Gluon puzzle of gapless superconductivity

Igor A. Shovkovy

Institut für Theoretische Physik

J. W. Goethe Universität

Frankfurt am Main

Collaborator(s)

- Mei Huang
  (ITP, Goethe-University & Tsinghua University, Beijing)

References

- I. Shovkovy and M. Huang, Phys. Lett. B **564** (2003) 205, `hep-ph/0302142`
- M. Huang and I. Shovkovy, Nucl. Phys. A **729** (2003) 835, `hep-ph/0307273`
- M. Huang and I. Shovkovy, in preparation

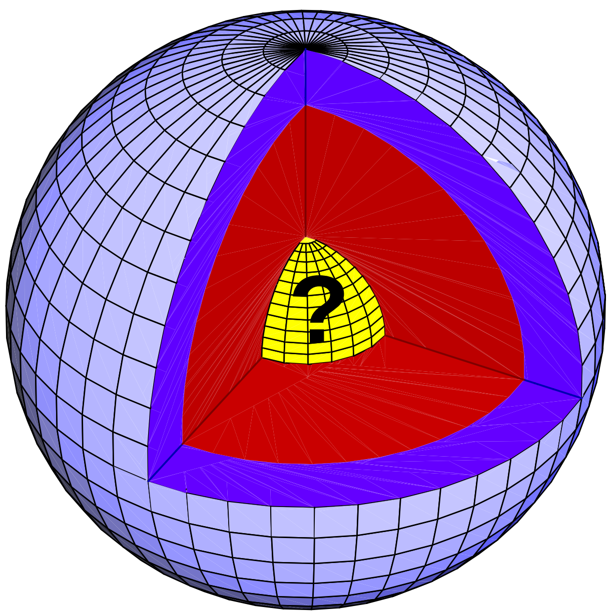
Matter at high density

We study this because we need to understand

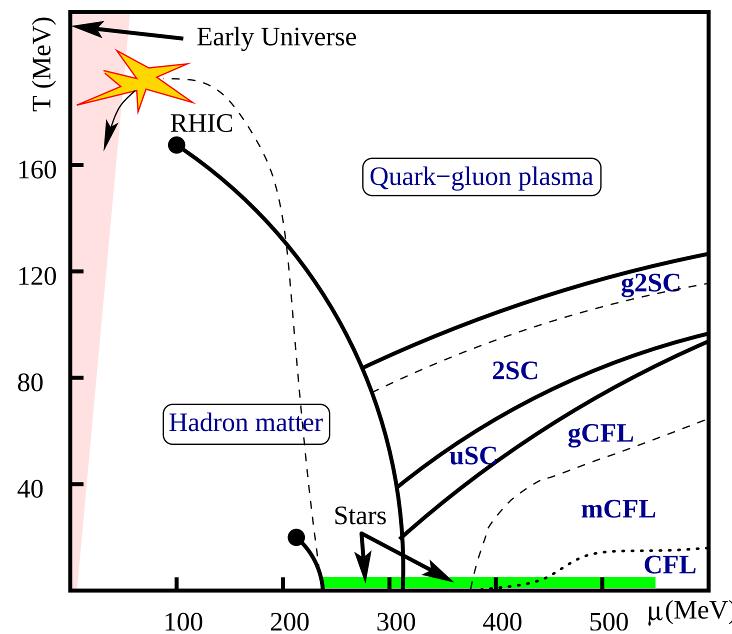
(i) properties of dense matter that exists in the Universe

(ii) fundamental properties of QCD

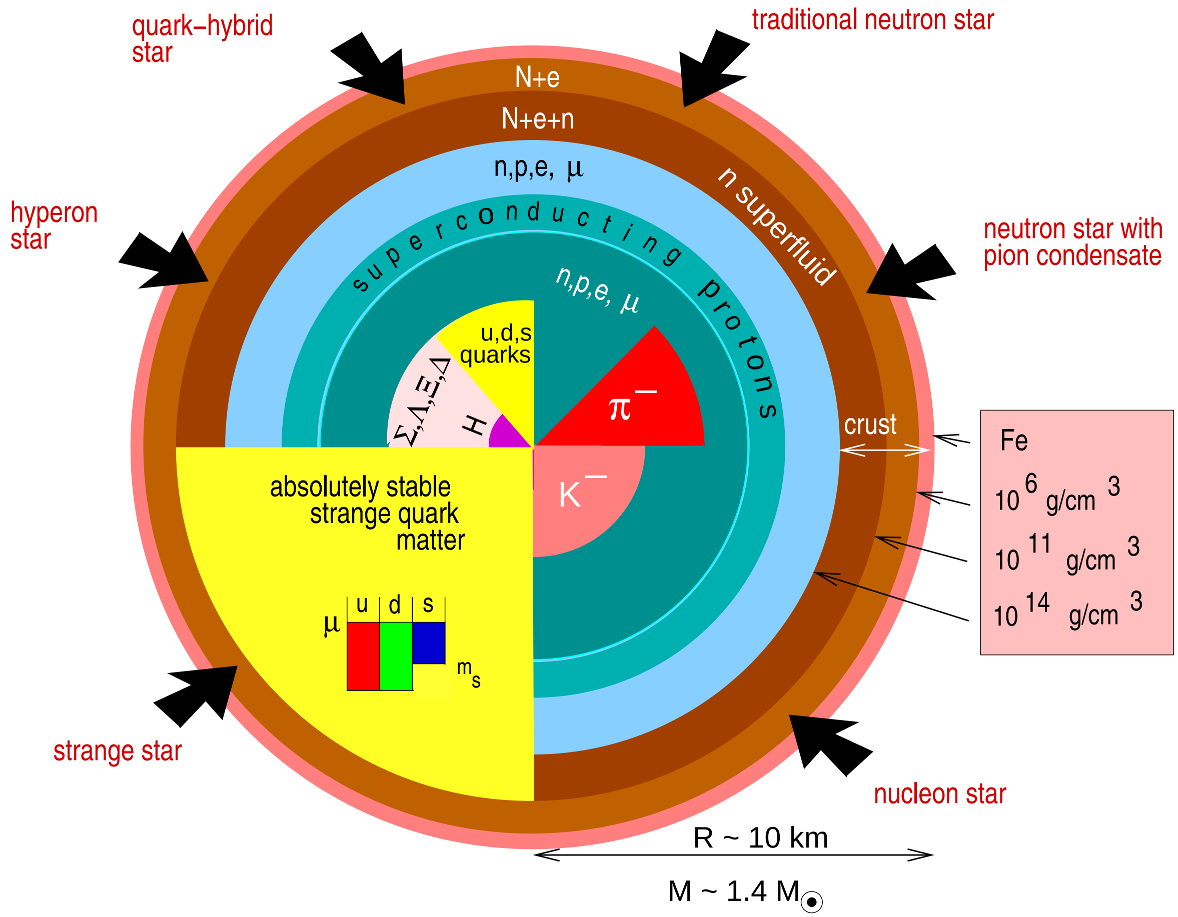
(densities in stars $\rho_c \gtrsim 5\rho_0$)



($\mu_q \gtrsim \Lambda_{QCD}$: no lattice results)



Neutron, quark or hybrid stars



[figure is taken from a talk by F. Weber]

Is there SC inside stars?

The answer is: **we do not know yet**

Arguments in favor:

- Relatively high densities in stars, $\rho_c \gtrsim 5\rho_0$, suggest that quarks may be deconfined
- An attractive diquark channel is likely to exist
- Temperatures are quite low, $T \lesssim 50$ keV, to allow pairing

Arguments against:

- Strongly coupled dynamics is not under control
- Matter may not necessarily be deconfined at existing densities
- Specific conditions inside stars (e.g., β-equilibrium) may not favor color superconductivity

The natural approach: **To give predictions and to test them**

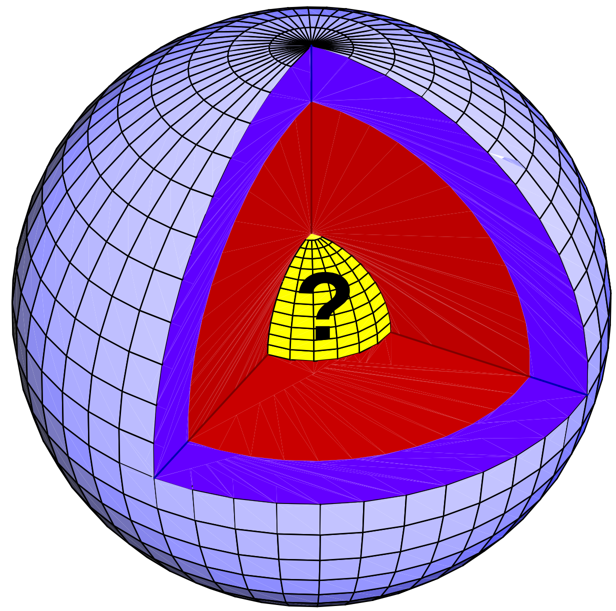
Specific conditions inside stars

Matter in the bulk of a star is

(i) β-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:

$$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$$



Otherwise, a star would **not** be stable!

- Coulomb energy (when $n_Q \neq 0$)

$$E_{\rm Coulomb} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/{\rm fm}^3}\right)^2 \left(\frac{R}{1~{\rm km}}\right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/{\rm fm}^3 \quad \Rightarrow \quad E_{\rm Coulomb}^{\rm 2SC} \gg M_\odot c^2$

Neutrality vs. color superconductivity

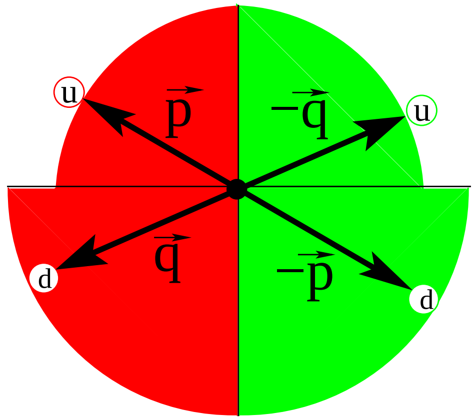
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in β-equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \quad \Rightarrow \quad \mu_d \approx 2^{1/3}\mu_u \ \Rightarrow \ \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{i.e.,} \qquad n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

The "best" Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Mismatch vs. coupling strength

Mismatch parameter μ_e is **not** a free model parameter,

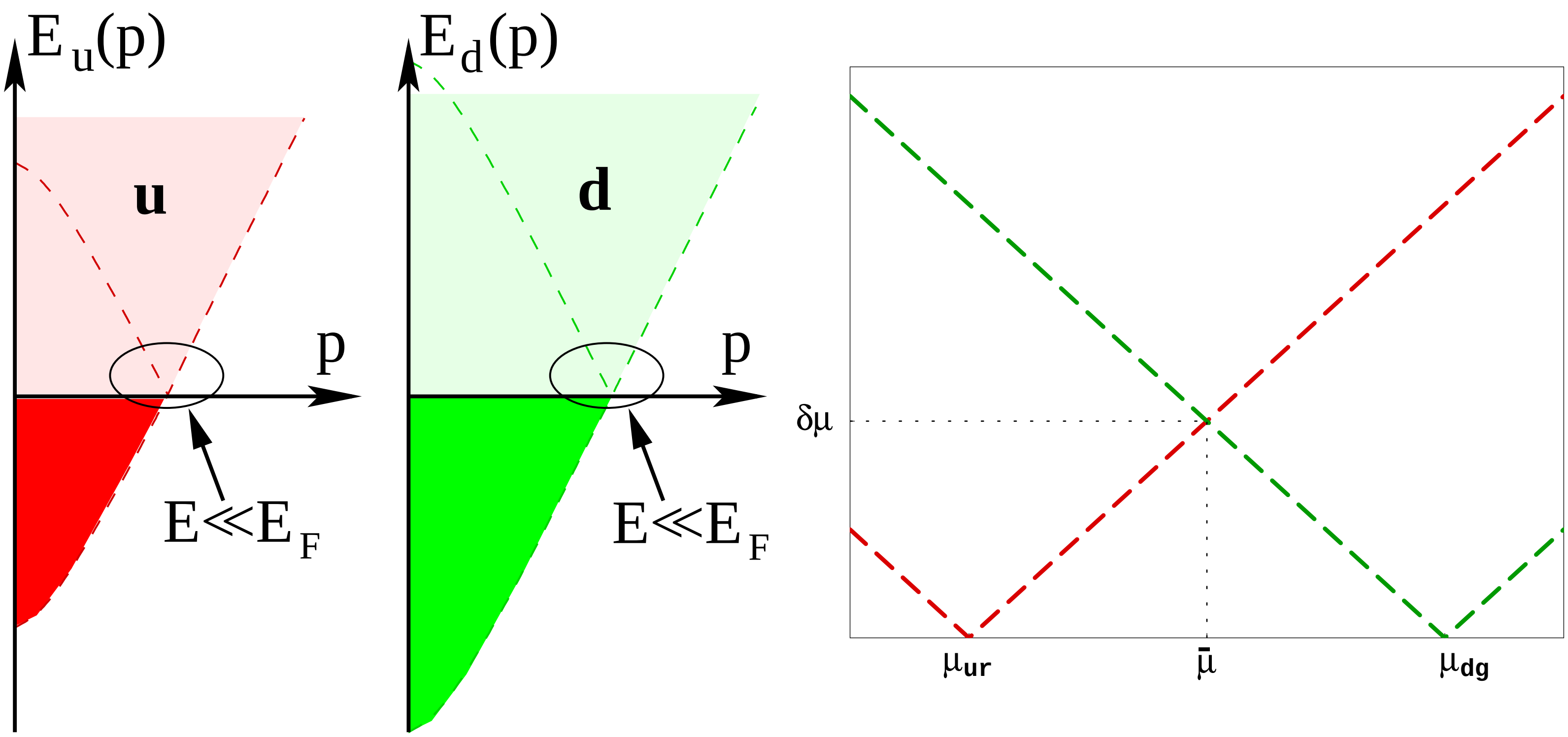
$$n_Q \equiv -\frac{\partial\Omega}{\partial\mu_e} = 0 \qquad \Rightarrow \qquad \mu_e = \mu_e(\bar{\mu}_q, \Delta)$$

However, the diquark coupling strength (η) **is** a model parameter:

1. Weak coupling, $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing: Normal phase is the ground state
2. Strong coupling, $\eta \gtrsim 0.8$ — pairing wins over the mismatch between the Fermi surfaces: 2SC is the ground state
3. Intermediate strength coupling, $0.7 \lesssim \eta \lesssim 0.8$ — the ground state is a new gapless color superconducting (g2SC) phase.

Quasiparticle spectrum in normal phase

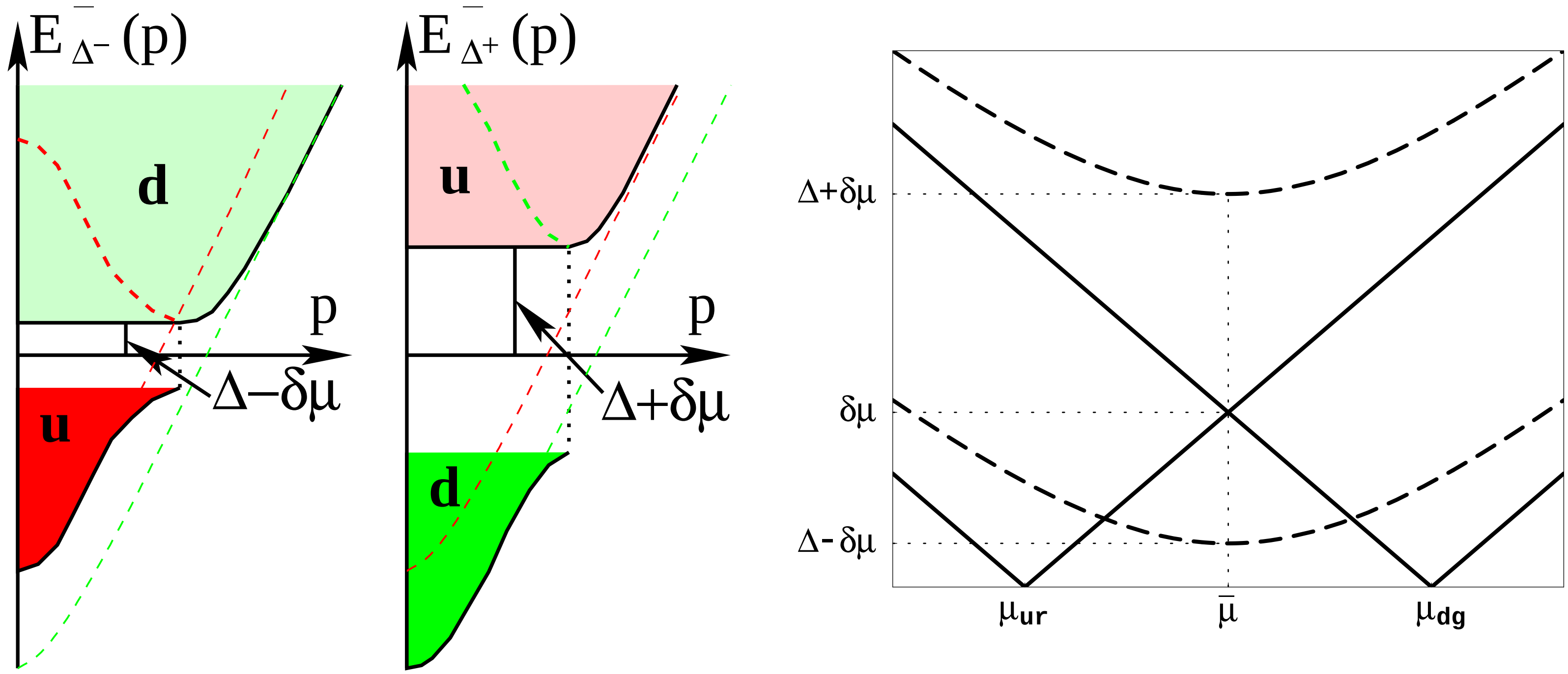
Weak coupling (normal phase)



How does this spectrum change when Cooper pairs are formed?

Quasiparticle spectrum in 2SC phase

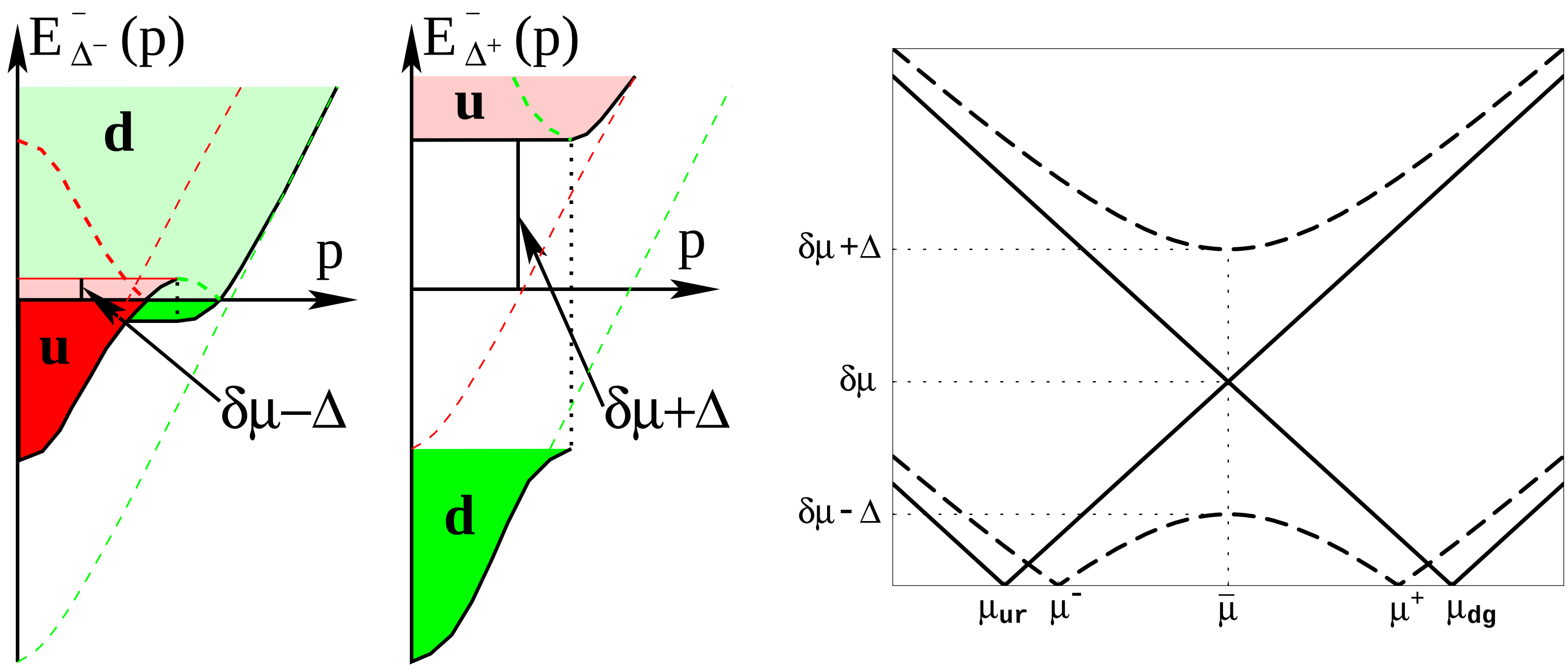
Strong coupling (2SC phase)



The energy gaps in the quasiparticle spectra are $\boxed{\Delta - \delta\mu}$ & $\boxed{\Delta + \delta\mu}$

Quasiparticle spectrum in g2SC phase

Intermediate coupling (gapless phase)



The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Sarma phase in condensed matter

Type II superconductors in a constant magnetic field:

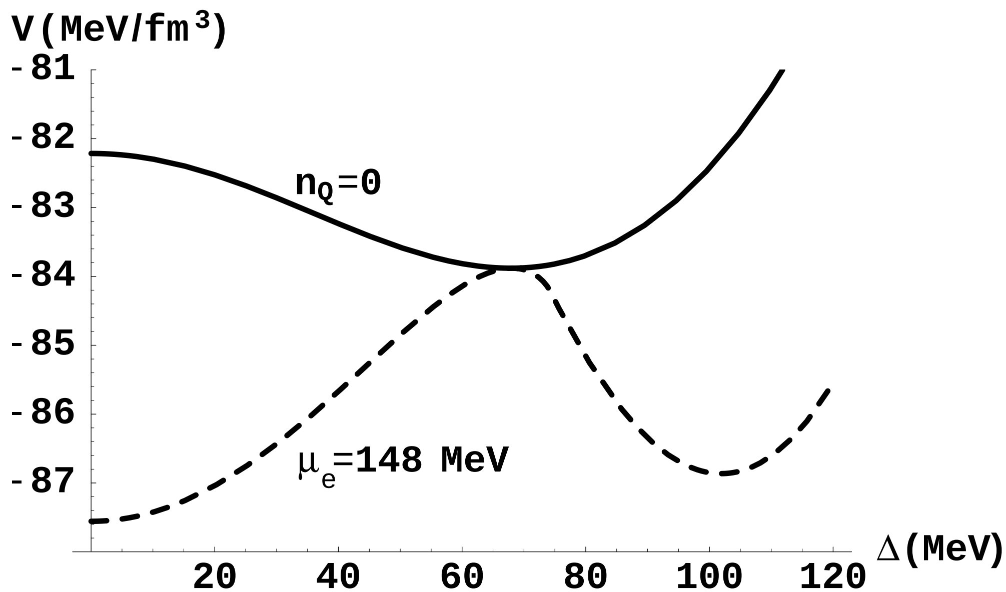
[G. Sarma, J. Phys. Chem. Solids **24** (1963) 1029.]

- Magnetic field originates from ferromagnetic order of impurities in $La_{1-x}Gd_x$ and $Y_{1-x}Gd_xOs_2$ [B.Matthias,H.Suhl & Corenzwit, Phys. Rev. Lett. **1** (1958) 92], [N.Phillips,B.Matthias, Phys. Rev. **121** (1961) 105]
- Pairing happens between spin-$\uparrow$ and spin-$\downarrow$ holes/electrons
- Fermi momenta of $\uparrow$- and $\downarrow$-quasiparticles are different
- The mismatch parameter $\delta\mu \sim H \sim n_{\text{impurity}}$

The gapless "Sarma" phase is **unstable**!

Stability of g2SC phase

Effective potential at $T = 0$ [I.S. & M.Huang, Phys. Lett. B564 (2003) 205]:

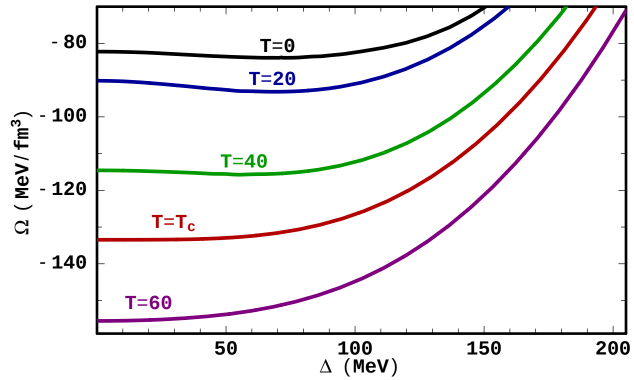


(Q.: Mixed phase? → A.: Unlikely if $\sigma \gtrsim 20$ MeV/fm^2 [I.S., Hanauske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054])

No Sarma instability in g2SC phase if $n_Q = 0$ is enforced *locally*!

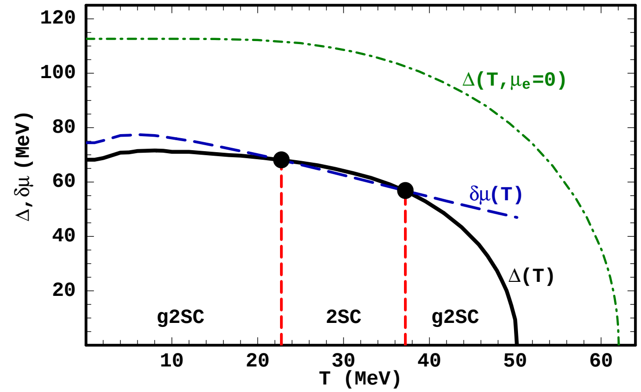
Finite temperature properties

1 Effective potential at $T \neq 0$:



i.e., 2nd order phase transition

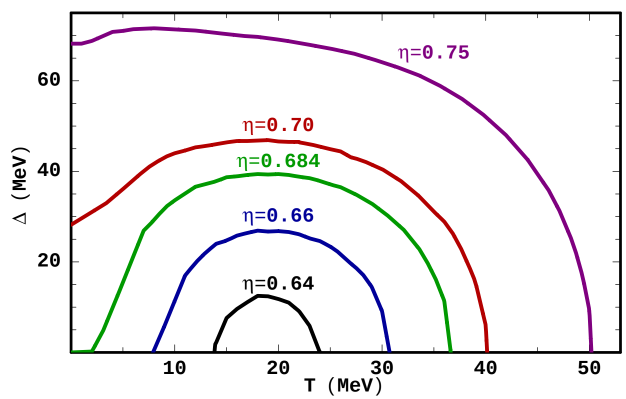
2 *Nonmonotonic* $\Delta(T)$ dependence:



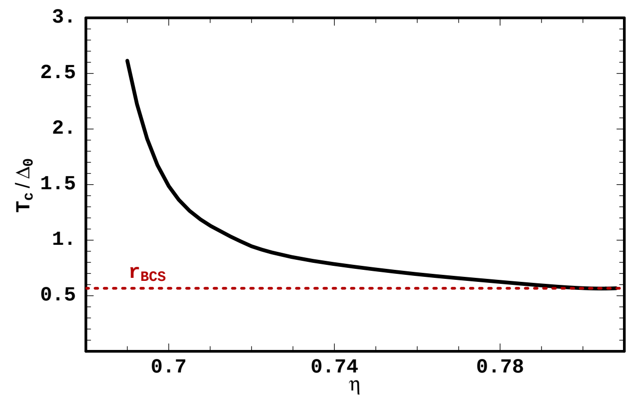
Note: g2SC $\rightarrow$ 2SC $\rightarrow$ g2SC $\rightarrow$ NQ

[M.Huang & I.S., Nucl. Phys. A 729 (2003) 835]

3 *Extreme nonmonotonic* temperature dependence at weaker couplings:

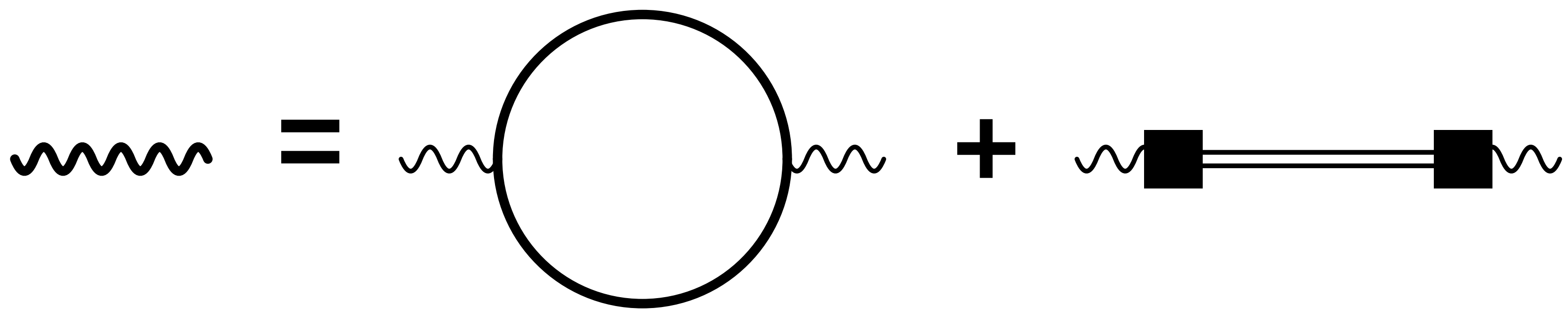


4 T_c/Δ_0 is *not universal* (unlike in BCS), and it can be arbitrarily large!



Higgs/Meissner effect in g2SC

- Higgs effect, i.e., $SU(3)_c \to SU(2)_c$ without NG bosons
  - there exists unitary gauge in which NG boson fields are "eaten" by 5 gluons
- Is there Meissner effect?
  - low energy spectrum looks like in normal quark matter
- Improved HDL approximation plus (NG) collective modes:



General structure of $\Pi^{AB,\mu\nu}$

$$\Pi^{AB,\mu\nu}(q) = \begin{cases} \delta^{AB}\Pi_1^{\mu\nu}, & \text{for} \quad A,B = 1,2,3, \\ \delta^{AB}\Pi_{4+}^{\mu\nu}, & \text{for} \quad A,B = (4+5i),(6+7i), \\ \delta^{AB}\Pi_{4-}^{\mu\nu}, & \text{for} \quad A,B = (4-5i),(6-7i), \\ \begin{pmatrix} \Pi_{88}^{\mu\nu} & \Pi_{8\gamma}^{\mu\nu} \\ \Pi_{\gamma 8}^{\mu\nu} & \Pi_{\gamma\gamma}^{\mu\nu} \end{pmatrix}, & \text{for} \quad A,B = 8,\gamma, \end{cases}$$

where
$$\Pi_a^{\mu\nu}(q) = \left(g^{\mu\nu} - u^\mu u^\nu + \frac{\vec{q}^\mu \vec{q}^\nu}{\vec{q}^2}\right) H_a(q) + u^\mu u^\nu K_a(q) - \frac{\vec{q}^\mu \vec{q}^\nu}{\vec{q}^2} L_a(q) + \frac{u^\mu \vec{q}^\nu + \vec{q}^\mu u^\nu}{|\vec{q}|} M_a(q)$$

Screening masses: $m_{M,a}^2 \equiv -H_a(0)$ and $m_{D,a}^2 \equiv -K_a(0)$

$\Pi^{AB,\mu\nu}$ with $A,B=1,2,3$

Meissner and Debye screening masses:

$$m^2_{M,1} \equiv -H_1(0) \simeq 0, \quad \text{(no Meissner effect)}$$

$$m^2_{D,1} \equiv -K_1(0) = \frac{2\alpha_s}{\pi}\left(\frac{(\mu^-)^2\delta\mu}{\sqrt{(\delta\mu)^2-\Delta^2}} + \frac{(\mu^+)^2\delta\mu}{\sqrt{(\delta\mu)^2-\Delta^2}}\right)\theta(\delta\mu-\Delta)$$

$$\simeq \frac{4\alpha_s\bar{\mu}^2\delta\mu}{\pi\sqrt{(\delta\mu)^2-\Delta^2}}\theta(\delta\mu-\Delta),$$

where

$$\bar{\mu} \equiv \tfrac{\mu_{gd}+\mu_{ru}}{2} \qquad \text{(average Fermi momentum)}$$

$$\delta\mu \equiv \tfrac{\mu_{gd}-\mu_{ru}}{2} \qquad \text{(mismatch between Fermi momenta)}$$

$$\mu^{\pm} \equiv \bar{\mu} \pm \sqrt{\delta\mu^2-\Delta^2} \qquad \text{(boundaries of “blocking” region)}$$

$\Pi^{AB,\mu\nu}$ with $A, B = 8, \gamma$ (Debye screening)

$$
\begin{aligned}
K_{88} &\simeq \frac{4\alpha_s \bar{\mu}^2}{\pi}, \\
K_{\gamma\gamma} &\simeq \frac{8\alpha \bar{\mu}^2}{3\pi} \left(1 + \frac{3\delta\mu\ \theta(\delta\mu - \Delta)}{2\sqrt{(\delta\mu)^2 - \Delta^2}} \right), \\
K_{8\gamma} &= K_{\gamma 8} \simeq 0
\end{aligned}
$$

There is no mixing (static, long-range Debye screening).
However, a mixing will appear in the "natural basis",

$$
\tilde{A}^8_\mu = A^8_\mu \cos\varphi + A^\gamma_\mu \sin\varphi,
$$
$$
\tilde{A}^\gamma_\mu = A^\gamma_\mu \cos\varphi - A^8_\mu \sin\varphi,
$$

How about gauge symmetry? — No problem.

$\Pi^{AB,\mu\nu}$ with $A,B=8,\gamma$ (Meissner screening)

$$H_{88} \simeq \frac{4\alpha_s\bar{\mu}^2}{9\pi}\left(1-\frac{\delta\mu\ \theta(\delta\mu-\Delta)}{\sqrt{(\delta\mu)^2-\Delta^2}}\right),$$

$$H_{\gamma\gamma} \simeq \frac{4\alpha\bar{\mu}^2}{27\pi}\left(1-\frac{\delta\mu\ \theta(\delta\mu-\Delta)}{\sqrt{(\delta\mu)^2-\Delta^2}}\right),$$

$$H_{8\gamma} = H_{\gamma 8} \simeq \frac{4\sqrt{\alpha\alpha_s}\bar{\mu}^2}{9\sqrt{3}\pi}\left(1-\frac{\delta\mu\ \theta(\delta\mu-\Delta)}{\sqrt{(\delta\mu)^2-\Delta^2}}\right)$$

This becomes diagonal in the new basis:

$$\tilde{A}^8_\mu = A^8_\mu \cos\varphi + A^\gamma_\mu \sin\varphi,$$
$$\tilde{A}^\gamma_\mu = A^\gamma_\mu \cos\varphi - A^8_\mu \sin\varphi,$$

where the mixing angle is determined by

$$\sin\varphi = \sqrt{\frac{\alpha}{3\alpha_s + \alpha}},$$
$$\cos\varphi = \sqrt{\frac{3\alpha_s}{3\alpha_s + \alpha}}.$$

Then, the Meissner screening masses are

$$m^2_{M,\tilde{8}} \equiv \frac{4(3\alpha_s + \alpha)\bar{\mu}^2}{27\pi}\left(1 - \frac{\delta\mu\,\theta(\delta\mu - \Delta)}{\sqrt{(\delta\mu)^2 - \Delta^2}}\right),$$
$$m^2_{M,\tilde{\gamma}} \equiv 0 \qquad \text{i.e., no Meissner effect connected with } \tilde{U}(1)_{\rm em}.$$

Note that $m^2_{M,\tilde{8}} < 0$ in the g2SC phase.

This means that there is a **plasma** (magnetic) type instability.

Note that spin-1 condensates around $\mu^{\pm}$ remove the instability.

$\Pi^{AB,\mu\nu}$ **with** $A,B=(4\pm)$

Meissner and Debye screening masses:

$$\begin{aligned} m^2_{M,4\pm} &\equiv -H_{4\pm}(0) \\ &\simeq \frac{4\alpha_s\bar{\mu}^2}{3\pi}\left[\frac{\Delta^2-2\delta\mu^2}{2\Delta^2}+\theta(\delta\mu-\Delta)\frac{\delta\mu\sqrt{\delta\mu^2-\Delta^2}}{\Delta^2}\right], \\ m^2_{D,4\pm} &\equiv -K_{4\pm}(0) \\ &\simeq \frac{4\alpha_s\bar{\mu}^2}{\pi}\left[\frac{\Delta^2+2\delta\mu^2}{2\Delta^2}-\theta(\delta\mu-\Delta)\frac{\delta\mu\sqrt{\delta\mu^2-\Delta^2}}{\Delta^2}\right] \end{aligned}$$

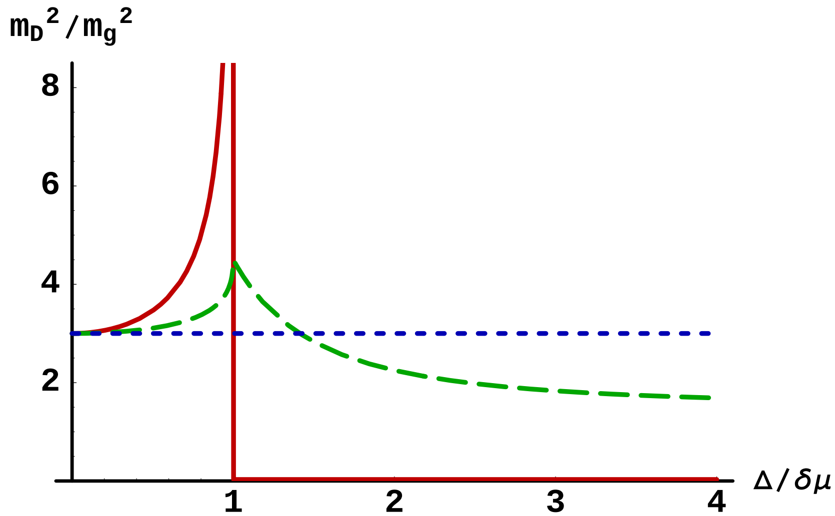
Note that $m^2_{M,4\pm}<0$ when

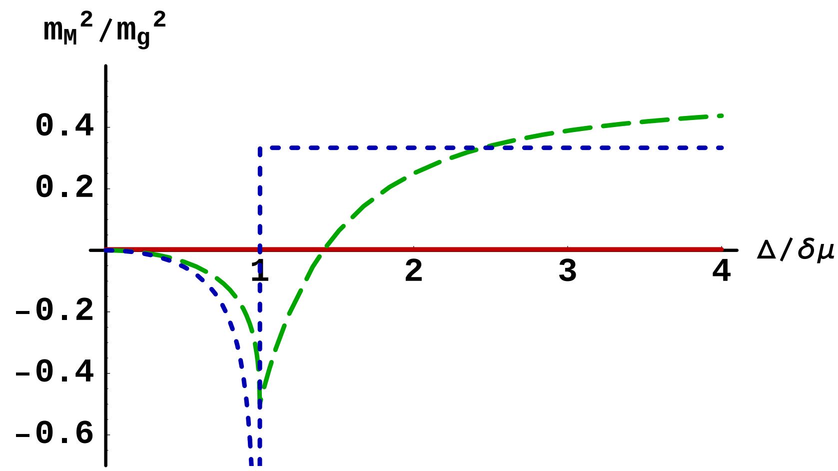
$$0<\Delta<\sqrt{2}\delta\mu \qquad \text{(i.e., in g2SC and 2SC phases)}$$

Thus, there is a **plasma** (magnetic) type instability

Overview of the screening properties

Without spin-1 condensates





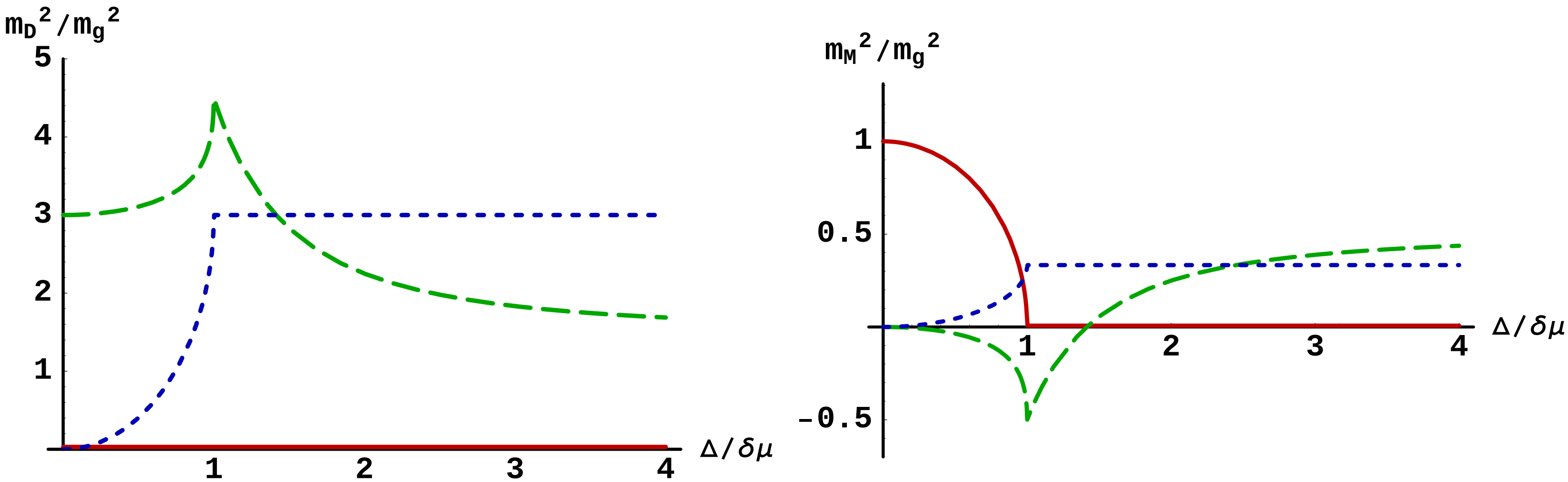
$A = 1, 2, 3$ — red solid line

$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

Overview of the screening properties

With spin-1 condensates



$A = 1, 2, 3$ — red solid line

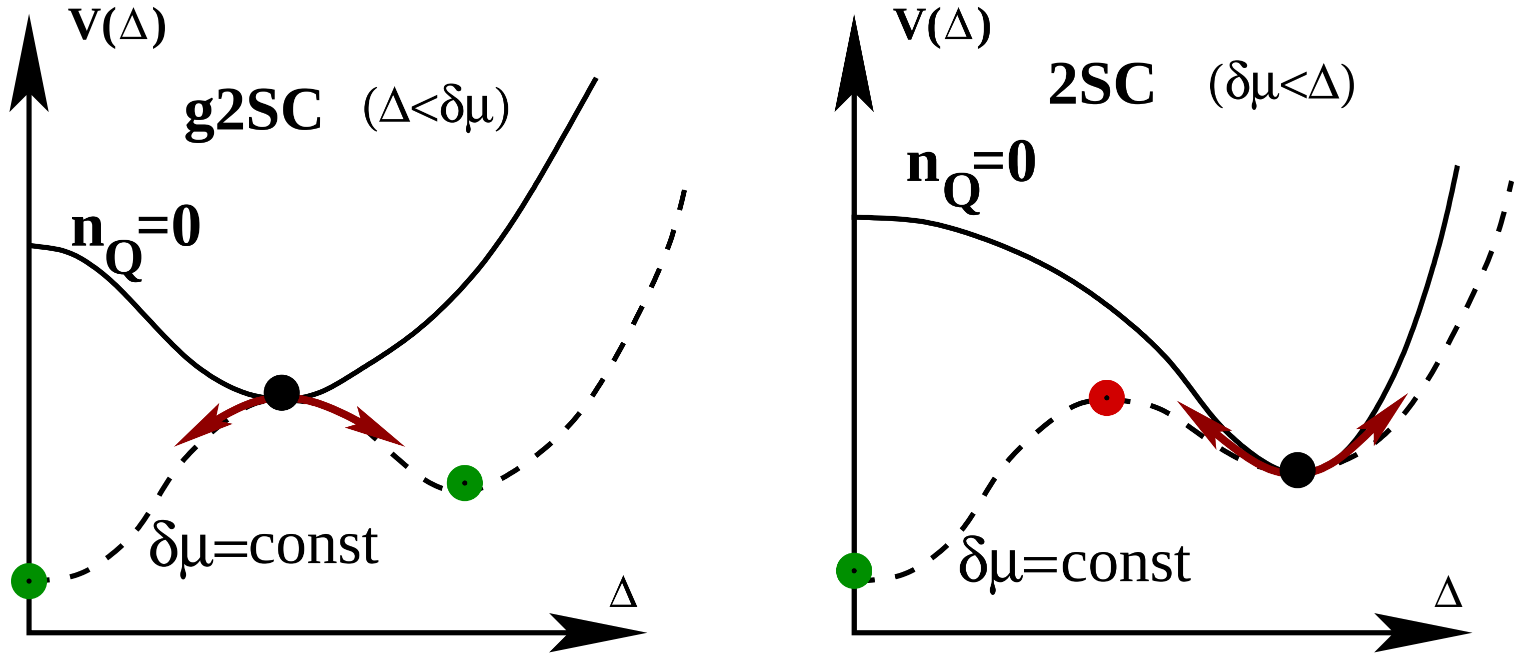
$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

Magnetic instability $m^2_{M,4\pm} < 0$

Could gluons "feel" the local maximum of the effective potential?

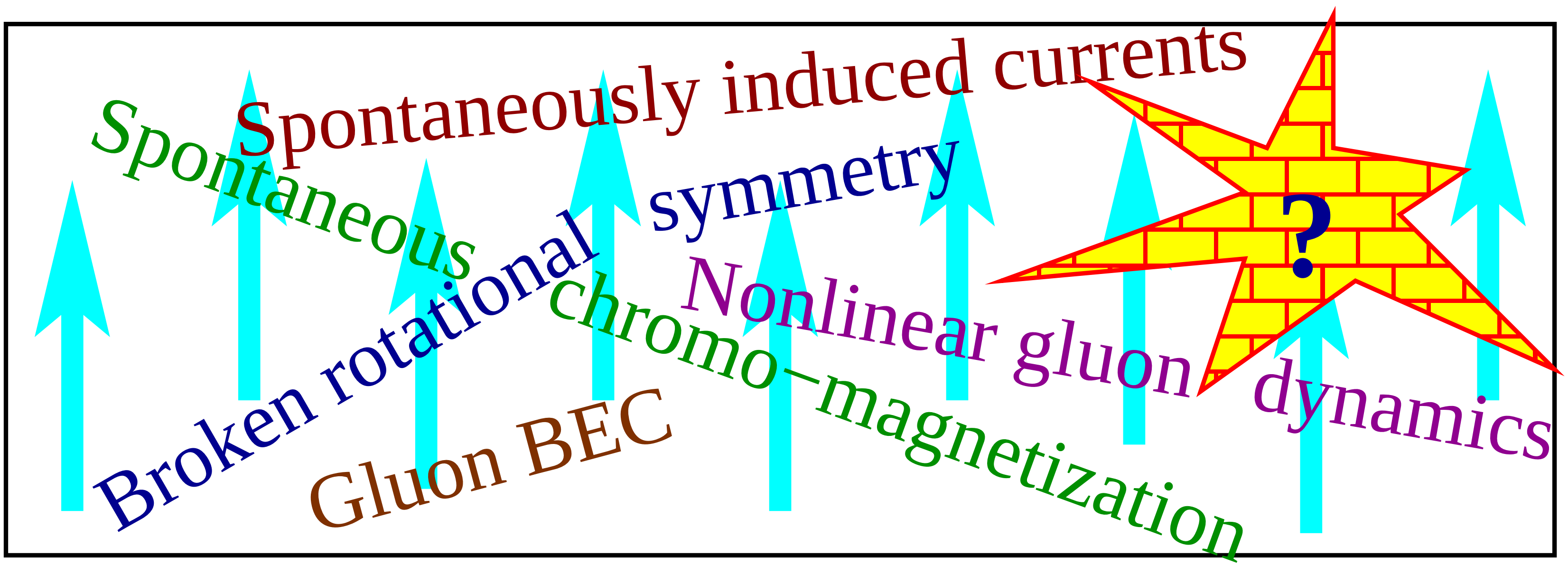
This is possible, but this is not everything ...



What is the origin of instability in 2SC phase when $\delta\mu < \Delta < \sqrt{2}\delta\mu$?

So, the ground state is . . .

something like this



Summary

- The g2SC phase is a new state of matter that may exist in cores of compact stars (gCFL → [Rajagopal, Kouvaris, Alford, hep-ph/0311286])
- There is no Sarma instability in g2SC phase if the neutrality is enforced locally
- The spectrum of low-energy excitations in the g2SC phase has extra gapless modes (these should affect transport properties)
- Finite temperature properties of the g2SC phase are rather unusual [$\Delta(T)$ is nonmonotonic; T_c/Δ_0 is nonuniversal]
- There is a new type plasma instability for a range of parameters in g2SC phase and even in 2SC phase!
- Part of plasma instability is removed by spin-1 condensates
- There are additional (BEC) gluon condensates in 2SC phase