Scalar boson emission from Magnetized relativistic plasma

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Jorge Jaber-Urquiza and I. S., arXiv:2310.00050

Seminar of Latin American network on electromagnetic effects in strongly interacting matter

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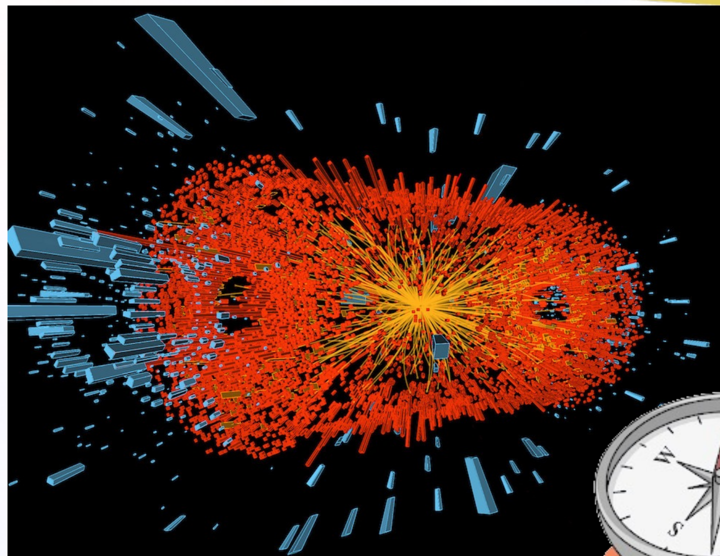
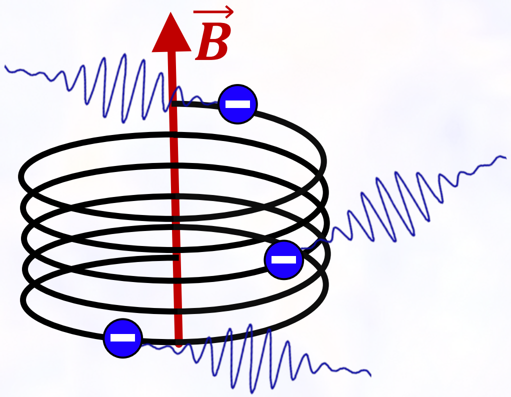


MOTIVATION

Magnetized plasma

- **Magnetized relativistic plasmas**
  - Cosmology
  - Astrophysics
  - Heavy-ion collisions
  - Topological semimetals

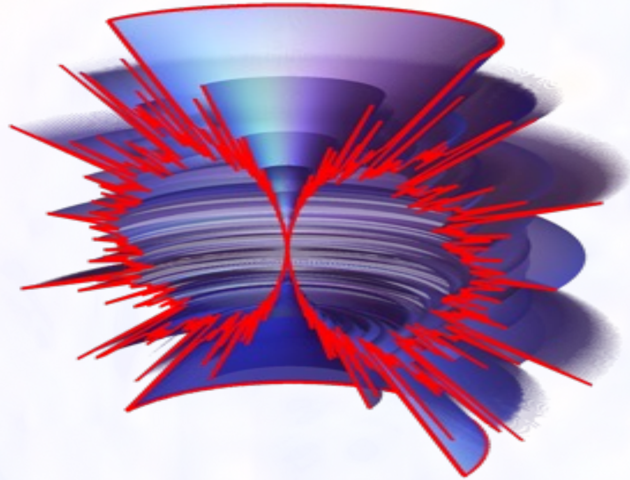


- **Photon & dilepton emission from magnetized QGP**
  - High anisotropy
  - Rate enhancement

[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020), arXiv:2006.16254]
[Wang, Shovkovy, Phys. Rev. D 104, 056017 (2021), arXiv:2103.01967]
[Wang, Shovkovy, Phys. Rev. D 106, 036014 (2022), arXiv:2205.00276]
[Wang, Shovkovy, arXiv:2307.07557]

- **Production of neutral scalar bosons as theoretical exercise**
  - offers insights into spin effects
  - may describe to sigma-boson production
  - can be useful in cosmology (inflation) and astrophysics (dark matter)

Scalar boson emission profile for $|qB| = 4m^2$,
$T = 15m$, $M = m/3$, $\Omega = 1.5m$ [arXiv:2310.00050]



MODEL

Model

- The model Lagrangian density

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi + \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - \frac{1}{2}M^{2}\phi^{2} - g\phi\bar{\psi}\psi$$

- The scalar bosons (ϕ) are neutral
- The fermions (ψ) carry an electric charge q
- The covariant derivative is

$$D^{\mu} = \partial^{\mu} + iqA^{\mu}(x)$$

and the background gauge field is

$$A^{\mu}(x) = -yB\delta_{1}^{\mu}$$

Scalar boson production rate

- The differential production rate is

$$\frac{d^3R}{d^3k} = -\frac{n_B(\Omega)}{(2\pi)^3\Omega}\mathrm{Im}\left[\Sigma^R(\Omega,\mathbf{k})\right]$$

where $\Sigma^R(\Omega,\mathbf{k})$ is the self-energy of the scalar field

$n_B(\Omega) = \left[e^{\Omega/T}+1\right]^{-1}$ is the Bose-Einstein distribution

- Note that $\mathrm{Im}[\Sigma^R(\Omega,\mathbf{k})]$ determines the boson decay rate

$$\Gamma = -\frac{\mathrm{Im}[\Sigma^R(\Omega,\mathbf{k})]}{\Omega}$$

where the mass-shell condition for the bosons is $\Omega = \sqrt{M^2+\mathbf{k}^2}$

Fermion propagator

- Fermion propagator has the structure

$$S(x,y) = \exp\left(-iq\int_y^x A_\mu(x)dx^\mu\right)\bar{S}(x-y)$$

- The translation invariant part is

$$\bar{S}(x) = \int \frac{d^2 p_\parallel}{(2\pi)^2}\tilde{S}(p_\parallel;\mathbf{r}_\perp)e^{-ip_\parallel\cdot x_\parallel}$$

- We use a mixed representation:

$$\tilde{S}(p_\parallel;\mathbf{r}_\perp) = i\frac{e^{-\xi/2}}{2\pi\ell^2}\sum_{n=0}^{\infty}\frac{\tilde{D}\left(p_\parallel;\mathbf{r}_\perp\right)}{p_\parallel^2 - m^2 - 2n|qB|}$$

where

$$\tilde{D}\left(p_\parallel;\mathbf{r}_\perp\right) \equiv \left(\not{p}_\parallel + m\right)\left[\mathcal{P}_+ L_n\left(\xi\right) + \mathcal{P}_- L_{n-1}\left(\xi\right)\right] - i\frac{\not{\mathbf{r}}_\perp}{\ell^2}L_{n-1}^1\left(\xi\right)$$

Scalar self-energy

- The self-energy of the scalar field at leading order in coupling

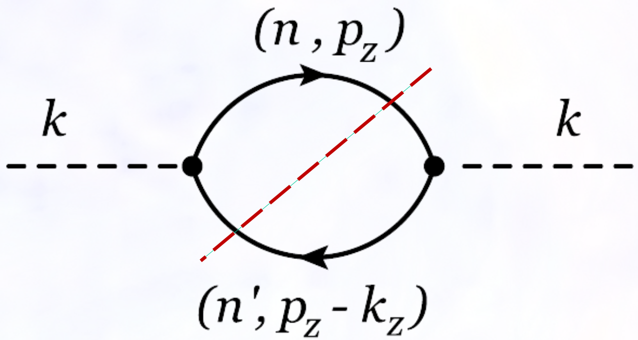
$$\Sigma(x-y) = ig^2 \mathrm{Tr}\left[\bar{S}(x-y)\bar{S}(y-x)\right]$$

- Thus, in momentum space

$$\Sigma(k) = ig^2 \int \frac{d^2 p_{\|}}{(2\pi)^2} \int d^2\mathbf{r}_\perp e^{-i\mathbf{r}_\perp\cdot\mathbf{k}_\perp} \mathrm{Tr}\left[\tilde{S}(p_{\|};\mathbf{r}_\perp)\tilde{S}(p_{\|}-k_{\|};-\mathbf{r}_\perp)\right]$$

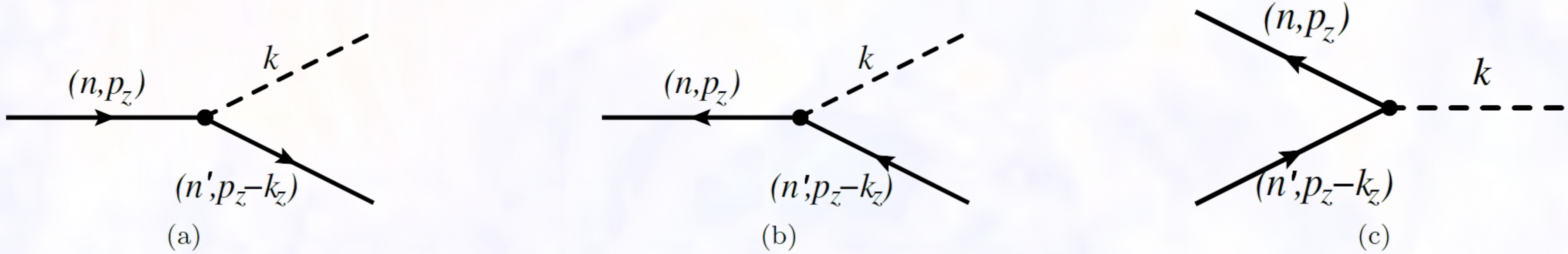
- Diagrammatic representation:

- Imaginary part:



Imaginary part of self-energy

- Three types of processes contributing:



- The energy conservation: $E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta\Omega = 0$
- Constraints:

$$\psi \to \psi + \phi : \qquad \sqrt{\Omega^2 - k_z^2} \le k_- \quad \text{and} \quad n > n',$$
$$\bar{\psi} \to \bar{\psi} + \phi : \qquad \sqrt{\Omega^2 - k_z^2} \le k_- \quad \text{and} \quad n < n',$$
$$\psi + \bar{\psi} \to \phi : \qquad \sqrt{\Omega^2 - k_z^2} \ge k_+,$$

where $k_\pm \equiv \left| \sqrt{m^2 + 2n|qB|} \pm \sqrt{m^2 + 2n'|qB|} \right|$

Imaginary part of self-energy

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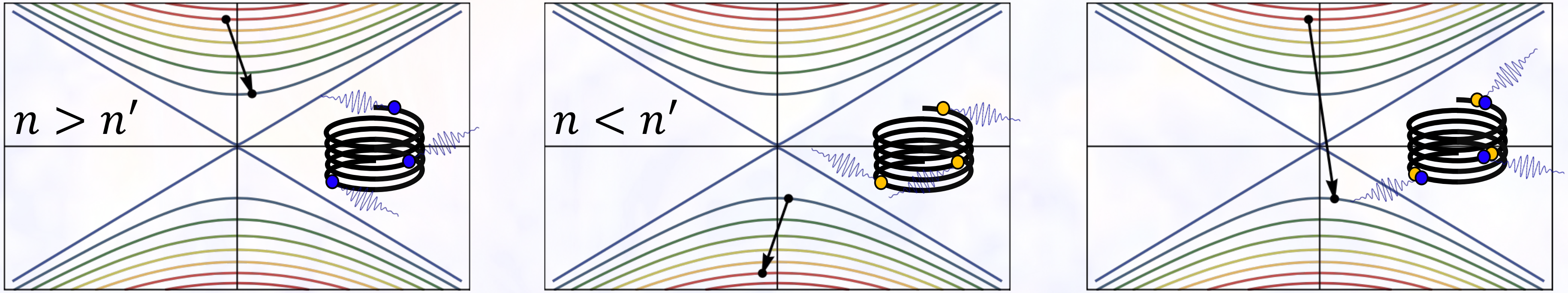

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where $k_\pm \equiv \left| \sqrt{m^2 + 2n|qB|} \pm \sqrt{m^2 + 2n'|qB|} \right|$

Analytical result for $\mathrm{Im}[\Sigma^R(\Omega,\mathbf{k})]$

- Final expression:

[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

$$
\begin{aligned}
\mathrm{Im}\left[\Sigma^R(\Omega,\mathbf{k})\right] = {} & \frac{g^2}{2\pi\ell^2}\sum_{n>n'}^{\infty}\frac{\theta\left(\Omega^2-k_z^2-k_+^2\right)-\theta\left(k_-^2+k_z^2-\Omega^2\right)}{\sqrt{\left(\Omega^2-k_z^2-k_-^2\right)\left(\Omega^2-k_z^2-k_+^2\right)}}h\left(n,n'\right) \\
& \times\left[\left(\left(n+n'\right)|qB|-\frac{1}{2}\left(\Omega^2-k_z^2\right)+2m^2\right)\left(\mathcal{I}_0^{n,n'}(\xi)+\mathcal{I}_0^{n-1,n'-1}(\xi)\right)-\frac{2}{\ell^2}\mathcal{I}_2^{n-1,n'-1}(\xi)\right] \\
& +\frac{g^2}{4\pi\ell^2}\sum_{n=0}^{\infty}\frac{\theta\left(\Omega^2-k_z^2-4m^2-8n|qB|\right)}{\sqrt{\left(\Omega^2-k_z^2\right)\left(\Omega^2-k_z^2-4m^2-8n|qB|\right)}}h_0\left(n\right) \\
& \times\left[\left(2n|qB|-\frac{1}{2}\left(\Omega^2-k_z^2\right)+2m^2\right)\left(\mathcal{I}_0^{n,n}(\xi)+\mathcal{I}_0^{n-1,n-1}(\xi)\right)-\frac{2}{\ell^2}\mathcal{I}_2^{n-1,n-1}(\xi)\right]
\end{aligned}
$$

where $h(n,n')$ and $h_0(n)$ are built from Fermi-Dirac distribution functions

- The result resembles but is *different from* the imaginary part of the photon polarization function

$$\mathrm{Im}\left[\Pi^{\mu}_{\mu}(\Omega,\mathbf{k})\right]$$

[Wang, Shovkovy, Yu, Huang, arXiv:2006.16254]
[Wang, Shovkovy, arXiv:2103.01967]

Special limit: $\mathbf{k}_\perp = 0$

- The scalar boson's momentum parallel to the $\vec{B}$-field

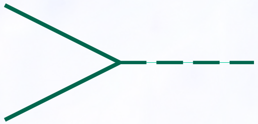
$$\mathrm{Im}\left[\Sigma^R(\Omega, \mathbf{0}, k_z)\right] = -\frac{g^2}{8\pi\ell^2}\frac{\left(\Omega^2 - k_z^2 - 4m^2\right)}{\sqrt{\Omega^2 - k_z^2}}\sum_{n=0}^{\infty}\alpha_n\frac{\theta\left(\Omega^2 - k_z^2 - 4m^2 - 8n|qB|\right)}{\sqrt{\Omega^2 - k_z^2 - 4m^2 - 8n|qB|}}h_0(n)$$

where $\alpha_n = 2 - \delta_0^n$

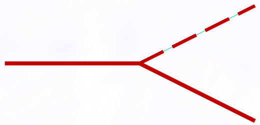
[Bandyopadhyay, Farias, Ramos, arXiv:1807.06515]

- This expression is much **simpler** than the general result

- Kinematics at $\mathbf{k}_\perp = 0$ is extremely **restrictive**
  - The boson production is allowed only when $\Omega > 2m$
  - Annihilation is the only process contributing ✓



  - Particle splitting processes are forbidden ✗



Zero field limit (**B**=0)

- We reproduce the B=0 limit from the expression at $\mathbf{k}_\perp = 0$

- $B \to 0$: the sum over n sum becomes an integral over $v = 2n|qB|$,

$$\mathrm{Im}\left[\Sigma^R(\Omega,\mathbf{k})\right] = -\frac{g^2}{8\pi}\frac{\left(\Omega^2-|\mathbf{k}|^2-4m^2\right)}{\sqrt{\Omega^2-|\mathbf{k}|^2}}\theta\left(\Omega^2-|\mathbf{k}|^2-4m^2\right)\int_0^{v_0}\frac{dv}{\sqrt{v_0-v}}\left[1-\sum_{s_2=\pm}n_F\left(\frac{\Omega}{2}+s_2\frac{|\mathbf{k}|\sqrt{v_0-v}}{\sqrt{\Omega^2-|\mathbf{k}|^2}}\right)\right]$$

where we replaced $|k_z| \to |\mathbf{k}|$ (Lorentz symmetry is restored at B=0!)

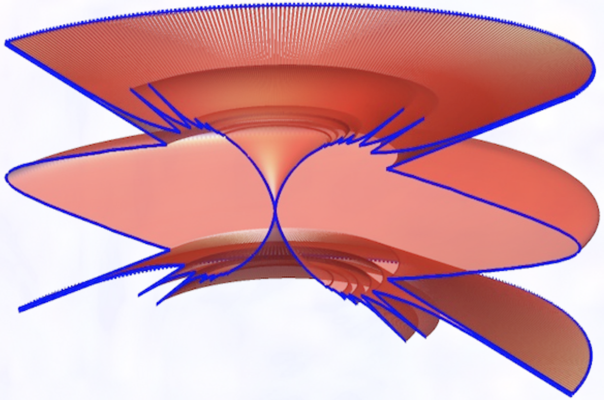
- After the integration, one derives

$$\mathrm{Im}\left[\Sigma^R(\Omega,\mathbf{k})\right] = -\frac{g^2}{8\pi}\left(\Omega^2-|\mathbf{k}|^2-4m^2\right)\left[\frac{\sqrt{\Omega^2-|\mathbf{k}|^2-4m^2}}{\sqrt{\Omega^2-|\mathbf{k}|^2}}+\frac{2T}{|\mathbf{k}|}\ln\frac{1+e^{-E_+/T}}{1+e^{-E_-/T}}\right]\theta\left(\Omega^2-|\mathbf{k}|^2-4m^2\right)$$

which is the correct result at B=0 !

[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

Scalar boson emission profile for $|qB| = 25m^2$, $T = 5m$, $M = m/3$, $\Omega = 1.5m$ [arXiv:2310.00050]



BOSON PRODUCTION RATE

Physics regimes

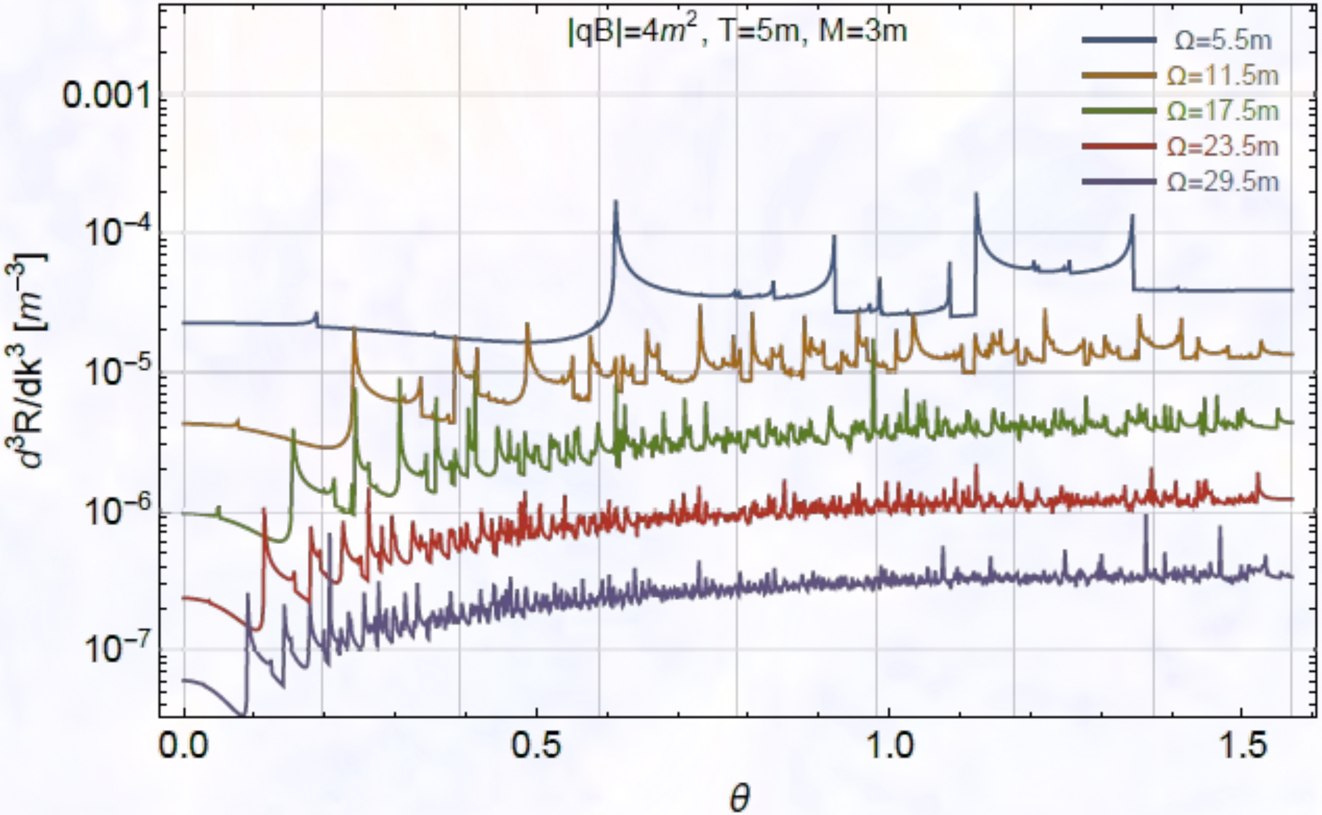
- Two very different physics regimes exist
  - Suprathreshold scalar boson mass $M > 2m$ (nonzero rate at B=0)
  - Subthreshold scalar boson mass $M < 2m$ (zero rate at B=0)
- Two temperatures
  - $T = 5m$ (relativistic plasma)
  - $T = 15m$ (ultra-relativistic plasma)
- Two magnetic field strengths
  - $|qB| = (2m)^2$ (moderately strong field)
  - $|qB| = (5m)^2$ (very strong field)
- Range of energies

$$M < \Omega \lesssim 50\, m$$

$$n_{\max} \gtrsim \left[\max\left\{\frac{T^2}{2|qB|}, \frac{\Omega^2}{2|qB|}\right\}\right]$$

Angular dependence ($M > 2m$)

- Typical data in the suprathreshold scalar boson mass



Angle θ is measured from the magnetic field direction, i.e.,

$k_z = k\cos\theta$

$k_\perp = k\sin\theta$

Nonzero rates at θ=0

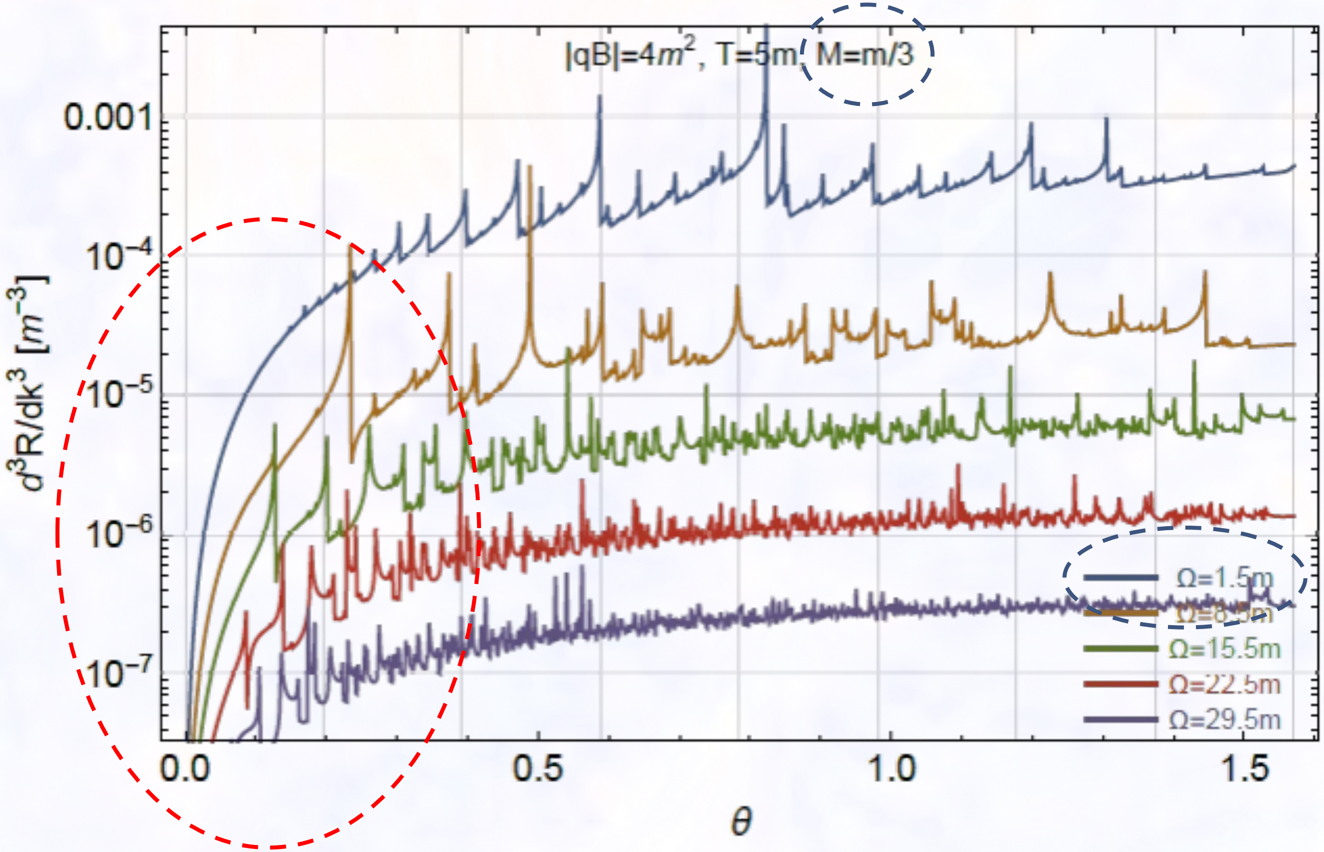
The rates grow with increasing θ

The rates decrease with increasing Ω

[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

Angular dependence ($M < 2m$)

- Typical data in the subthreshold scalar boson mass



The rates grow with increasing θ

The rates decrease with increasing Ω

The rates vanish at $\theta = 0$

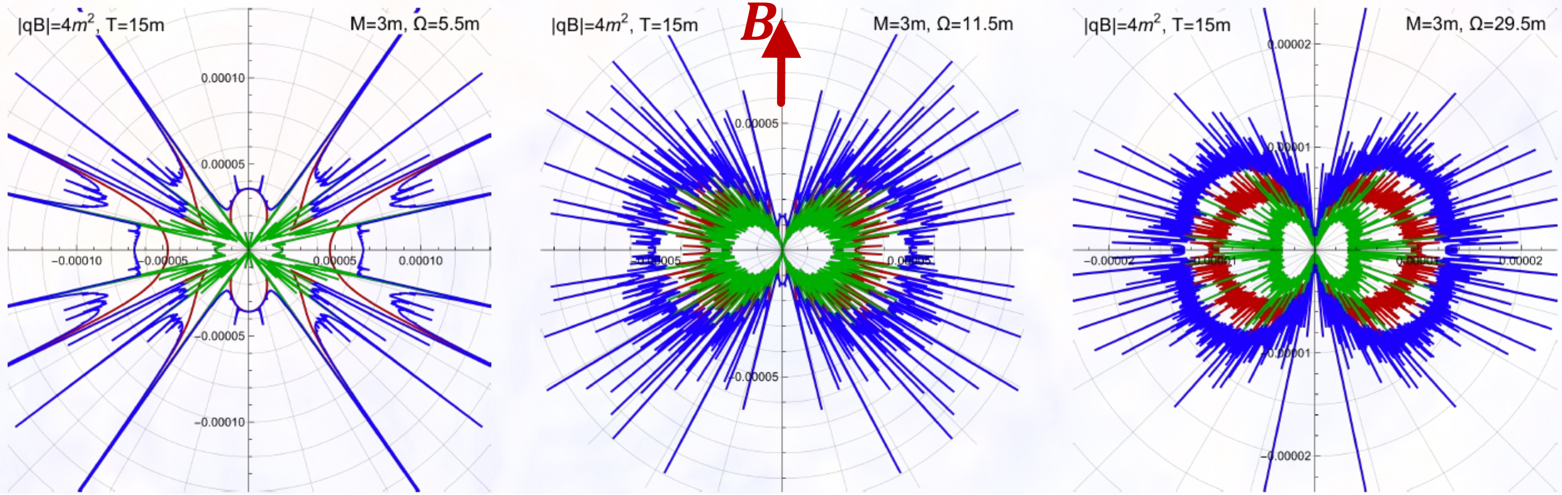
Particle-splitting processes give nonzero rate when

$$M < \Omega < 2m$$

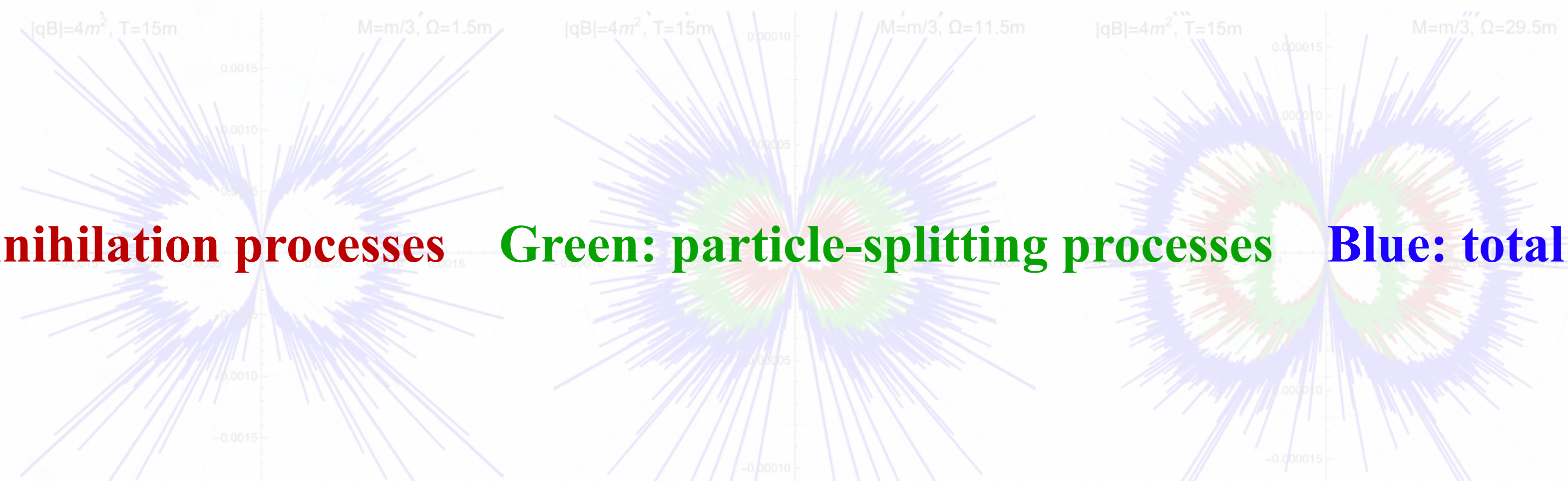
[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

Scalar boson production profiles

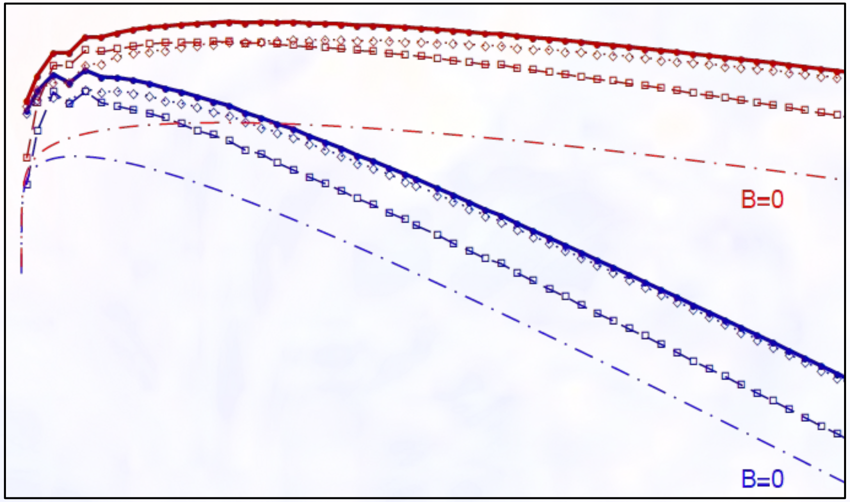
$M > 2m$:



$M < 2m$:



Red: annihilation processes **Green: particle-splitting processes** **Blue: total**



INTEGRATED RATE

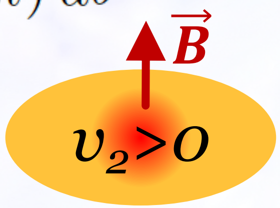
Integrated rates

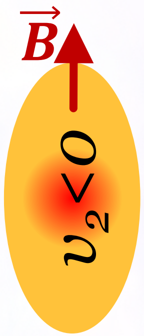
- Integrated rate

$$\frac{dR}{d\Omega} = -\int_0^\pi \frac{k\, n_B(\Omega)}{(2\pi)^2} \mathrm{Im}\left[\Sigma^R(\Omega, \mathbf{k})\right] \sin\theta d\theta$$

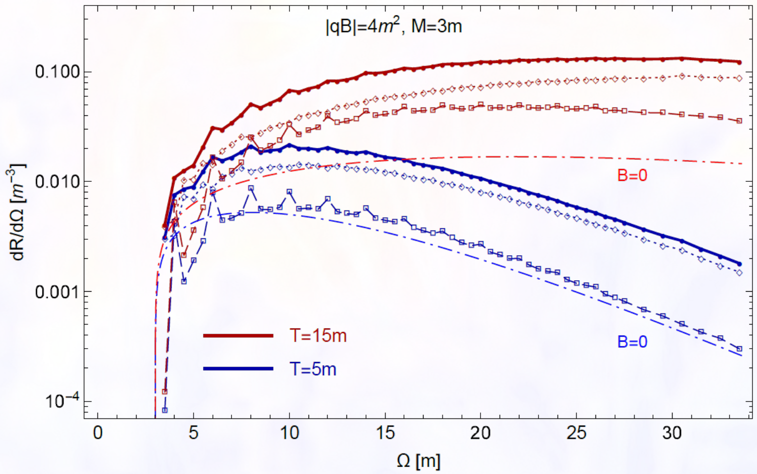
- Notice the phase factor *k* in the numerator
- The shape of production profile can be characterized by its ellipticity

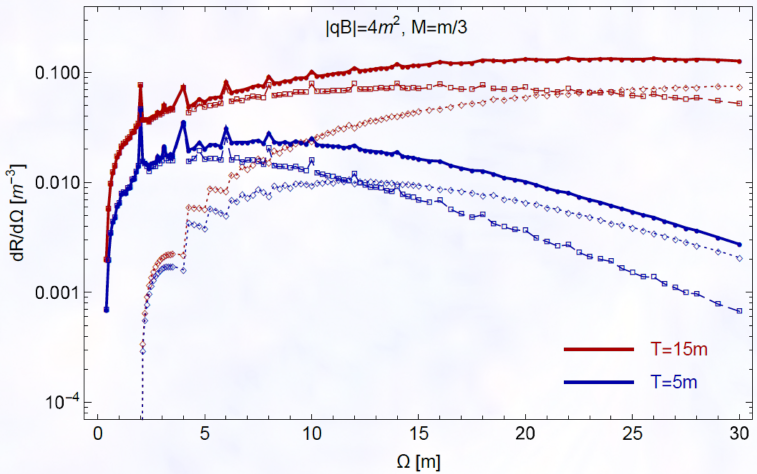
$$v_2 = -\frac{\int_0^\pi \left(d^3R/d^3k\right) \cos(2\theta) d\theta}{\int_0^\pi \left(d^3R/d^3k\right) d\theta}$$

- The sign convention for v_2: (i) 

(ii) 

Integrated rates, $|qB| = (2m)^2$





The rate grows with temperature

The peak rate is reached at some intermediate energy:

$$\Omega \sim 1.7\, T$$

(similar to black-body spectrum)

$M > 2m$: The rate is larger than the rate at $\vec{B} = 0$

$M < 2m$: The rate is nonzero at subthreshold energies:

$$M < \Omega < 2m$$

[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

Integrated rates, $|qB| = (2m)^2$

Generally, ellipticity is positive at large enough enrgies

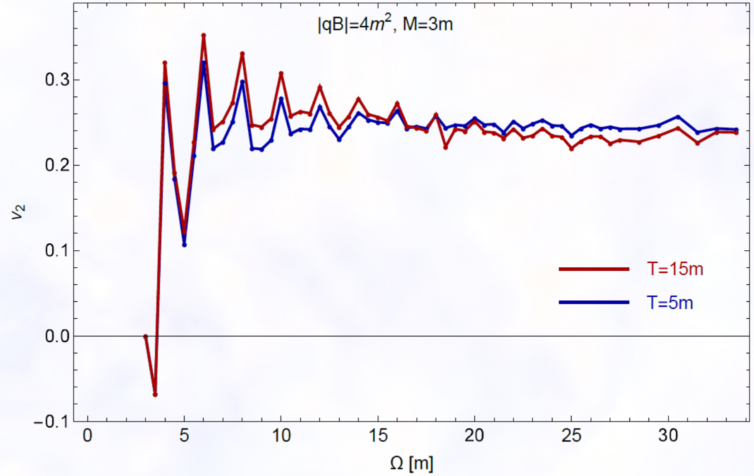
$$v_2 \sim 0.2 \text{ to } 0.3$$

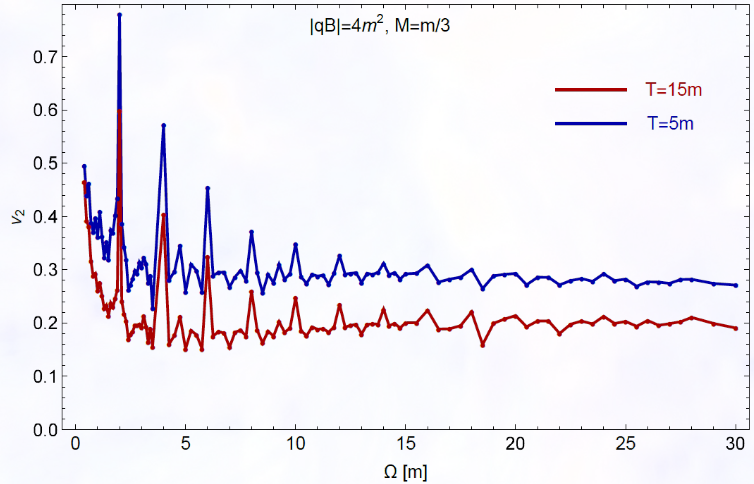
$$\boldsymbol{M > 2m}$$

- v_2 is small at small energies
- v_2 decreases with temperature

$$\boldsymbol{M < 2m}$$

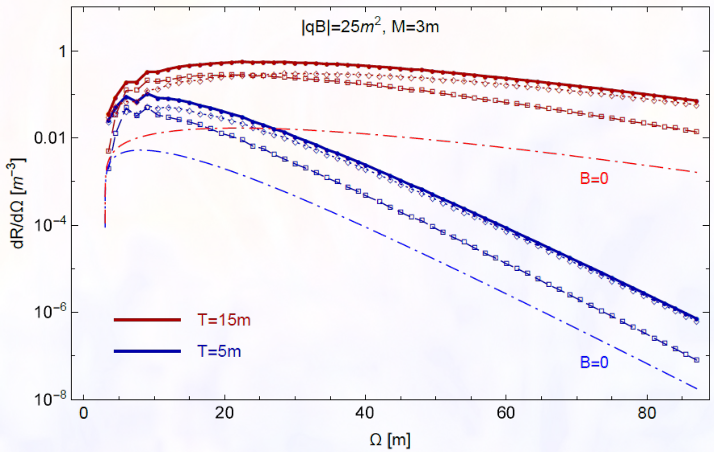
- v_2 is large at small energies
- v_2 decreases with temperature

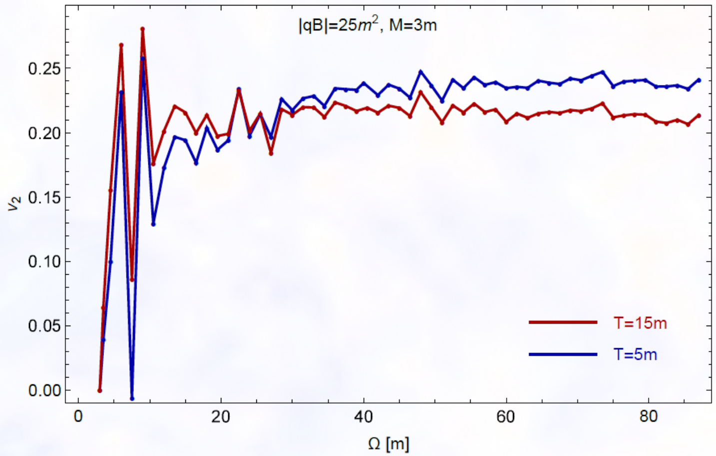


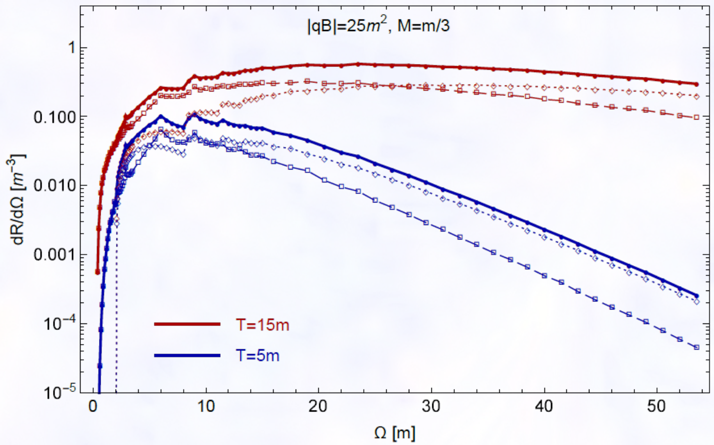


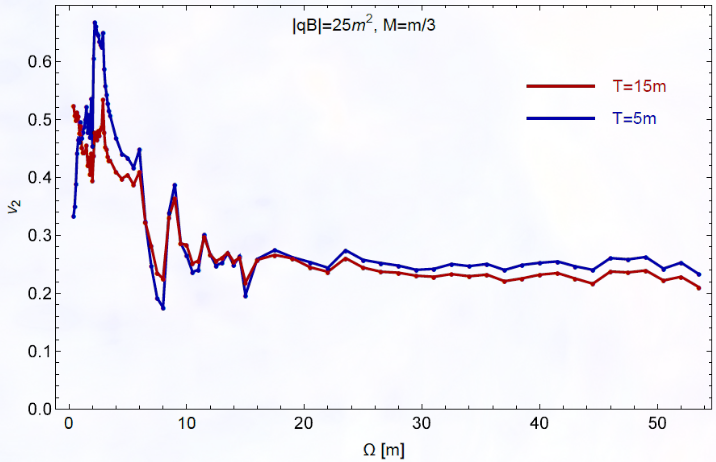
[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

Integrated rates, $|qB| = (5m)^2$



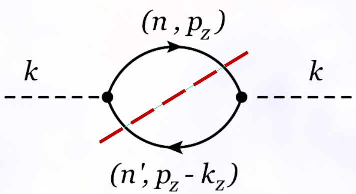






[Jorge Jaber-Urquiza and I. S., arXiv:2310.00050]

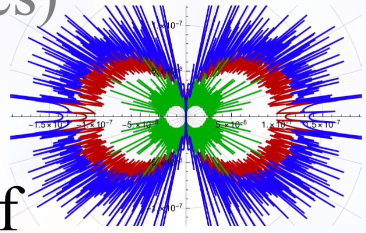
Summary (scalar bosons)



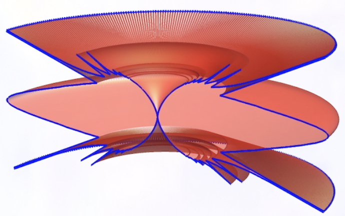
- At $\vec{B} \neq 0$, scalar bosons are produced via

  (i) $\psi \to \psi + \phi$, (ii) $\bar{\psi} \to \bar{\psi} + \phi$, (iii) $\psi + \bar{\psi} \to \phi$

- It is different from $\vec{B} = 0$ (only annihilation contributes)



- Scalar boson emission profile is anisotropic ($v_2 \gtrsim 0.2$)
- Particle-splitting does not contribute in the direction of the field ($\mathbf{k}_\perp = 0$)



- Production is allowed even for subthreshold energies $M < \Omega < 2m$ when $\vec{B} \neq 0$ (via particle-splitting)
- Magnetic field increases scalar boson production rate