ASU ARIZONA STATE UNIVERSITY

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Anisotropic emission from magnetized quark-gluon plasma

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[X. Wang and I. Shovkovy, arXiv:2205.00276]

[X. Wang and I. Shovkovy, Phys. Rev. D 104, 056017 (2021), arXiv:2103.01967]

[X. Wang and I. Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]

[X. Wang, I. Shovkovy, L. Yu, and M. Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]

SEWM 2022

Strong and ElectroWeak Matter Saclay - Paris, June 20-24, 2022

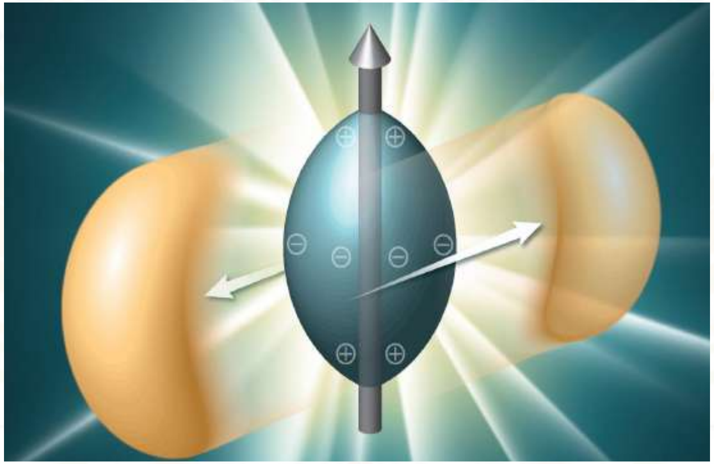


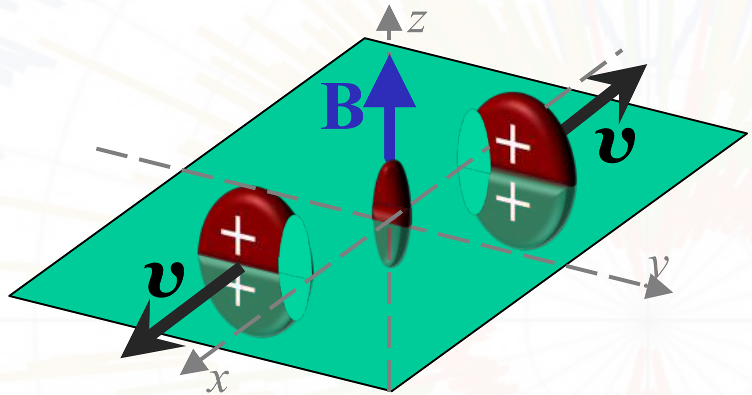
Image credit: Brookhaven National Laboratory

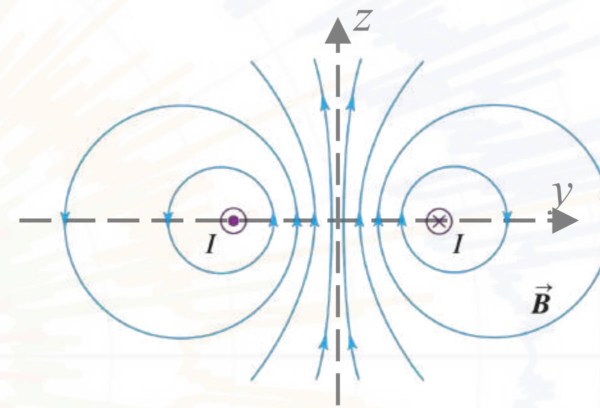
MAGNETIZED QUARK-GLUON PLASMA

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]

Heavy-ion collisions

- QGP produced at RHIC/LHC is **magnetized**
  - **10^{18} to 10^{19} G** $\sim m_\pi^2 \sim (100\ \mathrm{MeV})^2$





- Using Lienard-Wiechert potential, one finds

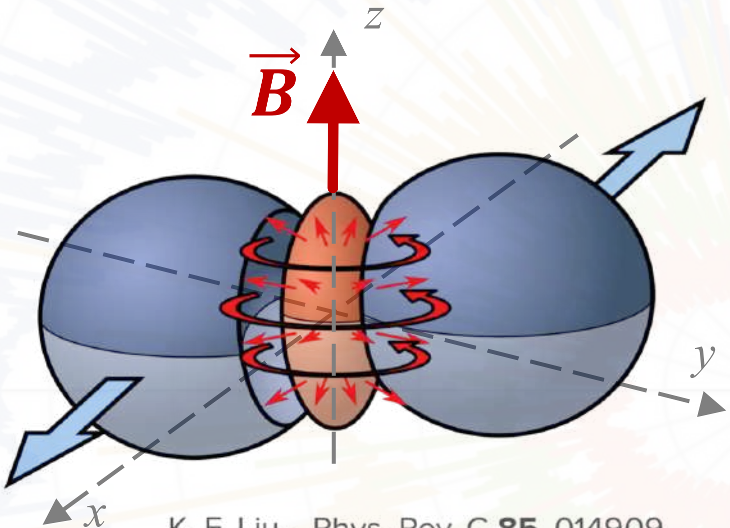
$$e\mathbf{E}(t,\mathbf{x}) = \alpha_{\mathrm{EM}} \sum_{n\in\mathrm{protons}} \frac{1-v_n^2}{R_n^3\left(1-[\mathbf{R}_n\times\mathbf{v}_n]^2/R_n^2\right)^{3/2}}\mathbf{R}_n$$

$$e\mathbf{B}(t,\mathbf{x}) = \alpha_{\mathrm{EM}} \sum_{n\in\mathrm{protons}} \frac{1-v_n^2}{R_n^3\left(1-[\mathbf{R}_n\times\mathbf{v}_n]^2/R_n^2\right)^{3/2}}\mathbf{v}_n\times\mathbf{R}_n$$

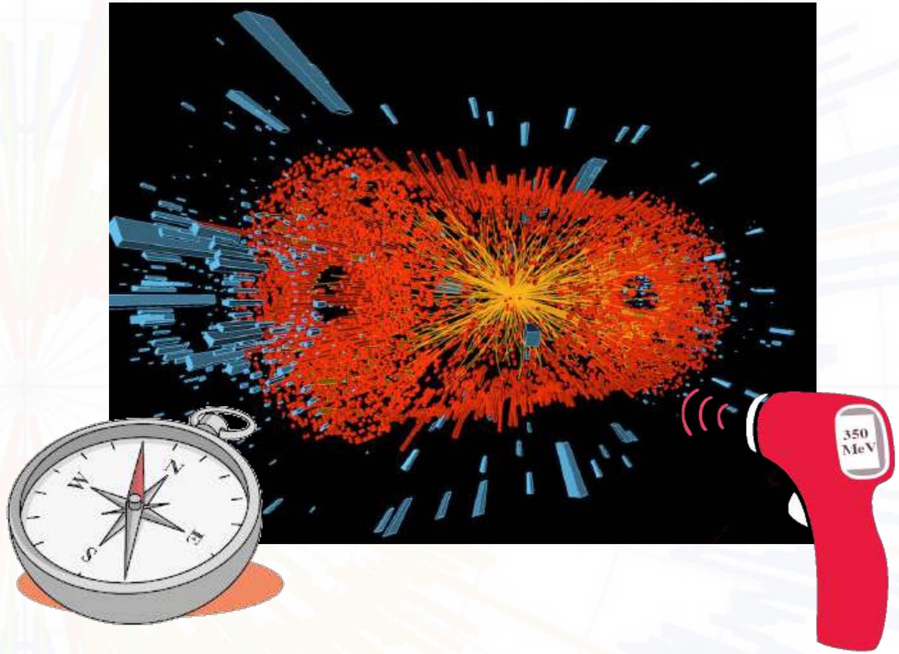
[Rafelski & Müller, PRL, 36, 517 (1976)]
[Kharzeev et al., arXiv:0711.0950]
[Skokov et al., arXiv:0907.1396]
[Voronyuk et al., arXiv:1103.4239]
[Bzdak &. Skokov, arXiv:1111.1949]
[Deng & Huang, arXiv:1201.5108]
[Bloczynski et al, arXiv:1209.6594]
...

Magnetometer for HICs

- How to measure the **magnetic field** of QGP in HICs?



K. F. Liu , Phys. Rev. C **85**, 014909

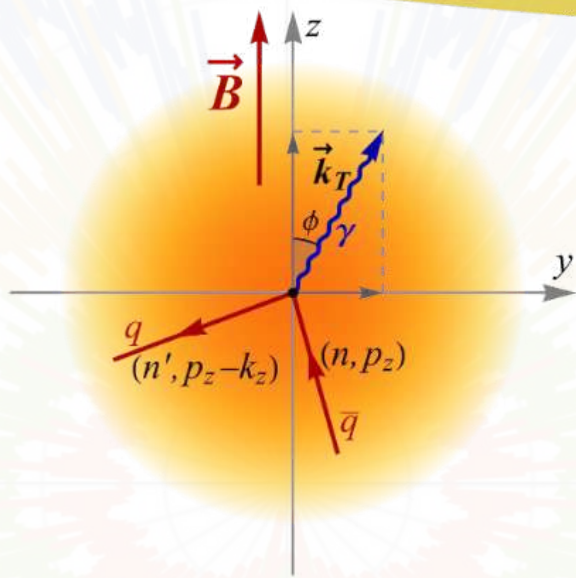


Old idea:

- Electromagnetic probes serve as a **thermometer** of QGP

New idea:

- They can be also used as a **magnetometer** **(direct photons or dileptons)**



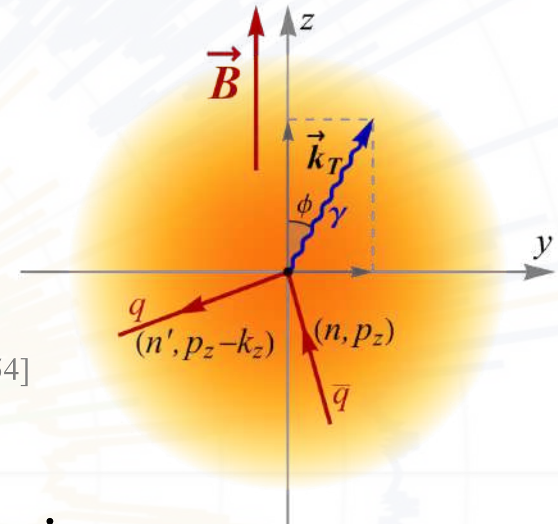
PHOTON RATE

[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]
[Wang, Shovkovy, Phys. Rev. D 104, 056017 (2021), arXiv:2103.01967]
[Wang, Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]

Photon emission rates

- The expression for the rate is

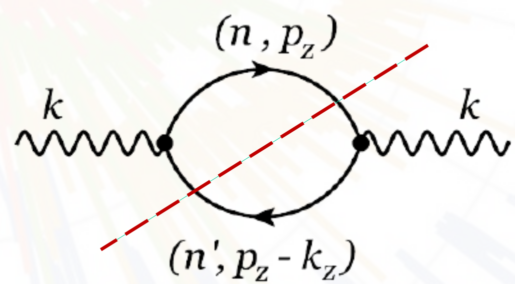
$$\Omega \frac{d^3R}{d^3\mathbf{k}} = -\frac{1}{(2\pi)^3} \frac{\mathrm{Im}[\Pi^{\mu}_{R,\mu}(\Omega, \mathbf{k})]}{\exp(\frac{\Omega}{T}) - 1}$$



[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]
[Wang, Shovkovy, Phys. Rev. D 104, 056017 (2021), arXiv:2103.01967]
[Wang, Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]

- At $\vec{B} \neq 0$, the imaginary part of the polarization tensor

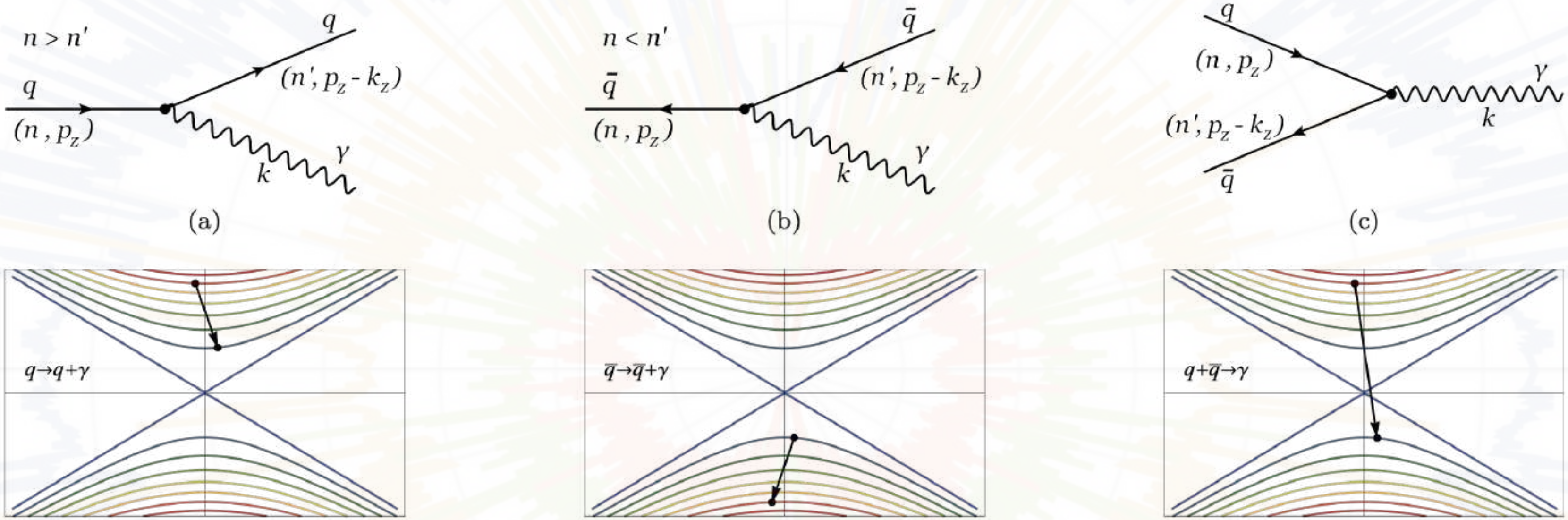
$$\mathrm{Im}[\Pi^{\mu}_{R,\mu}(\Omega, \mathbf{k})] \quad =$$



is nonzero at leading order in α_s!

Physics processes

- Relevant physics processes (0^{th} order in α_s):



The energy momentum conservation

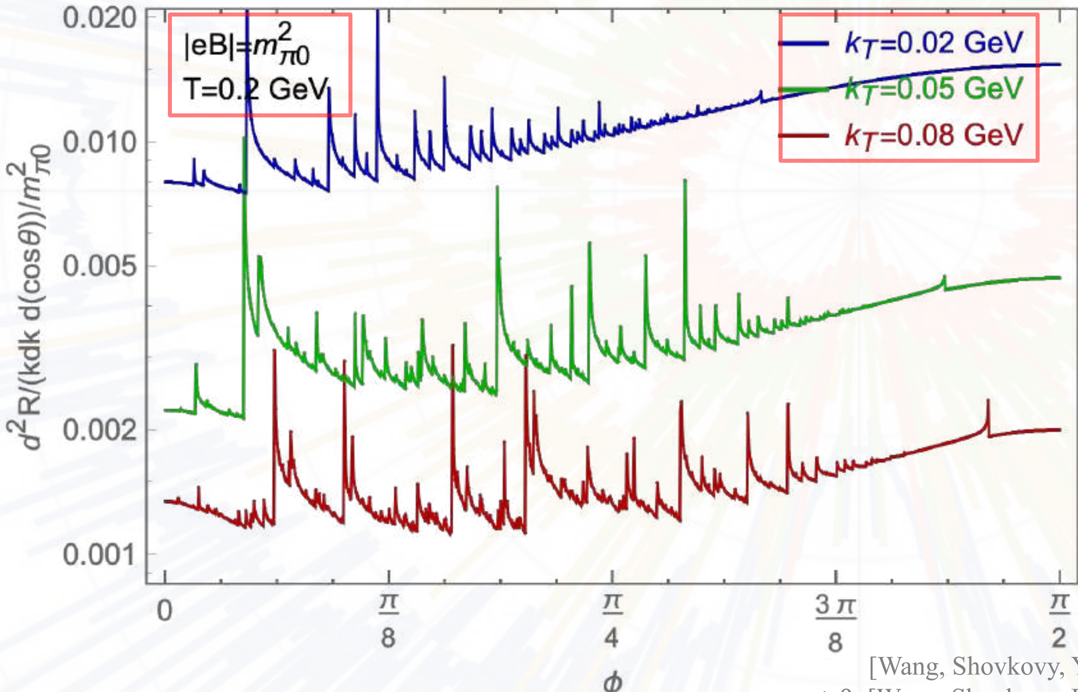
$$E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta\Omega = 0$$

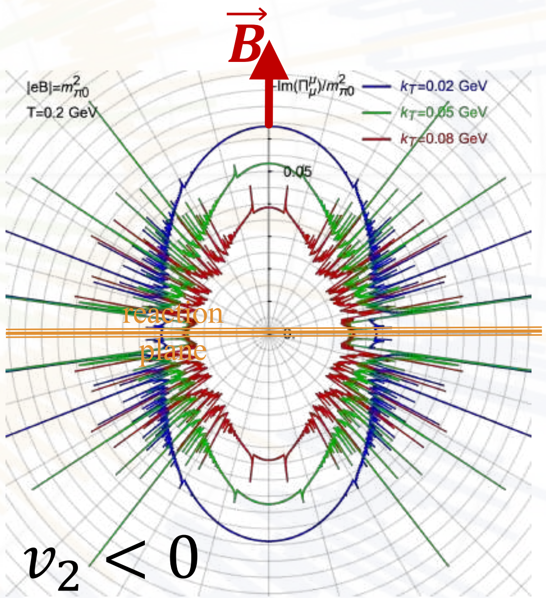
is satisfied for these $1 \to 2$ and $2 \to 1$ processes

[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]
$\mu \neq 0$: [Wang, Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]

Angular dependence (1)

- At very small k_T, the emission rate is maximal at $\phi = \frac{\pi}{2}$ (i.e., emission **perpendicular** to the reaction plane)
- Effectively, this gives photon "flow" with $v_2 < 0$

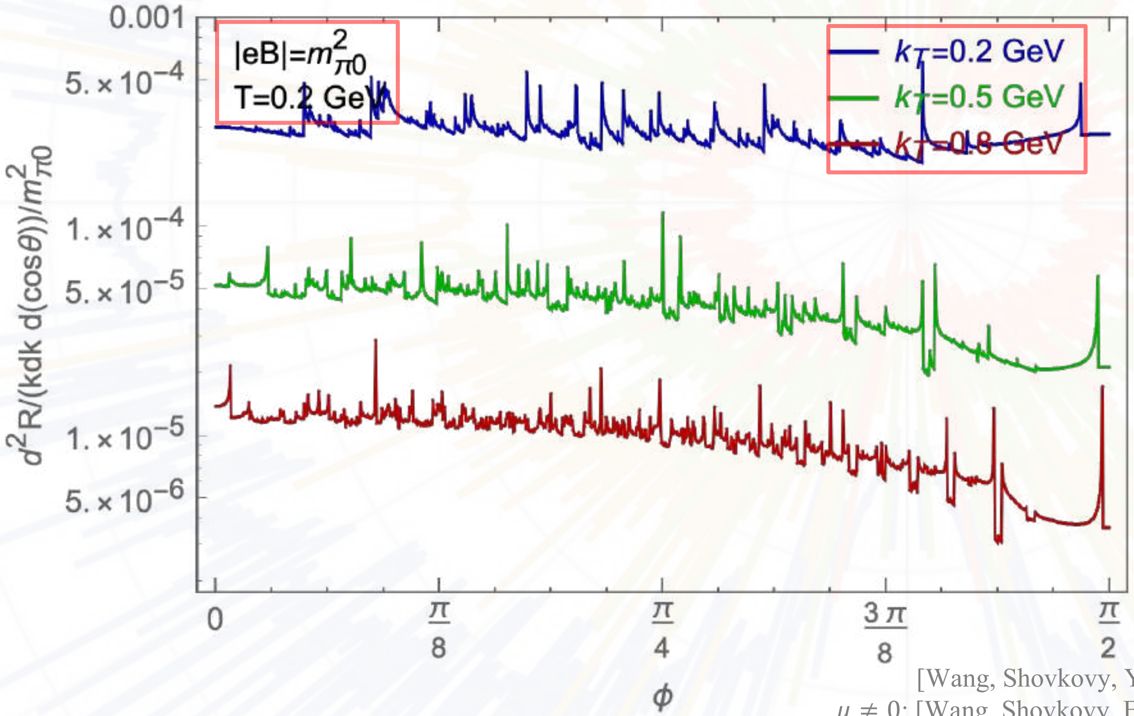


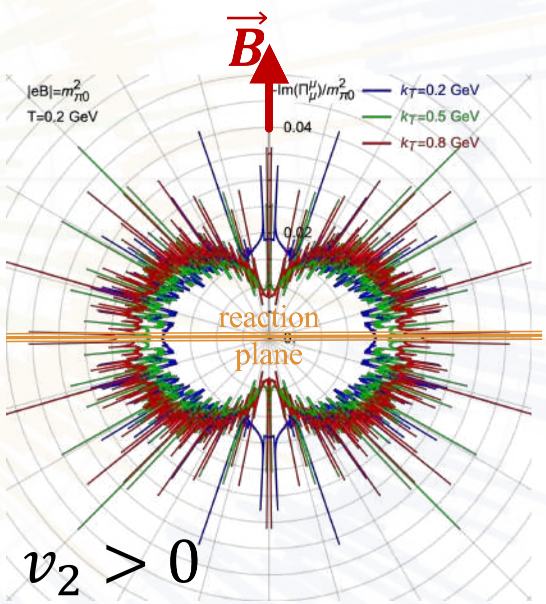


[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]
$\mu \neq 0$: [Wang, Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]

Angular dependence (2)

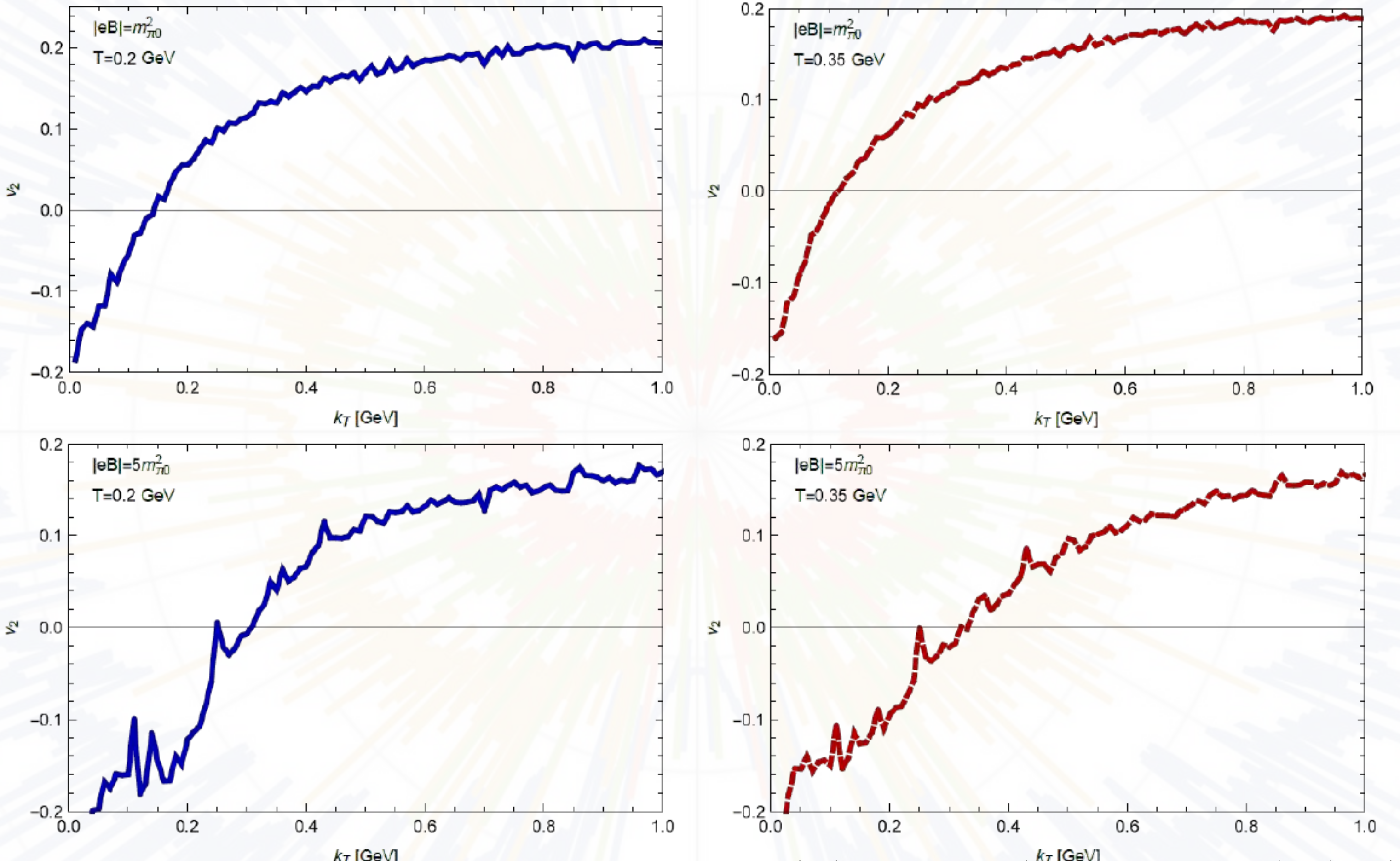
- At large k_T, the emission rate is maximal at $\phi = 0$ (i.e., **parallel** to the reaction plane)
- Effectively, this gives photon "flow" with $v_2 > 0$





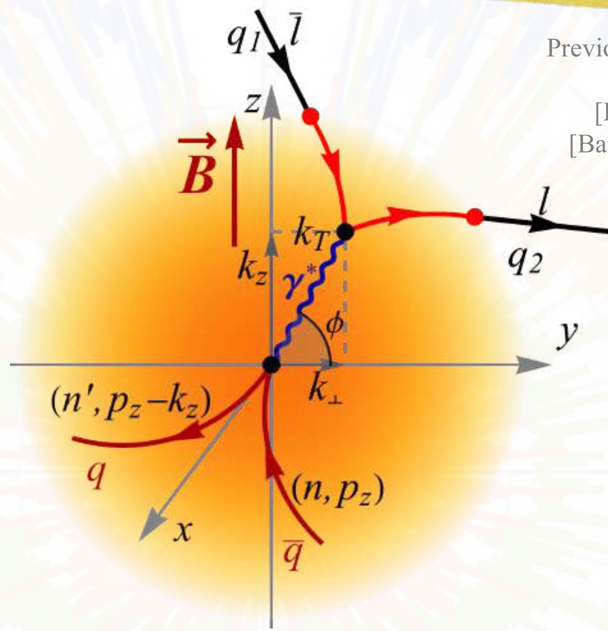
[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]
$\mu \neq 0$: [Wang, Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]

Nonzero elliptic "flow" (v_2)



[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]

$\mu \neq 0$: [Wang, Shovkovy, Eur. Phys. J. C 81 (2021), 901, arXiv:2106.09029]



Previous studies: [Tuchin, Phys. Rev. C 88, 024910 (2013)]
[Sadooghi, Taghinavaz, Annals Phys. 376, 218 (2017)]
[Bandyopadhyay et al., Phys. Rev. D 94, 114034 (2016)]
[Bandyopadhyay, Mallik, Phys. Rev. D 95, 074019 (2017)]
[Ghosh, Chandra, Phys. Rev. D 98, 076006 (2018)]
[Islam et al., Phys. Rev. D 99, 094028 (2019)]
[Das et al., Phys. Rev. D 99, 094022 (2019)]
[Ghosh et al., Phys. Rev. D 101, 096002 (2020)]
[Chaudhuri et al., Phys. Rev. D 103, 096021 (2021)]
[Das et al., arXiv:2109.00019]

DILEPTON RATE

[Wang and Shovkovy, arXiv:2205.00276]

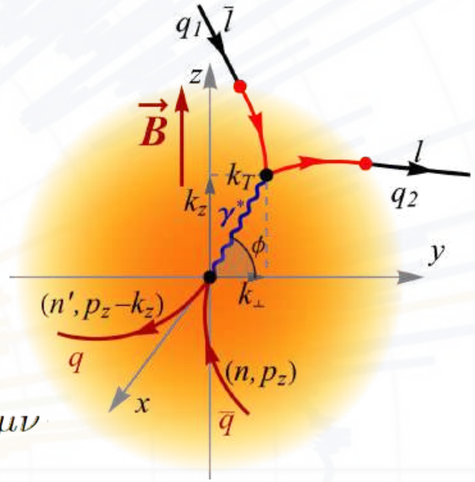
Dilepton rate (1)

- The differential lepton multiplicity per unit spacetime volume reads [Weldon, Phys. Rev. D 42, 2384 (1990)]

$$dR_{l\bar{l}} = 2\pi e^2 e^{-\beta\Omega} L_{\mu\nu}(Q_1, Q_2) \rho^{\mu\nu}(\Omega, \mathbf{k}) \frac{d^3\mathbf{q}_1}{(2\pi)^3 E_1} \frac{d^3\mathbf{q}_2}{(2\pi)^3 E_2}$$

where the leptonic tensor (plane-wave final states) is

$$L_{\mu\nu}(Q_1, Q_2) = Q_{1\mu} Q_{2\nu} + Q_{1\nu} Q_{2\mu} - \left(Q_1 \cdot Q_2 + m_l^2\right) g_{\mu\nu}$$



- **Note:** leptons are Landau-level states $|n_l\rangle$ inside QGP but turn into **plane waves** when leaving it, i.e.,

$$\sum |n_l\rangle \langle n_l | Q\rangle = \langle Q|$$

- The electromagnetic spectral function (to leading order in α) is

$$\rho^{\mu\nu}(\Omega, \mathbf{k}) = -\frac{1}{\pi} \frac{e^{\beta\Omega}}{e^{\beta\Omega} - 1} \frac{\mathrm{Im}\left[\Pi^{\mu\nu}(\Omega, \mathbf{k})\right]}{K^4}$$

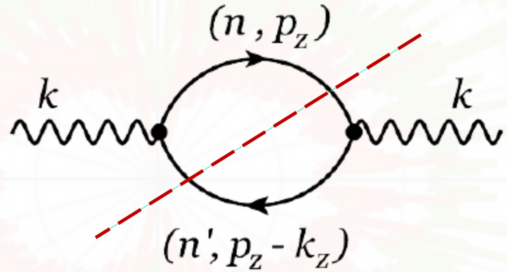
Dilepton rate (2)

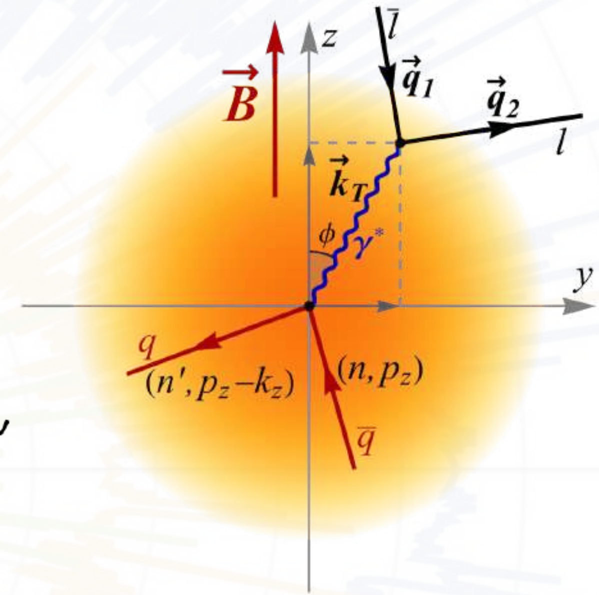
- The expression for the rate is [Wang, Shovkovy, arXiv:2205.00276]

$$\frac{dR_{l\bar{l}}}{d^4K} = \frac{\alpha}{12\pi^4}\frac{n_B(\Omega)}{M^2}\mathrm{Im}\left[\Pi^{\mu}_{\mu}(\Omega,\mathbf{k})\right]$$

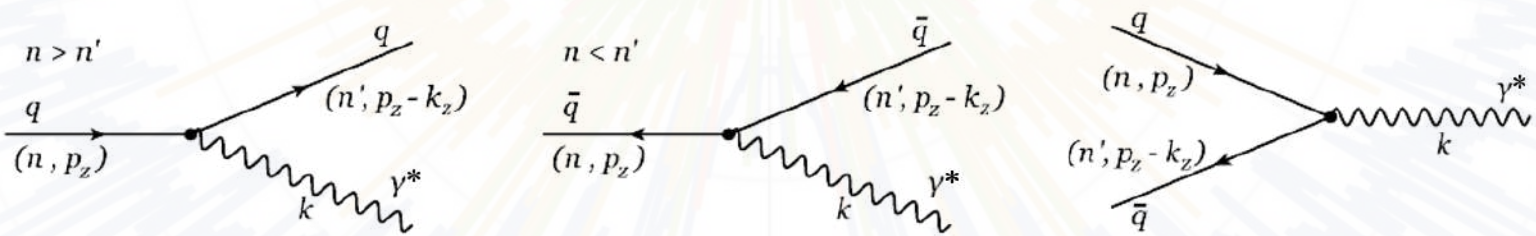
where $M^2 = \Omega^2 - k^2$ and

$$\mathrm{Im}[\Pi^{\mu}_{R,\mu}(\boldsymbol{\Omega},\mathbf{k})] =$$





- Three leading-order processes contribute:



[Wang, Shovkovy, Yu, Huang, Phys. Rev. D **102**, 076010 (2020), arXiv:2006.16254]
[Wang and Shovkovy, Phys. Rev. D **104**, 056017 (2021), arXiv:2103.01967]

Dilepton rate: explicit expression

- Explicit expression for the rate [Wang, Shovkovy, arXiv:2205.00276]

$$\frac{dR_{l\bar{l}}}{d^4K} = \frac{\alpha^2 N_c}{48\pi^5}\frac{n_B(\Omega)}{M^2}\sum_{f=u,d}\frac{q_f^2}{\ell_f^4}\left[\sum_{n=0}^{\infty}\frac{g_0(n)\theta\left(\sqrt{M^2+k_\perp^2}-k_+^f\right)}{\sqrt{(M^2+k_\perp^2)\left[M^2+k_\perp^2-(k_+^f)^2\right]}}\mathcal{F}_{n,n}^f(\xi)\right.$$

$$\left.-\ 2\sum_{n>n'}^{\infty}\frac{g(n,n')\left[\theta\left(k_-^f-\sqrt{M^2+k_\perp^2}\right)-\theta\left(\sqrt{M^2+k_\perp^2}-k_+^f\right)\right]}{\sqrt{\left[(k_-^f)^2-(M^2+k_\perp^2)\right]\left[(k_+^f)^2-(M^2+k_\perp^2)\right]}}\mathcal{F}_{n,n'}^f(\xi)\right]$$

where $g_0(n) = g(n,n)$ and

$$g(n,n') = 2 - \sum_{s_1,s_2=\pm} n_F\left(\frac{\Omega}{2} + s_1\frac{\Omega(n-n')|e_fB|}{M^2+k_\perp^2} + \frac{s_2|k_z|}{2(M^2+k_\perp^2)}\sqrt{\left(M^2+k_\perp^2-(k_-^f)^2\right)\left(M^2+k_\perp^2-(k_+^f)^2\right)}\right)$$

- $\mathcal{F}_{n,n'}^f(\xi)$ are given in terms of generalized Laguerre polynomials
- Notation: $\xi = k_\perp^2\ell_f^2/2$ and $k_\pm^f = \left|\sqrt{m^2+2n|e_fB|} \pm \sqrt{m^2+2n'|e_fB|}\right|$

Cross-check at k=0 & B=0

- The rate in the limit $k \to 0$ is related to optical conductivity

$$\left.\frac{dR_{l\bar{l}}}{d^4K}\right|_{|\boldsymbol{k}|\to 0} \simeq \frac{\alpha}{12\pi^4}\frac{n_B(M)}{M}\left[\sigma_{\parallel}(M) + 2\sigma_{\perp}(M)\right]$$

- The optical conductivity in the limit $B \to 0$ reads [Wang, Shovkovy, arXiv:2205.00276]

$$\left.\sigma_{\parallel}(\Omega)\right|_{B\to 0} = \left.\sigma_{\perp}(\Omega)\right|_{B\to 0} \simeq \frac{\alpha N_c(q_u^2 + q_d^2)}{3}\Omega \tanh\left(\frac{\Omega}{4T}\right)$$

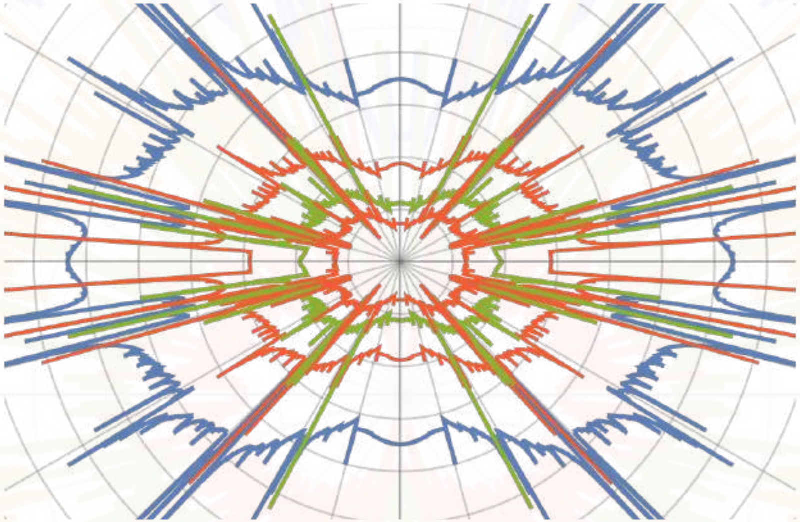
- Thus, at $k \to 0$ and $B \to 0$, one has

$$\left.\frac{dR_{l\bar{l}}}{d^4K}\right|_{|\boldsymbol{k}|\to 0, B\to 0} \simeq \frac{5\alpha^2}{36\pi^4} n_B(M) \tanh\left(\frac{M}{4T}\right)$$

- This agrees with the Born rate at $B = 0$, i.e.,

$$\frac{dR_{l\bar{l},\mathrm{Born}}}{d^4K} = \frac{5\alpha^2 T}{18\pi^4|\boldsymbol{k}|} n_B(\Omega) \ln\left(\frac{\cosh\frac{\Omega+|\boldsymbol{k}|}{4T}}{\cosh\frac{\Omega-|\boldsymbol{k}|}{4T}}\right)$$

[Cleymans, Fingberg, Redlich, Phys. Rev. D 35, 2153 (1987)]

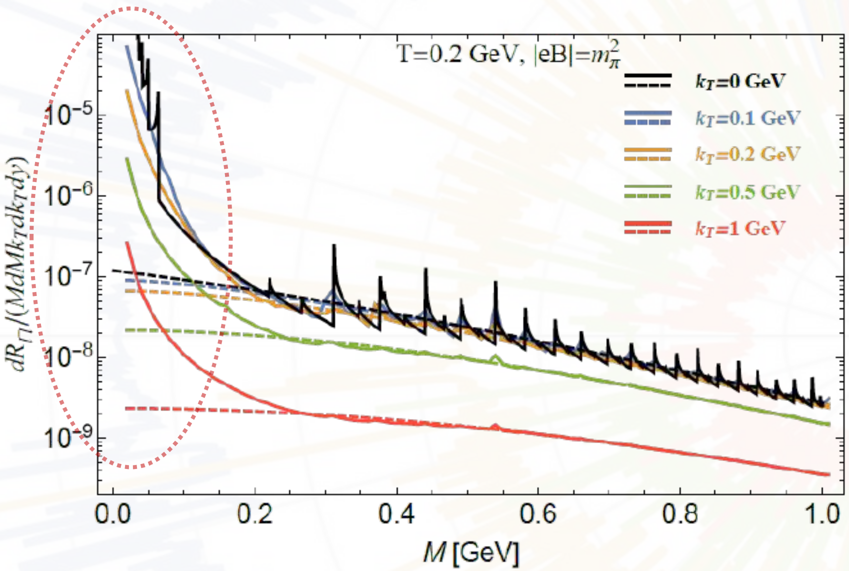


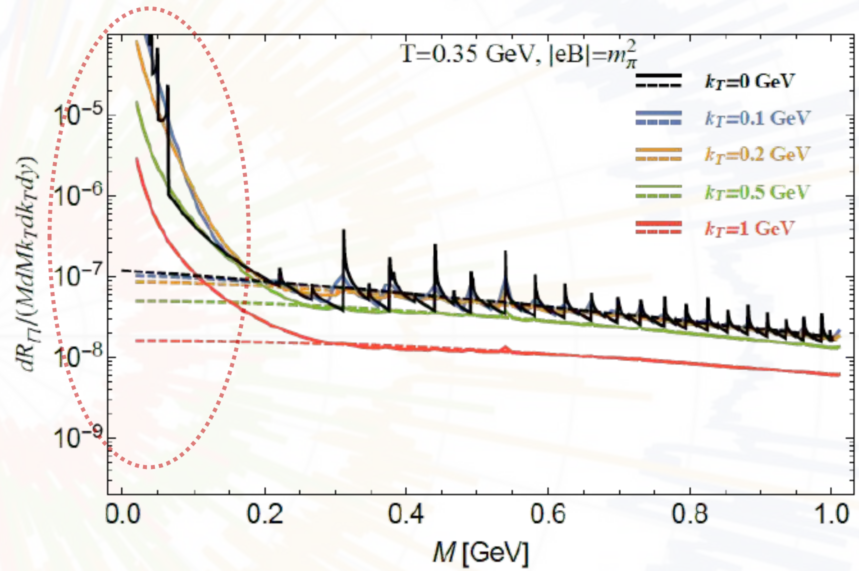
NUMERICAL RESULTS

[Wang and Shovkovy, arXiv:2205.00276]

Results: integrated rate

- Definition $\left(y=\frac{1}{2}\ln\frac{\Omega+k_x}{\Omega-k_x}\right)$: $$\frac{dR_{l\bar{l}}}{M\,dM\,k_T\,dk_T\,dy}=\int_0^{2\pi}d\phi\frac{dR_{l\bar{l}}}{d^4K}$$



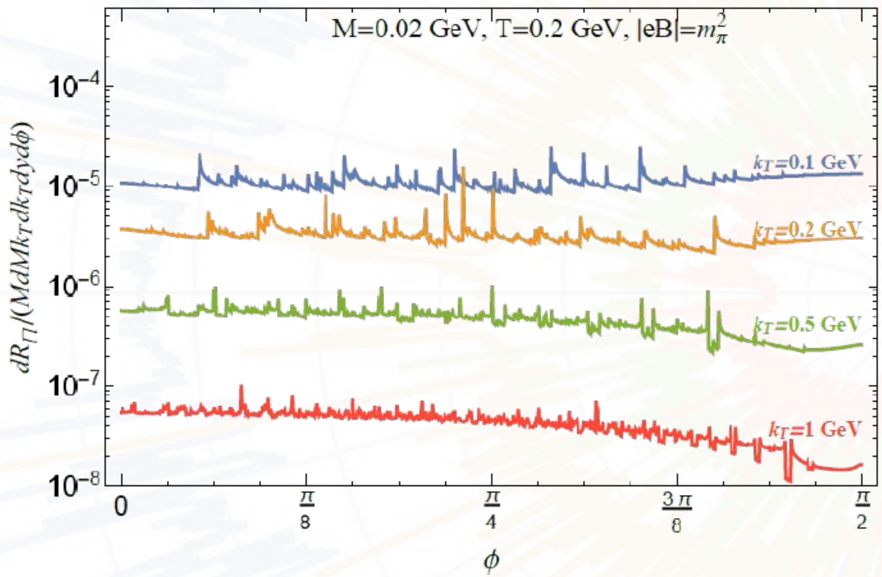


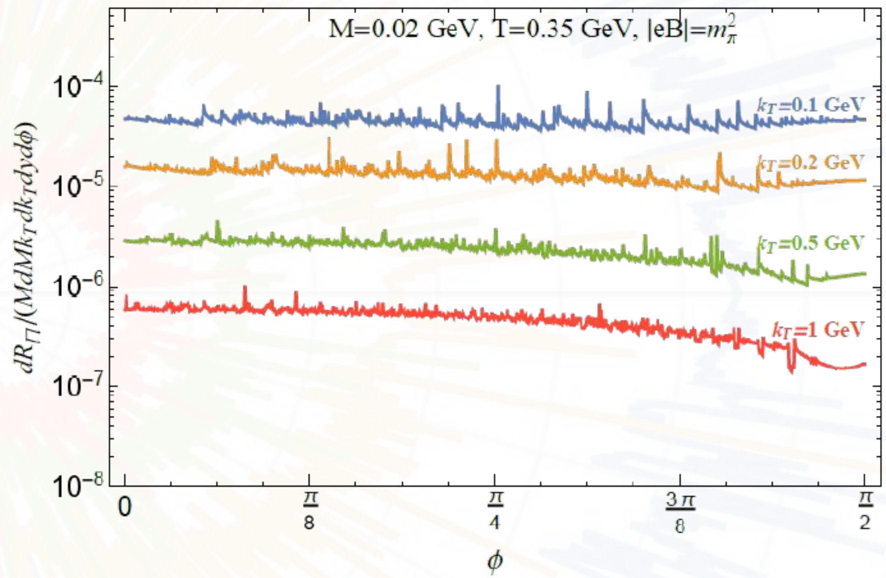
- Overall, dilepton rate grows with temperature
- Large enhancement is seen at **small invariant masses**, $M \lesssim \sqrt{|eB|}$

[Wang, Shovkovy, arXiv:2205.00276]

Angular dependence @ small M

- Dilepton rate tends to decrease with increasing k_T



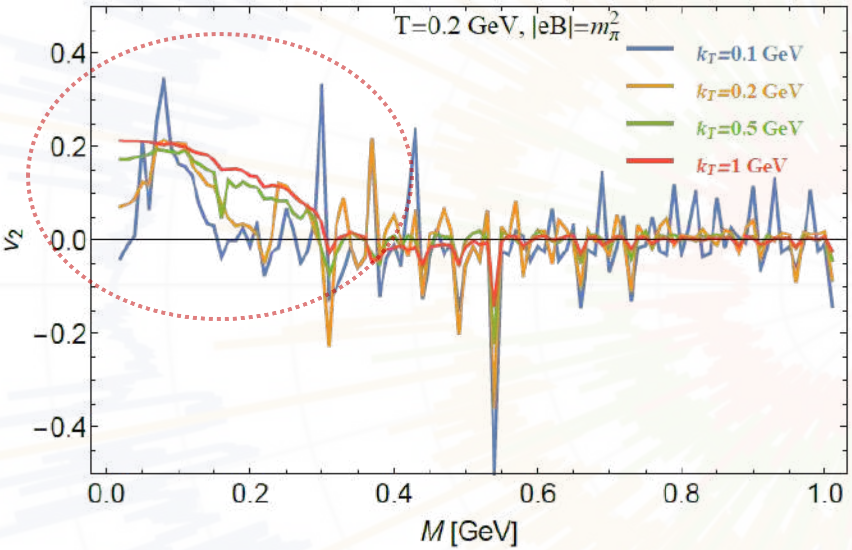


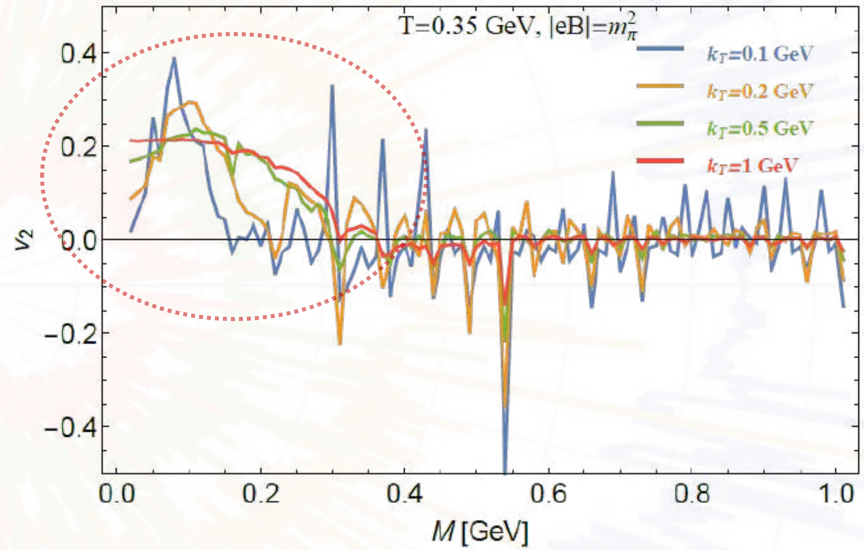
- The angular dependence indicates a possible nonzero v_2
- A nonvanishing v_2 is most prominent at small M and large k_T

[Wang, Shovkovy, arXiv:2205.00276]

Ellipticity of dilepton emission

- Definition: $$v_2(M,k_T) = \frac{\int_0^{2\pi} d\phi \cos(2\phi)\,(dR_{l\bar{l}}/d^4k)}{\int_0^{2\pi} d\phi\,(dR_{l\bar{l}}/d^4k)}$$





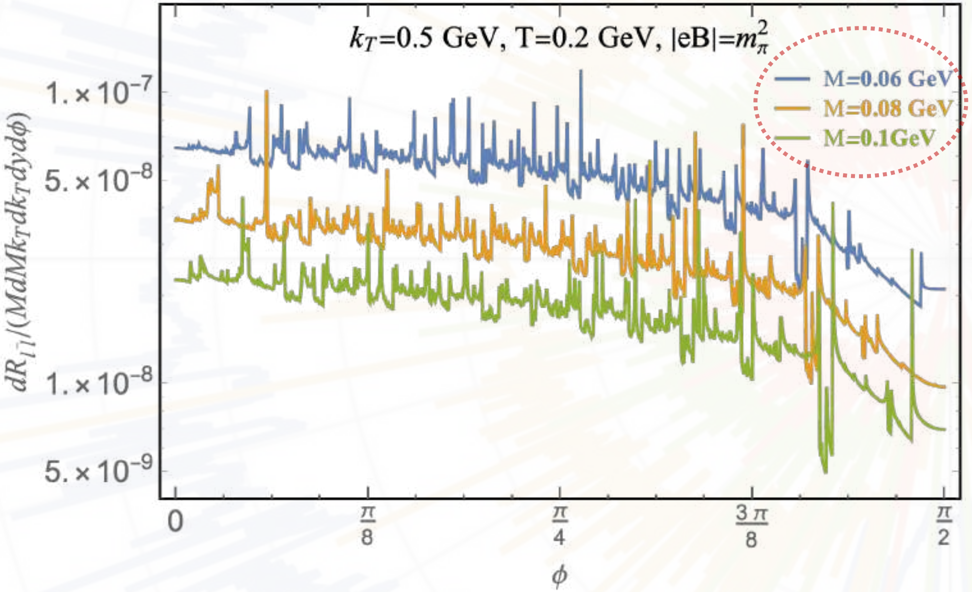
- Ellipticity is large ($v_2 \lesssim 0.2$) for $M \lesssim \sqrt{|eB|}$ and $k_T \gg \sqrt{|eB|}$
- On the other hand, $v_2 \approx 0$ for $M \gg \sqrt{|eB|}$ and all k_T

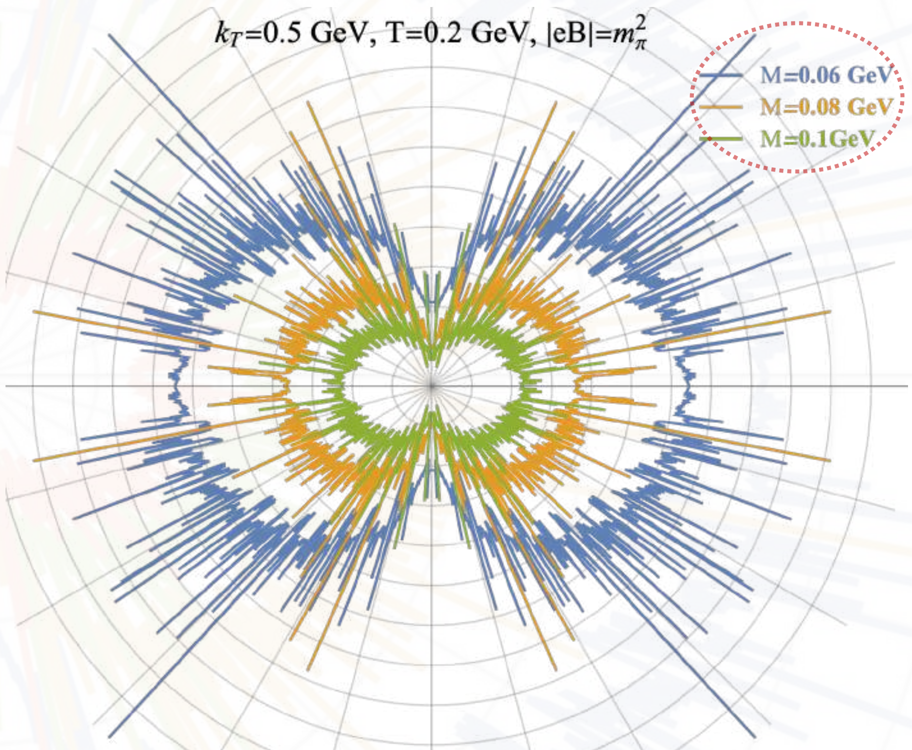
[Wang, Shovkovy, arXiv:2205.00276]

Angular dependence @ small M

- The ellipticity is well pronounced at small M and large k_T

$|eB| = m_\pi^2$



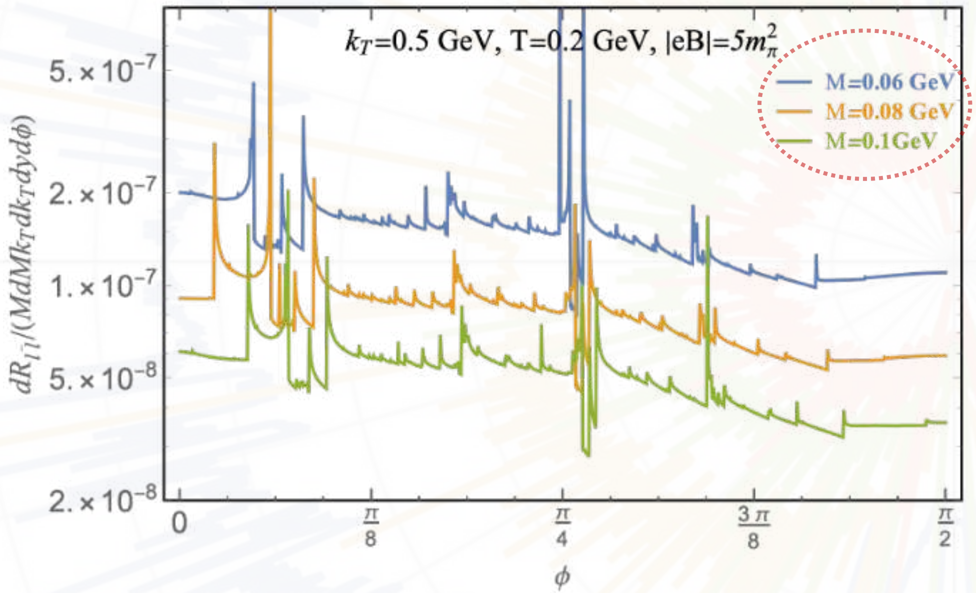


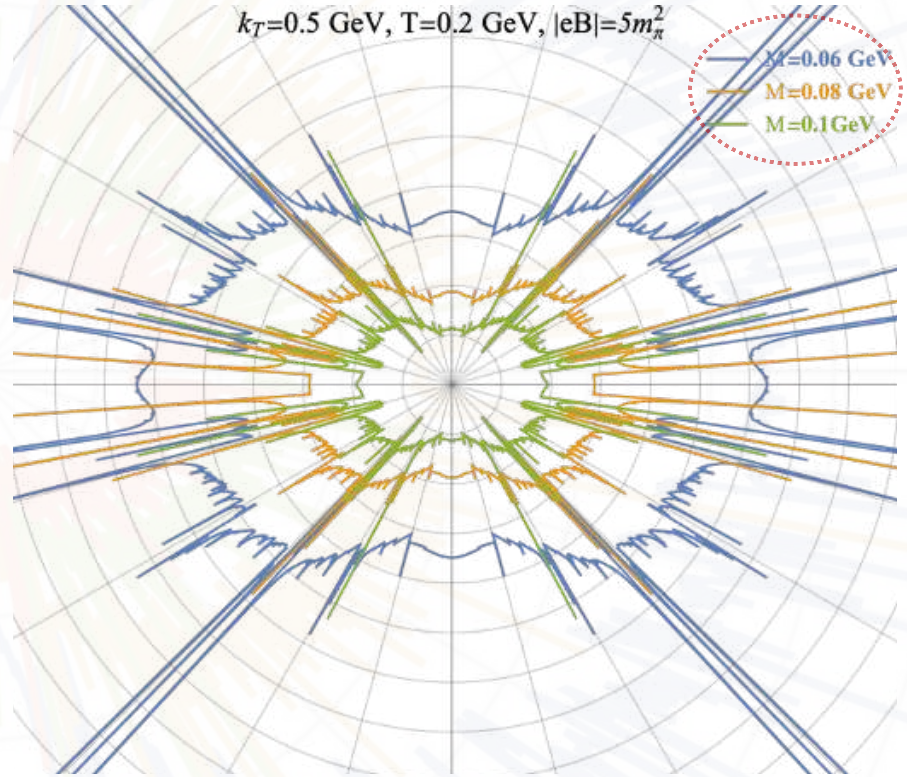
- **Note**: magnetic field strongly enhances the rate at small M

Angular dependence @ small M

- The ellipticity is well pronounced at small M and large k_T

$|eB| = 5m_\pi^2$



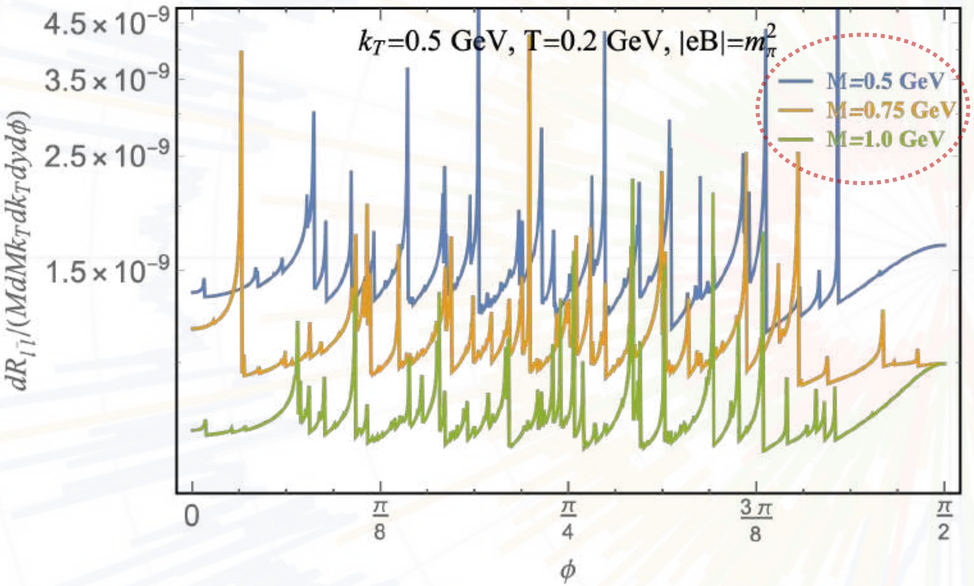


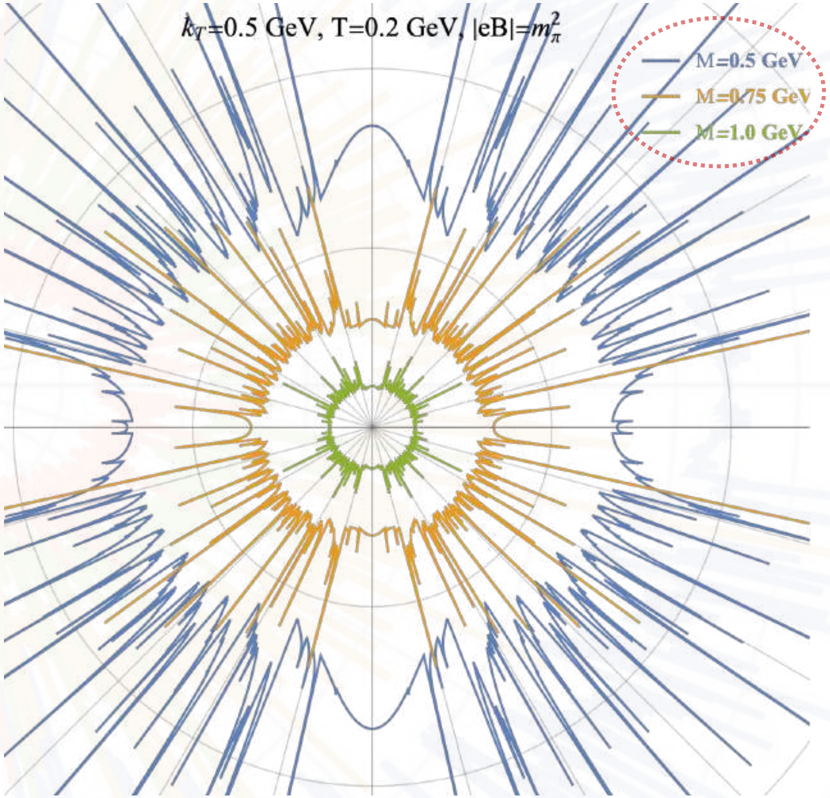
- **Note**: magnetic field strongly enhances the rate at small M

Angular dependence @ large M

- The ellipticity is approximately vanishing at large M

$|eB| = m_\pi^2$



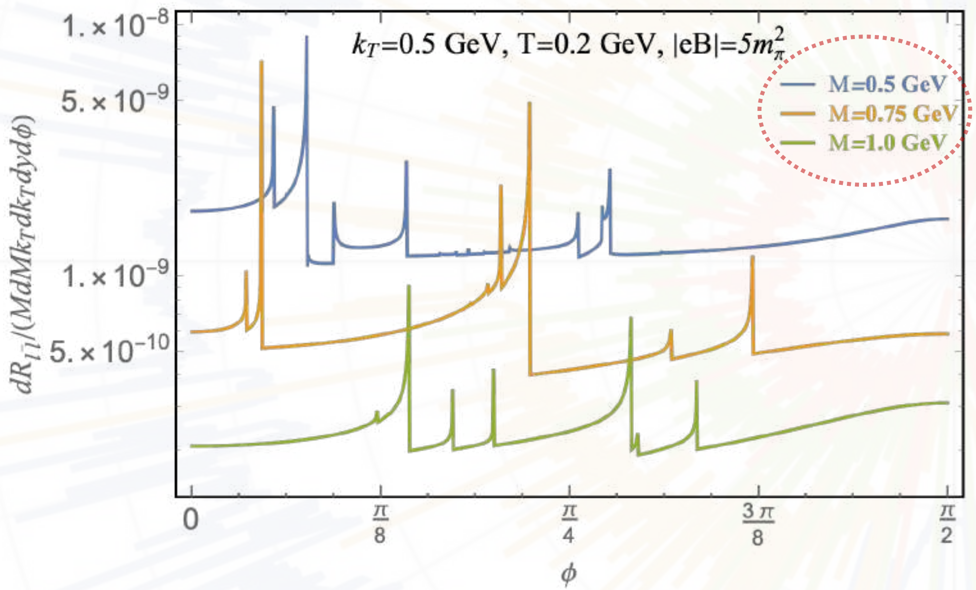


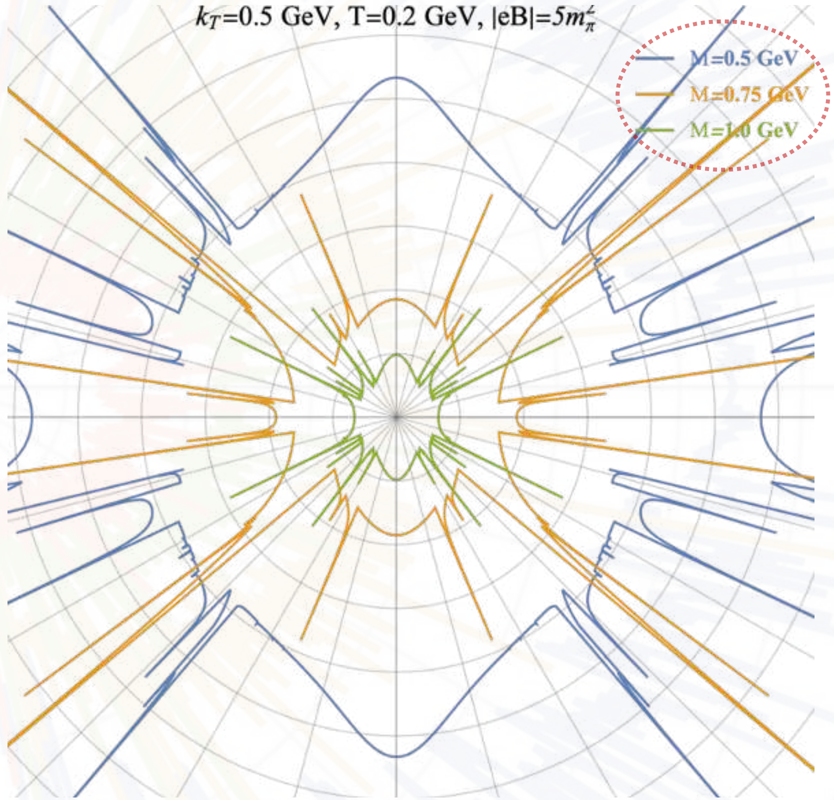
- **Note**: magnetic field does not affect much dilepton rate M

Angular dependence @ large M

- The ellipticity is approximately vanishing at large M

$|eB| = 5m_\pi^2$

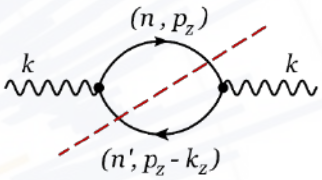




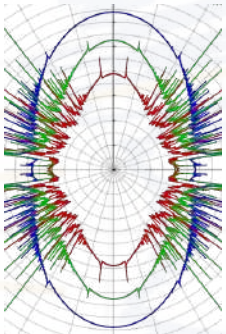
- **Note**: magnetic field does not affect much dilepton rate at large M

Summary (photons)

- $\vec{B} \neq 0$: photons are produced at 0^{th} order in α_s

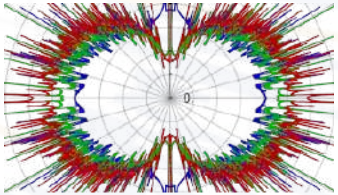


(i) $q \to q + \gamma$, (ii) $\bar{q} \to \bar{q} + \gamma$, (iii) $q + \bar{q} \to \gamma$



- Photon emission at $B \neq 0$ has a well pronounced ellipticity

$$v_2 < 0 \quad \text{for} \quad k_T \lesssim \sqrt{|eB|}$$

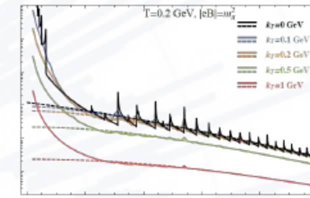


$$v_2 > 0 \quad \text{for} \quad k_T \gtrsim \sqrt{|eB|}$$

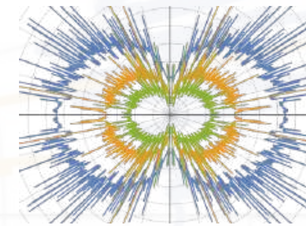
- Nonzero ellipticity of photon emission measures indirectly the magnetic field in HICs

Summary (dileptons)

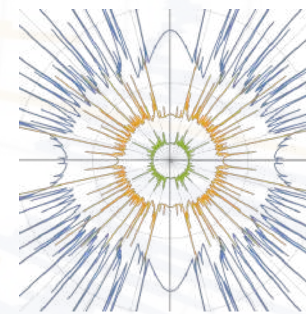
- Magnetic field strongly enhances the dilepton rate at **small invariant masses**, $M \lesssim \sqrt{|eB|}$



- Dilepton emission rate is non-isotropic when $B \neq 0$



$v_2 \lesssim 0.2$ when $M \lesssim \sqrt{|eB|}$ and $k_T \gg \sqrt{|eB|}$



$v_2 \simeq 0$ when $M \gg \sqrt{|eB|}$ all k_T

- Dilepton rate and ellipticity together can also provide indirect measurements of the magnetic field in HICs