College of Integrative Sciences and Arts
ARIZONA STATE UNIVERSITY

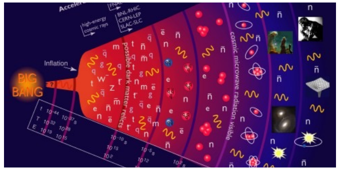
Anomalous chiral transport in nuclear physics and beyond

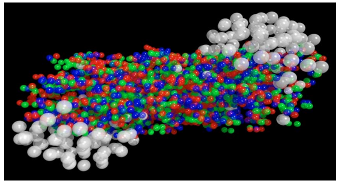
Igor Shovkovy

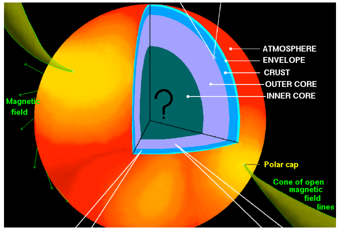
The 2023 Fall Meeting of the Division of Nuclear Physics of the American Physical Society and Physical Society of Japan

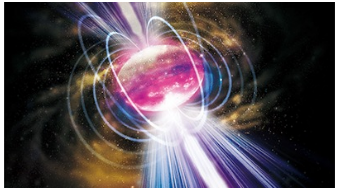
(Quasi-) Relativistic Matter

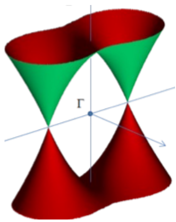
- **Heavy-ion collisions**
  (temperature $\sim$ 100 MeV)
- **Early Universe**
  (extremely high temperature $\sim$ 100 GeV)
- **Magnetospheres of magnetars**
  (electron-positron plasma at temperatures $\lesssim$ 10 MeV)
- **Super-dense matter in compact stars**
  (high densities $\lesssim 10^{17}$ kg/m^3)
- **Electron plasma in Dirac/Weyl (semi-)metals**
  (chiral quasiparticle plasma at temperatures $\lesssim$ 10 meV)
- **Other: cold atoms, superfluid ^{3}He-A, etc.**
  (chiral quasiparticles at temperatures $\sim 10^{-3}$ K)

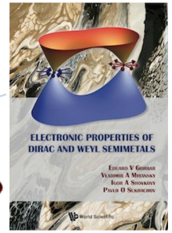


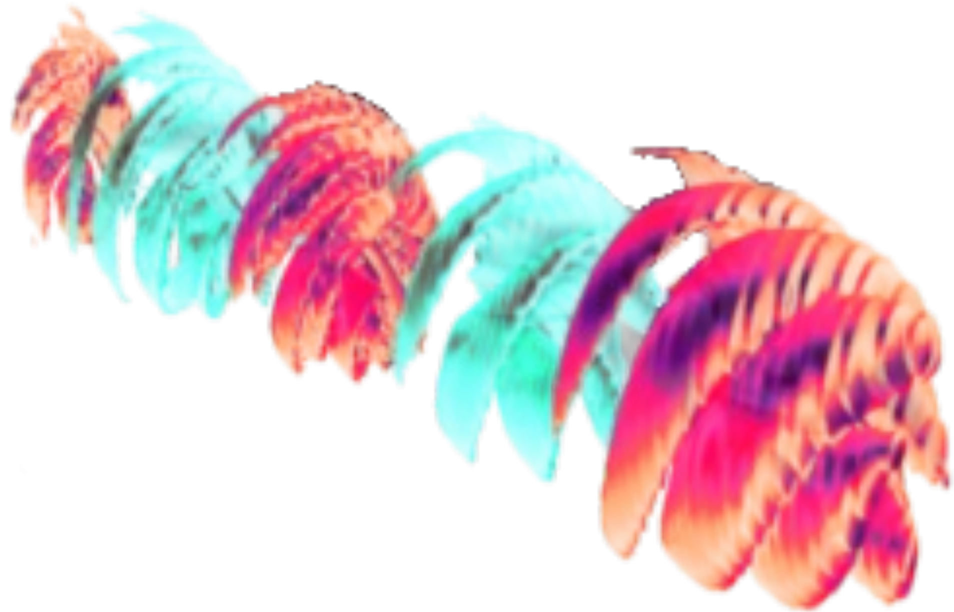










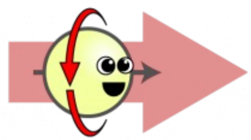


ANOMALOUS MATTER

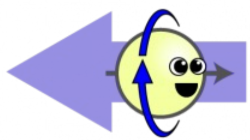
[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Becattini, Liao, Lisa, Lect. Notes Phys. **987**, 1 (2021)]

Chirality

- Only *massless* Dirac/Weyl fermions have a well-defined chirality ($\gamma^5\psi = \pm\psi$)*:



Right-handed (spin parallel to momentum)



Left-handed (spin opposite to momentum)

- The chirality of *massive* Dirac fermions is *almost* well-defined in the *ultra-relativistic* regime*
  - High temperature: $T \gg m$
  - High density: $\mu \gg m$

*Note: like the particle spin, chirality is a quantum property

Anomalous chiral matter

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ existing on *macroscopic* time/distance scales

- The spacetime dynamics of $n = n_R + n_L$ and $n_5 = n_R - n_L$ is governed by continuity equations

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

chiral anomaly

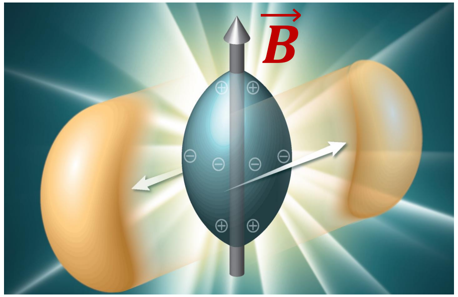
$$\frac{\partial n_5}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \boxed{\frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2}} - \mathit{\Gamma}_{\mathrm{m}} n_5$$

where the chirality flip rate: $\mathit{\Gamma}_{\mathrm{m}} \propto \alpha^2 T (m/T)^2$

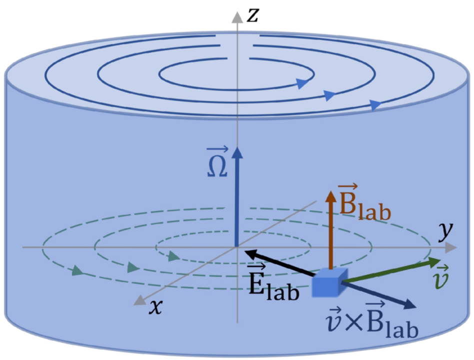
- Chiral anomaly can produce *macroscopic* effects in plasmas

Anomalous effects

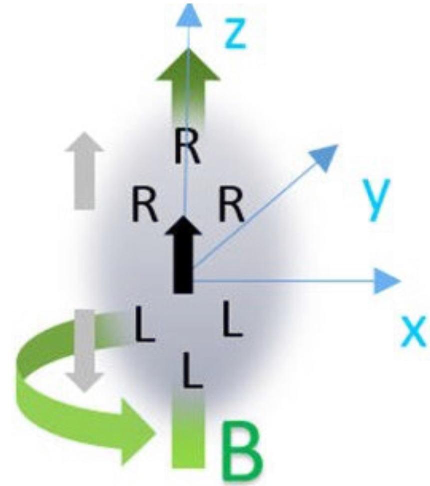
- **Theory**: Many *macroscopic* chiral anomalous effects were proposed
- Some are triggered by an external magnetic field
  - Chiral magnetic effect
  - Chiral separation effect
  - Chiral magnetic wave
  - Negative magnetoresistance
  - ...



- Others are triggered by vorticity
  - Chiral vortical effect
  - Chiral vortical wave
  - …



Review: [Becattini, Liao, Lisa, Lect. Notes Phys. **987**, 1 (2021)]



CHIRAL ANOMALOUS EFFECT

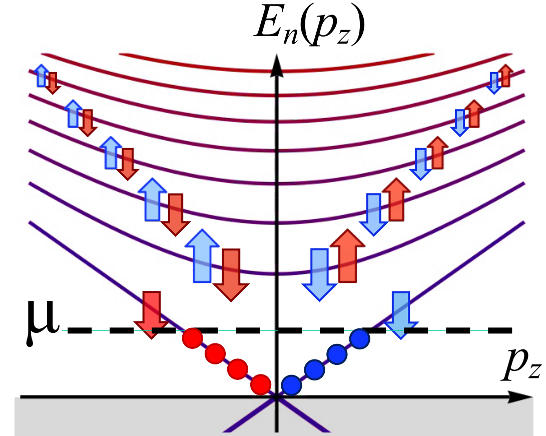
$$\langle \vec{J}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu \quad \& \quad \langle \vec{J} \rangle = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

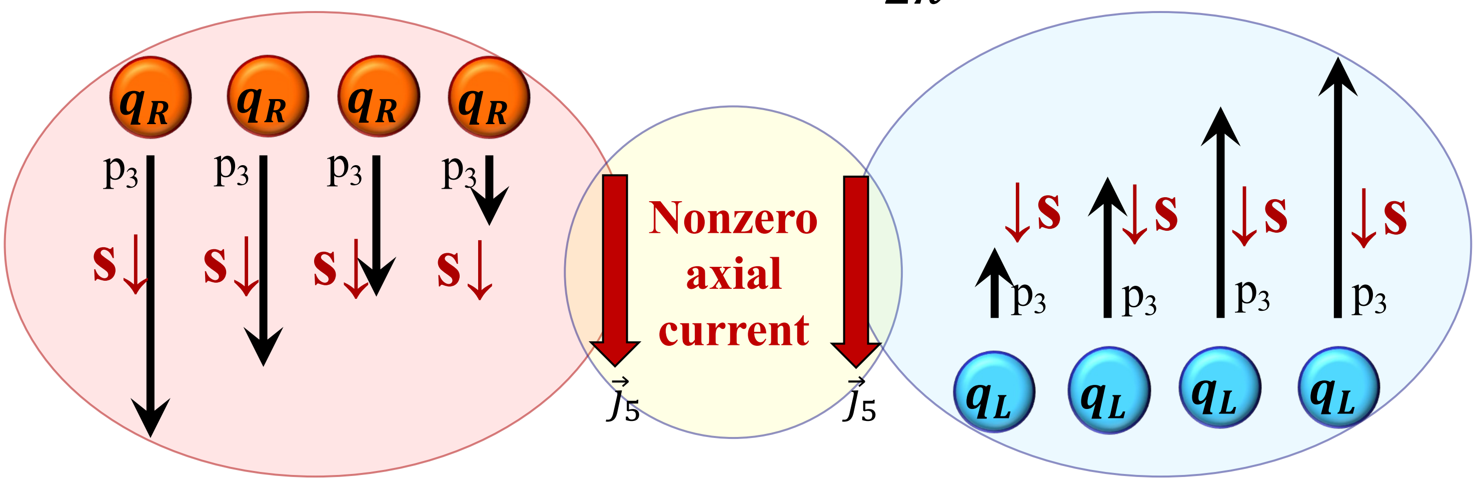
Chiral Separation Effect ($\mu \neq 0$)

- **Spin polarized** LLL is chirally asymmetric
  - – states with $p_3<0$ (and s=↓) are R-handed
  - – states with $p_3>0$ (and s=↓) are L-handed

  i.e., a nonzero **chiral** current is induced



$$\langle \vec{j}_5 \rangle = -tr[\vec{\gamma}\gamma^5 S(x,x)] = -\frac{e\vec{B}}{2\pi^2}\mu$$

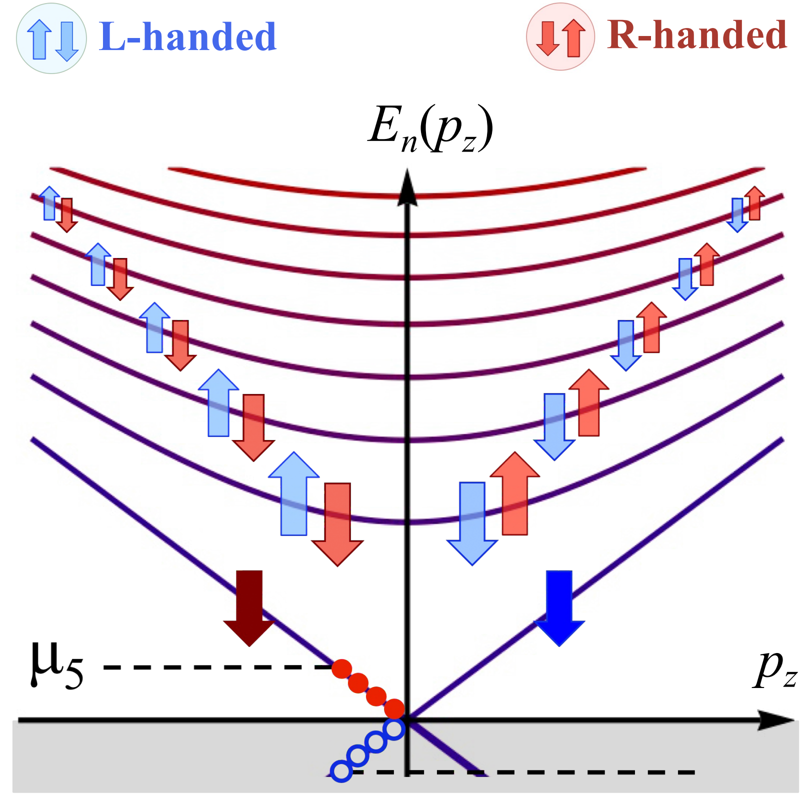


Chiral Magnetic Effect ($\mu_5 \neq 0$)

Assume a *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

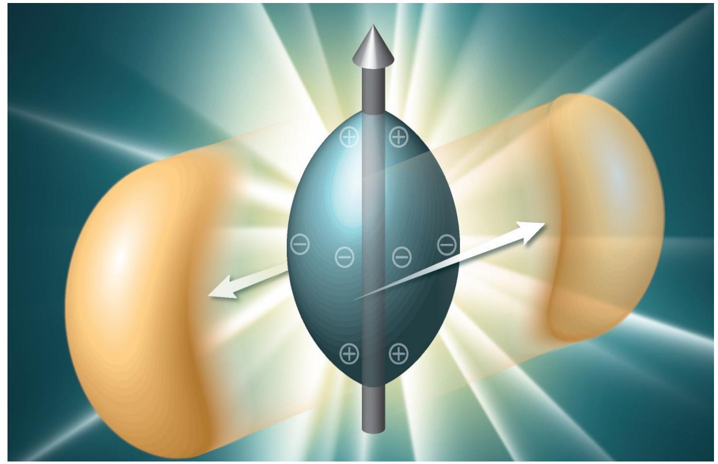
Spin polarized LLL (s=↓ for particles of a *negative* charge):

- Some R-handed states ($p_3 < 0$ & $E < \mu_5$) are occupied
- Some L-handed states ($p_3 < 0$ & $|E| < \mu_5$) are empty (i.e., holes with $p_3 > 0$)



CME current: $$\langle \vec{j} \rangle = -tr[\vec{\gamma} S(x,x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

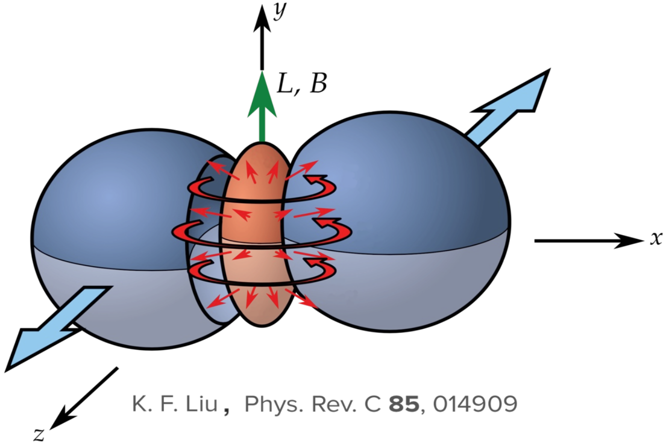
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]



HEAVY-ION COLLISIONS

$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields are induced by the currents of passing charged ions



K. F. Liu, Phys. Rev. C **85**, 014909

[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108], …

- Vorticity estimate: [Adamczyk et al. (STAR), Nature 548, 62 (2017)]

$$\omega \sim 9 \times 10^{21} s^{-1} \ (\sim 10\ \mathrm{MeV})$$

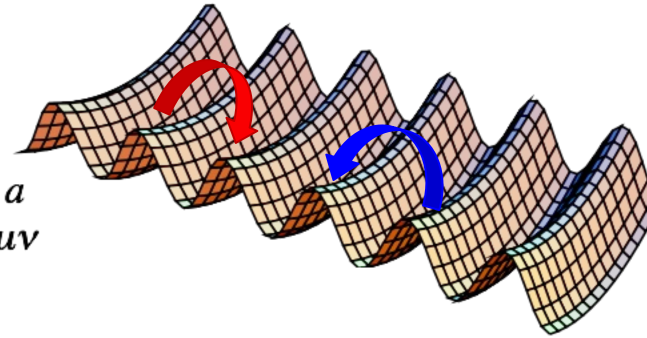
- Magnetic field estimate:

$$B \sim 10^{18} \text{ to } 10^{19}\ \mathrm{G}\ (\sim 100\ \mathrm{MeV})$$

Source of chirality in QCD

- Chiral charge can be produced by topological configurations in QCD

$$\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2}\int d^3x\, F_a^{\mu\nu}\tilde{F}^a_{\mu\nu}$$

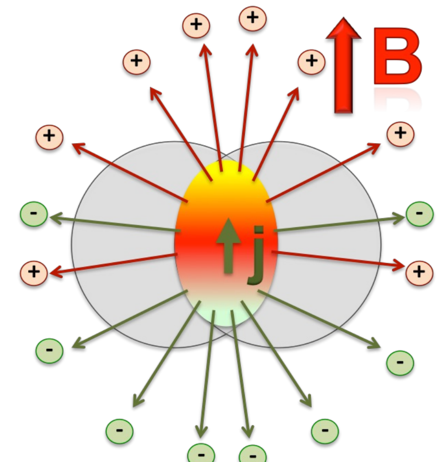


- A random fluctuation with nonzero chirality should produce

$$N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0$$

- The latter leads to an electric CME current

$$\langle \vec{j} \rangle = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

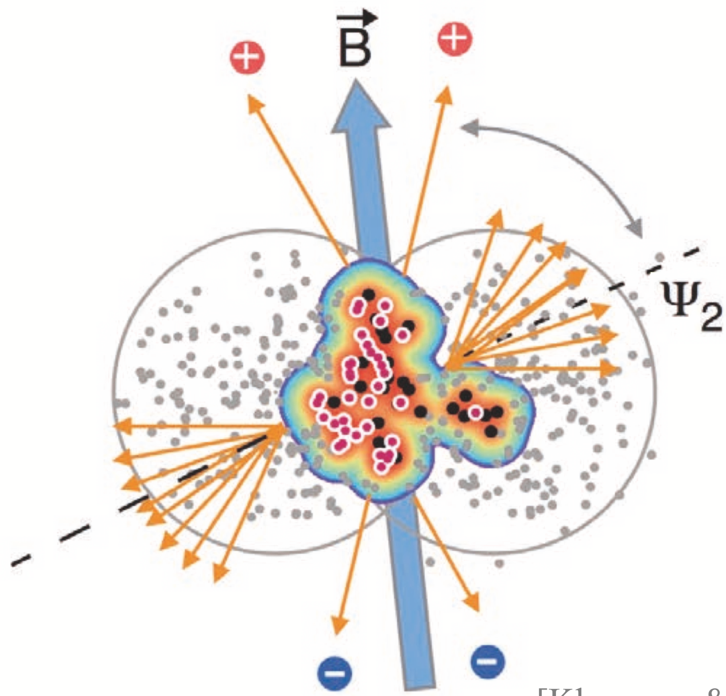


Dipole CME

- Dipole pattern of *charged particle correlations* in heavy-ion collisions

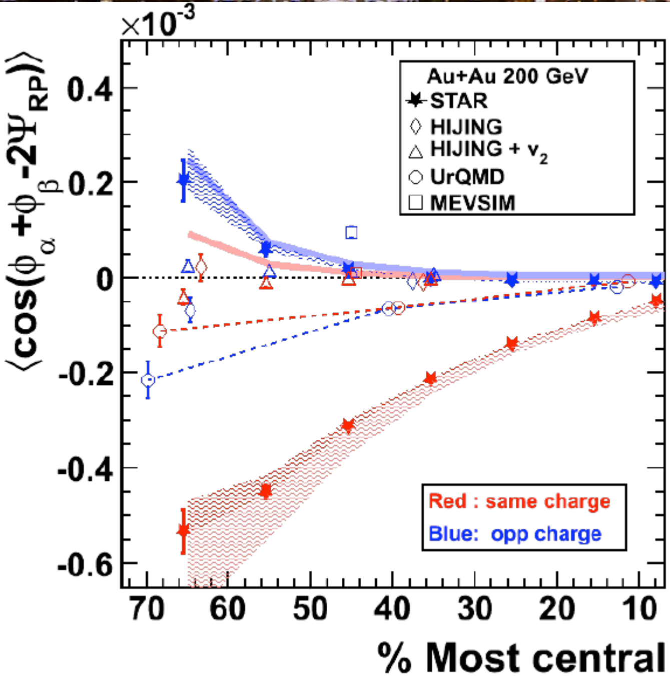
$$\langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\mp} - 2\Psi_{RP}) \rangle > 0 \quad \& \quad \langle \cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\pm} - 2\Psi_{RP}) \rangle < 0$$

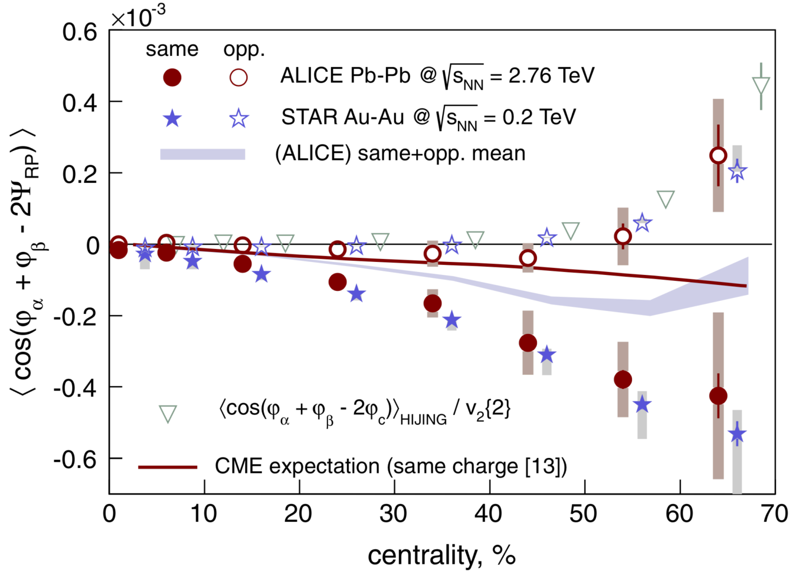
[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]
[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]



[Kharzeev & Liao, Nucl. Phys. News **29**, 1 (2019)]

CME: Experimental evidence





Correlations of same & opposite charge particles:
$$\begin{cases} \langle\cos(\varphi_\alpha^{\pm}+\varphi_\beta^{\mp}-2\Psi_{RP})\rangle > 0 \\ \langle\cos(\varphi_\alpha^{\pm}+\varphi_\beta^{\pm}-2\Psi_{RP})\rangle < 0 \end{cases}$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]
[Adamczyk et al. (STAR), PRL **113**, 052302 (2014)]
[Khachatryan et al. (CMS), PRL **118**, 122301 (2017)]

Large background effects!

[Belmont & Nagle, PRC **96**, 024901 (2017)]
[ALICE Collaboration, Phys. Lett. B**777**, 151 (2018)]

Isobar collisions

Utilize collisions of isobars, e.g.,

[Voloshin, PRL **105**, 172301 (2010)]
[Deng et al. PRC **94**, 041901(R) (2016)]

$$ {}^{96}_{40}\mathrm{Zr} + {}^{96}_{40}\mathrm{Zr} \text{ vs. } {}^{96}_{44}\mathrm{Ru} + {}^{96}_{44}\mathrm{Ru} $$

$$ {}^{130}_{52}\mathrm{Te} + {}^{130}_{52}\mathrm{Te} \text{ vs. } {}^{130}_{56}\mathrm{Ba} + {}^{130}_{56}\mathrm{Ba} $$

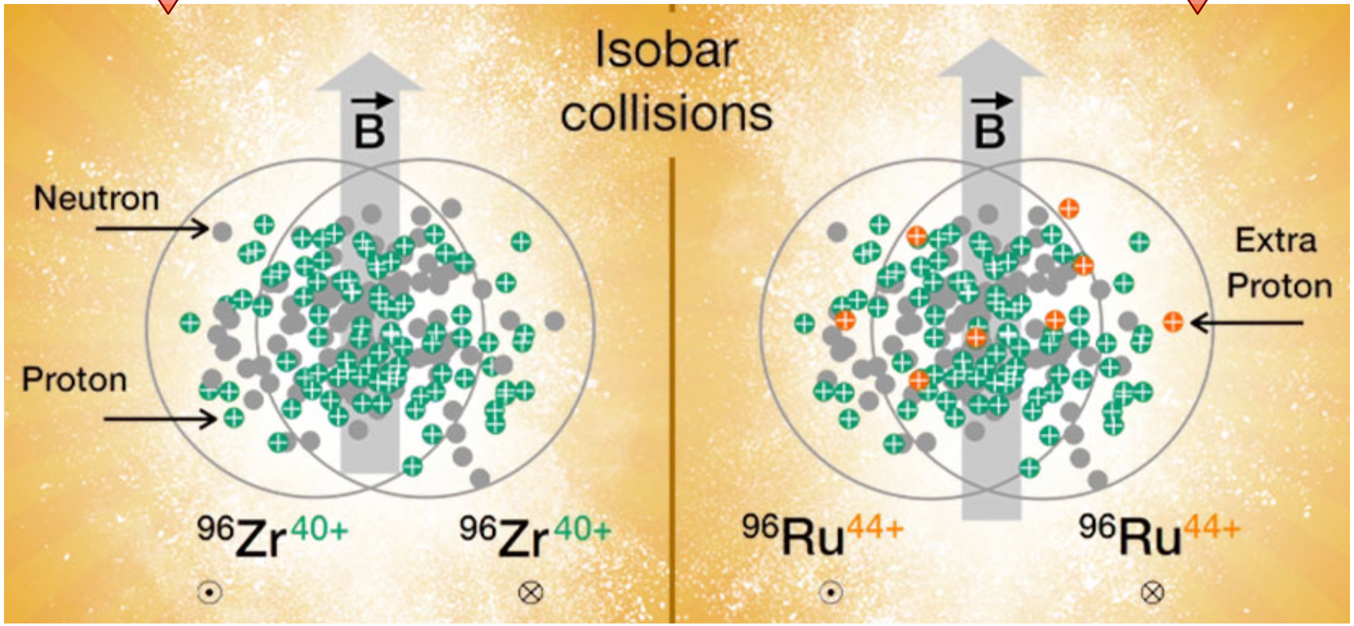


Image credit: Brookhaven National Laboratory, https://www.bnl.gov/newsroom/news.php?a=119062

Isobar collisions (experiment)

- Isobar run was completed by STAR in 2018
- $\approx$ 3.8 billion collisions of ^{96}Ru+ ^{96}Ru and ^{96}Zr+ ^{96}Zr at $\sqrt{s} = 200$ GeV
- Blind analysis by 5 groups of the STAR Collaboration (2021)
- Under the pre-defined criteria, **no CME** signature observed

[STAR Collaboration, Phys.Rev.C **105**, 014901 (2022)]

- The results are also inconsistent with the existing theoretical models
- Difference in backgrounds (RuRu and ZrZr) could make results consistent with a **finite CME signal** $\sim (6.8 \pm 2.6)\%$.

[Kharzeev, Liao, Shi, Phys.Rev.C 106, L051903 (2022)]

- Improved background estimates are still consistent with **no CME** (and set the upper limit of CME fraction ~ 10%)

[STAR Collaboration, arXiv:2310.13096]

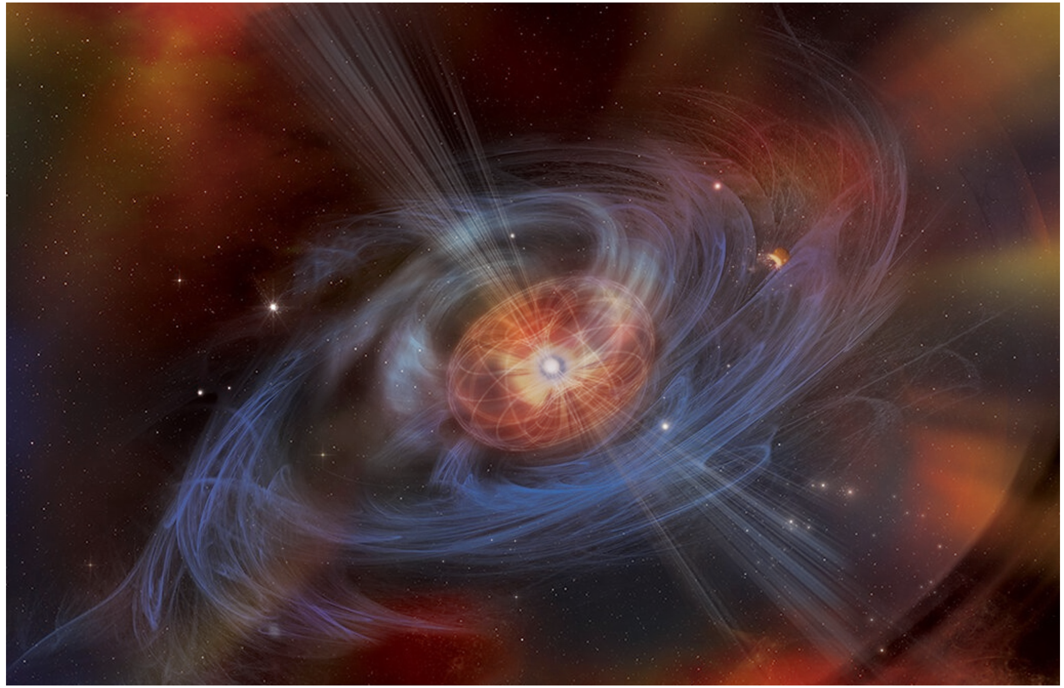


Image credit: Aurore Simonnet, Sonoma State University

MAGNETARS

Gaps in magnetosphere

- Gaps can develop in magnetosphere
- **Electric field** in the gap

$$E_{\parallel} \simeq Bh/R_{LC}$$

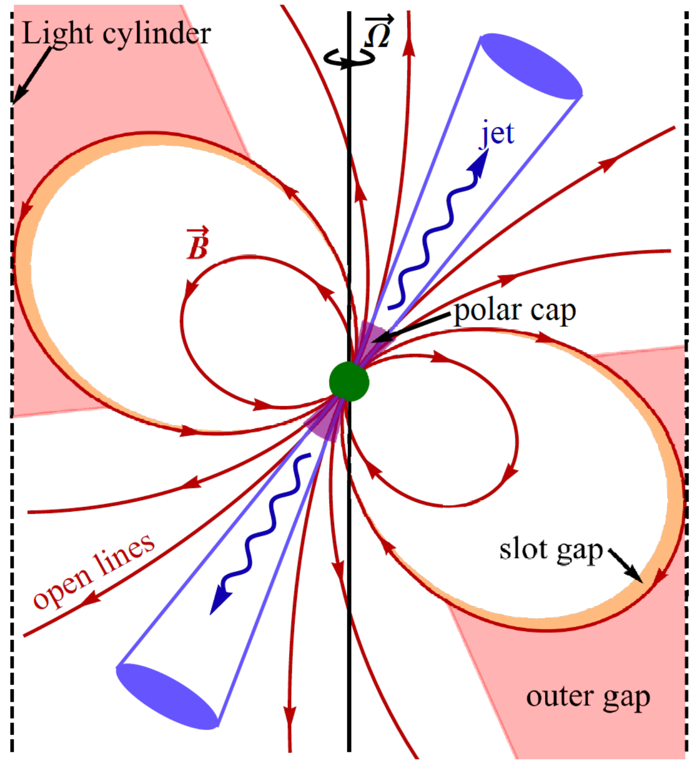
$$h \simeq 3.6\ \mathrm{m} \left(\frac{R}{10\ \mathrm{km}}\right)^{2/7} \left(\frac{\Omega}{1\ \mathrm{s}^{-1}}\right)^{-3/7} \left(\frac{B}{10^{14}\ \mathrm{G}}\right)^{-4/7}$$

$R_{LC} = c/\Omega$ is the light cylinder radius

- The estimate for the field

$$E_{\parallel} \approx 2.7\times10^{-8} E_c \left(\frac{R}{10\ \mathrm{km}}\right)^{2/7} \left(\frac{\Omega}{1\ \mathrm{s}^{-1}}\right)^{4/7} \left(\frac{B}{10^{14}\ \mathrm{G}}\right)^{3/7}$$

where $E_c = m_e^2/e = 1.3 \times 10^{18}$ V/m.



[Ruderman & Sutherland, Astrophys. J. **196**, 51 (1975)]

Chiral charge production

- The evolution of the chiral charge is governed by

$$\frac{\partial n_5}{\partial t} \cancel{+ \vec{\nabla} \cdot \vec{j}_5} = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_{\mathrm{m}} n_5$$

- The chiral anomaly produces n_5 and chirality flipping destroys it
- The steady-state value is quickly $(t^\star \sim 1/\Gamma_{\mathrm{m}} \sim 10^{-17}\ \mathrm{s})$ achieved:

$$n_5 = \frac{e^2}{2\pi^2 \Gamma_{\mathrm{m}}} \vec{E} \cdot \vec{B}$$

- where estimate for the chirality flipping rate is

$$\Gamma_{\mathrm{m}} \simeq \frac{\alpha^2 m_e^2}{T} \qquad (T \lesssim m_e/\sqrt{\alpha})$$

[Boyarsky, Cheianov, Ruchayskiy, Sobol, Phys. Rev. Lett. **126**, 021801 (2021)]

Chiral plasma instability

[Joyce & Shaposhnikov, PRL 79, 1193 (1997)]
[Boyarsky, Frohlich, Ruchayskiy, PRL 108, 031301 (2012)]
[Tashiro, Vachaspati, Vilenkin, PRD 86, 105033 (2012)]
[Akamatsu & Yamamoto, PRL 111, 052002 (2013)]
[Tuchin, PRC 91, 064902 (2015)]
[Manuel & Torres-Rincon, PRD 92, 074018 (2015)]
[Hirono, Kharzeev, Yin, PRD 92, 125031(2015)]
[Sigl & Leite, JCAP 01, 025 (2016)]

- Collective modes of a chiral plasma

$$\omega_{1,2} \simeq \begin{cases} -i\left(\sigma + \dfrac{k(\lambda k_\star - k)}{\sigma}\right) \\ i\,\dfrac{k(\lambda k_\star - k)}{\sigma} \end{cases}$$

- The 1st mode is damped by charge screening:

$$B_{k,1} \propto B_0 e^{-\sigma t}$$

- The 2nd mode is unstable when $k < \lambda k_\star$:

Note: $k_\star = \dfrac{2\alpha\mu_5}{\pi}$

$$B_{k,2} \propto B_0 e^{+tk(\lambda k_\star - k)/\sigma}$$

- The momentum of the fastest growing mode $B_{k,2}$ is

$$\frac{1}{2}k_\star$$

Observational consequences

- Unstable plasma in the gaps produces **helical** (circularly polarized) **modes** in the frequency range

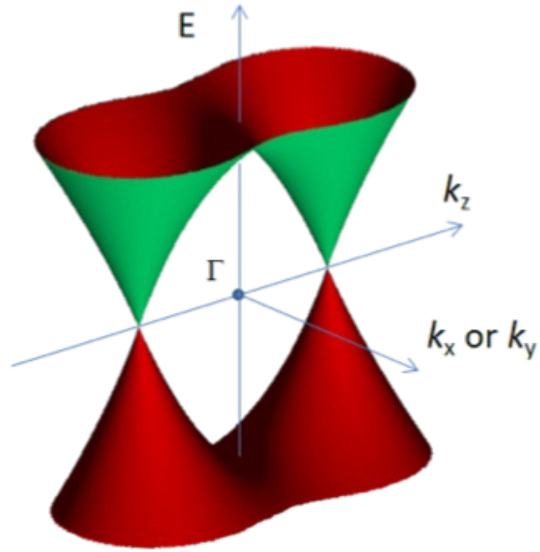
$$0 \lesssim \omega \lesssim k_\star$$

- For magnetars, these span **radio frequencies** and may reach into the **near-infrared** range

- Available energy is of the order of $\Delta\mathcal{E} \sim \mu_5^2 T^2 h^3$, i.e.,

$$\Delta\mathcal{E} \simeq 2.1\times 10^{25}\ \mathrm{erg}\left(\frac{T}{1\ \mathrm{MeV}}\right)\left(\frac{R}{10\ \mathrm{km}}\right)^{6/7} \times \left(\frac{\Omega}{1\ \mathrm{s}^{-1}}\right)^{-9/7}\left(\frac{B}{10^{14}\ \mathrm{G}}\right)^{2/7}$$

- The energy may be sufficient to feed some **radio bursts**



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL SEMIMETALS

[Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]

Dirac/Weyl fermions

- Electron quasiparticles with a wide range of properties are possible
- They may even have the emergent spinor structure of *massless* Weyl fermions,

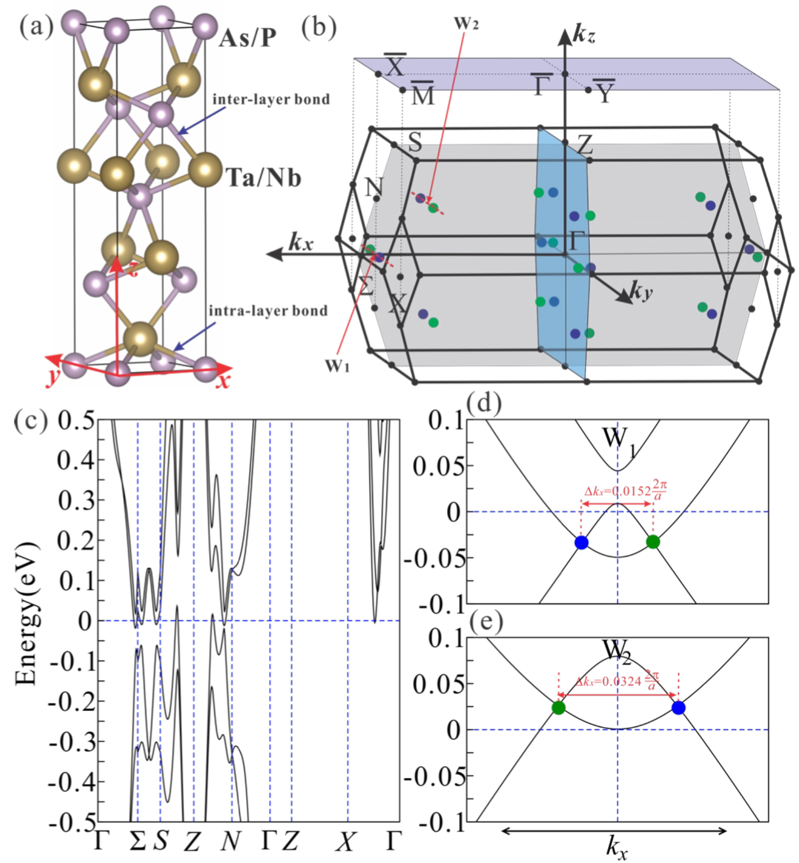
$$H_W \approx \pm v_F (\vec{\sigma} \cdot \vec{\boldsymbol{k}})$$

Such nodes are not uncommon!

Na_3Bi, Cd_3As_2, $ZrTe_5$, TaAs, NbAs, …

[Liu et al., Science **343**, 864 (2014)]
[Neupane et al., Nature Commun. **5**, 3786 (2014)]
[Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
[Li et al., Nature Physics **12**, 550 (2016)]
[S.-Y. Xu et al., Science 349, 613 (2015)]
[B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
[S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
[S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
[F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]

Weyl semimetals TaAs, TaP, NbAs, and NbP

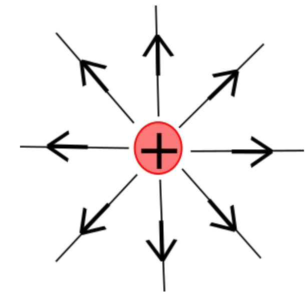


Sun, Wu & Yan, Phys. Rev. B **92**, 115428 (2015)

Berry curvature & topology

- For Weyl eigenstates, the Berry curvature is

$$\boldsymbol{\Omega}_k \equiv \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}} = \lambda \frac{\vec{\boldsymbol{k}}}{2k^3}$$



- The Chern number (topological charge)

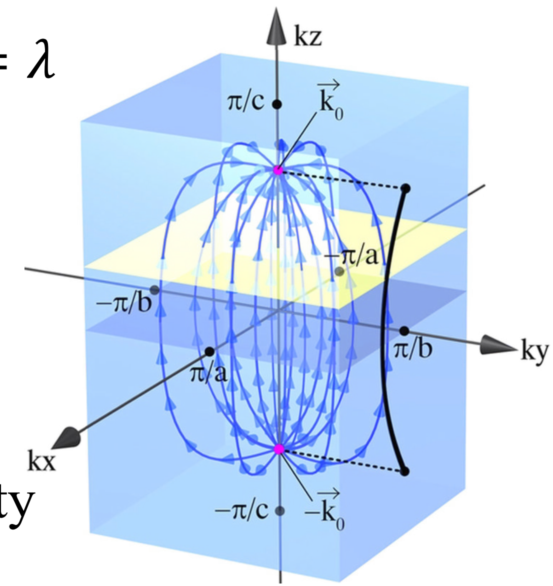
$$C = \frac{1}{2\pi} \oint \boldsymbol{\Omega}_k \cdot d\boldsymbol{S}_{\boldsymbol{k}} = \frac{\lambda}{2\pi} \oint \frac{\vec{\boldsymbol{k}}}{2k^3} \cdot \frac{\vec{\boldsymbol{k}}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda$$

- In a solid state material, the Brillouin zone is compact

- A closed surface around a node at $\vec{\boldsymbol{k}}_{\mathbf{0}}$ is also a closed surface around the rest of the Brillouin zone

- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]



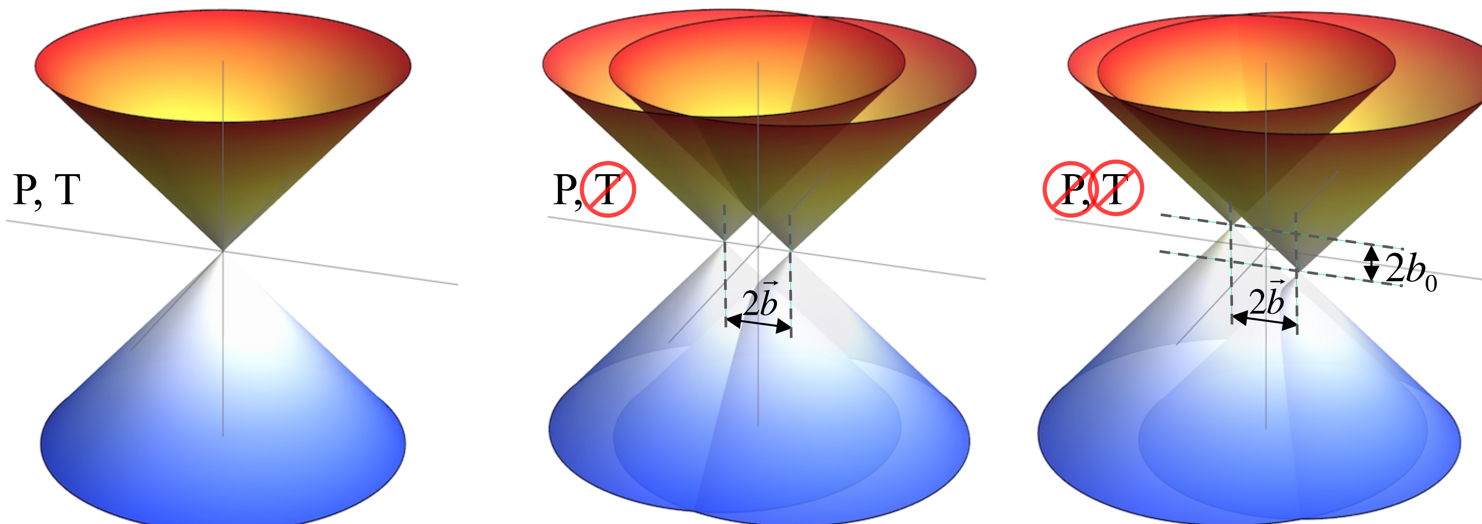
[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

Idealized Dirac and Weyl model

- Low-energy Hamiltonians of a Dirac and Weyl materials

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \overset{\cancel{T}}{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5} + \overset{\cancel{P}}{b_0\gamma^0\gamma^5}\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) Weyl (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Anomalous effects in semimetals

- Observable properties of Dirac/Weyl semimetals are sensitive to (i) the chiral anomaly, (ii) the values of b_0 and $\vec{b}$, and (iii) nontrivial topology
- Partial list of potential anomalous effects:
  - Negative magnetoresistance ($\rho_{\|}$ decreasing with B)
  - New types of collective modes (anomalous Hall waves, pseudo-magnetic helicons, chiral zero sound, etc.)
  - Anomalous thermoelectric effects (e.g., $\vec{J}_Q \propto \vec{b}\times\vec{E}$ and $\vec{J}_Q \propto \vec{b}\times\vec{\nabla}T$)
  - Strain/torsion induced CME ($\vec{J} \propto u_{33}\vec{B}$ and $\vec{J} \propto \mu\vec{B}_5$)
  - Strain/torsion dependent conductivity/resistance
  - Quantum oscillations in thin films $[T \propto v_F/(\mu b)]$
  - Strain/torsion induced quantum oscillations (pseudo-Landau levels)
  - Nonlocal anomalous transport

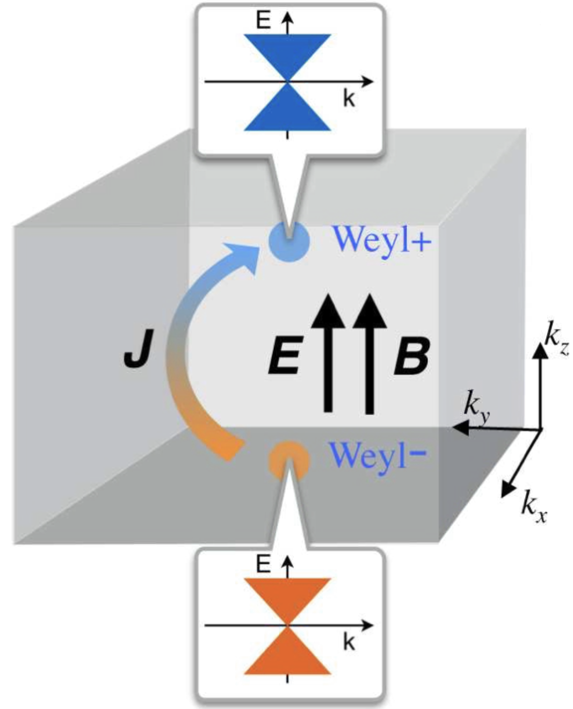


Image credit [Zhang et al., Nat. Commun. 7, 10735 (2016)]

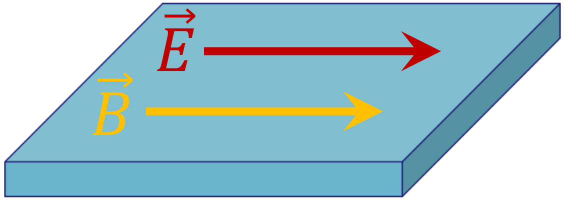
NEGATIVE MAGNETORESISTANCE & MORE

Steady CME current

- Homogeneous chiral plasma:

[Nielsen & Ninomiya, Phys. Lett. B **130**, 390 (1983)]
[Son & Spivak, Phys. Rev. B **88**, 104412 (2013)]

$$\frac{\partial n_5}{\partial t} + \cancel{\vec{\nabla}\cdot\vec{J}_5} = \frac{e^2\vec{E}\cdot\vec{B}}{2\pi^2} - \frac{n_5}{\tau_{\rm ch}}$$



- Steady state $(\tau_{\rm ch} \sim 1\ \text{ps to } 1\ \text{ns})$

$$n_5 = \frac{e^2\vec{E}\cdot\vec{B}}{2\pi^2}\tau_{\rm ch} \quad\rightarrow\quad \mu_5 = \frac{n_5}{\chi_5} \approx \frac{3v^3 n_5}{T^2+\mu^2/\pi^2}$$

- The CME current

$$J_i = \frac{e^2}{2\pi^2}\mu_5 B_i = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{\rm ch}\frac{B_i B_k}{\chi_5}E_k \quad\rightarrow\quad \sigma_{CME}^{\parallel} = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{\rm ch}\frac{B^2}{\chi_5}$$

i.e.,

$$\rho_{\rm total}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}$$

Negative Magnetoresistance

- Experimental confirmation

$$\rho_{\text{total}}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}$$

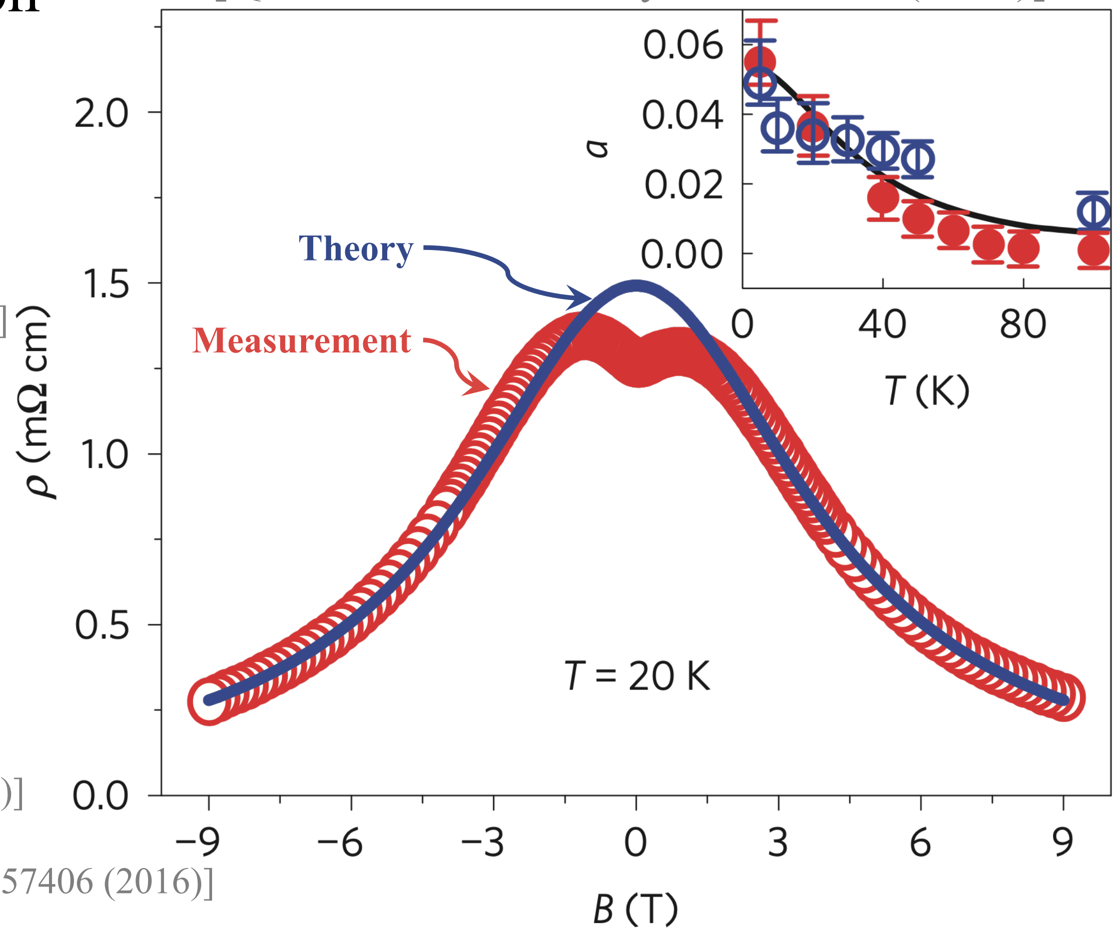
Dirac semimetals:

[Kim et al, Phys. Rev. Lett. **111**, 246603 (2013)]
[Li et al., Nat. Mater. **12**, 550 (2016)]
[Xiong et al., Science **350**, 413 (2015)]
[Feng, et al., Phys. Rev. B **92**, 081306 (2015)]
[Li et al., Nat. Commun. **6**, 10137 (2015)]
[Li et al., Nat. Commun. **7**, 10301 (2016)]

Weyl semimetals:

[Huang et al., Phys. Rev. X **5**, 031023 (2015)]
[Zhang et al., Nat. Commun. **7**, 10735 (2016)]
[Hirschberger et al., Nat. Mater. **15**, 1161 (2016)]
[Wang et al., Phys. Rev. B **93**, 121112 (2016)]
[Du et al., Sci. China Phys. Mech. Astron. **59**, 657406 (2016)]
[Li et al., Front. Phys. **12**, 127205 (2017)]

[Q. Li et al, Nature Physics **12**, 550 (2016)]

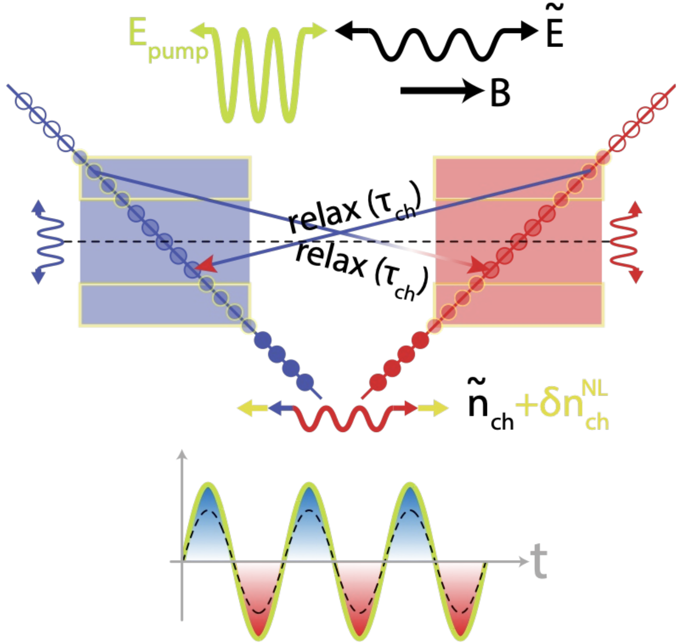


Chiral charge pumping (theory)

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

- Weyl semimetal TaAs
  - $\vec{B} \neq 0$ & oscillating $\vec{E} \parallel \vec{B}$
- The nonlinear contribution to chiral charge-pumping conductivity

$$\delta\sigma_{\mathrm{ch}}^{\mathrm{NL}} = i\frac{9\alpha^2 e^5 v^3}{8h^2\omega^3}\left(\frac{\tilde{\mathbf{E}}_{\mathrm{pump}}\cdot\mathbf{B}}{B}\right)^2 B$$

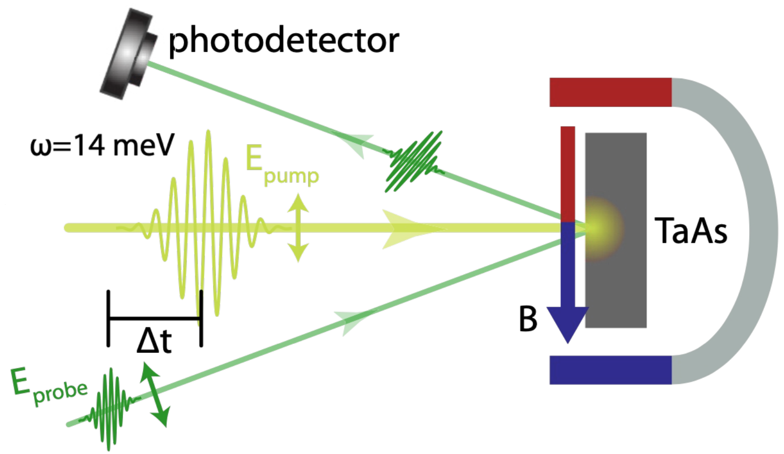


- The reflection coefficient

$$R(T) = \left|\frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}}\right|^2 \quad \text{where} \quad \epsilon = \epsilon_\infty + i\frac{\sigma}{\omega\epsilon_0}$$

Chiral charge pumping (data)

- Experimental setup

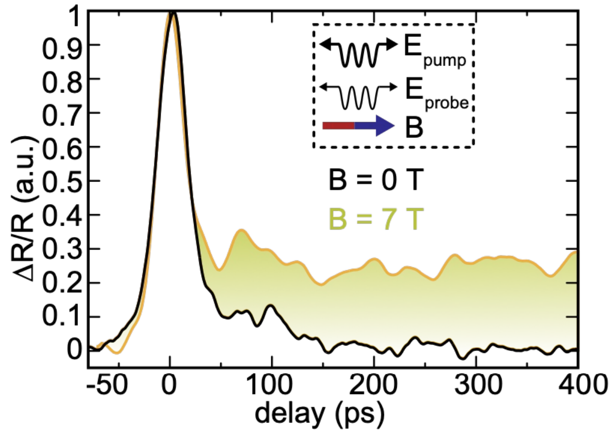


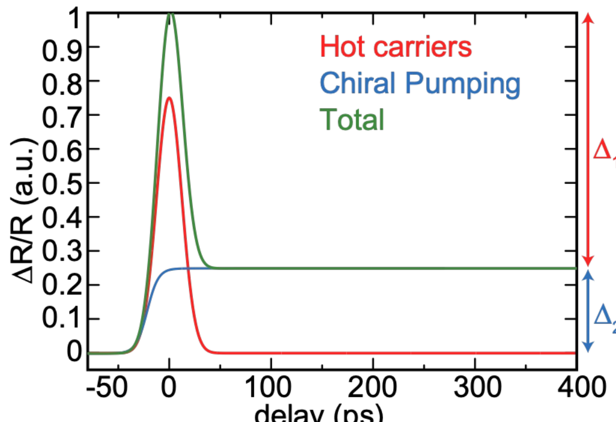
- Chiral charge relaxation time

$$1\ \text{ns} \ll \tau_{\text{ch}} < 77\ \text{ns}$$

[Jadidi et al., Phys. Rev. B **102**, 245123 (2020)]

- Measurements:

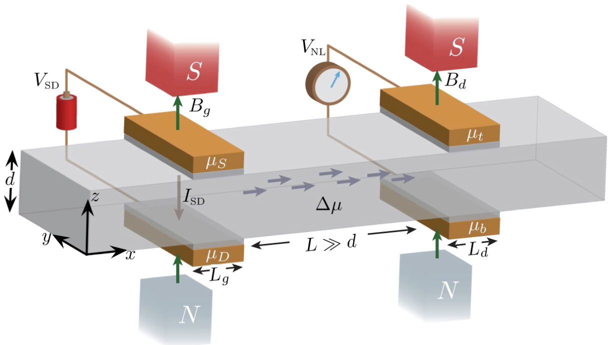




Nonlocal anomalous transport

- Theory

[Parameswaran, Grover, Abanin, Pesin, Vishwanath, PRX 4, 031035 (2014)]



$$\alpha_{NL} = \frac{R_{NL}}{R_L} \propto e^{-L/L_V}$$

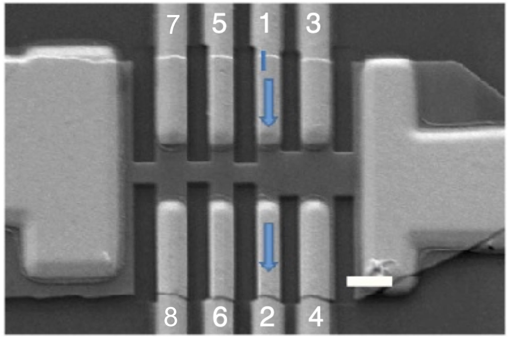
- Measurements:

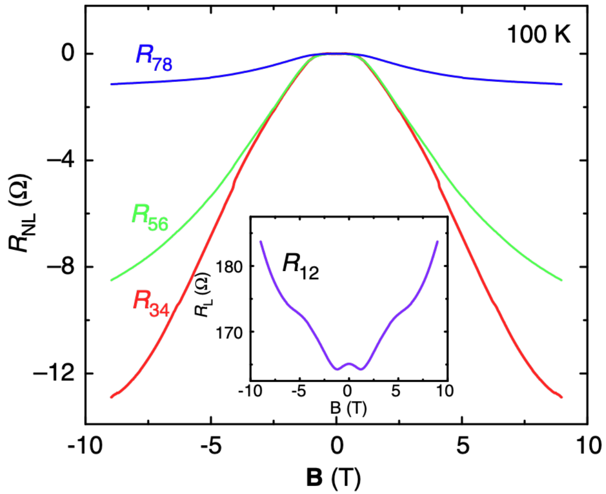
$$L_V \sim 2\ \mu\text{m}$$

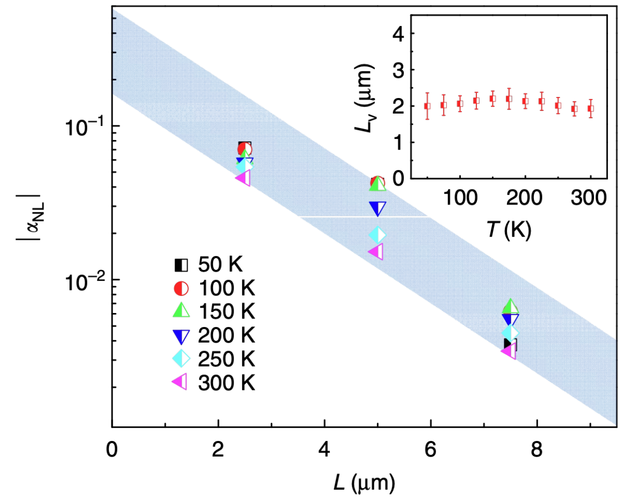
- Experiment (challenge: Ohmic diffusion)

[Zhang et al., Nat. Commun. 8, 13741 (2017)]

Cd_3As_2







Summary

- Chiral anomaly can have macroscopic implications in relativistic plasmas
- (Dipole) chiral magnetic effect can be seen via charged particle correlations in heavy-ion collisions
- Chiral anomaly may affect activity of magnetars
- Chiral anomaly can be realized and tested in Dirac/Weyl semimetals
- Chiral charge is relatively long-lived and can be optically pumped and manipulated (promising new technologies)