College of Integrative Sciences and Arts
ARIZONA STATE UNIVERSITY

Anomalous effects in chiral plasmas

Igor Shovkovy

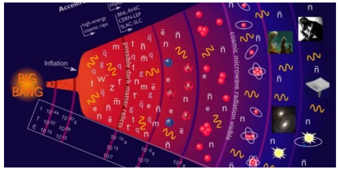
Happy Birthday, Gordon!

Gravity, Strings and Fields: A Conference in Honour of Gordon Semenoff

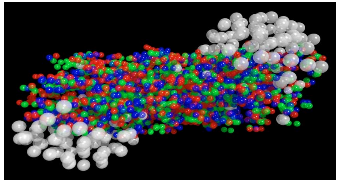
JULY 24 - 28, 2023

(Quasi-) Relativistic Matter

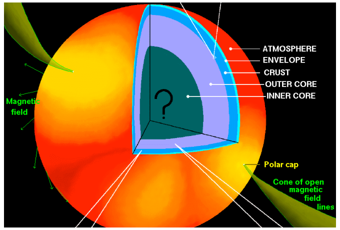
- **Early Universe**
  (extremely high temperature $> 10^{15}$ K)



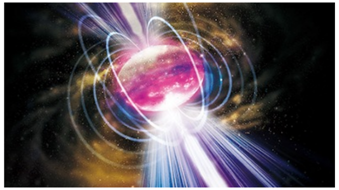
- **Heavy-ion collisions**
  (high temperature $\lesssim 4\times10^{12}$ K)



- **Super-dense matter in compact stars**
  (high densities $\lesssim 10^{17}$ kg/m^3)

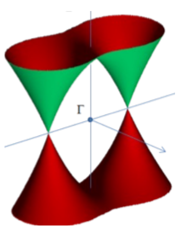


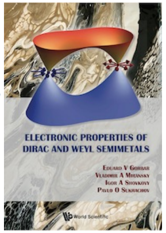
- **Magnetospheres of magnetars**
  (electron-positron plasma at temperatures $\lesssim 10^{11}$ K)

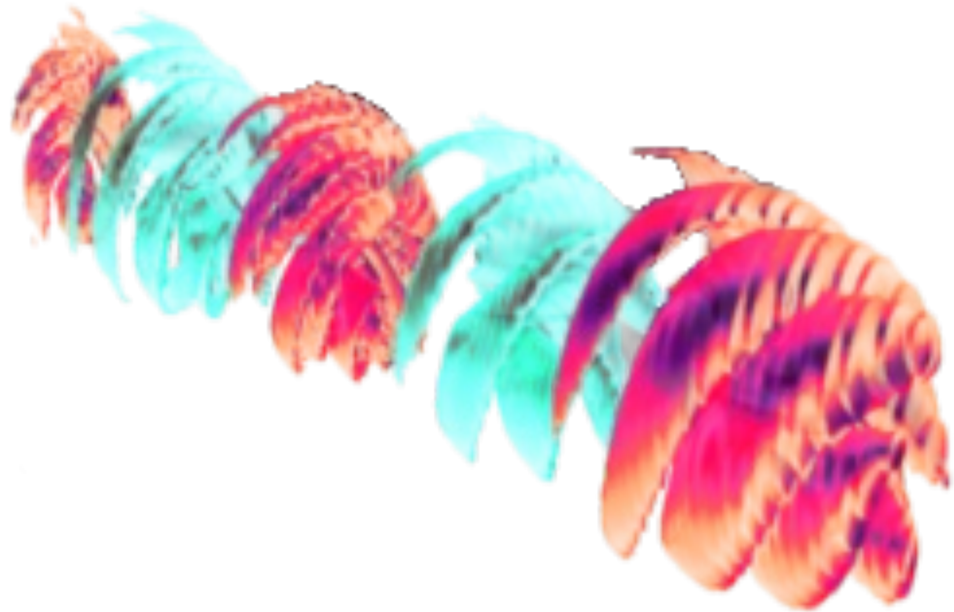


- **Electron plasma in Dirac/Weyl (semi-)metals**
  (chiral quasiparticle plasma at temperatures $\lesssim 10^{2}$ K)

- **Other: cold atoms, superfluid ^{3}He-A, etc.**
  (chiral quasiparticles at temperatures $\sim 10^{-3}$ K)







CHIRAL PLASMAS

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Becattini, Liao, Lisa, Lect. Notes Phys. **987**, 1 (2021)]

Anomalous chiral matter

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ existing on *macroscopic* time/distance scales

- The spacetime dynamics of $n = n_R + n_L$ and $n_5 = n_R - n_L$ is governed by continuity equations, i.e.,

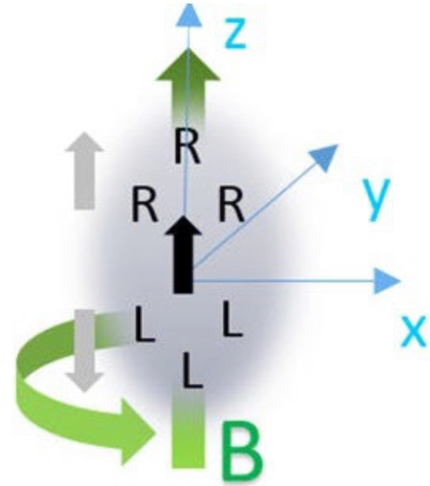
$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial n_5}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_{\mathrm{m}} n_5$$

chiral anomaly

where the chirality flip rate: $\Gamma_{\mathrm{m}} \propto \alpha^2 T (m/T)^2$

- Chiral anomaly can produce *macroscopic* effects in plasmas



CHIRAL ANOMALOUS EFFECT

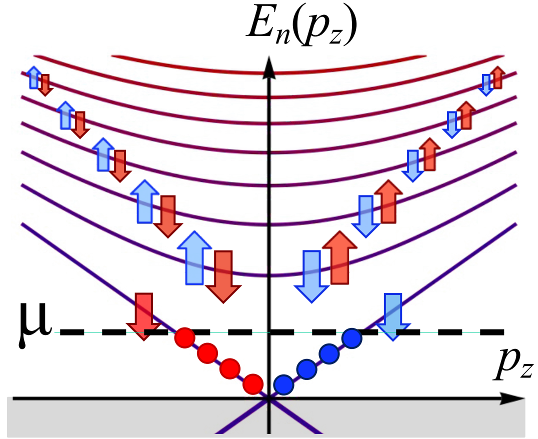
$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu \quad \& \quad \langle \vec{j} \rangle = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Chiral Separation Effect ($\mu \neq 0$)

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_z<0$ (and $s=\downarrow$) are R-handed
  - states with $p_z>0$ (and $s=\downarrow$) are L-handed

  i.e., a nonzero **axial** current is induced

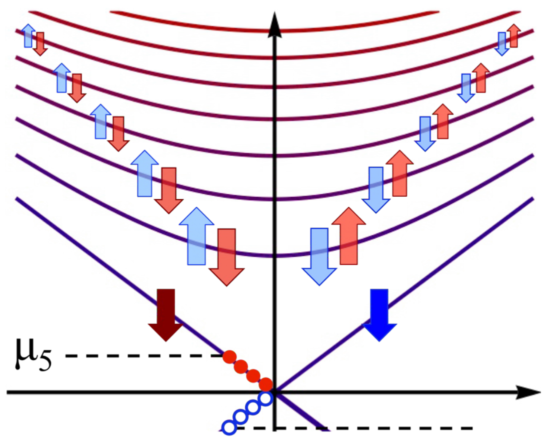


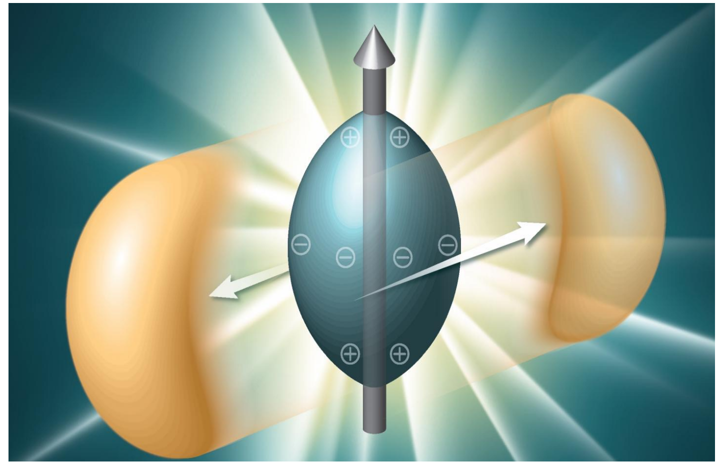
- CSE current:

$$\langle \vec{j}_5 \rangle = -tr[\vec{\gamma}\gamma^5 S(x,x)] = -\frac{e\vec{B}}{2\pi^2}\mu$$

- CME current:

$$\langle \vec{j} \rangle = -tr[\vec{\gamma} S(x,x)] = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

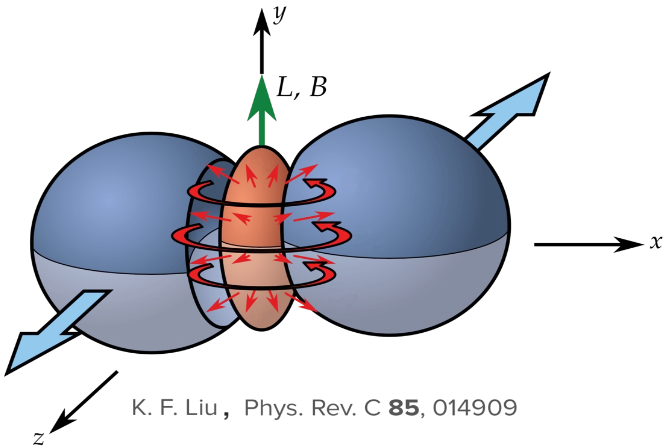




HEAVY-ION COLLISIONS (THEORY)

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields are induced by the currents of passing charged ions



K. F. Liu, Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108], …

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

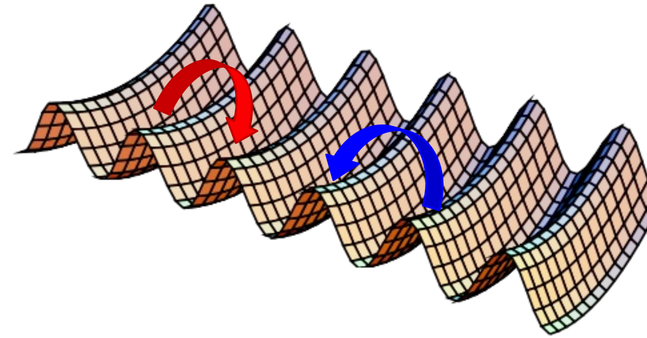
$$\omega \sim 9 \times 10^{21} s^{-1} \; (\sim 10\text{ MeV})$$

- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19}\text{ G} \; (\sim 100\text{ MeV})$$

Source of chirality in QCD

- Chiral charge can be produced by topological configurations in QCD

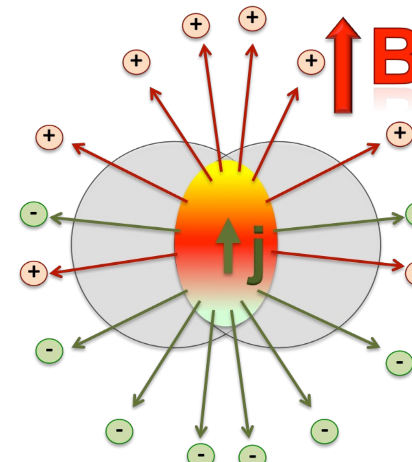


$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2}\int d^3x\, F_a^{\mu\nu}\tilde{F}^a_{\mu\nu}$$

- A random fluctuation with nonzero chirality should produce

$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- Dipole CME effect in heavy-ion colllisions



$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2}\mu_5$$

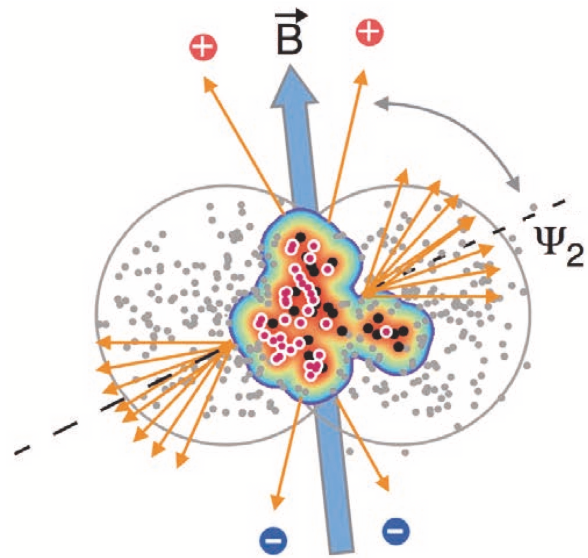
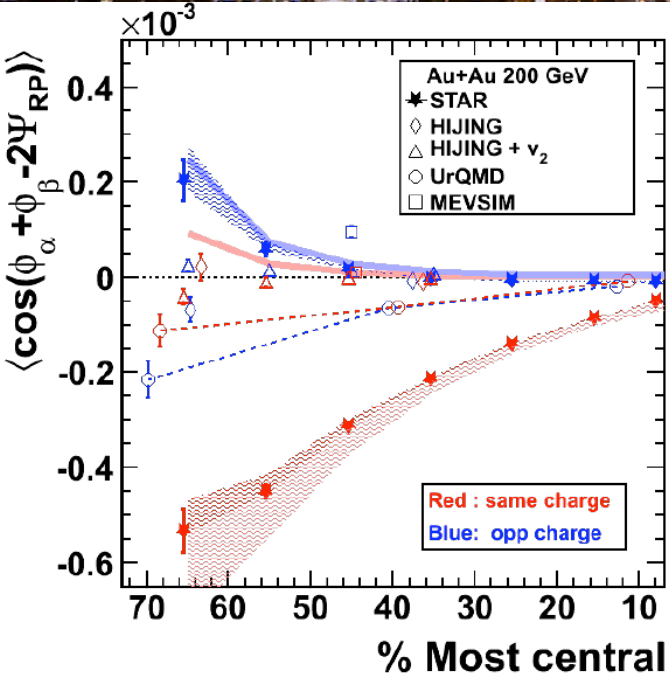
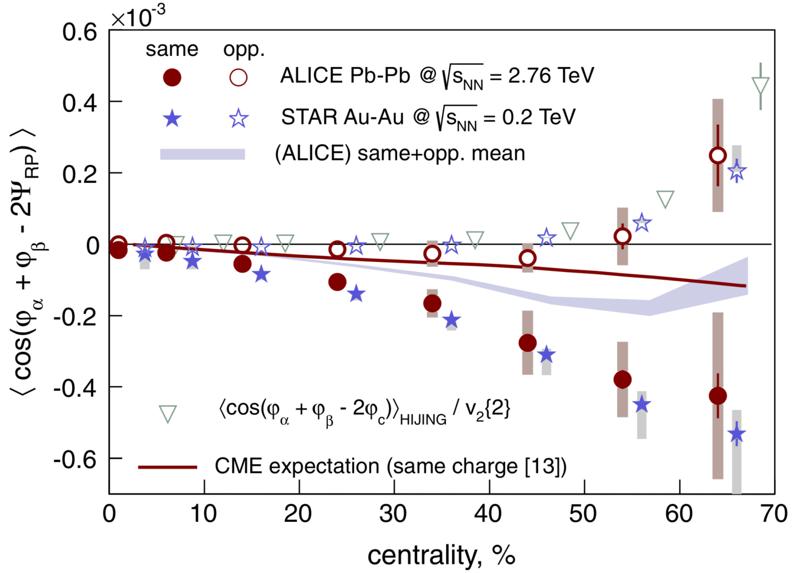


Image credit [Kharzeev & Liao, Nucl. Phys. News **29**, 1 (2019)]

HEAVY-ION COLLISIONS (EXPERIMENT)

CME: Experimental evidence





Correlations of same & opposite charge particles:

$$\left\langle \cos\left(\varphi_\alpha^\pm + \varphi_\beta^\mp - 2\Psi_{RP}\right)\right\rangle > 0$$
$$\left\langle \cos\left(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}\right)\right\rangle < 0$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]
[Adamczyk et al. (STAR), PRL **113**, 052302 (2014)]
[Khachatryan et al. (CMS), PRL **118**, 122301 (2017)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)]
[ALICE Collaboration, Phys. Lett. B**777**, 151 (2018)]

Brilliant idea: Isobar collisions

Utilize collisions of isobars, e.g.,

[Voloshin, PRL **105**, 172301 (2010)]
[Deng et al. PRC **94**, 041901(R) (2016)]

$^{96}_{40}\mathrm{Zr} + ^{96}_{40}\mathrm{Zr}$ vs. $^{96}_{44}\mathrm{Ru} + ^{96}_{44}\mathrm{Ru}$

$^{130}_{52}\mathrm{Te} + ^{130}_{52}\mathrm{Te}$ vs. $^{130}_{56}\mathrm{Ba} + ^{130}_{56}\mathrm{Ba}$

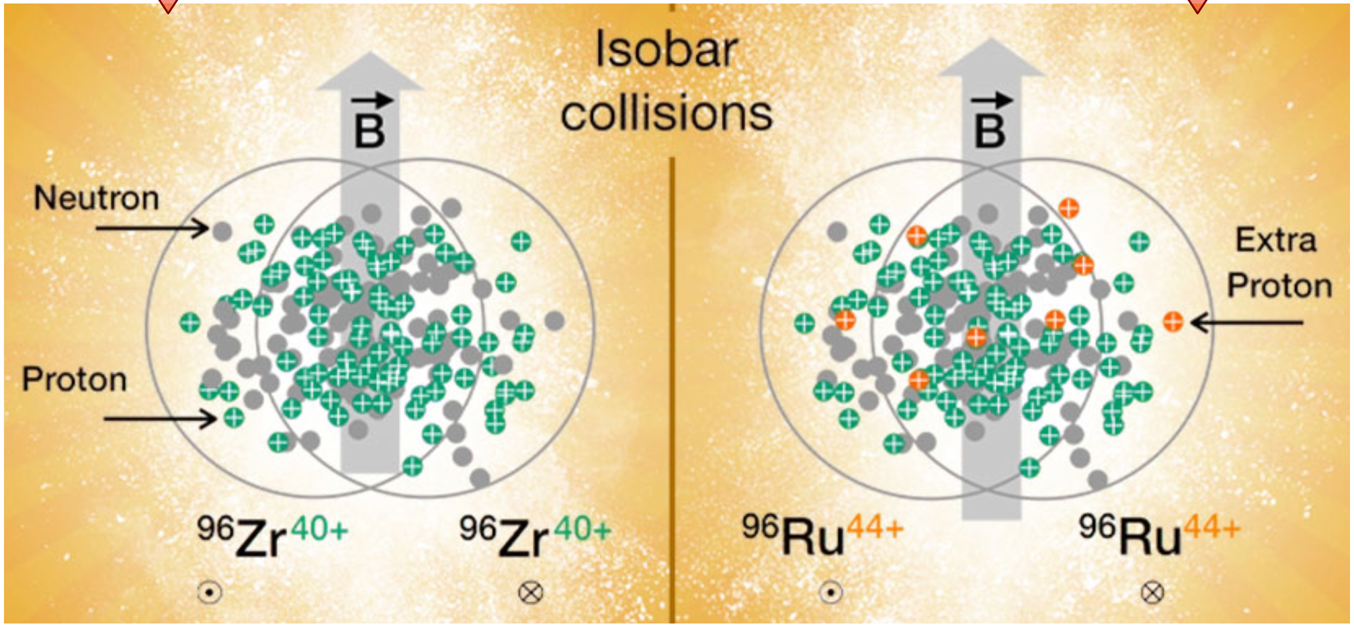


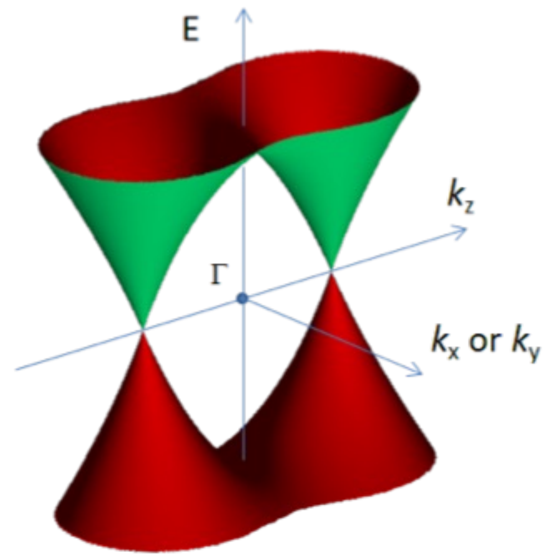
Image credit: Brookhaven National Laboratory, https://www.bnl.gov/newsroom/news.php?a=119062

Isobar collisions (experiment)

- Isobar run was completed by STAR in May 2018
- $\approx$ 3.8 billion collisions of ^{96}Ru+ ^{96}Ru and ^{96}Zr+ ^{96}Zr at $\sqrt{s} = 200$ GeV
- Blind analysis by five groups of the STAR Collaboration
- Report announced on Aug. 31, 2021 (online event @ BNL)
- Paper published on Jan. 3, 2022

[STAR Collaboration, Phys.Rev.C **105**, 014901 (2022); arXiv:2109.00131]

influence of signal and backgrounds, the STAR Collaboration performed a blind analysis of a large data sample of approximately 3.8 billion isobar collisions of $^{96}_{44}$Ru + $^{96}_{44}$Ru and $^{96}_{40}$Zr + $^{96}_{40}$Zr at $\sqrt{s_{NN}} = 200$ GeV. Prior to the blind analysis, the CME signatures are predefined as a significant excess of the CME-sensitive observables in Ru + Ru collisions over those in Zr + Zr collisions, owing to a larger magnetic field in the former. A precision down to 0.4% is achieved, as anticipated, in the relative magnitudes of the pertinent observables between the two isobar systems. Observed differences in the multiplicity and flow harmonics at the matching centrality indicate that the magnitude of the CME background is different between the two species. No CME signature that satisfies the predefined criteria has been observed in isobar collisions in this blind analysis.



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL SEMIMETALS

[Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]

Dirac/Weyl fermions

- Electron quasiparticles with a wide range of properties are possible
- They can even include *massless* Weyl fermions,

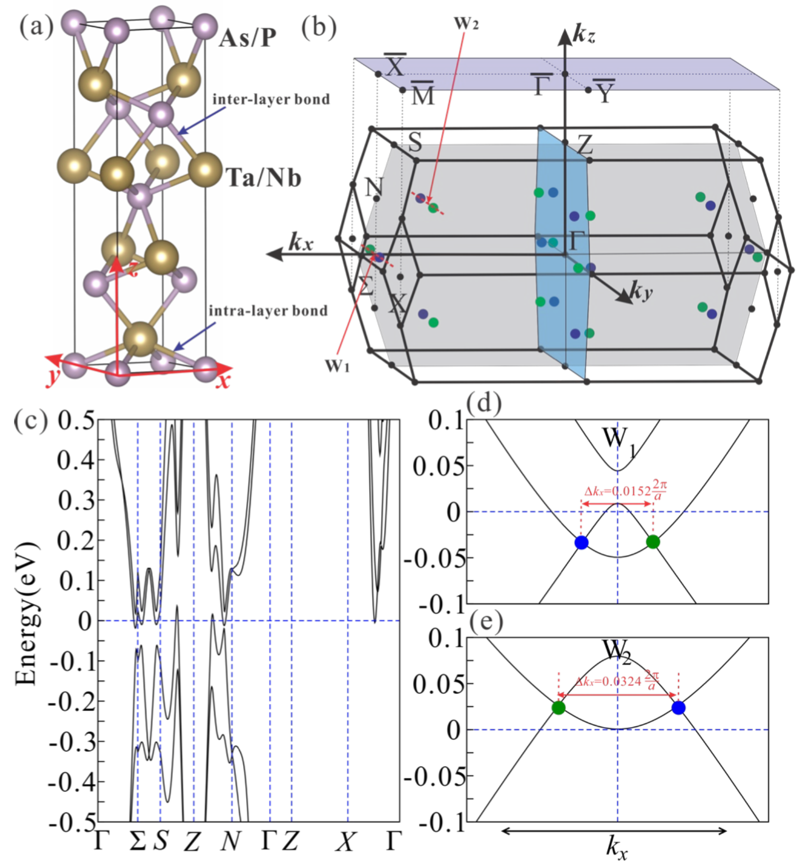
$$H_W \approx \pm v_F(\vec{\sigma} \cdot \vec{\boldsymbol{k}})$$

It is not uncommon!

Na_3Bi, Cd_3As_2, $ZrTe_5$, TaAs, NbAs, …

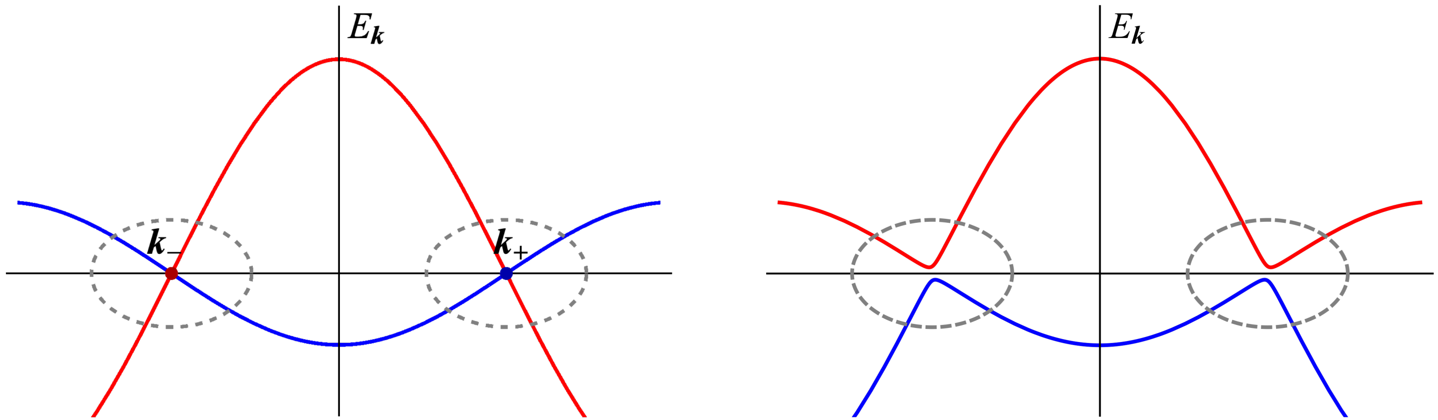
[Liu et al., Science **343**, 864 (2014)]
[Neupane et al., Nature Commun. **5**, 3786 (2014)]
[Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
[Li et al., Nature Physics **12**, 550 (2016)]
[S.-Y. Xu et al., Science 349, 613 (2015)]
[B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
[S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
[S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
[F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]

Weyl semimetals TaAs, TaP, NbAs, and NbP



Sun, Wu & Yan, Phys. Rev. B **92**, 115428 (2015)

Relativistic-like band crossing



A generic 2-band Hamiltonian reads

$$H_{\boldsymbol{k}} = a_{\boldsymbol{k}} + \vec{b}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{\sigma}} \quad \Longrightarrow \quad E_{\boldsymbol{k}} = a_{\boldsymbol{k}} \pm \sqrt{\left(\vec{b}_{\boldsymbol{k}}\right)^2}$$

The bands cross when [Witten, Riv. Nuovo Cimento **39**, 313 (2016)]

$$\vec{b}_{\boldsymbol{k}} = 0$$

These 3 equations can be solved by adjusting $\vec{\boldsymbol{k}}$ in 3D

Weyl quasiparticles

- The eigenstates of Weyl Hamiltonian are

$$\psi_{\boldsymbol{k}}^{\lambda} = \frac{1}{\sqrt{2}\sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}} \begin{pmatrix} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{pmatrix}$$

- The relativistic-like energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$
- The mapping $k \rightarrow \psi_k^{\lambda}$ has a nontrivial topology
- Adiabatic evolution of the wave function from $\psi_{\boldsymbol{k}}$ to $\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}$:

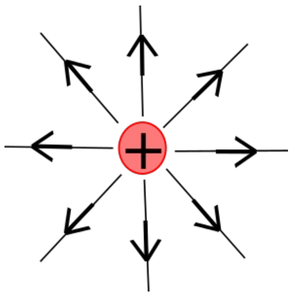
$$\langle \psi_{\boldsymbol{k}} | \psi_{\boldsymbol{k}+\delta\boldsymbol{k}} \rangle \approx 1 + \delta\boldsymbol{k} \cdot \langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle \approx e^{i\boldsymbol{a}_{\boldsymbol{k}} \cdot \delta\boldsymbol{k}}$$

where $\boldsymbol{a}_{\boldsymbol{k}} = -i \langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle$ is the Berry connection

Berry curvature & topology

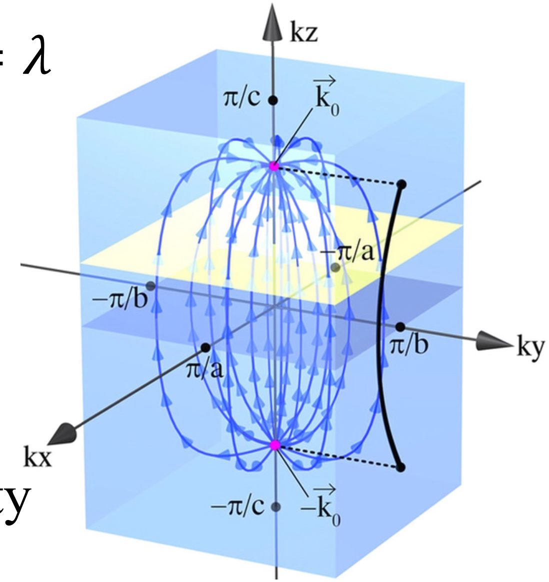
- For Weyl eigenstates, the Berry curvature is

$$\boldsymbol{\Omega}_k \equiv \boldsymbol{\nabla}_k \times \boldsymbol{a}_k = \lambda \frac{\vec{\boldsymbol{k}}}{2k^3}$$



- The Chern number (topological charge)

$$C = \frac{1}{2\pi} \oint \boldsymbol{\Omega}_k \cdot d\boldsymbol{S}_k = \frac{\lambda}{2\pi} \oint \frac{\vec{\boldsymbol{k}}}{2k^3} \cdot \frac{\vec{\boldsymbol{k}}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda$$



- In solid state materials, the Brillouin zone is compact
- A closed surface around a node at $\vec{\boldsymbol{k}}_0$ is also a closed surface around the rest of the Brillouin zone
- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]

[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

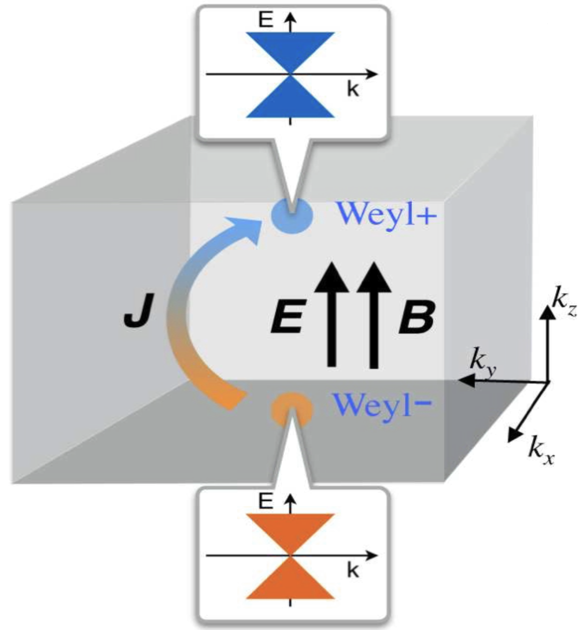


Image credit [Zhang et al., Nat. Commun. 7, 10735 (2016)]

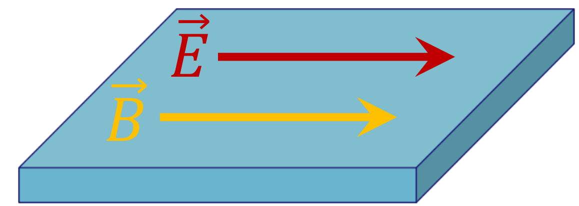
OBSERVATION OF ANOMALOUS EFFECTS

Steady CME current

- Homogeneous chiral plasma:

[Nielsen & Ninomiya, Phys. Lett. B **130**, 390 (1983)]
[Son & Spivak, Phys. Rev. B **88**, 104412 (2013)]

$$\frac{\partial n_5}{\partial t} + \vec{\nabla}\cdot\vec{j}_5 = \frac{e^2 \vec{E}\cdot\vec{B}}{2\pi^2} - \frac{n_5}{\tau_{\rm ch}}$$



- Steady state ($\tau_{\rm ch} \sim 1$ ps to 1 ns)

$$n_5 = \frac{e^2 \vec{E}\cdot\vec{B}}{2\pi^2}\tau_{\rm ch} \quad \rightarrow \quad \mu_5 = \frac{n_5}{\chi_5} \approx \frac{3v^3 n_5}{T^2+\mu^2/\pi^2}$$

- The CME current

$$J_i = \frac{e^2}{2\pi^2}\mu_5 B_i = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{\rm ch}\frac{B_i B_k}{\chi_5}E_k \quad \rightarrow \quad \sigma_{CME}^{\parallel} = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{\rm ch}\frac{B^2}{\chi_5}$$

i.e.,

$$\rho_{\rm total}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}$$

Negative Magnetoresistance

- Experimental confirmation

$$\rho^{\parallel}_{\text{total}} = \frac{1}{\sigma_0 + a(T)B^2}$$

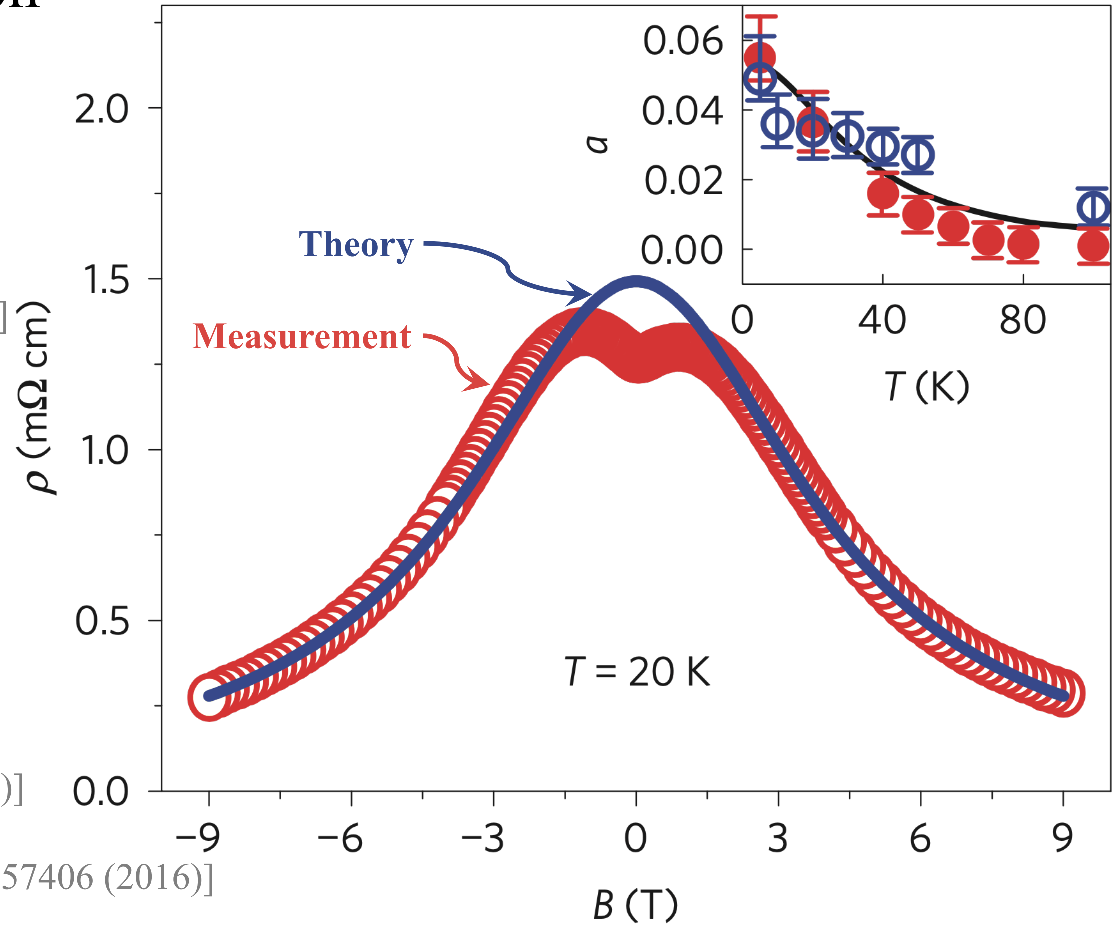
Dirac semimetals:

[Kim et al, Phys. Rev. Lett. **111**, 246603 (2013)]
[Li et al., Nat. Mater. **12**, 550 (2016)]
[Xiong et al., Science **350**, 413 (2015)]
[Feng, et al., Phys. Rev. B **92**, 081306 (2015)]
[Li et al., Nat. Commun. **6**, 10137 (2015)]
[Li et al., Nat. Commun. **7**, 10301 (2016)]

Weyl semimetals:

[Huang et al., Phys. Rev. X **5**, 031023 (2015)]
[Zhang et al., Nat. Commun. **7**, 10735 (2016)]
[Hirschberger et al., Nat. Mater. **15**, 1161 (2016)]
[Wang et al., Phys. Rev. B **93**, 121112 (2016)]
[Du et al., Sci. China Phys. Mech. Astron. **59**, 657406 (2016)]
[Li et al., Front. Phys. **12**, 127205 (2017)]

[Q. Li et al, Nature Physics **12**, 550 (2016)]

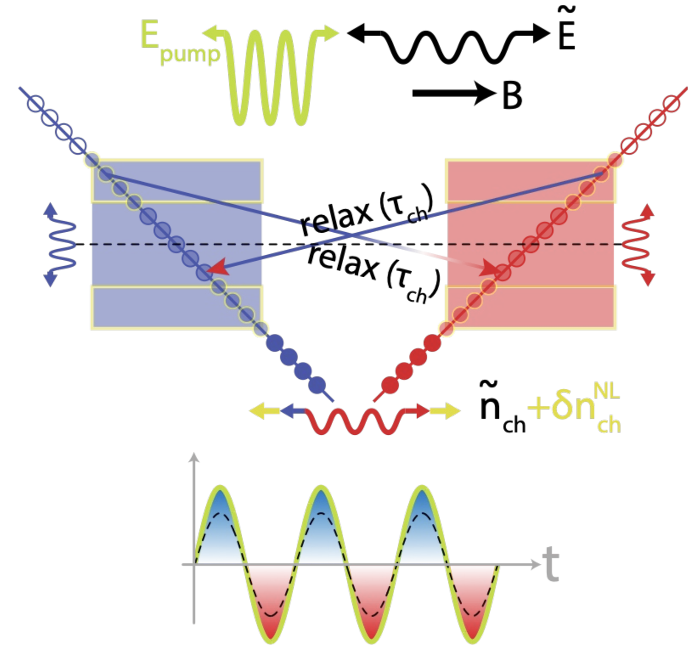


Chiral charge pumping (theory)

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

- Weyl semimetal TaAs
  - $\vec{B} \neq 0$ & $\vec{E}_{\text{pump}} \parallel \vec{B}$
- The extra contribution to chiral charge-induced conductivity

$$\delta\sigma_{\text{ch}}^{\text{NL}} = i\frac{9\alpha^2 e^5 v^3}{8h^2\omega^3}\left(\frac{\tilde{\mathbf{E}}_{\text{pump}}\cdot\mathbf{B}}{B}\right)^2 B$$

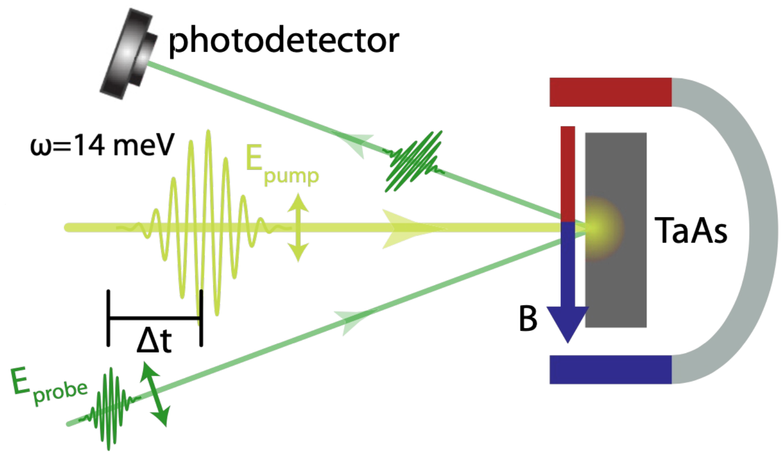


- The reflection coefficient

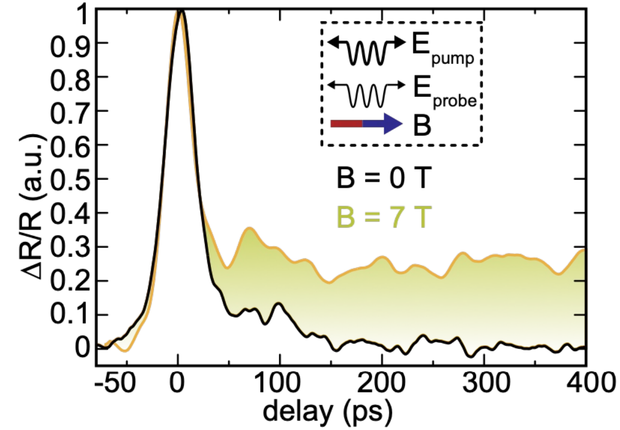
$$R(T) = \left|\frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}}\right|^2 \quad \text{where} \quad \epsilon = \epsilon_\infty + i\frac{\sigma}{\omega\epsilon_0}$$

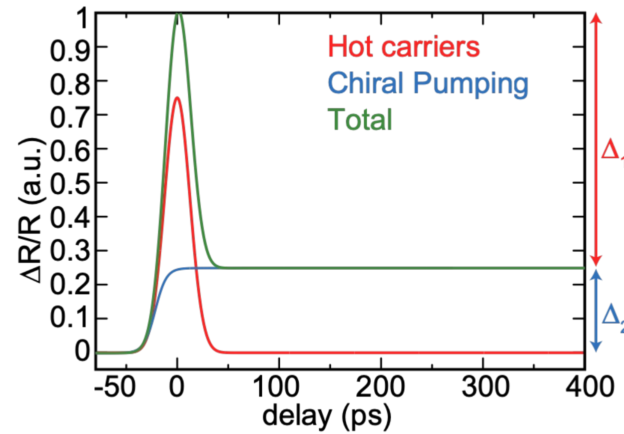
Chiral charge pumping (data)

- Experimental setup



- Measurements:

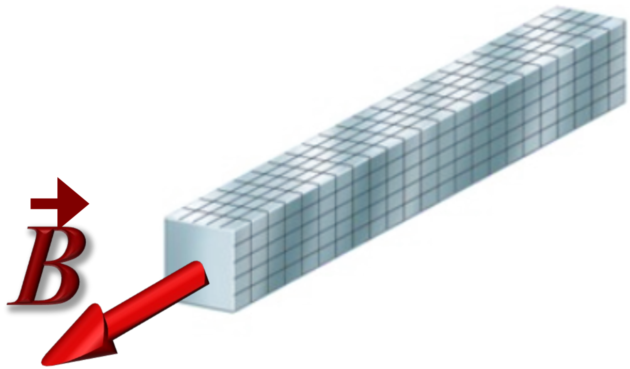


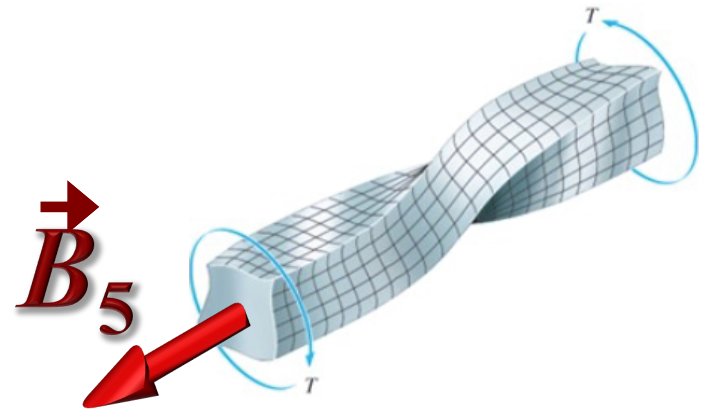


- Chiral charge relaxation time

$$1\ \text{ns} \ll \tau_{\text{ch}} < 77\ \text{ns}$$

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]





CHIRAL STRAINTRONICS

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]
[Grushin, Venderbos,Vishwanath, Ilan, Phys. Rev. X **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B **94**, 241405 (2016)]
[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Pseudo-electromagnetic fields

- **Strains** modify the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

via the emergent **chiral gauge fields** are

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

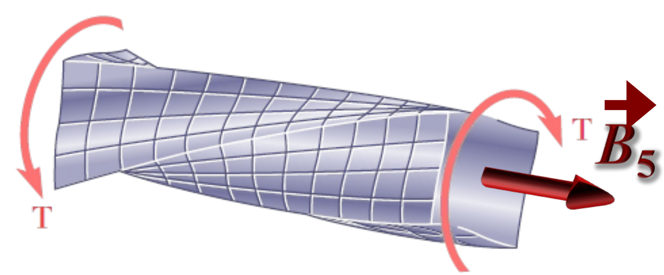
$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
[Grushin, Venderbos,Vishwanath, Ilan, PRX **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]

leading to the **pseudo-EM** fields

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$



Continuity relations (with $\vec{B}_5$ & $\vec{E}_5$)

- Naïve continuity relations from chiral kinetic theory:

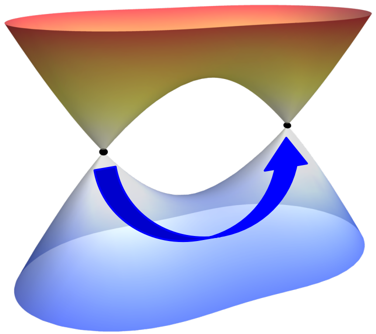
$$\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B}) + (\mathbf{E}_5\cdot\mathbf{B}_5)\Big] \quad ✔$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B}_5) + (\mathbf{E}_5\cdot\mathbf{B})\Big] \quad ✘$$

- Extra Bardeen-Zumino (Chern-Simons) terms are needed, i.e.

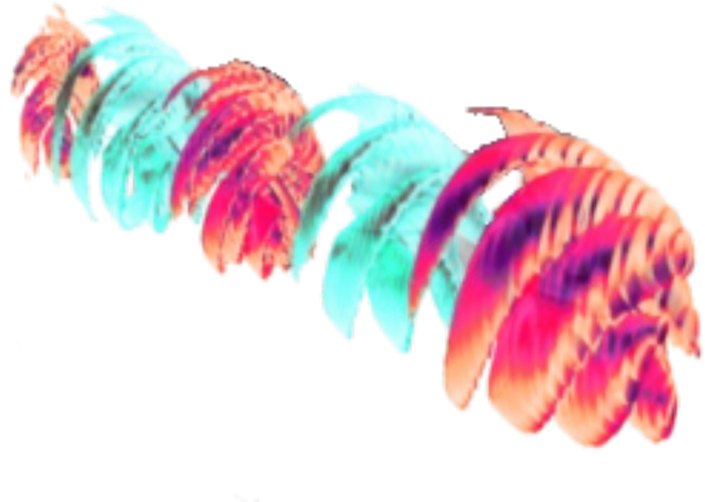
$$\delta\rho = -\frac{e^3}{2\pi^2\hbar^2 c^2}\left((\vec{b}+\vec{A}_5)\cdot\vec{B}\right)$$

$$\delta\vec{j} = -\frac{e^3}{2\pi^2\hbar^2 c^2}\left(b_0 + A_{5,0}\right)\vec{B} + \frac{e^3}{2\pi^2\hbar^2 c^2}\left[(\vec{b}+\vec{A}_5)\times\vec{E}\right]$$



  - Electric charge is conserved ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - No CME in equilibrium ($\mu_5 = -eb_0$)

[Landsteiner, PRB **89**, 075124 (2014)]
[Landsteiner, Acta Phys. Polon. B **47**, 2617 (2016)]
[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **96**, 085130 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]



UNUSUAL COLLECTIVE MODES

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **97**, 121105(R) (2018)]

Strain-induced helicons ($B = 0$)

- Strain-induced pseudo-magnetic field $B_{0,5}$ leads to

$$\omega_h\big|_{B_0\to 0,\mu\to 0} \overset{b_0\to -\mu_5/e}{=} \frac{eB_{0,5}c^3\hbar^2\pi v_F^2 k^2}{\pi\hbar^2 c^2\Omega_e^2\mu_5 + 2B_{0,5}e^4 v_F^2 b_\parallel} + O(k^3)$$

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

- Properties:
  - Gapless electromagnetic wave propagates in metals **without magnetic field**!
  - Chiral shift modifies effective helicon dispersion
  - In equilibrium, i.e., $\mu_5 = -eb_0$, the term linear in the wave vector is **absent**

Summary

- Chiral anomaly can have macroscopic implications in relativistic plasmas
- (Dipole) chiral magnetic effect can be seen via charged particle correlations in heavy-ion collisions
- Latest isobar measurements are promising but inconclusive (more studies are underway)
- Chiral anomaly can be realized and tested in Dirac/Weyl semimetals
- Chiral charge, which is relatively long-lived, can be optically pumped and manipulated (promising new technologies)