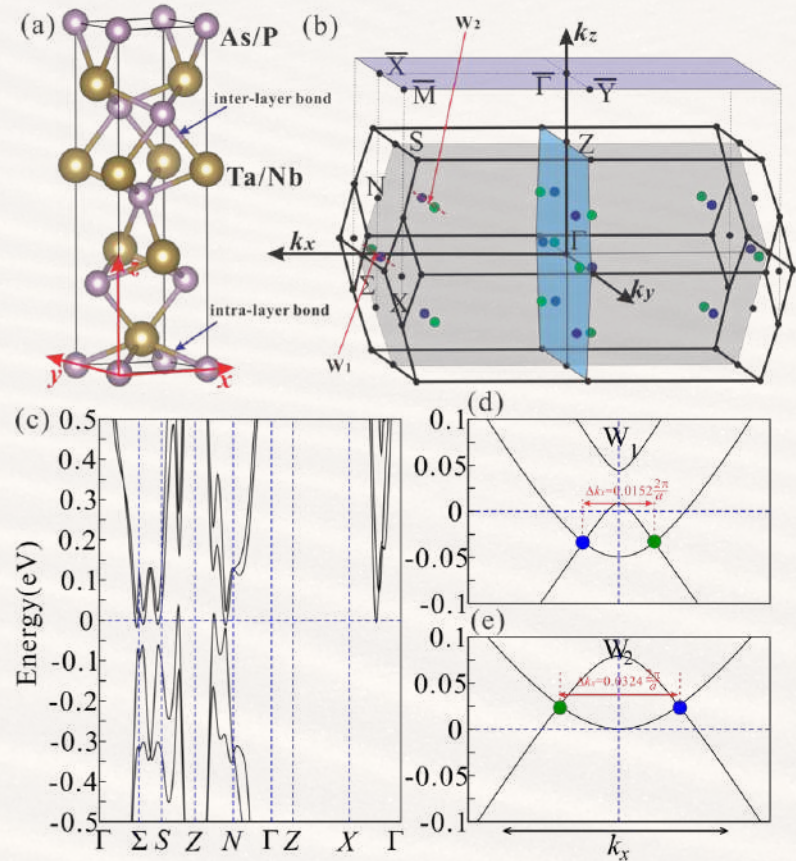
$$H_W \approx \pm v_F (\vec{\sigma} \cdot \vec{k})$$

Such nodes are not uncommon!

Na_3Bi , Cd_3As_2 , ZrTe_5 , TaAs , NbAs , ...

- [Liu et al., Science **343**, 864 (2014)]
- [Neupane et al., Nature Commun. **5**, 3786 (2014)]
- [Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
- [Li et al., Nature Physics **12**, 550 (2016)]
- [S.-Y. Xu et al., Science **349**, 613 (2015)]
- [B. Q. Lv et al., Phys. Rev. X **5**, 031013 (2015)]
- [S.-Y. Xu et al., Nature Physics **11**, 748 (2015)]
- [S.-Y. Xu et al., Science Adv. **1**, 1501092 (2015)]
- [F. Y. Bruno et al., Phys. Rev. B **94**, 121112 (2016)]

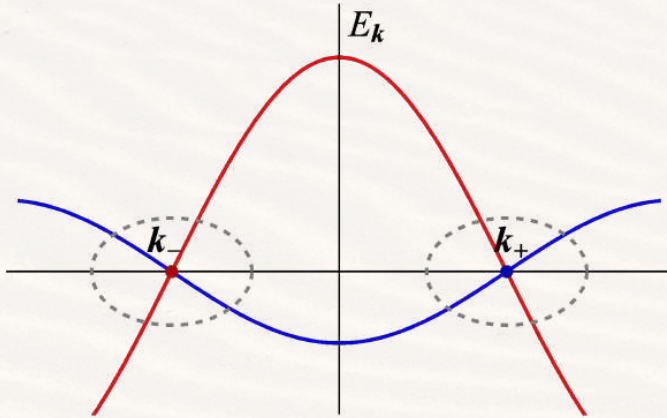
Weyl semimetals TaAs, TaP, NbAs, and NbP



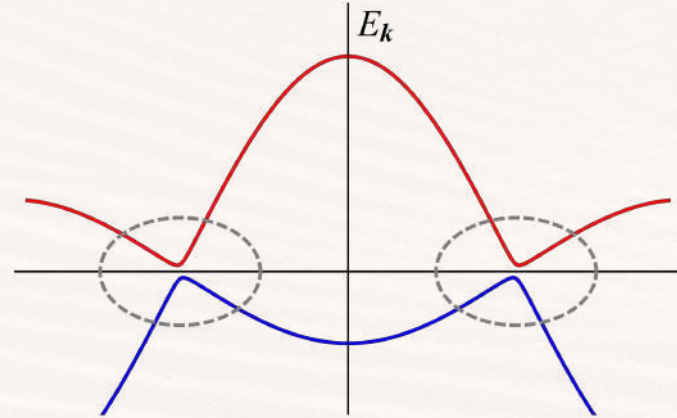
Sun, Wu & Yan, Phys. Rev. B **92**, 115428 (2015)

Relativistic-like band crossing

Do energy levels cross?



Or do they repel?



A generic 2-band Hamiltonian reads

$$H_{\mathbf{k}} = a_{\mathbf{k}} + \vec{b}_{\mathbf{k}} \cdot \vec{\sigma} \quad \Rightarrow \quad E_{\mathbf{k}} = a_{\mathbf{k}} \pm \sqrt{(\vec{b}_{\mathbf{k}})^2}$$

The bands cross when [Witten, Riv. Nuovo Cimento 39, 313 (2016)]

$$\vec{b}_{\mathbf{k}} = 0$$

These 3 equations can be solved by adjusting $\vec{\mathbf{k}}$ in 3D

Emergent chirality in solids

Near a band crossing (e.g., $\vec{k} \approx \vec{k}_+$)

$$H_{\vec{k}} = a_{\vec{k}_+} + \cancel{(\vec{\nabla}_{\vec{k}} a_{\vec{k}} \cdot \delta \vec{k})} + \sum_{i,j} \sigma_i b_{ij} \delta k_j$$

cone tilting

After an orthogonal transformation

$$b_{ij} \equiv \partial b_i / \partial k_j \rightarrow \hbar v_i \delta_{ij}$$

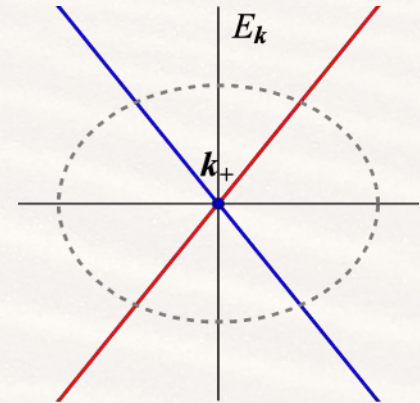
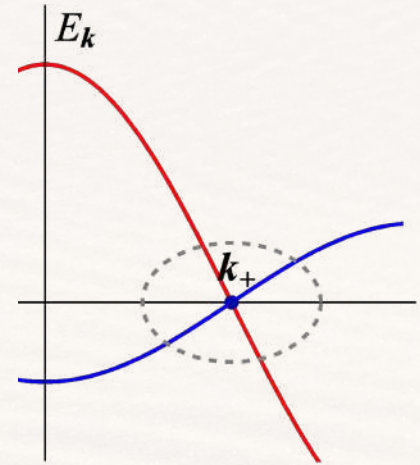
Assuming *isotropy* & a suitable *reference point*,

$$H_{\vec{k}} = \pm v_F (\vec{\sigma} \cdot \vec{k})$$

which is the Weyl Hamiltonian

The *chirality* is defined by

$$\lambda = \text{sign}[\det(b_{ij})]$$



Weyl quasiparticles

- The quasiparticle eigenstates for Weyl Hamiltonian $H_\lambda = \lambda v_F (\vec{k} \cdot \vec{\sigma})$ are

$$\psi_{\mathbf{k}}^\lambda = \frac{1}{\sqrt{2} \sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

- The quasiparticle energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$ is relativistic-like
- Function $k \rightarrow \psi_{\mathbf{k}}^\lambda$ has a nontrivial topology
- Consider adiabatic evolution of the wave function from $\psi_{\mathbf{k}}$ to $\psi_{\mathbf{k}+\delta\mathbf{k}}$:

$$\langle \psi_{\mathbf{k}} | \psi_{\mathbf{k}+\delta\mathbf{k}} \rangle \approx 1 + \delta\mathbf{k} \cdot \langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle \approx e^{i\mathbf{a}_{\mathbf{k}} \cdot \delta\mathbf{k}}$$

where $\mathbf{a}_{\mathbf{k}} = -i \langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle$ is the Berry connection

Berry curvature & topology

- For Weyl eigenstates, the Berry curvature is

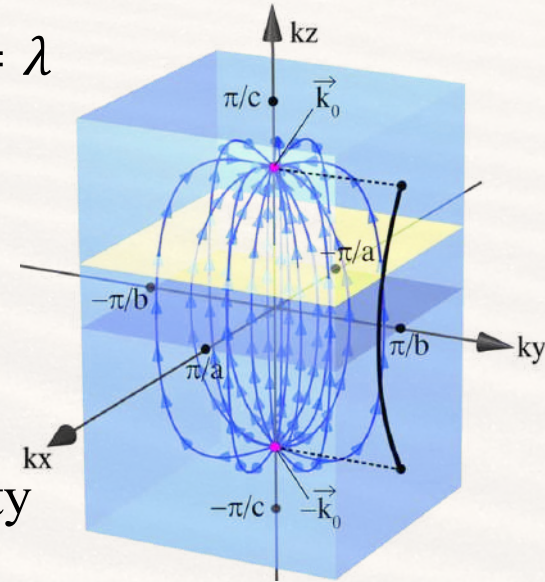
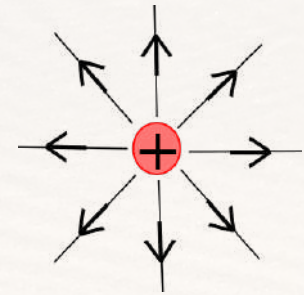
$$\mathbf{\Omega}_k \equiv \nabla_k \times \mathbf{a}_k = \lambda \frac{\vec{k}}{2k^3}$$

- The Chern number (topological charge)

$$C = \frac{1}{2\pi} \oint \mathbf{\Omega}_k \cdot d\mathbf{S}_k = \frac{\lambda}{2\pi} \oint \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin \theta d\theta d\varphi = \lambda$$

- In a solid state material, the Brillouin zone is compact
- A closed surface around a node at \vec{k}_0 is also a closed surface around the rest of the Brillouin zone
- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

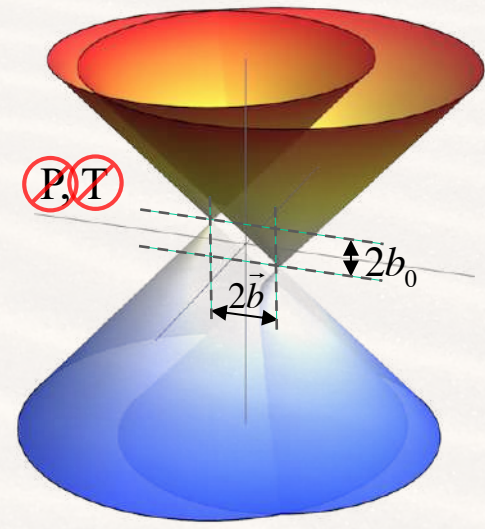
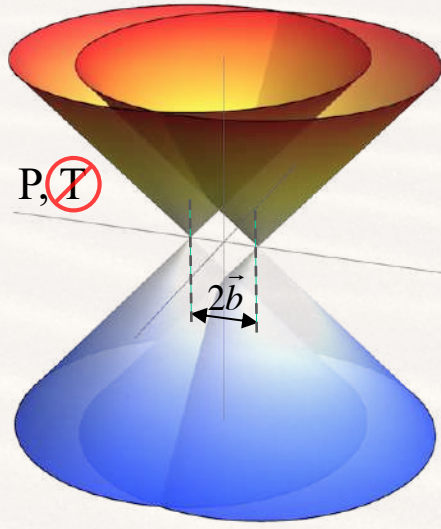
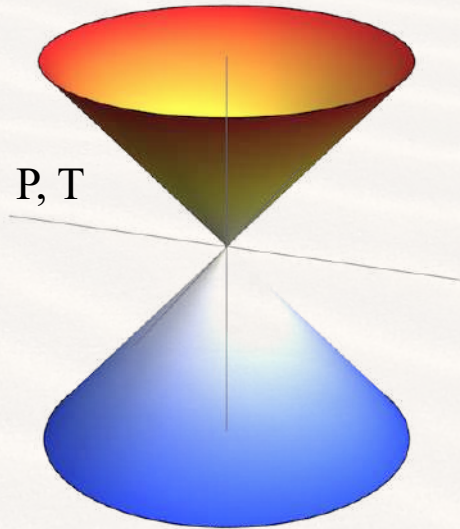
Idealized Dirac and Weyl model

- Low-energy Hamiltonians of a Dirac and Weyl materials

$$H = \int d^3\mathbf{r} \bar{\psi} \left[-iv_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - \underbrace{(\vec{b} \cdot \vec{\gamma})}_{\text{P}} \gamma^5 + \underbrace{b_0 \gamma^0 \gamma^5}_{\text{T}} \right] \psi$$

Dirac (e.g., Na₃Bi, Cd₃As₂, ZrTe₅)

Weyl (e.g., TaAs, NbAs, TaP, NbP, WTe₂)



- Observable properties of Dirac/Weyl semimetals are sensitive to (i) the chiral anomaly, (ii) the values of b_0 and \vec{b} , and (iii) nontrivial topology
- Partial list of potential anomalous effects:
 - Negative magnetoresistance (ρ_{\parallel} decreasing with B)
 - New types of collective modes (anomalous Hall waves, pseudo-magnetic helicons, chiral zero sound, etc.)
 - Anomalous thermoelectric effects (e.g., $\vec{J}_Q \propto \vec{b} \times \vec{E}$ and $\vec{J}_Q \propto \vec{b} \times \vec{\nabla}T$)
 - Strain/torsion induced CME ($\vec{J} \propto u_{33}\vec{B}$ and $\vec{J} \propto \mu\vec{B}_5$)
 - Strain/torsion dependent conductivity/resistance
 - Quantum oscillations in thin films [$T \propto v_F/(\mu b)$]
 - Strain/torsion induced quantum oscillations (pseudo-Landau levels)
 - Nonlocal anomalous transport

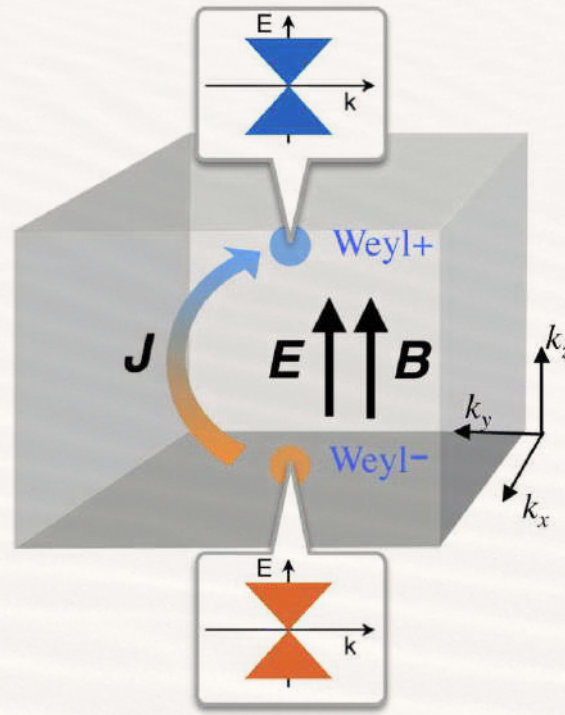


Image credit [Zhang et al., Nat. Commun. 7, 10735 (2016)]

NEGATIVE MAGNETORESISTANCE & MORE

Steady CME current

- Homogeneous chiral plasma:

$$\frac{\partial n_5}{\partial t} + \cancel{\vec{\nabla} \cdot \vec{j}_5} = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \frac{n_5}{\tau_{\text{ch}}}$$

- Steady state ($\tau_{\text{ch}} \sim 1 \text{ ps to } 1 \text{ ns}$)

$$n_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} \tau_{\text{ch}} \quad \rightarrow \quad \mu_5 = \frac{n_5}{\chi_5} \approx \frac{3v^3 n_5}{T^2 + \mu^2/\pi^2}$$

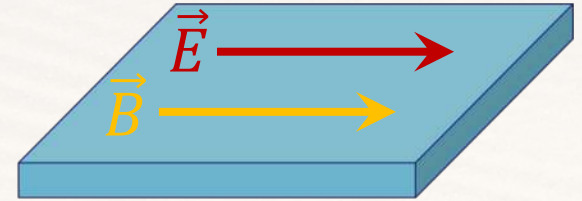
- The CME current

$$J_i = \frac{e^2}{2\pi^2} \mu_5 B_i = \left(\frac{e^2}{2\pi^2} \right)^2 \tau_{\text{ch}} \frac{B_i B_k}{\chi_5} E_k \quad \rightarrow \quad \sigma_{\text{CME}}^{\parallel} = \left(\frac{e^2}{2\pi^2} \right)^2 \tau_{\text{ch}} \frac{B^2}{\chi_5}$$

i.e.,

$$\rho_{\text{total}}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}$$

[Nielsen & Ninomiya, Phys. Lett. B **130**, 390 (1983)]
 [Son & Spivak, Phys. Rev. B **88**, 104412 (2013)]



Negative Magnetoresistance

- Experimental confirmation

$$\rho_{\text{total}}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}$$

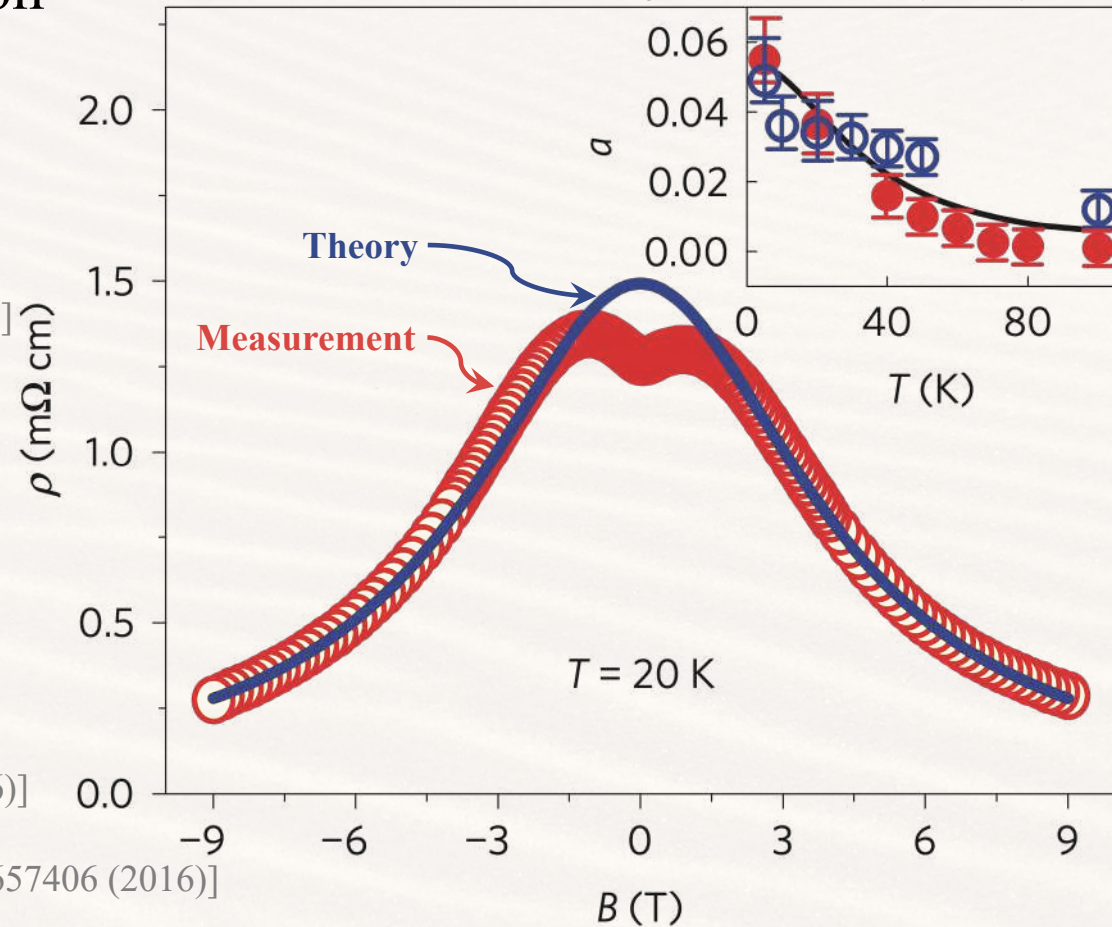
Dirac semimetals:

- [Kim et al, Phys. Rev. Lett. **111**, 246603 (2013)]
- [Li et al., Nat. Mater. **12**, 550 (2016)]
- [Xiong et al., Science **350**, 413 (2015)]
- [Feng, et al., Phys. Rev. B **92**, 081306 (2015)]
- [Li et al., Nat. Commun. **6**, 10137 (2015)]
- [Li et al., Nat. Commun. **7**, 10301 (2016)]

Weyl semimetals:

- [Huang et al., Phys. Rev. X **5**, 031023 (2015)]
- [Zhang et al., Nat. Commun. **7**, 10735 (2016)]
- [Hirschberger et al., Nat. Mater. **15**, 1161 (2016)]
- [Wang et al., Phys. Rev. B **93**, 121112 (2016)]
- [Du et al., Sci. China Phys. Mech. Astron. **59**, 657406 (2016)]
- [Li et al., Front. Phys. **12**, 127205 (2017)]

[Q. Li et al, Nature Physics **12**, 550 (2016)]



Chiral charge pumping (theory)

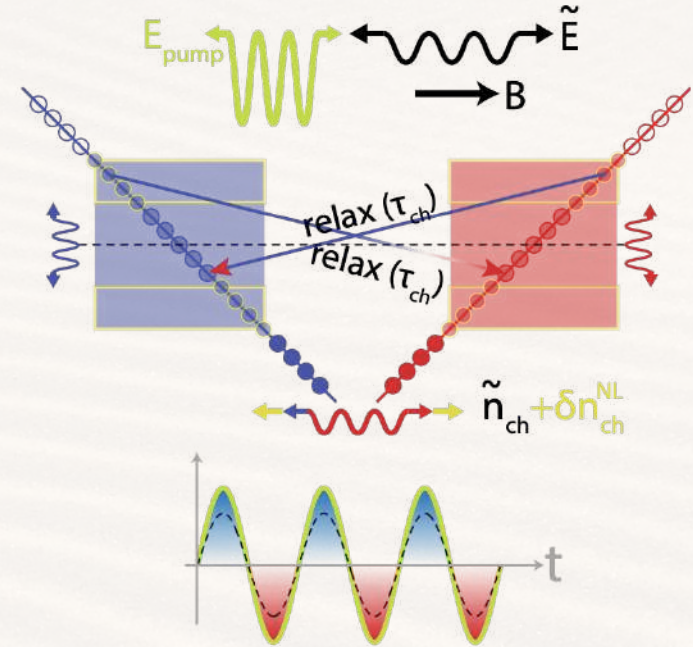
- Weyl semimetal TaAs
 - $\vec{B} \neq 0$ & oscillating $\vec{E} \parallel \vec{B}$
- The nonlinear contribution to chiral charge-pumping conductivity

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

$$\delta\sigma_{\text{ch}}^{\text{NL}} = i \frac{9\alpha^2 e^5 v^3}{8h^2 \omega^3} \left(\frac{\vec{\mathbf{E}}_{\text{pump}} \cdot \mathbf{B}}{B} \right)^2 B$$

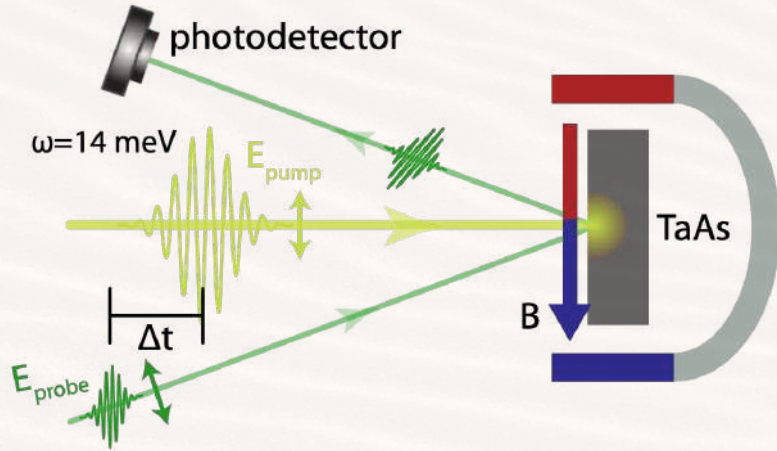
- The reflection coefficient

$$R(T) = \left| \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}} \right|^2 \quad \text{where} \quad \epsilon = \epsilon_{\infty} + i \frac{\sigma}{\omega \epsilon_0}$$



Chiral charge pumping (data)

- Experimental setup

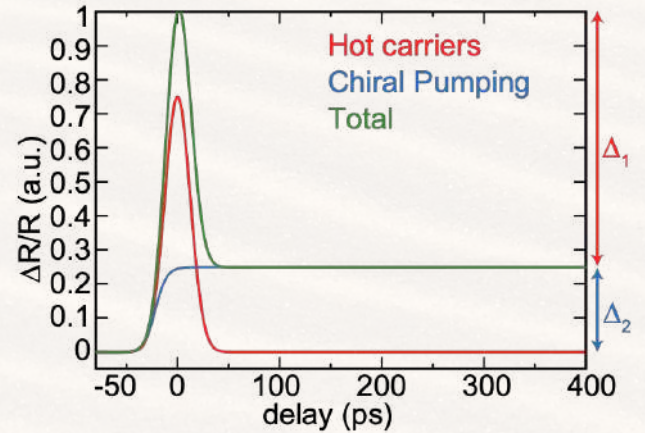
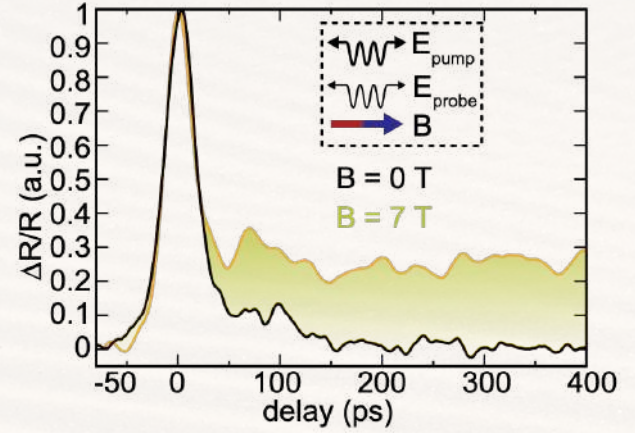


- Chiral charge relaxation time

$$1 \text{ ns} \ll \tau_{\text{ch}} < 77 \text{ ns}$$

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

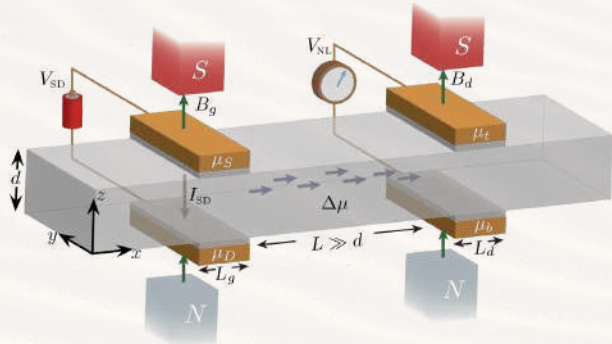
- Measurements:



Nonlocal anomalous transport

Theory

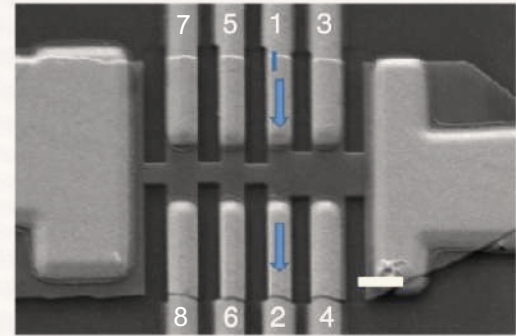
[Parameswaran, Grover, Abanin, Pesin, Vishwanath, PRX 4, 031035 (2014)]



Experiment (challenge: Ohmic diffusion)

[Zhang et al., Nat. Commun. 8, 13741 (2017)]

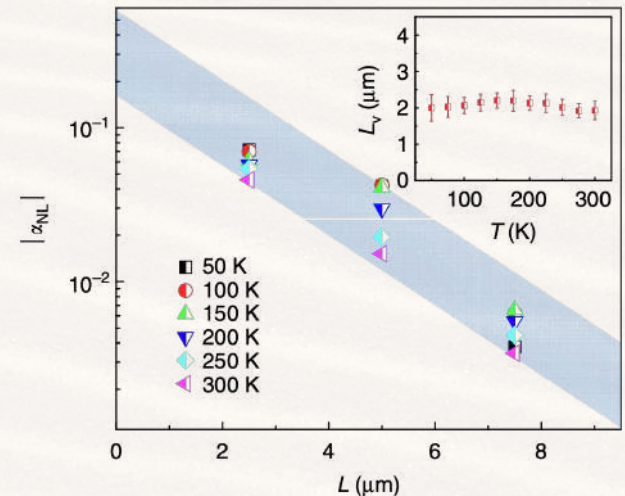
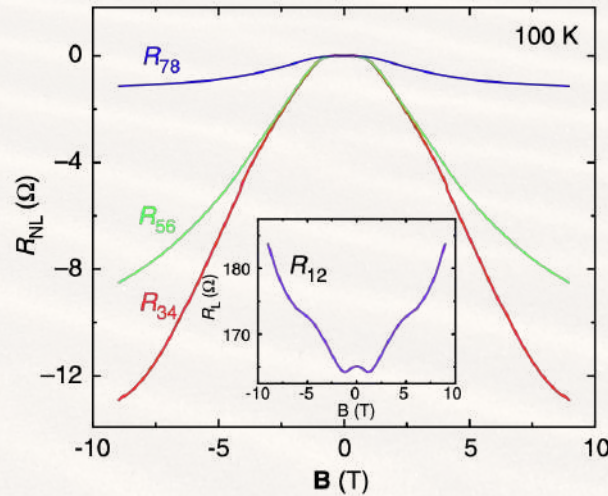
Cd_3As_2

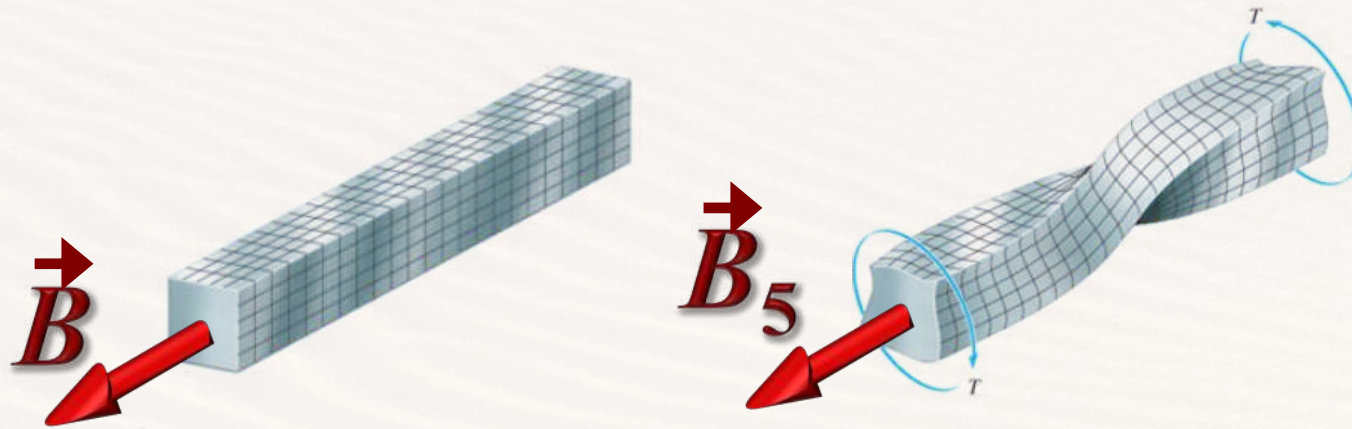


$$\alpha_{NL} = \frac{R_{NL}}{R_L} \propto e^{-L/L_V}$$

Measurements:

$$L_V \sim 2 \mu\text{m}$$





CHIRAL STRAINTRONICS

[Zubkov, Annals Phys. **360**, 655 (2015)]

[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]

[Grushin, Venderbos, Vishwanath, Ilan, Phys. Rev. X **6**, 041046 (2016)]

[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B **94**, 241405 (2016)]

[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Pseudo-electromagnetic fields

- **Strains** modify the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r} \bar{\psi} \left[-iv_F (\vec{\gamma} \cdot \vec{\mathbf{p}}) - (\vec{b} + \vec{A}_5) \cdot \vec{\gamma} \gamma^5 + (b_0 + A_{5,0}) \gamma^0 \gamma^5 \right] \psi$$

via the emergent **chiral gauge fields** are

$$A_{5,0} \propto b_0 \left| \vec{b} \right| \partial_{||} u_{||}$$

[Zubkov, Annals Phys. **360**, 655 (2015)]
 [Cortijo, Ferreira, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]

$$A_{5,\perp} \propto \left| \vec{b} \right| \partial_{||} u_{\perp}$$

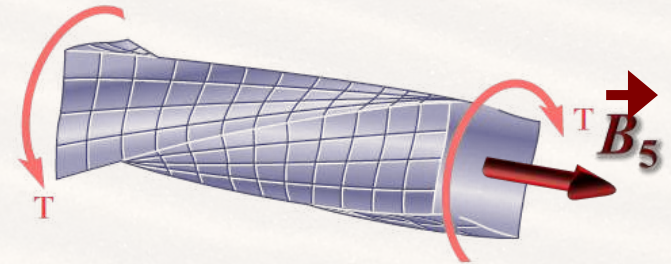
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
 [Grushin, Venderbos, Vishwanath, Ilan, PRX **6**, 041046 (2016)]

$$A_{5,||} \propto \alpha \left| \vec{b} \right|^2 \partial_{||} u_{||} + \beta \sum_i \partial_i u_i$$

[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

leading to the **pseudo-EM** fields

$$\vec{B}_5 = \vec{\nabla} \times \vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla} A_0 - \partial_t \vec{A}_5$$



Continuity relations (with \vec{B}_5 & \vec{E}_5)

- Naïve continuity relations from chiral kinetic theory:

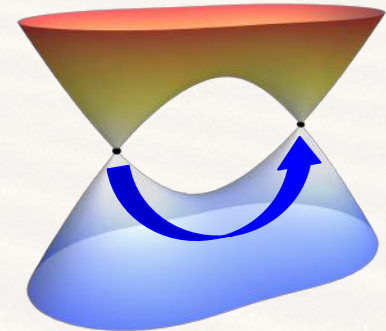
$$\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2 \hbar^2 c} [(\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5)] \quad \checkmark$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2 \hbar^2 c} [(\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B})] \quad \times$$

- Extra Bardeen-Zumino (Chern-Simons) terms are needed, i.e.

$$\delta \rho = -\frac{e^3}{2\pi^2 \hbar^2 c^2} ((\vec{b} + \vec{A}_5) \cdot \vec{B})$$

$$\delta \vec{j} = -\frac{e^3}{2\pi^2 \hbar^2 c^2} (b_0 + A_{5,0}) \vec{B} + \frac{e^3}{2\pi^2 \hbar^2 c^2} [(\vec{b} + \vec{A}_5) \times \vec{E}]$$



- Electric charge is conserved ($\partial_\mu J^\mu = 0$)
- Anomalous Hall effect is reproduced
- No CME in equilibrium ($\mu_5 = -eb_0$)

[Landsteiner, PRB **89**, 075124 (2014)]

[Landsteiner, Acta Phys. Polon. B **47**, 2617 (2016)]

[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **96**, 085130 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]



INSTRUCTIVE EXAMPLE: COLLECTIVE MODES

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

Maxwell equations

Faraday's law:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

or in momentum space:

$$\frac{c}{\omega} \vec{k} \times \vec{E} = \vec{B}$$

Ampere-Maxwell's law:

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

or in momentum space:

$$\frac{c}{\omega} \vec{k} \times \left(\frac{c}{\omega} \vec{k} \times \vec{E} \right) = - \left(4\pi \frac{i}{\omega} \vec{J} + \vec{E} \right)$$

Gauss's law constrains \vec{E} as follows: $i\vec{k} \cdot \vec{E} = 4\pi\rho$

$$\vec{P} = \frac{i}{\omega} \vec{J}$$

Anomalous contributions enter here

Chiral magnetic plasmons

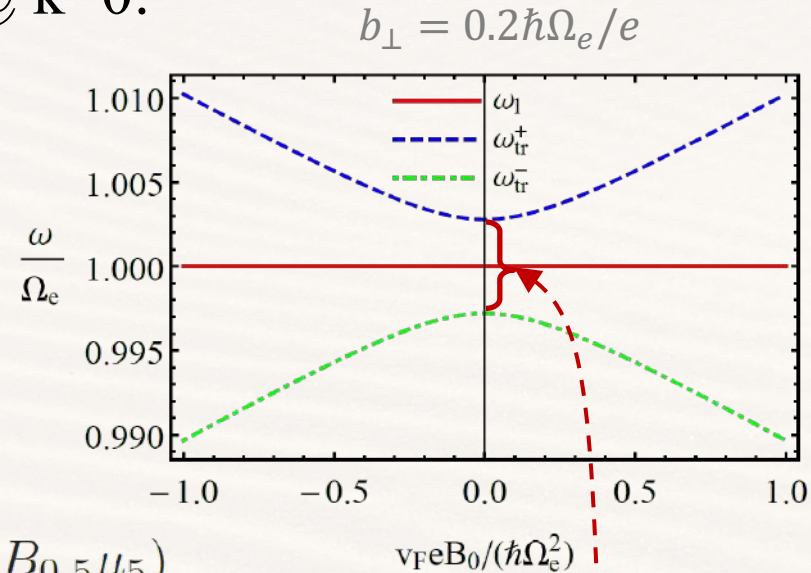
Non-degenerate plasmon frequencies @ $k=0$:

$$\omega_l = \Omega_e, \quad \omega_{\text{tr}}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2} \left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right)}$$

and
$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2} \left\{ 9\hbar^2 b_{\perp}^2 + \left[\frac{2v_F}{\Omega_e^2} (B_0\mu + B_{0,5}\mu_5) - 3\hbar b_{\parallel} - \frac{v_F\hbar^2}{4T} \sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu\lambda}{T}\right) \right]^2 \right\}^{1/2}$$



$$\omega_{\text{tr}}^+ - \omega_{\text{tr}}^- \approx \frac{2e\alpha v_F b_{\perp}}{\pi c\hbar}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

- Strain-induced pseudo-magnetic field $B_{0,5}$ leads to

$$\omega_h |_{B_0 \rightarrow 0, \mu \rightarrow 0} \stackrel{b_0 \rightarrow -\mu_5/e}{=} \frac{e B_{0,5} c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu_5 + 2 B_{0,5} e^4 v_F^2 b_{\parallel}} + O(k^3)$$

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

- Properties:
 - Gapless electromagnetic wave propagates in metals **without magnetic field!**
 - Chiral shift modifies effective helicon dispersion
 - In equilibrium, i.e., $\mu_5 = -eb_0$, the term linear in the wave vector is **absent**

- Chiral anomaly can have macroscopic implications in relativistic plasmas
- (Dipole) chiral magnetic effect can be seen via charged particle correlations in heavy-ion collisions
- Latest isobar measurements are promising but inconclusive (more studies are underway)
- Chiral anomaly can be realized and tested in Dirac/Weyl semimetals
- Chiral charge, which is relatively long-lived, can be optically pumped and manipulated (promising new technologies)