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Chiral plasma instability in the magnetosphere of magnetars

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WORKSHOP ON ELECTROMAGNETIC EFFECTS IN STRONGLY INTERACTING MATTER

Gorbar & Shovkovy, Eur. Phys. J. C 82, 625 (2022)

October 27, 2022

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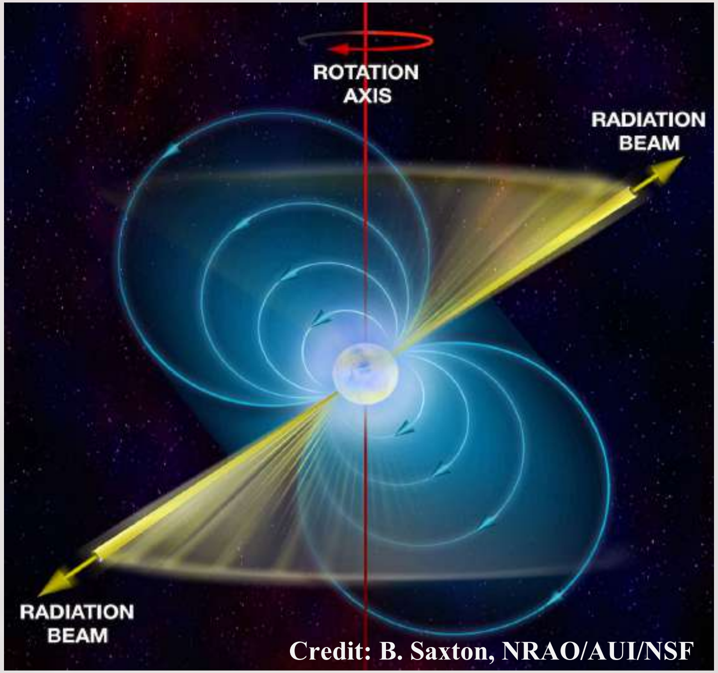
Pulsars

- **Neutron stars** are laboratories of matter under extreme conditions



Credit: B. Saxton, NRAO/AUI/NSF

- Prediction

[Baade & Zwicky, *Proc. Nat. Acad. Sci.* **20**, 259 (1934)]

- Observation

[Hewish, Bell, Pilkington, Scott & Collins, *Nature* **217**, 709 (1968)]

- **Pulsars** are neutron stars that are
  - rapidly rotating ($P \sim 1$ ms to 10 s)
  - strongly magnetized ($B \sim 10^8$ to 10^{15} G)
- Pulsar radiation is beamed along the magnetic field direction (the “lighthouse” effect)

Pulsars in P-$\dot{P}$ plane

- Characteristic age

$$\tau \simeq \frac{P}{2\dot{P}}$$

- Spin-down luminosity

$$-\dot{E} \simeq 4\pi^2 I \frac{\dot{P}}{P^3}$$

- Characteristic magnetic field

$$B \simeq 3\times 10^{19} \left(\frac{P\dot{P}}{s}\right)^{1/2} \mathrm{G}$$

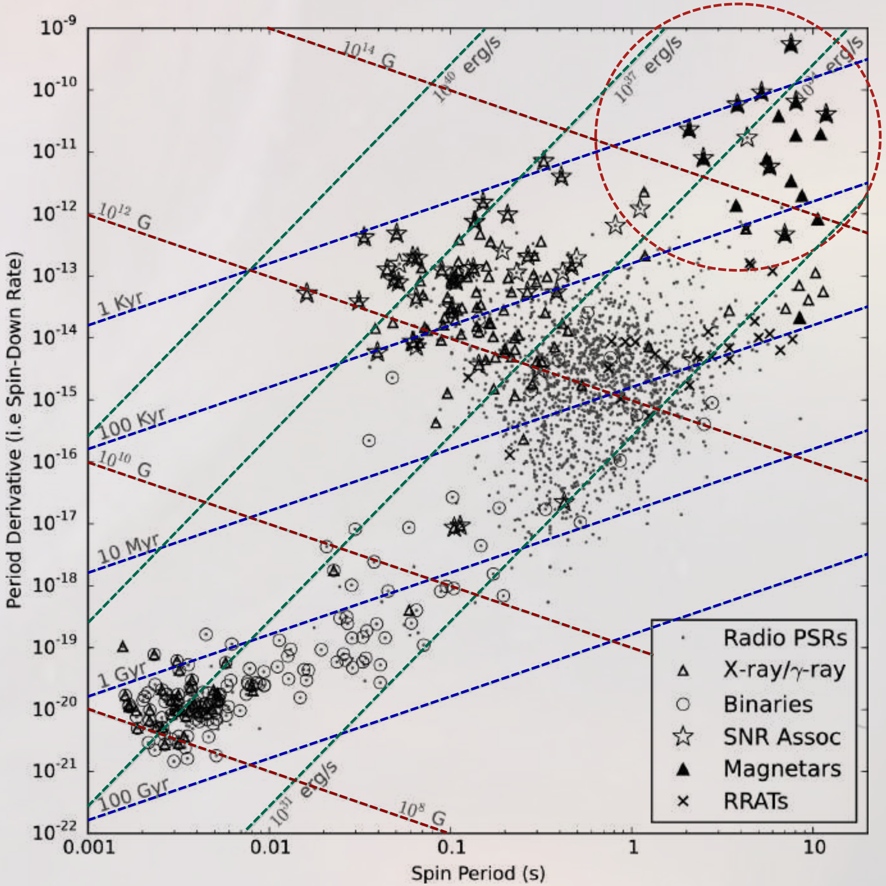


Image credit: Condon & Ransom, "Essential Radio Astronomy" (2016)

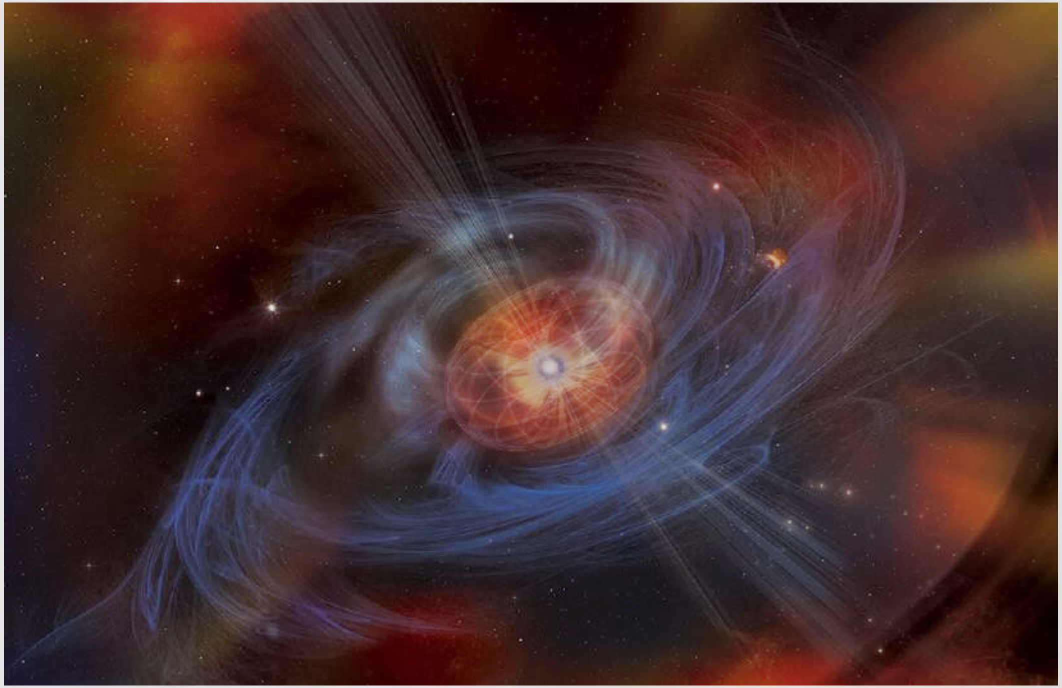


Image credit: Aurore Simonnet, Sonoma State University

MAGNETOSPHERES

Pulsar electrodynamics (VDM)

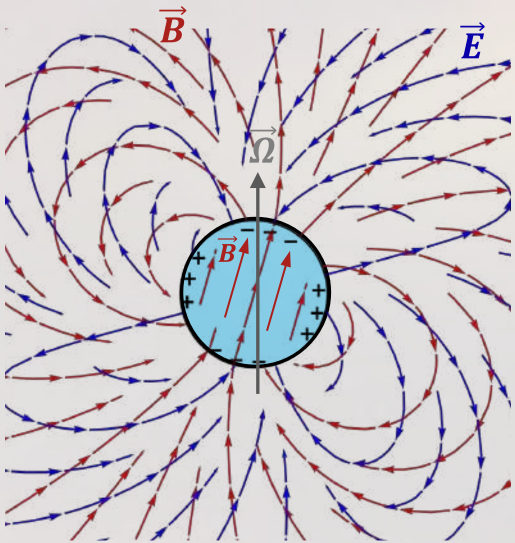
- **Vacuum dipole model (VDM)** ($\rho = 0$ & $J = 0$ outside the star)
- Stellar interior (good conductor):

$$\vec{\boldsymbol{E}}'_{in} = \vec{\boldsymbol{E}}_{in} + \frac{\vec{\boldsymbol{\Omega}}\times\vec{\boldsymbol{r}}}{c}\times\vec{\boldsymbol{B}}_{in} = 0$$

- Fields outside the pulsar are

$$\vec{\boldsymbol{B}} = \frac{B_0 R^3}{2r^3}\left(3(\widehat{\boldsymbol{m}}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \widehat{\boldsymbol{m}}\right)$$

$$\vec{\boldsymbol{E}} = \cdots$$ [see Deutsch, Ann. Astrophys. **18**, 1 (1955)]



where $\boldsymbol{m}$ is the magnetic moment and $\boldsymbol{\Omega}$ is the angular frequency

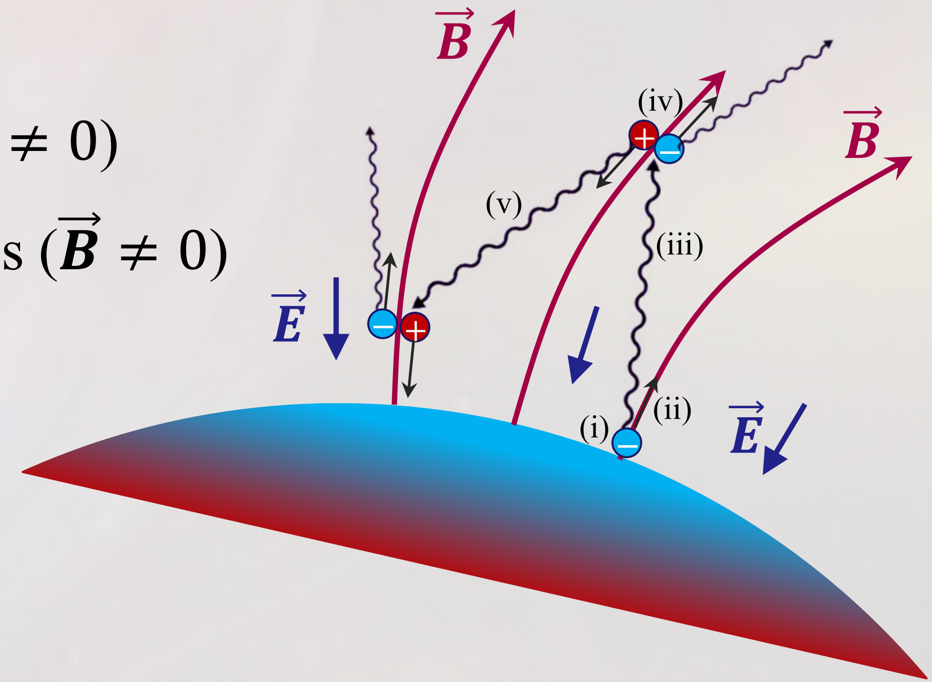
- There is a nonzero charge density and a strong electric field on the surface ($E_{\rm surf} \sim \Omega R B_0 \sim 10^{12}$ to 10^{15} V/m)

Pulsar electrodynamics (VDM)

- Charged particles
  i. pulled up from the surface ($\vec{E} \neq 0$)
  ii. move along curved trajectories ($\vec{B} \neq 0$)
  iii. produce curvature radiation
  iv. γ-quanta produce e^+e^- pairs

$$l_\gamma \simeq \frac{2R_C}{15}\frac{B_C}{B}\frac{m_e}{\varepsilon_\gamma}$$

  v. Secondary particles produce synchrotron & curvature radiation



- **End result:** (I) magnetized vacuum is nontransparent for photons with $\varepsilon_\gamma \gtrsim 2m_e$; (II) vacuum turns into plasma

Pulsar electrodynamics (RMM)

- **Rotating magnetosphere model (RMM)** (assuming a highly conducting plasma outside the star)

$$\vec{\boldsymbol{E}}' = \vec{\boldsymbol{E}} + \frac{\vec{\boldsymbol{\Omega}}\times\vec{\boldsymbol{r}}}{c}\times\vec{\boldsymbol{B}} = 0$$

i.e., $E_{\parallel}=0$

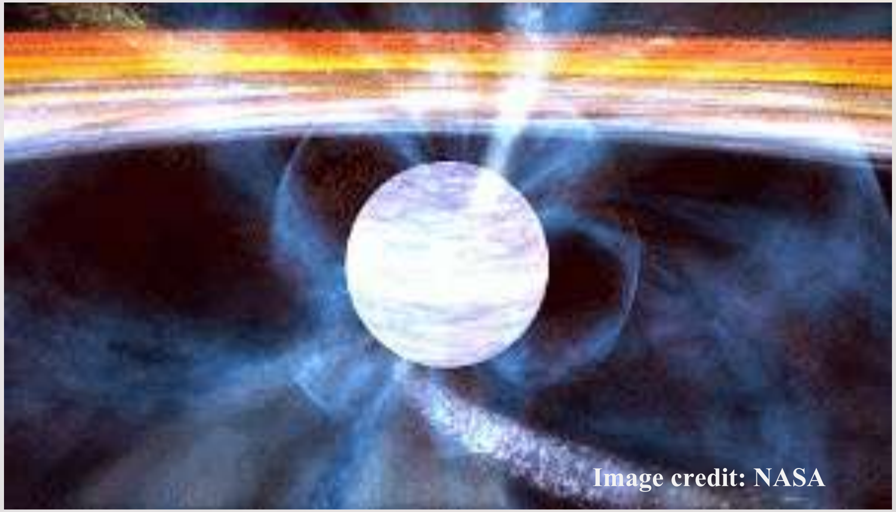
- Plasma motion is determined by

$$\vec{\boldsymbol{v}}_{\mathrm{drift}} = c\,\frac{\vec{\boldsymbol{E}}\times\vec{\boldsymbol{B}}}{B^2} = \vec{\boldsymbol{\Omega}}\times\vec{\boldsymbol{r}} + j_{\parallel}\vec{\boldsymbol{B}}$$

- Corotating plasma is charged

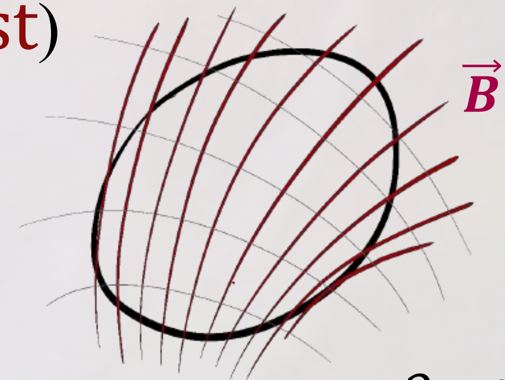
$$\rho_{\mathrm{GJ}} = \vec{\boldsymbol{\nabla}}\cdot\vec{\boldsymbol{E}} = -\frac{2}{c}\vec{\boldsymbol{\Omega}}\cdot\vec{\boldsymbol{B}}$$

[Goldreich & Julian, Astrophys. J. **157**, 869 (1969)]



Gaps in magnetosphere

- If one assumes that $E_{\parallel}$=0 everywhere, the magnetic field lines are equipotential ($V = \text{const}$)



- Then,

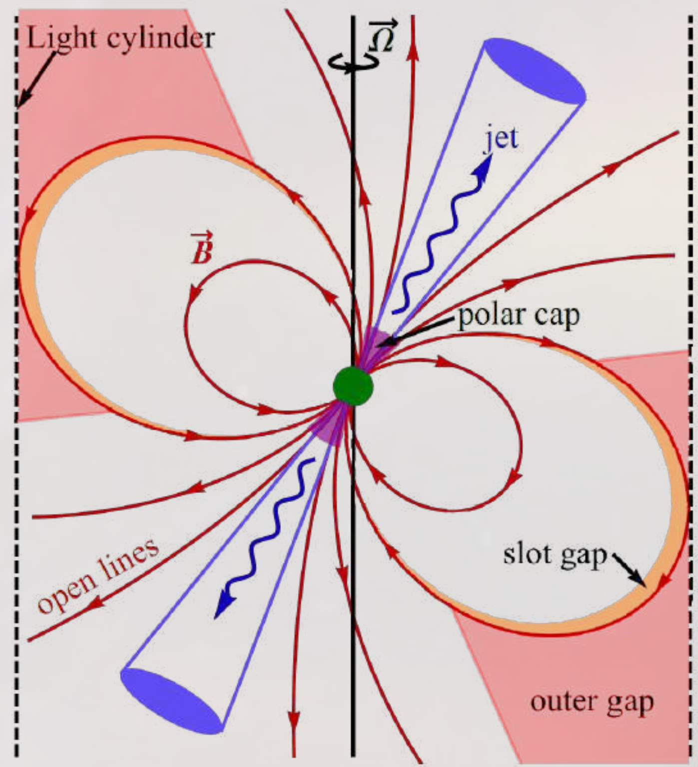
$$0 = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = \int \left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}}\right) \cdot d\vec{\boldsymbol{s}} = -\frac{\partial}{\partial t} \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}}$$

- Thus, $E_{\parallel}$=0 cannot be enforced everywhere if $\vec{\boldsymbol{B}}$ changes in time
- Regions ("gaps") with unscreened $E_{\parallel}$ will necessarily develop (they result from dynamical charge/current starvation)

[Ruderman & Sutherland, Astrophys. J. **196**, 51 (1975)]

Gaps in magnetosphere

- Gaps can develop at various locations
- Intermittent gaps are caused by rapid outflow of charge
- The **gap size** h grows at a speed close to the speed of light
- Electric **potential** difference grows like $\Delta V = E_{\|} h \propto h^2$
- ΔV & photon flux cause an avalanche production of **electron-positron** pairs



[Ruderman & Sutherland, Astrophys. J. **196**, 51 (1975)]

- Since $B \propto 1/r^3$, anomalous effects are strongest near **polar caps**

Pulsar gaps

- Estimates for the **electric field** and the **gap size**

$$E_{\|} \simeq Bh/R_{LC}$$

$$h \simeq 3.6 \text{ m} \left(\frac{R}{10 \text{ km}}\right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}}\right)^{-3/7} \left(\frac{B}{10^{14} \text{ G}}\right)^{-4/7}$$

where $R_{LC} = c/\Omega$ is the radius of light cylinder

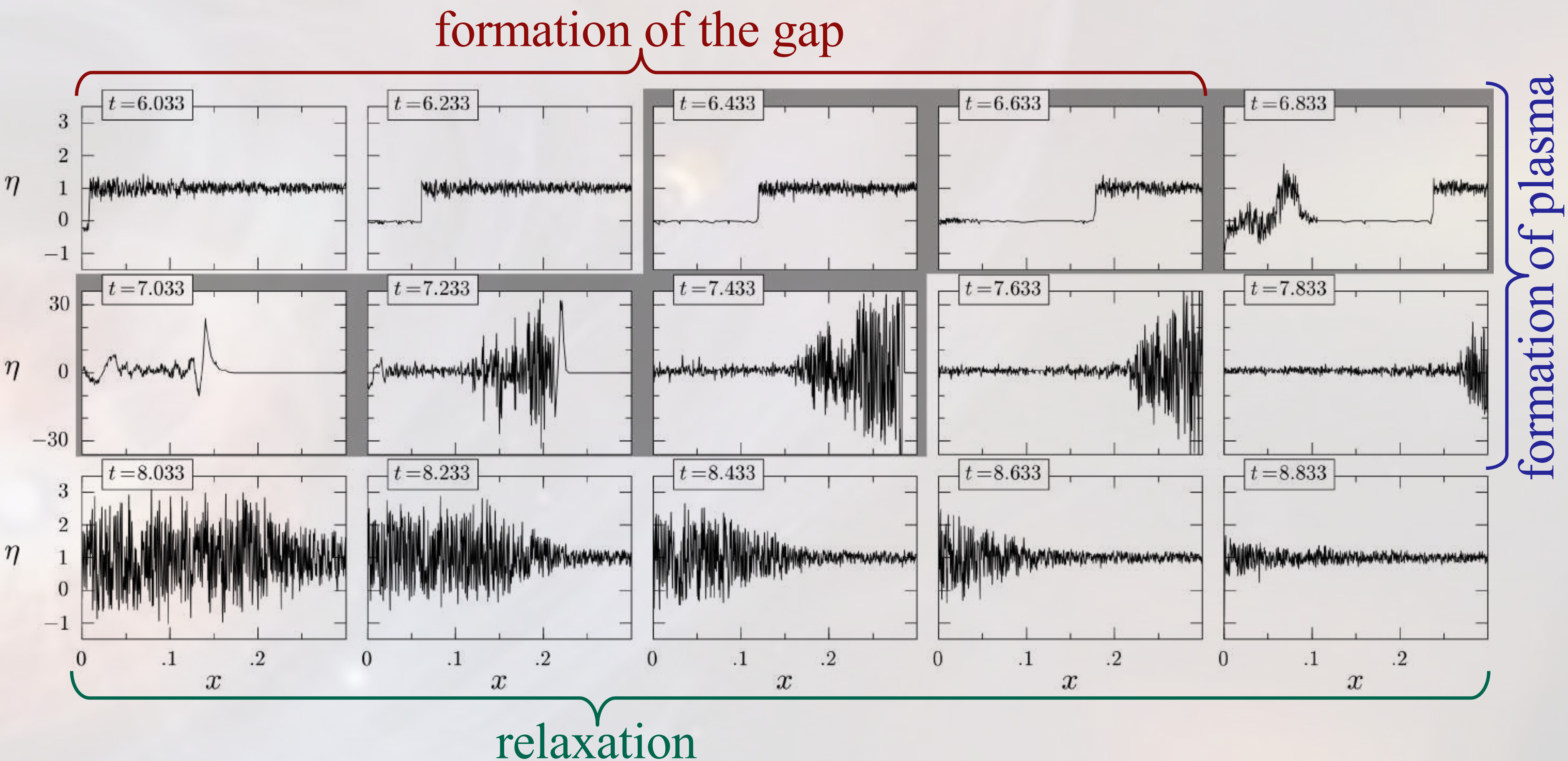
The field scales with pulsar parameters as follows

$$E_{\|} \approx 2.7\times 10^{-8} E_c \left(\frac{R}{10 \text{ km}}\right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14} \text{ G}}\right)^{3/7}$$

where $E_c = m_e^2/e = 1.3 \times 10^{18}$ V/m.

[Ruderman & Sutherland, Astrophys. J. **196**, 51 (1975)]

Charge in the gap



[Timokhin, Mon. Not. R. Astron. Soc. 408, 2092–2114 (2010)]

Gap parameters

- Quantitative estimate of the gap size and fields

B	10^{12} G	10^{13} G	10^{14} G	10^{15} G
h	50 m	13.4 m	3.6 m	0.97 m
$\frac{E_\parallel}{E_c}$	3.8×10^{-9}	1.0×10^{-8}	2.7×10^{-8}	7.3×10^{-8}
$\frac{\boldsymbol{E}\cdot\boldsymbol{B}}{E_c B_c}$	8.6×10^{-11}	2.3×10^{-9}	6.2×10^{-8}	1.7×10^{-6}

where

$$E_c = m_e^2/e = 1.3 \times 10^{18}\ \mathrm{V/m}$$

$$B_c = m_e^2/e = 4.4 \times 10^{13}\ \mathrm{G}$$

Chiral charge production

- The evolution of the chiral charge is determined by

$$\frac{\partial n_5}{\partial t} \cancel{+ \vec{\nabla} \cdot \vec{j}_5} = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_\mathrm{m} n_5$$

- While the chiral anomaly produces n_5, the chirality flipping tries to wash it away
- The chiral charge n_5 approaches the following steady-state value:

$$n_5 = \frac{e^2}{2\pi^2 \Gamma_\mathrm{m}} \vec{E} \cdot \vec{B}$$

- The estimates for the chirality flip rate in a hot plasma

$$\Gamma_\mathrm{m} \simeq \frac{\alpha^2 m_e^2}{T} \quad (T \lesssim m_e/\sqrt{\alpha}) \quad \text{and} \quad \Gamma_\mathrm{m} \simeq \frac{\alpha\, m_e^2}{T} \quad (T \gg m_e/\sqrt{\alpha})$$

[Boyarsky, Cheianov, Ruchayskiy, Sobol, Phys. Rev. Lett. **126**, 021801 (2021)]

Time scales

- The gap formation time

$$t_h \sim h/c \sim 10^{-8}\ \mathrm{s}$$

- Timescale for chiral charge production

$$t^{\star} \sim 1/\Gamma_{\mathrm{m}} \sim 10^{-17}\ \mathrm{s}$$

- Note that

$$t_h \gg t^{\star}$$

- Thus, the chirality production is nearly instantaneous

Estimate for n_5 in magnetars

- The estimate for the chiral charge is given by

$$n_5 \simeq \frac{e^2 E_{\parallel} B}{2\pi^2 \Gamma_m} \simeq 1.5 \times 10^{-5}\ \mathrm{MeV}^3 \left(\frac{T}{1\ \mathrm{MeV}}\right) \times \left(\frac{R}{10\ \mathrm{km}}\right)^{2/7} \left(\frac{\Omega}{1\ \mathrm{s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14}\ \mathrm{G}}\right)^{10/7}$$

- The corresponding chiral chemical potential is

$$\mu_5 \simeq \frac{3n_5}{T^2} \simeq 4.6 \times 10^{-5}\ \mathrm{MeV} \left(\frac{T}{1\ \mathrm{MeV}}\right)^{-1} \times \left(\frac{R}{10\ \mathrm{km}}\right)^{2/7} \left(\frac{\Omega}{1\ \mathrm{s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14}\ \mathrm{G}}\right)^{10/7}$$

Values of n_5 and μ_5

- The corresponding numerical values for chiral charge and chiral chemical potential are

B	10^{12} G	10^{13} G	10^{14} G	10^{15} G
h	50 m	13.4 m	3.6 m	0.97 m
$\frac{E_\parallel}{E_c}$	3.8×10^{-9}	1.0×10^{-8}	2.7×10^{-8}	7.3×10^{-8}
$\frac{\boldsymbol{E}\cdot\boldsymbol{B}}{E_c B_c}$	8.6×10^{-11}	2.3×10^{-9}	6.2×10^{-8}	1.7×10^{-6}
$\frac{n_5}{m_e^3}$	1.6×10^{-7}	4.3×10^{-6}	1.1×10^{-4}	3.1×10^{-3}
$\frac{\mu_5}{m_e}$	1.2×10^{-7}	3.4×10^{-6}	9.0×10^{-5}	2.4×10^{-3}

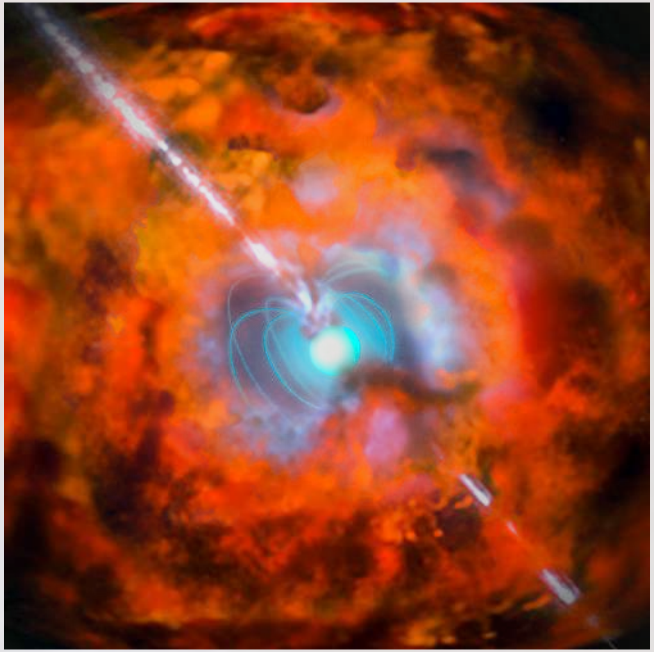


Image credit: European Southern Observatory

CHIRAL PLASMA INSTABILITY

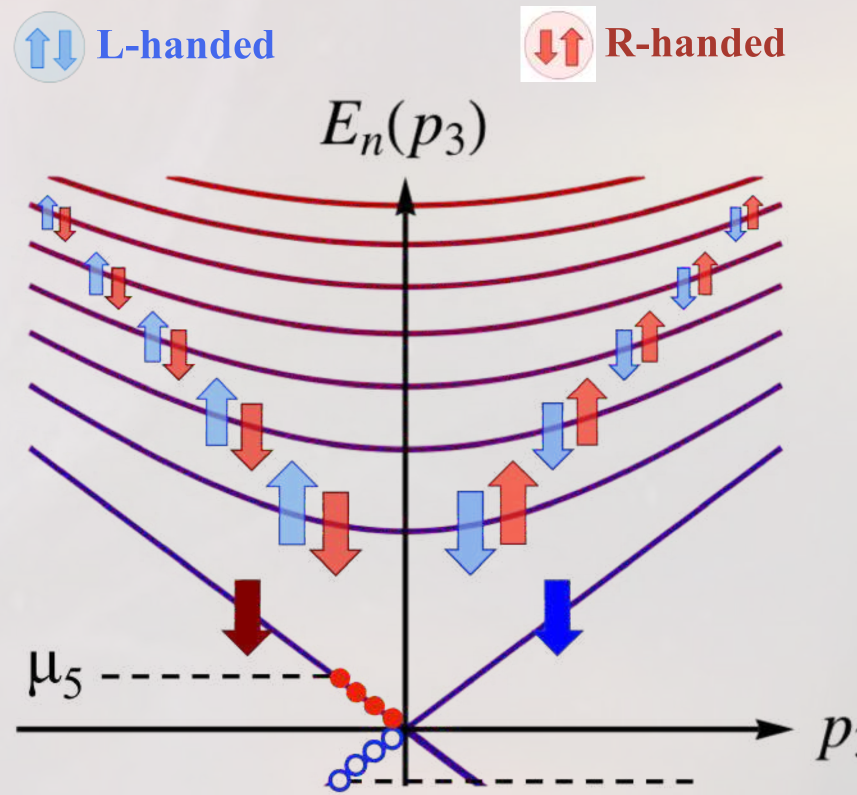
[]

Plasma with $\mu_5 \neq 0$

- Nonzero μ_5 and $\vec{B}$ drive the chiral magnetic effect (CME)

$$\vec{j} = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

- The effect comes from the spin-polarized LLL (s=↓)
  - L-handed states ($p_3 < 0$ & $|E| < \mu_5$) are empty (holes with $p_3 > 0$)
  - R-handed states ($p_3 < 0$ & $E < \mu_5$) are occupied



- However, plasma at $\mu_5 \neq 0$ is unstable

Maxwell equations at $\mu_5 \neq 0$

- The total current (CME + Ohm)

$$j = \boxed{\frac{2\alpha}{\pi}\mu_5}\mathbf{B} + \sigma\mathbf{E}$$

- By substituting ***j*** into Ampere's law

$$\nabla \times \mathbf{B} = j + \frac{\partial \mathbf{E}}{\partial t}$$

and solving for the electric field, one derives

$$\mathbf{E} = \frac{1}{\sigma}\left(\nabla \times \mathbf{B} - k_\star \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t}\right)$$

where $\boxed{k_\star = \frac{2\alpha\mu_5}{\pi}}$

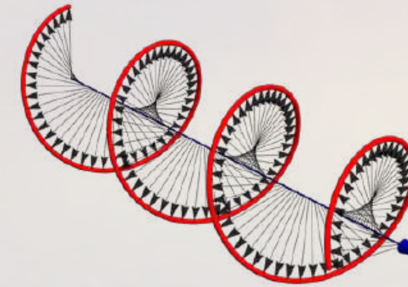
- Finally, by calculating the curl and using Faraday's law,

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{\sigma}\left(\nabla \times (\nabla \times \mathbf{B}) - k_\star \nabla \times \mathbf{B} + \frac{\partial^2 \mathbf{B}}{\partial t^2}\right)$$

Helical modes at $\mu_5 \neq 0$

- Search for a solution as a superposition of helical eigenstates

$$\boldsymbol{\nabla} \times \mathbf{B}_{\lambda,k} = \lambda k \mathbf{B}_{\lambda,k}$$



e.g.,

$$\mathbf{B}_{\lambda,k} = B_0 \left(\hat{\boldsymbol{x}} + i\lambda\hat{\boldsymbol{y}}\right) e^{-i\omega t + ikz}$$

Then, for a fixed eigenmode, the evolution equation reads

$$\frac{d\mathbf{B}_{\lambda,k}}{dt} = \frac{1}{\sigma}\left(\lambda k_\star k - k^2 - \frac{\partial^2}{\partial t^2}\right)\mathbf{B}_{\lambda,k}$$

- The two solutions for the frequency are

$$\omega_{1,2} \;=\; -\frac{i}{2}\left(\sigma \pm \sqrt{\sigma^2 + 4k(\lambda k_\star - k)}\right)$$

Long-wavelength modes

[Joyce & Shaposhnikov, PRL 79, 1193 (1997)]
[Boyarsky, Frohlich, Ruchayskiy, PRL 108, 031301 (2012)]
[Tashiro, Vachaspati, Vilenkin, PRD 86, 105033 (2012)]
[Akamatsu & Yamamoto, PRL 111, 052002 (2013)]
[Tuchin, PRC 91, 064902 (2015)]
[Manuel & Torres-Rincon, PRD 92, 074018 (2015)]
[Hirono, Kharzeev, Yin, PRD 92, 125031(2015)]
[Sigl & Leite, JCAP 01, 025 (2016)]

- For a plasma with high conductivity

$$\omega_{1,2} \simeq \begin{cases} -i\left(\sigma + \frac{k(\lambda k_\star - k)}{\sigma}\right) \\ i\frac{k(\lambda k_\star - k)}{\sigma} \end{cases}$$

- The 1st mode is damped by charge screening:

$$B_{k,1} \propto B_0 e^{-\sigma t}$$

- The 2nd mode is unstable when $k < \lambda k_\star$:

$$B_{k,2} \propto B_0 e^{+tk(\lambda k_\star - k)/\sigma}$$

- The momentum of the fastest growing mode $B_{k,2}$ is

$$\frac{1}{2} k_\star$$

Instability in pulsars

- The estimate for $k_\star$

$$k_\star \simeq 2.2\times10^{-7}\ \text{MeV}\left(\frac{T}{1\ \text{MeV}}\right)^{-1}\left(\frac{R}{10\ \text{km}}\right)^{2/7}\left(\frac{\Omega}{1\ \text{s}^{-1}}\right)^{4/7}\left(\frac{\text{B}}{10^{14}\ \text{G}}\right)^{10/7}$$

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$\frac{n_5}{m_e^3}$	1.6×10^{-7}	4.3×10^{-6}	1.1×10^{-4}	3.1×10^{-3}
$\frac{\mu_5}{m_e}$	1.2×10^{-7}	3.4×10^{-6}	9.0×10^{-5}	2.4×10^{-3}
$\frac{k_\star}{m_e}$	5.8×10^{-10}	1.6×10^{-8}	4.2×10^{-7}	1.1×10^{-5}

Observational consequences

- Unstable plasma in the gaps produces **helical** (circularly polarized) **modes** in the frequency range

$$0 \lesssim \omega \lesssim k_{\star}$$

- For magnetars, these span **radio frequencies** and may reach into the **near-infrared** range
- Available energy is of the order of $\Delta\mathcal{E} \sim \mu_5^2 T^2 h^3$, i.e.,

$$\Delta\mathcal{E} \simeq 2.1 \times 10^{25}\ \mathrm{erg} \left(\frac{T}{1\ \mathrm{MeV}}\right)\left(\frac{R}{10\ \mathrm{km}}\right)^{6/7} \times \left(\frac{\Omega}{1\ \mathrm{s}^{-1}}\right)^{-9/7}\left(\frac{B}{10^{14}\ \mathrm{G}}\right)^{2/7}$$

- The energy is sufficient to feed the **fast radio bursts** (FRB)

Outstanding problems

- Interplay of chiral charge and electron-positron pair **production** induced by energetic photons should be studied in detail
- The modification of the **chiral flip rate** $\Gamma_{\mathrm{m}} \simeq \frac{\alpha^2 m_e^2}{T}$ by the strong magnetic field (extra suppression?)
- The role of the **inverse magnetic cascade** and the **chiral-magnetic turbulence** should be quantified
- Self-consistent **dynamics** of chiral plasma in the gap regions should be simulated in detail
- Detailed mechanism of the **energy transfer** from unstable helical modes to radio emission in FRBs

Summary

- Chiral anomaly can have *macroscopic* implications in pulsars
- It leads to a *significant* chiral charge production (up to 10^{34} m^{-3}) in strongly magnetized magnetospheres
- The chiral chemical potential μ_5 can be up to 10^{-3} MeV
- This is sufficient to trigger emission of helical waves with frequencies up to about $k_\star \simeq \frac{2}{\pi}\alpha\mu_5$ (radio to infrared range)
- Helical waves can affect the pulsar jets and observable features of the fast radio bursts
- For quantitative effects, further detailed studies are needed