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POLYTECHNIC CAMPUS

Huada School on QCD
JUN 1, 2017
CCNU, Wuhan

Lecture #5 Magnetic catalysis in graphene

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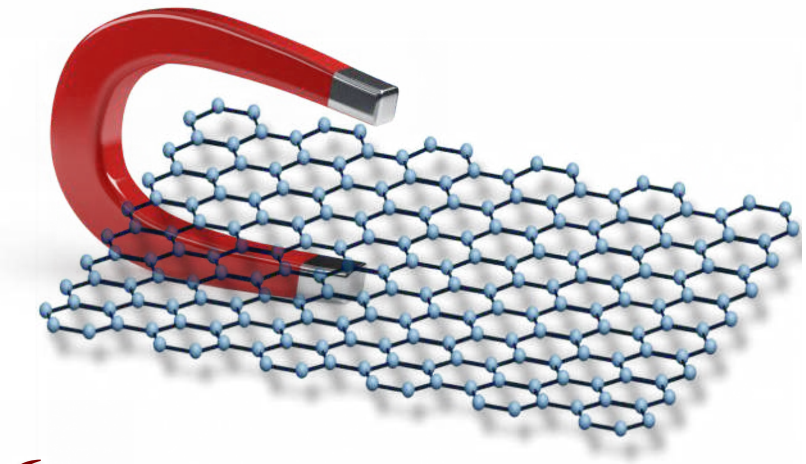
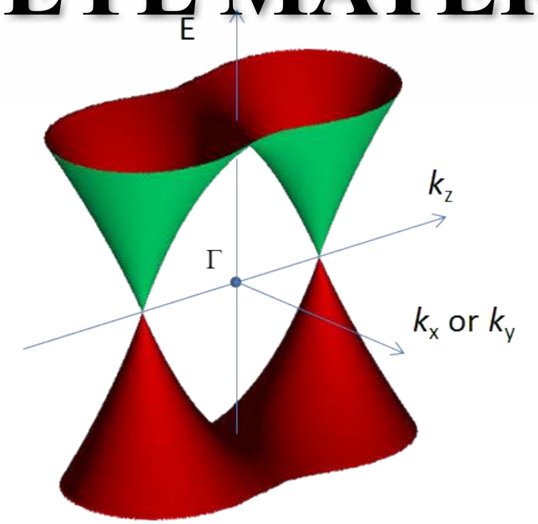
Reading material: V.A. Miransky & I.A. Shovkovy, Physics Reports **576** (2015)

LECTURE #5
MAGNETIC CATALYSIS IN GRAPHENE



LECTURE #6
3D DIRAC & WEYL MATERIALS

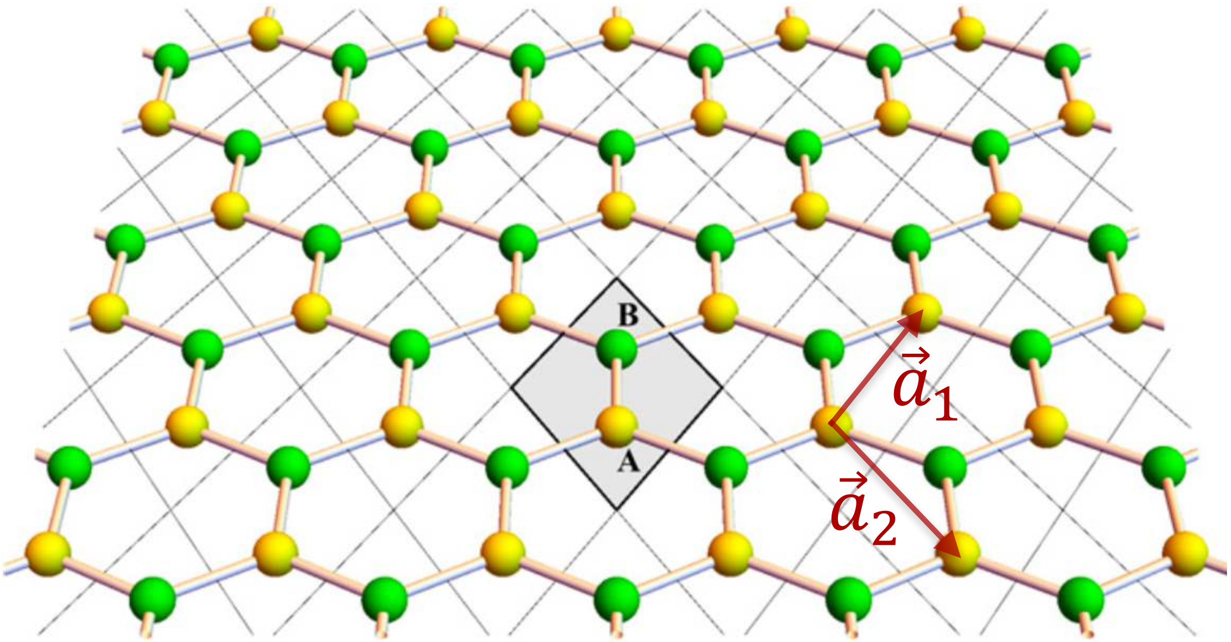


Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

Graphene

- It is a single atomic layer of graphite with interesting basic physics [Novoselov et al., Science **306**, 666 (2004)]

- 2D crystal with hexagonal lattice of carbon atoms



$$\vec{a}_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\vec{a}_2 = a\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

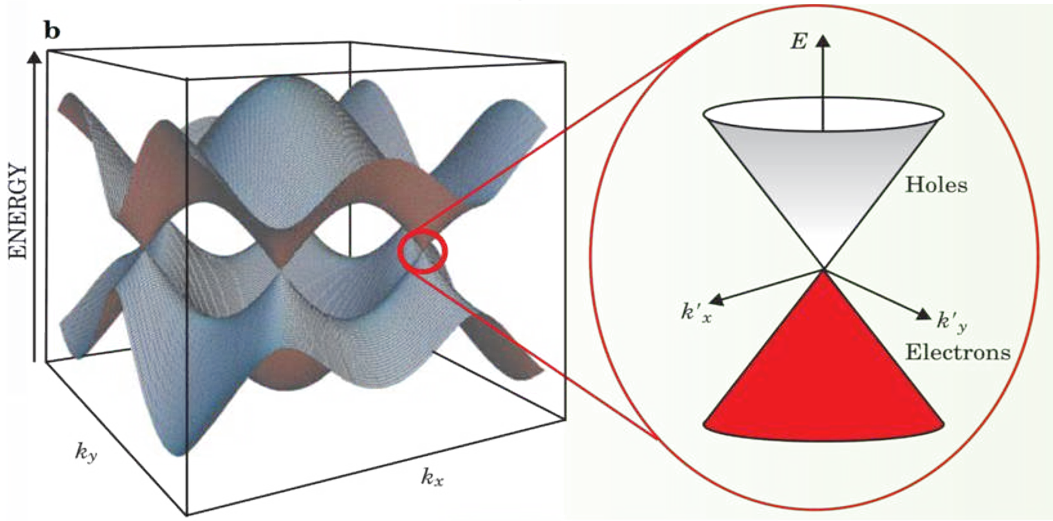
- Tight-binding model

$$H = -t \sum_{\mathbf{n},\boldsymbol{\delta}_i,\sigma} \left[a^{\dagger}_{\mathbf{n},\sigma} \exp\left(\frac{ie}{\hbar c} \boldsymbol{\delta}_i \mathbf{A}\right) b_{\mathbf{n}+\boldsymbol{\delta},\sigma} + \text{c.c.} \right]$$

$a^{+}_{\mathbf{n}}$ and $b_{\mathbf{n}}$ are creation/annihilation operators on sublattice A/B

DISPERSION RELATION

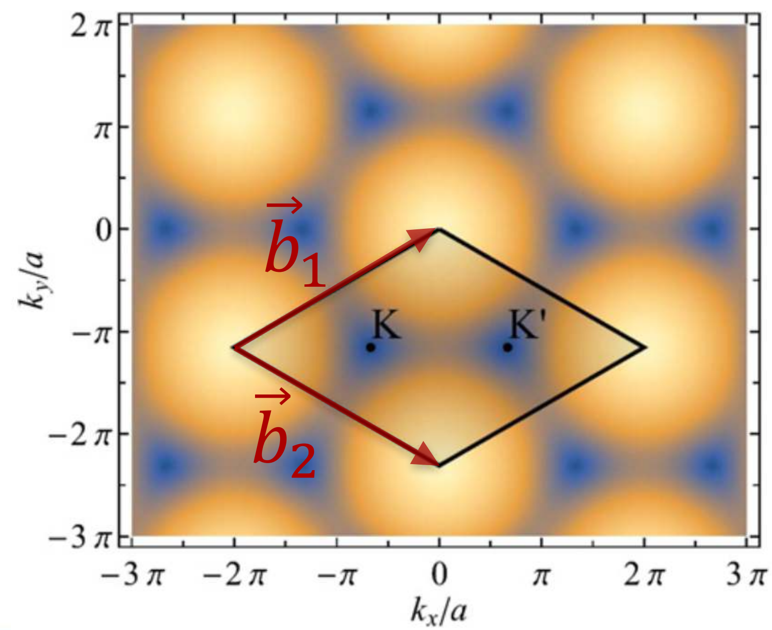
- Energy spectrum $\epsilon(\mathbf{k}) = t\sqrt{1 + 4\cos^2\frac{k_x a}{2} + 4\cos\frac{k_x a}{2}\cos\frac{\sqrt{3}k_y a}{2}}$



- Brillouin zone in momentum space

$$\vec{b}_1 = \frac{2\pi}{a}\left(1, \frac{1}{\sqrt{3}}\right)$$

$$\vec{b}_2 = \frac{2\pi}{a}\left(1, -\frac{1}{\sqrt{3}}\right)$$



Dirac Fermions in Graphene

- Low energy quasiparticles in the vicinity of K and K' points are **massless** Dirac fermions ($v_F \approx c/300$) [Semenoff, PRL **53**, 2449 (1984)]
- The components of two Dirac spinors ($s = \pm {}^1/_2$):

$$\Psi_s^T = (\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, -\psi_{K'As})$$

- The low-energy Hamiltonian has $U_\downarrow(2) \times U_\uparrow(2)$ symmetry

$$H_{\text{tot}} \equiv H + \int d^2r \, (\underbrace{\mu_B B \Psi^\dagger \sigma^3 \Psi}_{\text{Zeeman energy}} - \mu_0 \Psi^\dagger \Psi)$$

where $H = H_0 + H_C$ and

$$H_0 = v_F \int d^2r \, \overline{\Psi} \left(\gamma^1 \pi_x + \gamma^2 \pi_y\right) \Psi \quad \text{\} Free Hamiltonian}$$

$$H_C = \frac{1}{2} \int d^2r d^2r' \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) U_C(\mathbf{r} - \mathbf{r}') \Psi^\dagger(\mathbf{r}') \Psi(\mathbf{r}') \quad \text{\} Coulomb}$$

- Note that H_0 and H_C have $U(4)$ symmetry with the generators

$$\frac{\sigma_\alpha}{2}, \quad \frac{\sigma_\alpha}{2i}\gamma^3, \quad \frac{\sigma_\alpha}{2}\gamma^5, \quad \frac{\sigma_\alpha}{2}[\gamma^3, \gamma^5]$$

Magnetic catalysis in graphene (?)

- Graphene is a perfect playground for magnetic catalysis
  - Charge carriers are spin-½ Dirac fermions with $m=0$
  - Strong magnetic field limit is easy to achieve even at room temperature (!)

$$\epsilon_B \equiv \sqrt{2\hbar|eB_\perp|v_F^2/c} \simeq 424\sqrt{|B_\perp[\mathrm{T}]|}\ \mathrm{K}$$

  - Dimensional reduction from planar (2+1)D down to (0+1)D makes pairing perturbative (!)
  - Dynamical generation of a relatively large $m_{\mathrm{dyn}}\neq 0$ is expected even at weak coupling
  - Large global symmetry suggests a rich dynamics
  - Direct tabletop experiments are relatively easy to design
- In fact, it was predicted before graphene was discovered (!)

[Khveshchenko, PRL **87**, 206401 (2001)]

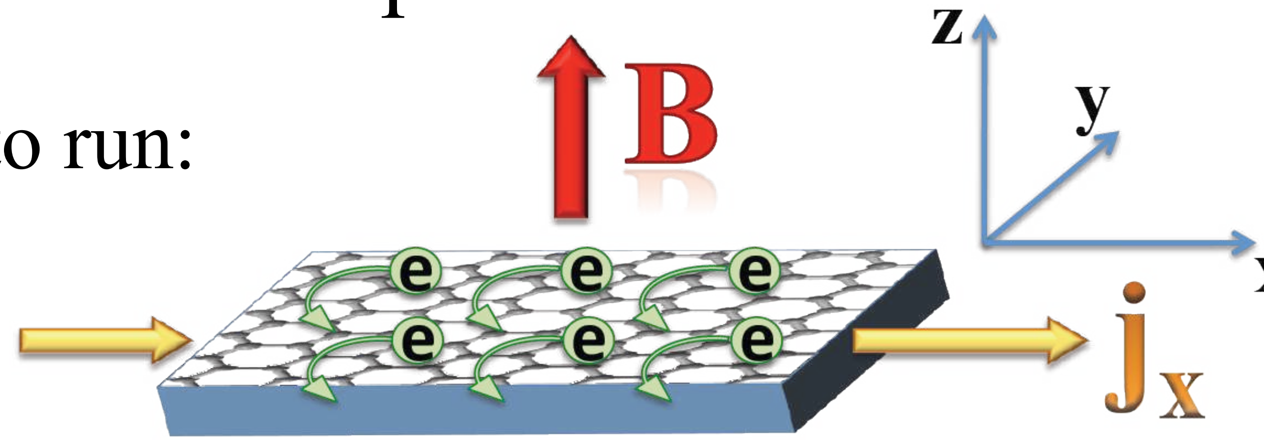
[Gorbar, Gusynin, Miransky, Shovkovy, PRB **66** (2002) 045108]

POTENTIAL COMPLICATIONS

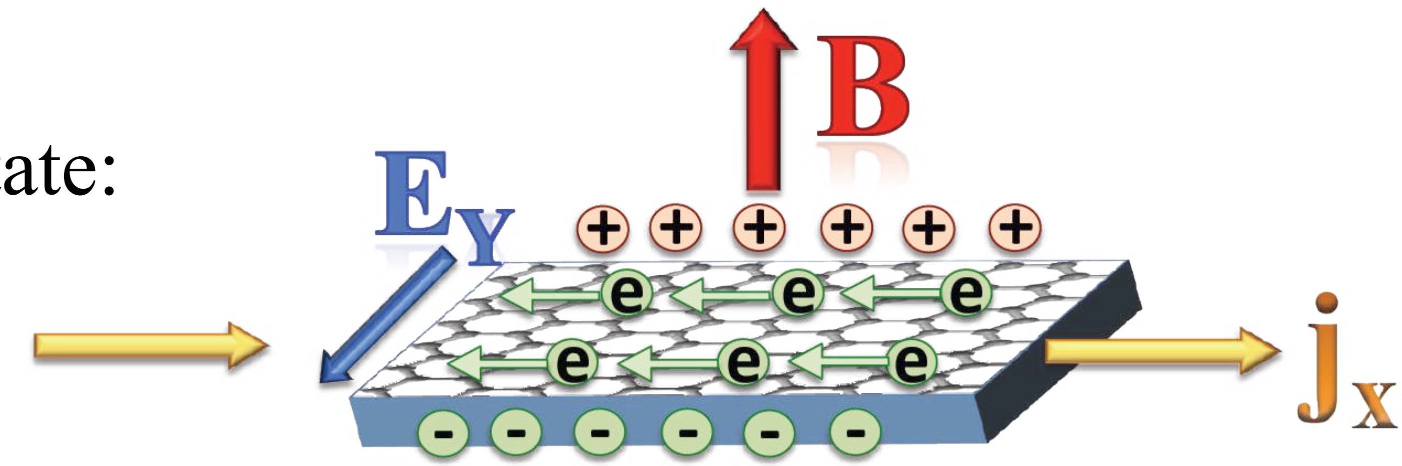
- Graphene is a real condensed matter material
  - it is not exactly flat in 2D (ripples, bending, etc.)
  - various lattice defects may exist
  - it not perfectly clean (impurities)
  - boundary effects play a role in finite size samples
  - non-uniform charge distribution (electron/hole puddles)
  - nonzero average electron/hole density
  - high symmetry implies many possible order parameters
  - Zeeman energy effects, $\epsilon_Z = \mu_B B \approx 0.67\, B[T]\, K$
  - competition with quantum Hall ferromagnetism

How to set an experiment?

- Quantum Hall effect setup
  - Current starts to run:



  - Steady state:



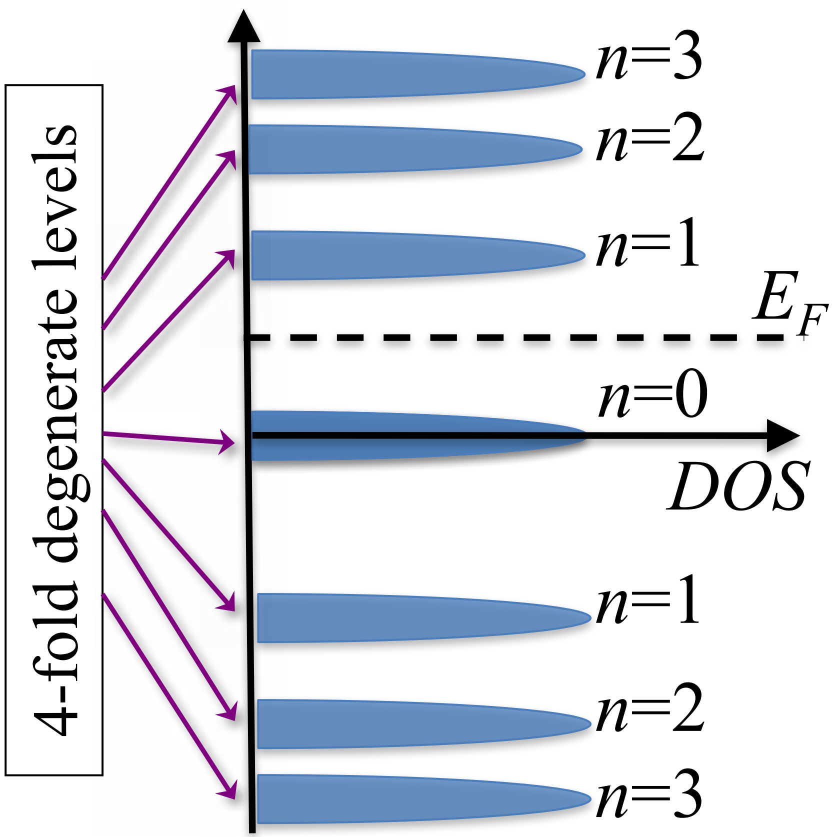
  - Hall conductivity:

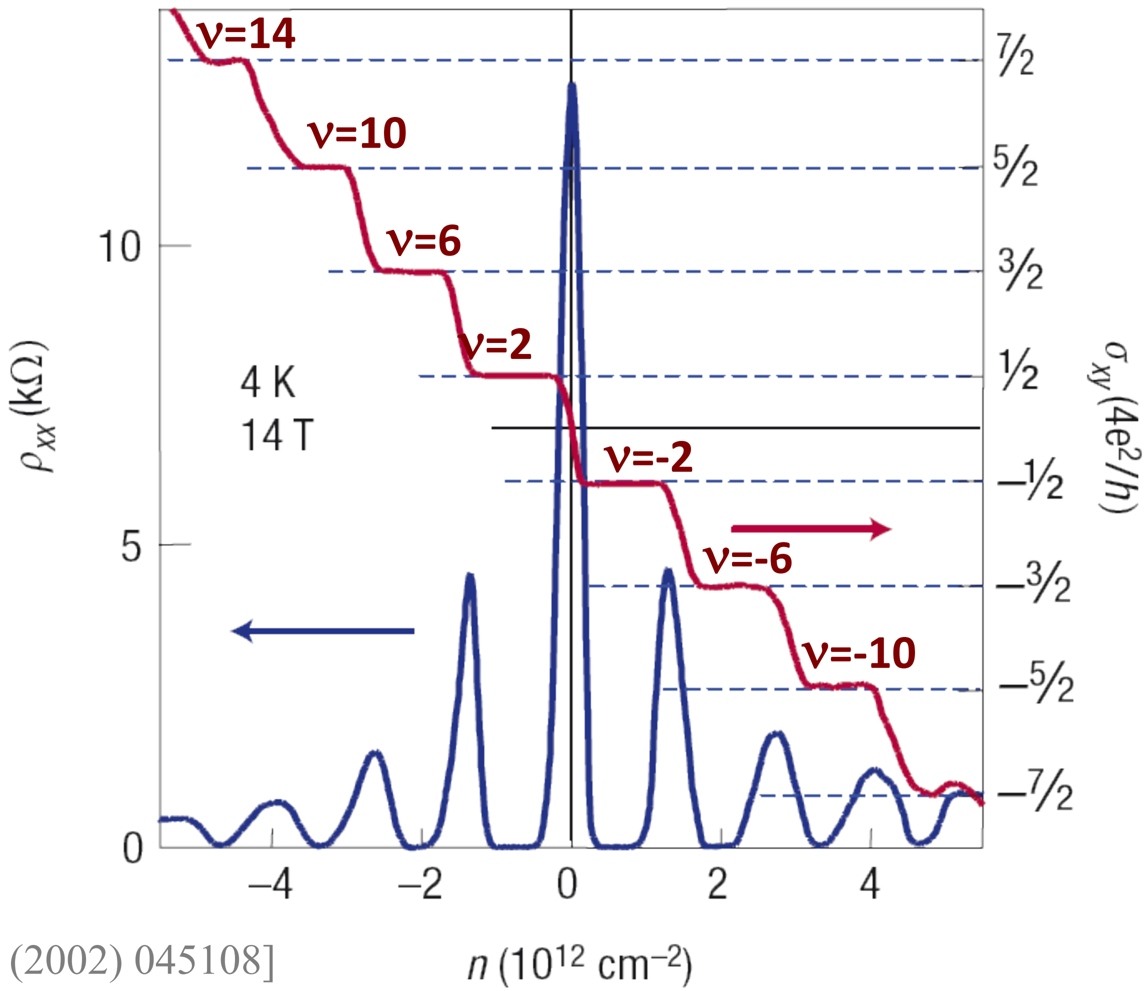
$$j_x = \sigma_{xy} E_y$$

QHE in graphene: weak B

$$E_n^{\pm} = \pm\sqrt{2\hbar v_F^2 n|eB|}$$

$$\sigma_{xy} = \nu \frac{e^2}{h} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$$





[Gorbar, Gusynin, Miransky, Shovkovy, PRB **66** (2002) 045108]
[Gusynin, Sharapov, Phys. Rev. Lett. **95**, 146801 (2005)]
[Peres, Guinea, Castro Neto, Phys. Rev. B **73**, 125411 (2006)]
[Novoselov et al., Nature **438**, 197 (2005)], [Zhang et al., Nature **438**, 201 (2005)]

ANOMALOUS QHE: STRONG B

- New plateaus are observed at filling factors

  $\nu=0$

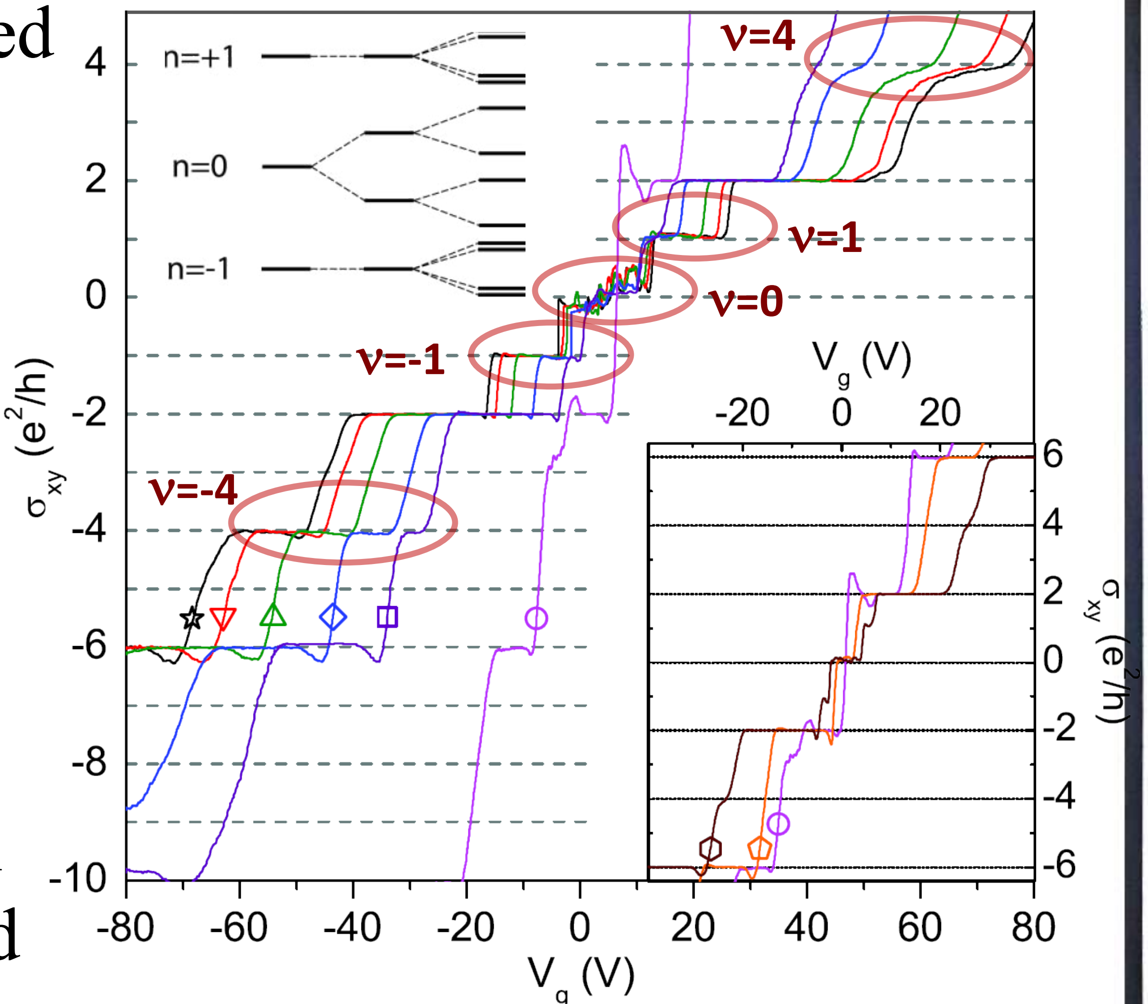
  $\nu=\pm1$

  $\nu=\pm3$

  $\nu=\pm4$

- Degeneracies of some Landau level are lifted in sufficiently strong B field

Zhang et al., PRL **96,** 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]
[Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)]
[Jiang et al., Phys. Rev. Lett. **99**, 106802 (2007)]
[Checkelsky et al., Phys. Rev. Lett. **100**, 206801 (2008)]
[Xu Du et al., Nature **462**, 192 (2009)]

[Bolotin et al., Nature **462**, 196 (2009)]
[Young et al., Nat. Phys. **8**, 550 (2012)]
[Young et al., Nature **505**, 528 (2014)]
[Chiappini et al., Phys. Rev. B **92**, 201412(R) (2015)]

ORDER PARAMETERS

- Many order parameters may be generated (due to various types of pairing from different valleys/sublattices)
- Parity odd (Haldane) and even (Dirac) masses are possible

$$\Delta_s: \quad \bar{\Psi}\gamma^3\gamma^5 P_s \Psi = \psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} + \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

$$\tilde{\Delta}_s: \quad \bar{\Psi} P_s \Psi = \psi^\dagger_{KAs}\psi_{KAs} + \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

- The former (latter) are singlets (triplets) under $U(2)_s$
- Similar singlet (triplet) types of (pseudo-)spin densities

$$\mu_3: \quad \Psi^\dagger \sigma^3 \Psi = \frac{1}{2}\sum_{\kappa=K,K'}\sum_{a=A,B}\left(\psi^\dagger_{\kappa a+}\psi_{\kappa a+} - \psi^\dagger_{\kappa a-}\psi_{\kappa a-}\right)$$

$$\tilde{\mu}_s: \quad \Psi^\dagger\gamma^3\gamma^5 P_s \Psi = \psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} + \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Rev. B **78** (2008) 085437]
[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scr. T **146** (2012) 014018]

- A rather flexible form of the fermion propagator is needed

Running dynamical parameters?

- One approach is to use the Ritus method

$$\bar{S}(x,y)=\sum_{n=0}^{\infty}\int\frac{dp_0dp_2}{(2\pi)^3}E_p(x)\frac{1}{(\bar{\mathbf{p}}\cdot\gamma)+\Sigma(p)}\bar{E}_p(y)$$

where

$$\bar{\mathbf{p}}=\left(p_0,0,-\sqrt{2|eB|n}\right)\quad\text{and}\quad\bar{E}_p(x)=\gamma^0E_p^{+}(x)\gamma^0$$

$$(\pi_\nu\gamma^\nu)E_p(x)=E_p(x)(\bar{\mathbf{p}}\cdot\gamma)$$

- Shortcomings:
  - Schwinger phase is never fully factorized
  - Eigenstates $E_p(x)$ are Dirac matrices
  - Obscure meaning of parameters in Σ(p)
- There is a better way

A better representation?

- The general form of the full propagator

$$G(x,y) = i\left\langle x\left|\left[k_0\gamma^0 + \hat{F}^+\left(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}\right) + \hat{\Sigma}^+\right]^{-1}\right|y\right\rangle$$

where $\hat{F}^{\pm}$ & $\hat{\Sigma}^{\pm}$ are functions of $\gamma^0, i\gamma^1\gamma^2, \& \left(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}\right)^2$

3 mutually commuting operators

$$\hat{F}^{\pm} = f \pm \gamma^0 g \pm i\gamma^1\gamma^2\tilde{g} + i\gamma^0\gamma^1\gamma^2\tilde{f}$$

$$\hat{\Sigma}^{\pm} = m \pm \gamma^0\mu \pm i\gamma^1\gamma^2\tilde{\mu} + i\gamma^0\gamma^1\gamma^2\Delta$$

Dirac mass

Haldane mass

and $f, g, \tilde{g}, \tilde{f}, m, \mu, \tilde{\mu}, \Delta$ depend on $\left(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}\right)^2$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

COMPLETE SET OF EIGENSTATES

- As before, the eigenstates for orbital motion:

$$\boldsymbol{\pi}^2 \left| N\, p \right\rangle = \left| eB \right| (2N+1) \left| N\, p \right\rangle$$

E.g., in the Landau gauge **A**=(0,*Bx*),

$$\left\langle \mathbf{r} \middle| N\, p \right\rangle = C_N H_N \left(\frac{x}{l} + pl \right) \exp\left(-\frac{(x+pl^2)^2}{2l^2} + ipy \right)$$

They satisfy

$$\int d^2\mathbf{r} \left\langle N\, p \middle| \mathbf{r} \right\rangle \left\langle \mathbf{r} \middle| N'\, p' \right\rangle = \delta_{NN'} \delta\left(p - p' \right)$$

$$\sum_{N=0}^{\infty} \int_{-\infty}^{\infty} dp \left\langle \mathbf{r} \middle| N\, p \right\rangle \left\langle N\, p \middle| \mathbf{r}' \right\rangle = \delta\left(\mathbf{r} - \mathbf{r}' \right)$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

GENERALIZED LL REPRESENTATION

- Then, the new representation in (2+1)D

$$G(x,y) = \exp\left(-e\int_y^x A_\mu \, dz^\mu\right)\bar{G}(x-y)$$

Schwinger phase

$$\bar{G}(t,\mathbf{r}) = \int \frac{dk_0}{2\pi} e^{-ik_0 t}\bar{G}(k_0,\mathbf{r})$$

where

$$\bar{G}(k_0,\mathbf{r}) = i\frac{e^{-\xi/2}}{2\pi l^2}\sum_{n,\sigma,s}\left(\frac{s(k_0+\mu_{n,\sigma})-m_{n,\sigma}}{(k_0+\mu_{n,\sigma})^2-E_{n,\sigma}^2}\left[\delta^s_{-\sigma}L_n(\xi)+\delta^s_{\sigma}L_{n-1}(\xi)\right]\right.$$

$$\left.+\frac{i(f_{n,\sigma}-s g_{n,\sigma})}{(k_0+\mu_{n,\sigma})^2-E_{n,\sigma}^2}\frac{(\gamma\cdot\mathbf{r})}{l^2}\right)P_{s,s\sigma}, \qquad \xi = \mathbf{r}^2/(2l^2)$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

LL ENERGIES

- Landau-level energies

$$n=0: \quad E_{0,\sigma}=\Delta_0+\sigma m_0$$

$$n\geq 1: \quad E_{n,\sigma}=\sqrt{2(f_{n,\sigma}^2-g_{n,\sigma}^2)n|eB|+m_{n,\sigma}^2}$$

where $\quad m_{n,\sigma}=m_n+\sigma\Delta_n, \quad \mu_{n,\sigma}=\mu_n+\sigma\tilde{\mu}_n, \quad$ etc.

By definition, $n=N+(1+s_{12})/2$ and

$$m\left((\boldsymbol{\pi}\cdot\boldsymbol{\gamma})^2\right)\left|N,s_{12}\right\rangle=m_n\left|N,s_{12}\right\rangle,$$

$$f\left((\boldsymbol{\pi}\cdot\boldsymbol{\gamma})^2\right)\left|N,s_{12}\right\rangle=f_n\left|N,s_{12}\right\rangle, \quad \text{etc.}$$

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

REFINED THE MODEL

- Model Hamiltonian

U(4)

$SU(2)_{\uparrow} \times SU(2)_{\downarrow}$

$$H = H_0 + H_{\text{Coulomb}} + \mu_B B \int d^2 r \Psi^+ \sigma_3 \Psi$$

- Now, with multiple MC and QHF order parameters

$m_{n,s}$:	$\bar{\Psi}_s \Psi_s$	Dirac mass (CDW)
$\Delta_{n,s}$:	$\bar{\Psi}_s (i\gamma^0\gamma^1\gamma^2) \Psi_s$	Haldane mass
$\tilde{\mu}_{n,s}$:	$\bar{\Psi}_s (i\gamma^1\gamma^2) \Psi_s$	pseudospin density
$\mu_{n,3}$:	$\bar{\Psi}_{\uparrow}\gamma^0\Psi_{\uparrow} - \bar{\Psi}_{\downarrow}\gamma^0\Psi_{\downarrow}$	spin polarization

- Recall that the spinors are defined by

$$\Psi_s^T = (\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, -\psi_{K'As})$$

NORMAL GROUND STATE (WEAK B)

- No symmetry breaking parameters
- Possible renormalization of v_F:

$$n \geq 1: \quad E_n = v_F \sqrt{2n|eB|} \quad \longrightarrow \quad E_n = f_n v_F \sqrt{2n|eB|}$$

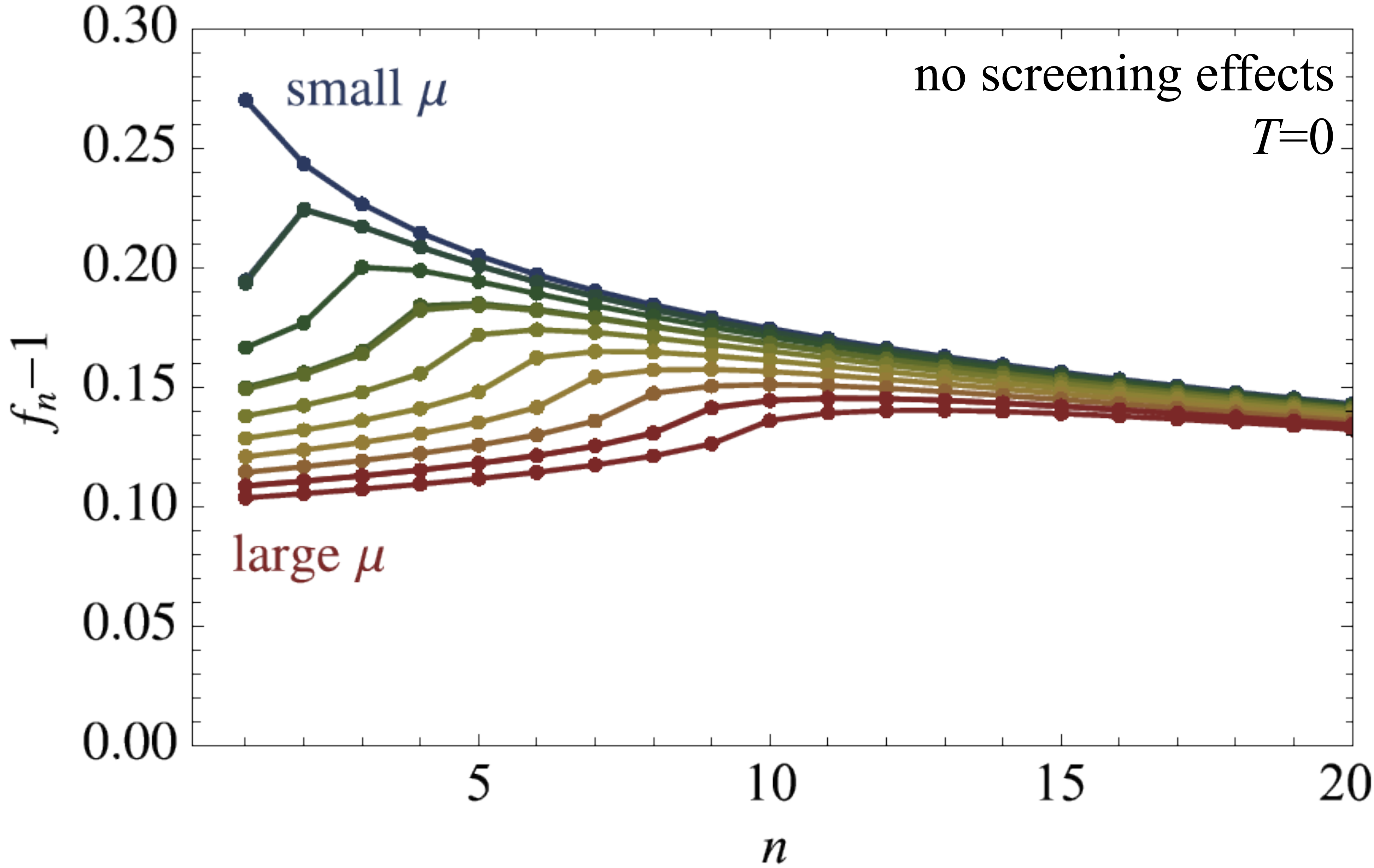
- Schwinger-Dyson equations for f_n

$$f_n = 1 + \frac{\alpha}{2} \sum_{n=1}^{n_{\max}} \frac{\kappa^{(1)}_{n'-1,n-1}}{n\sqrt{2n'}} \left[1 - n_F\left(E_{n'} - \mu\right) - n_F\left(E_{n'} + \mu\right)\right]$$

$$\kappa^{(i)}_{m,n} = \int_0^\infty \frac{dk}{2\pi} \frac{kl\ L^{(i)}_{m,n}(kl)}{k + \Pi(0,k)}, \quad \text{where} \quad l = 1/\sqrt{|eB|}$$

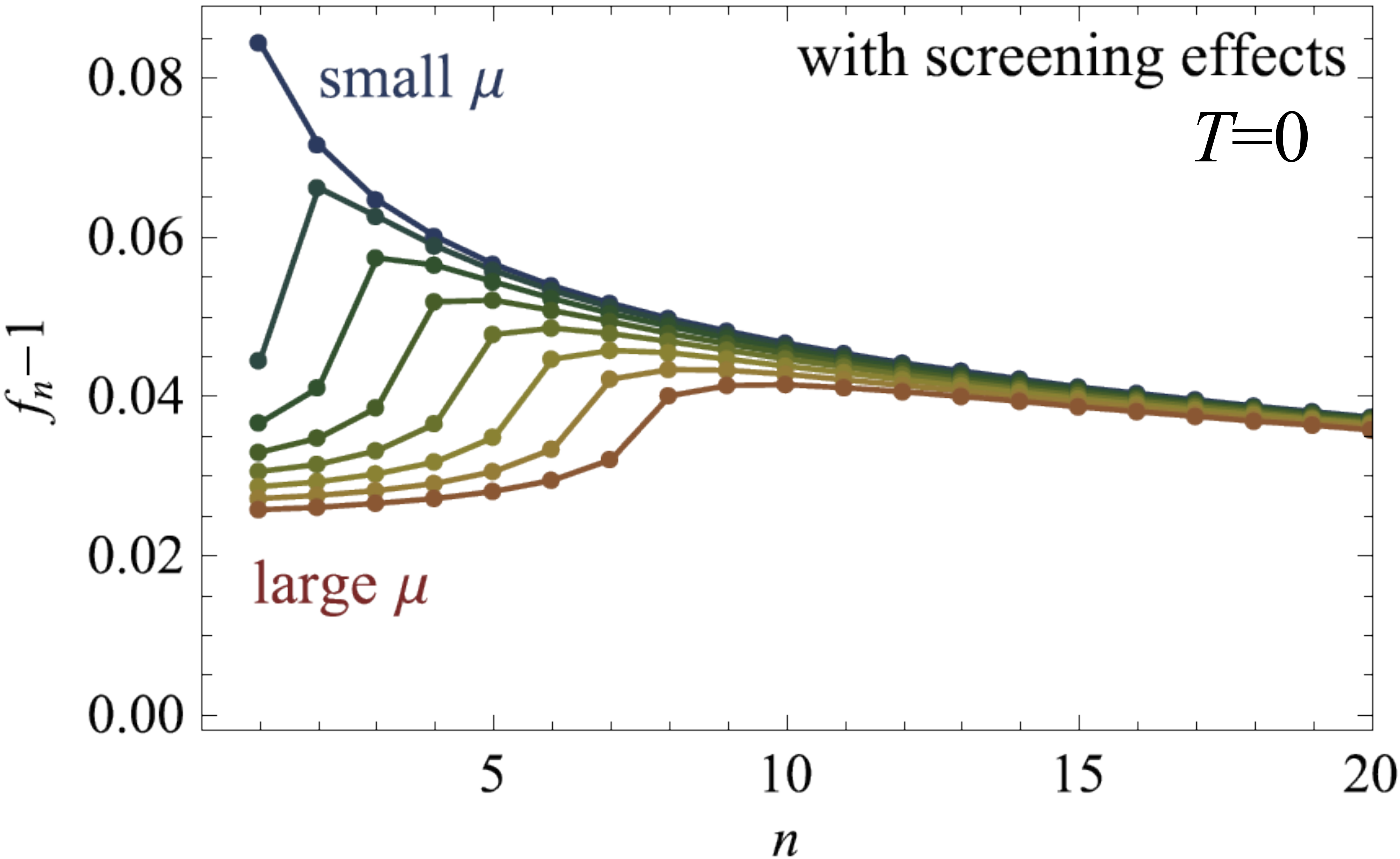
RENORMALIZATION OF v_F

[Gorbar, Gusynin, Miransky, Shovkovy, Phys. Scripta T146, 014018 (2012)]

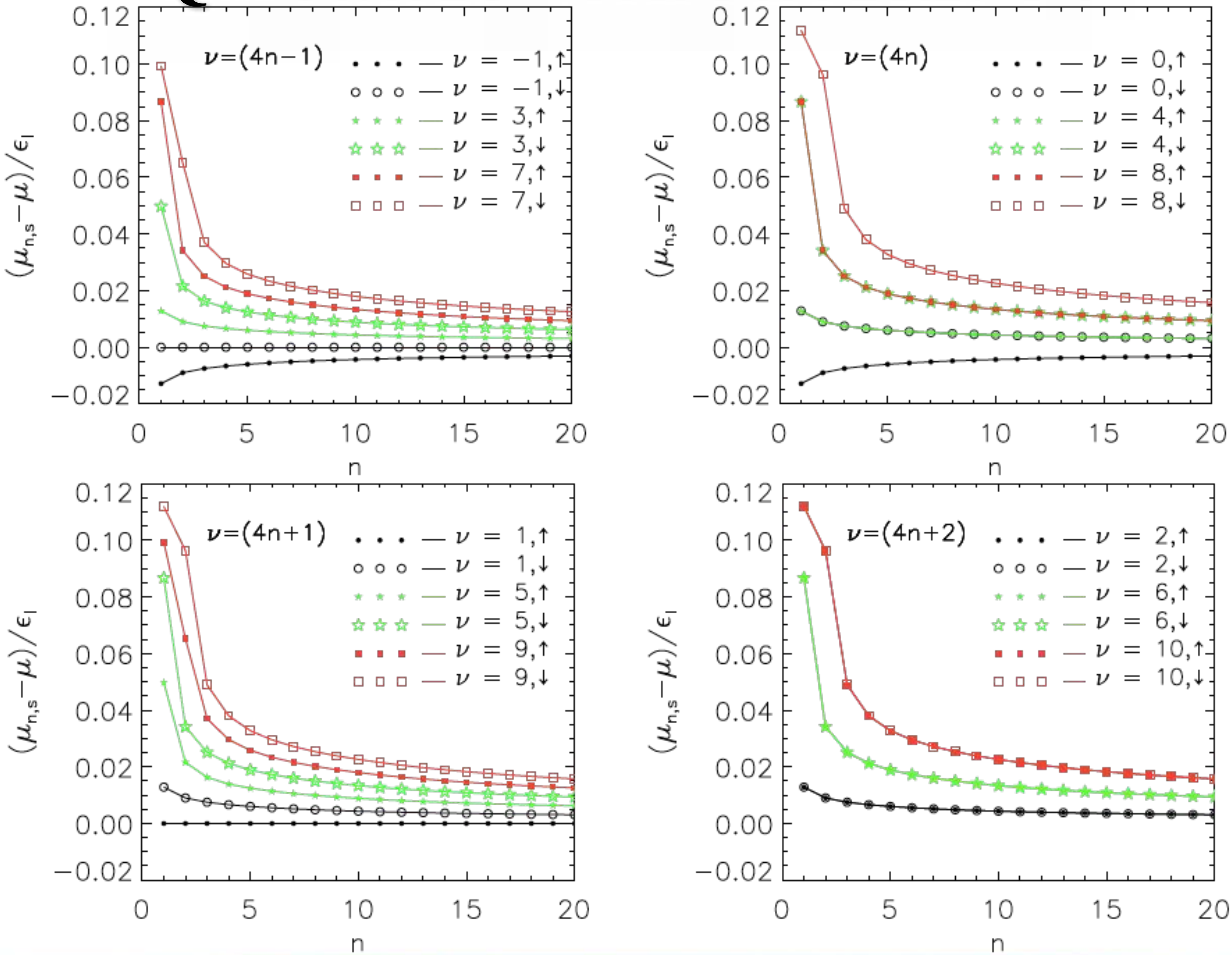


RENORMALIZATION OF v_F

[Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]

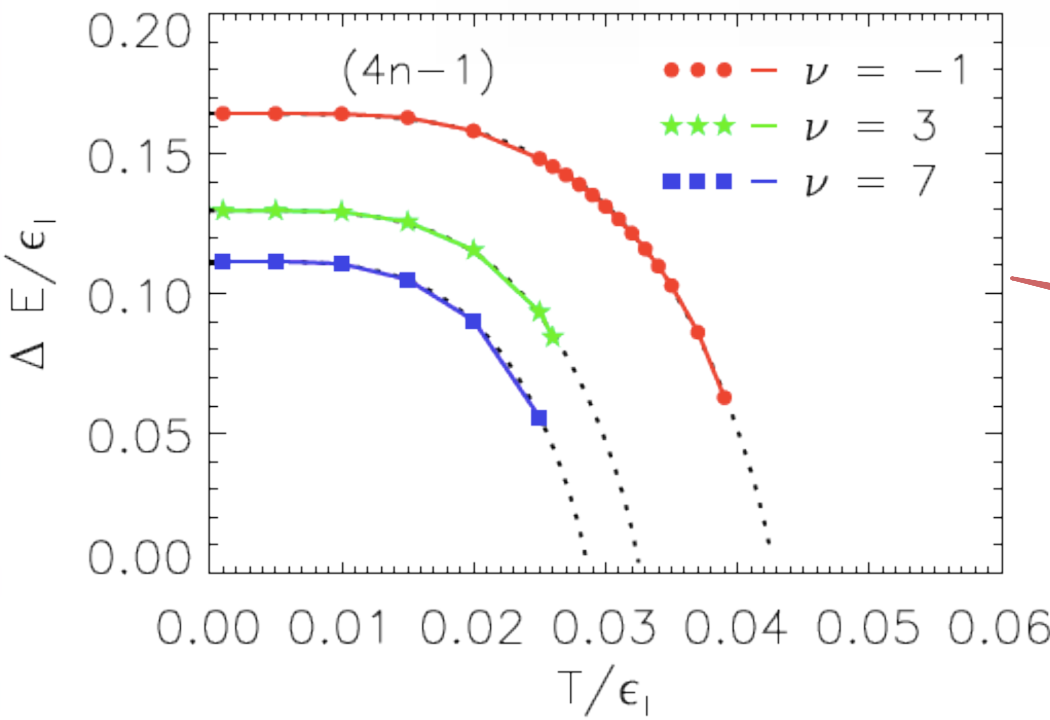


QH STATES WITH DIFFERENT ν



[Shovkovy & Lifang Xia, Phys. Rev. B **93**, 035454)]

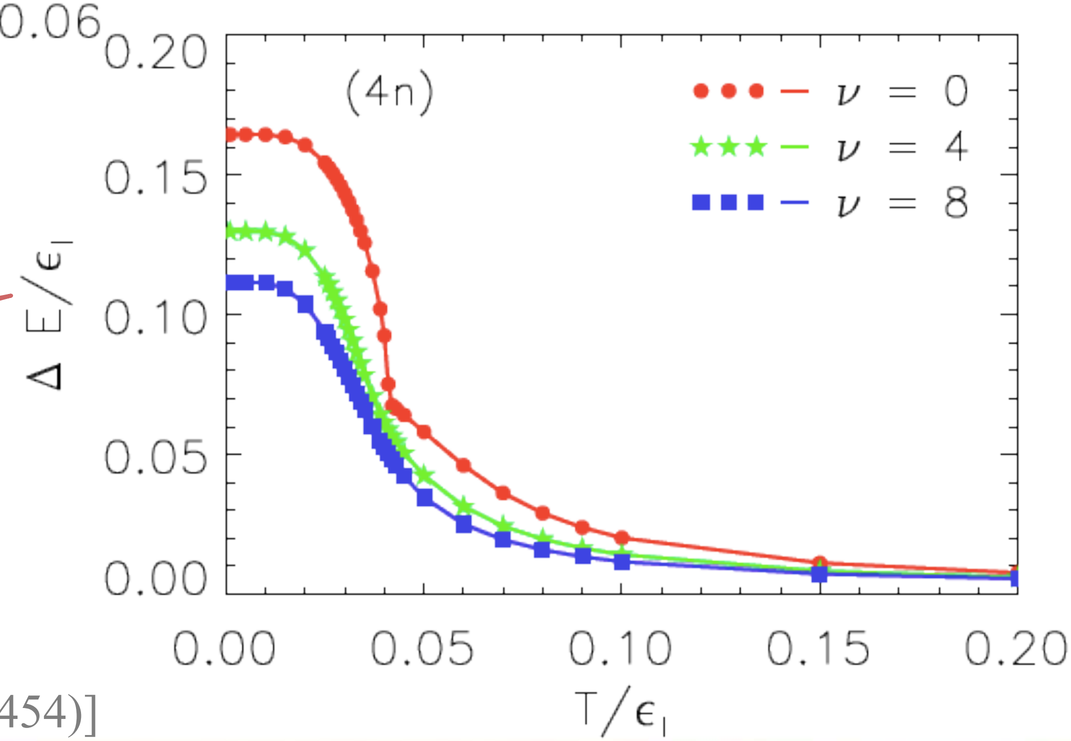
Some results at T≠0



$$\Delta E = \Delta E^{(0)}\left[1-\left(\frac{T}{T_c}\right)^4\right]^{0.8}$$

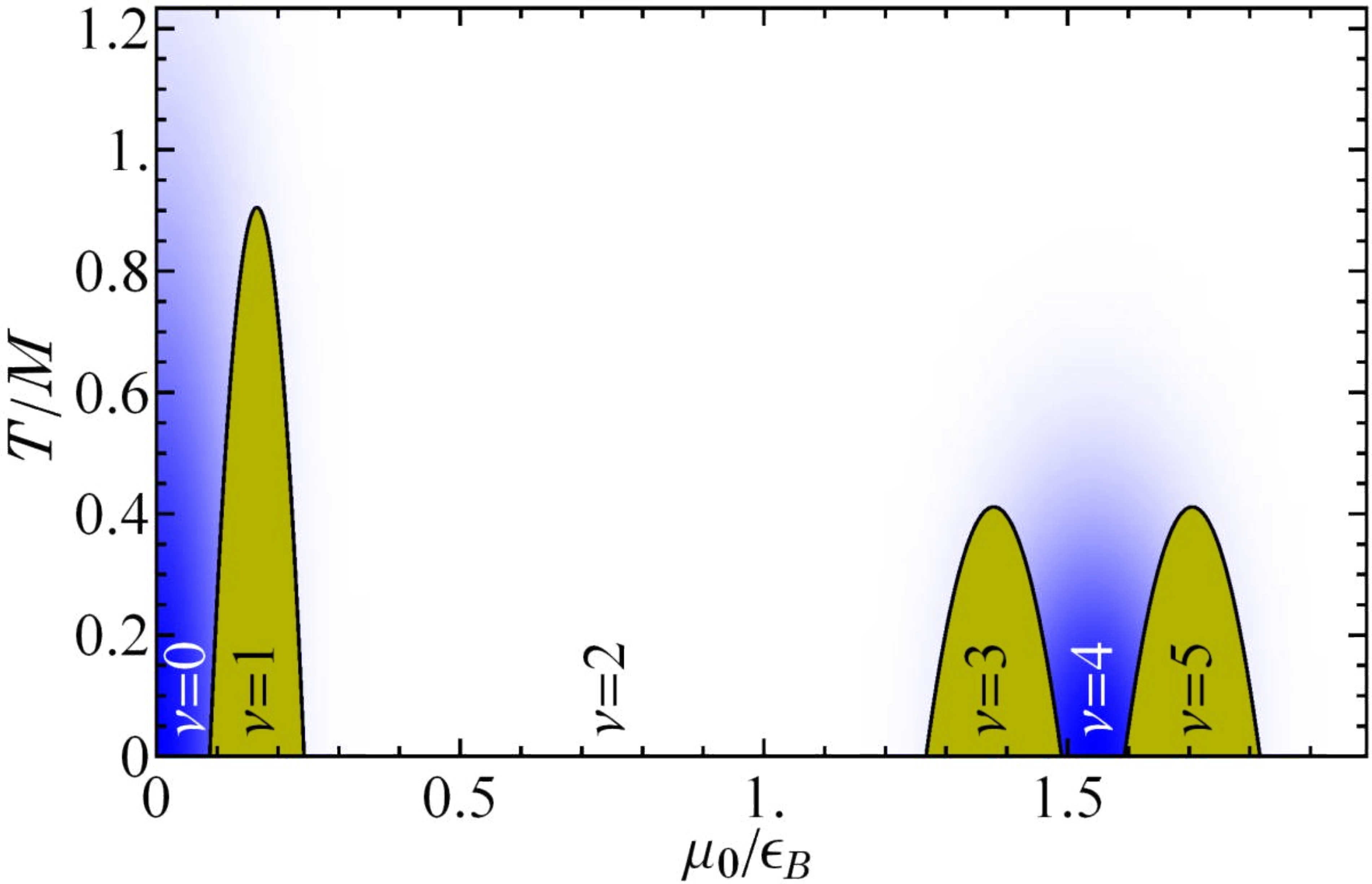
$U(1)_{\uparrow}\times SU(2)_{\downarrow}$

$SU(2)_{\uparrow}\times SU(2)_{\downarrow}$



[Shovkovy & Lifang Xia, Phys. Rev. B **93**, 035454)]

SCHEMATIC PHASE DIAGRAM



ARIZONA STATE UNIVERSITY

POLYTECHNIC CAMPUS

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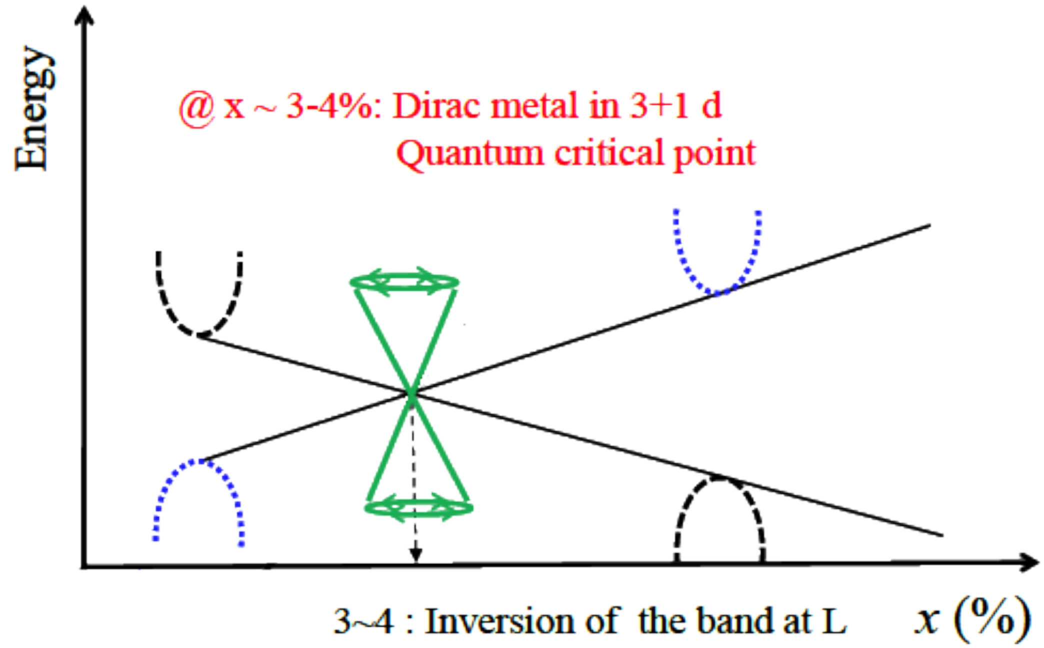
LECTURE #6
3D DIRAC & WEIL MATERIALS

Igor Shovkovy
Arizona State University

Summer School on Frontiers in Theoretical Physics and Huada School on QCD: Chiral Anomaly and Strong Magnetic Fields in Heavy-ion Collisions

DIRAC SEMIMETALS

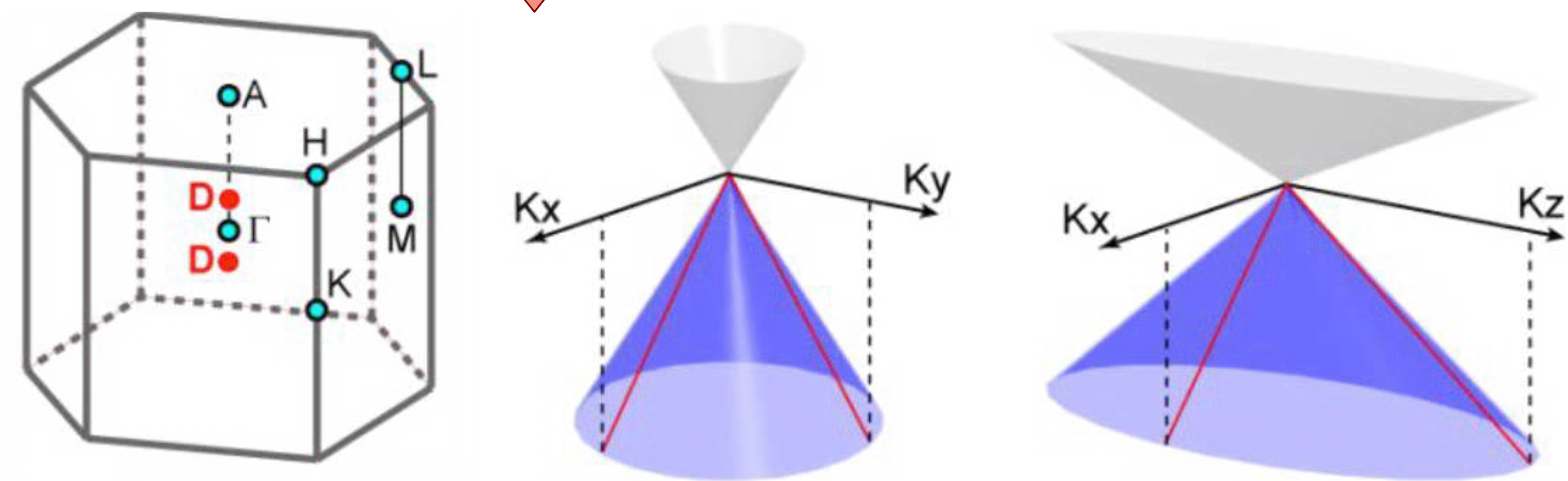
- Solid state materials with Dirac quasiparticles:
  - $Bi_{1-x}Sb_x$ alloy



- "New" 3D Dirac materials (ARPES):
  - Na_3Bi (Potassium bismuthide) [Liu et al., Science **343**, 864 (2014)]
  - Cd_3As_2 (Cadmium arsenide) [Neupane et al., Nature Commun. **5**, 3786 (2014)] [Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]

DIRAC MATERIALS

- $Bi_{1\text{-}x}Sb_x$ alloy (at $x \approx 4\%$)
- Na_3Bi → [Liu et al., Science **343**, 864 (2014)]
- Cd_3As_2 → [Neupane et al., Nature Commun. **5**, 3786 (2014)]; [Borisenko et al., Phys. Rev. Lett. **113**, 027603 (2014)]
- $ZrTe_5$ → [Li et al., Nature Physics **12**, 550 (2016)]



$$E = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + v_z^2 k_z^2}, \quad v_x \approx v_y \approx 3.74\times10^5\ m/s, \quad v_y \approx 2.89\times10^4\ m/s$$

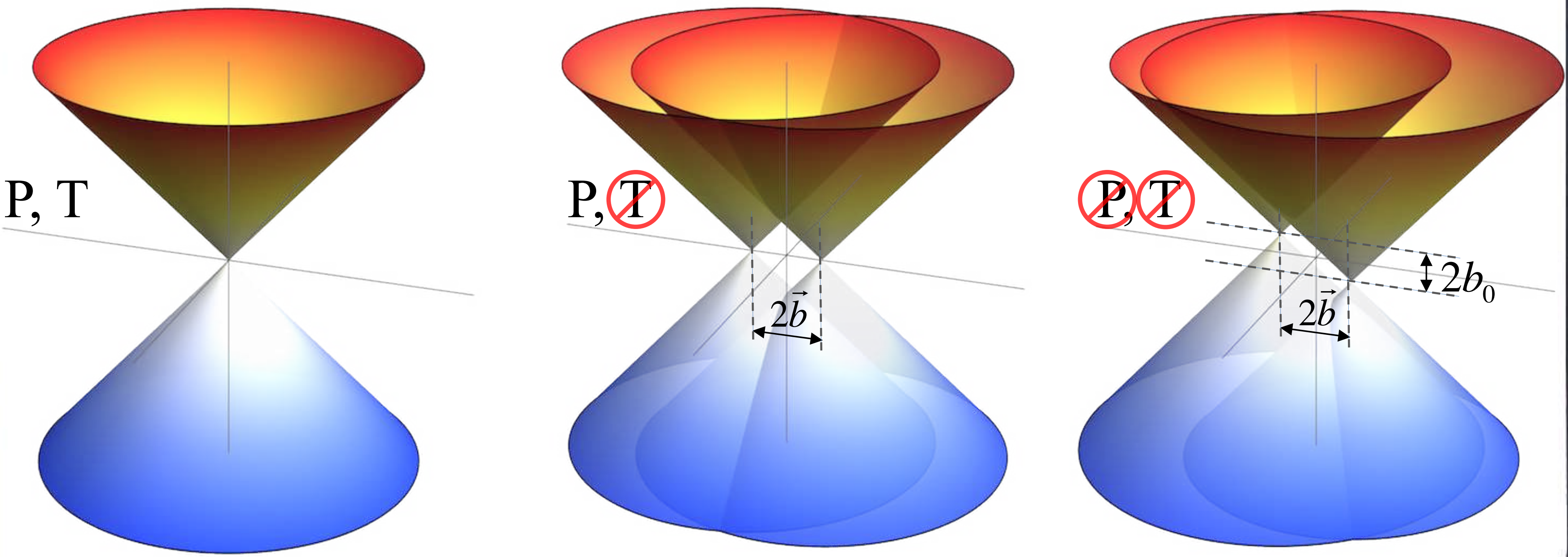
DIRAC VS. WEYL MATERIALS

- Low-energy Hamiltonian of a Dirac/Weyl material

T P

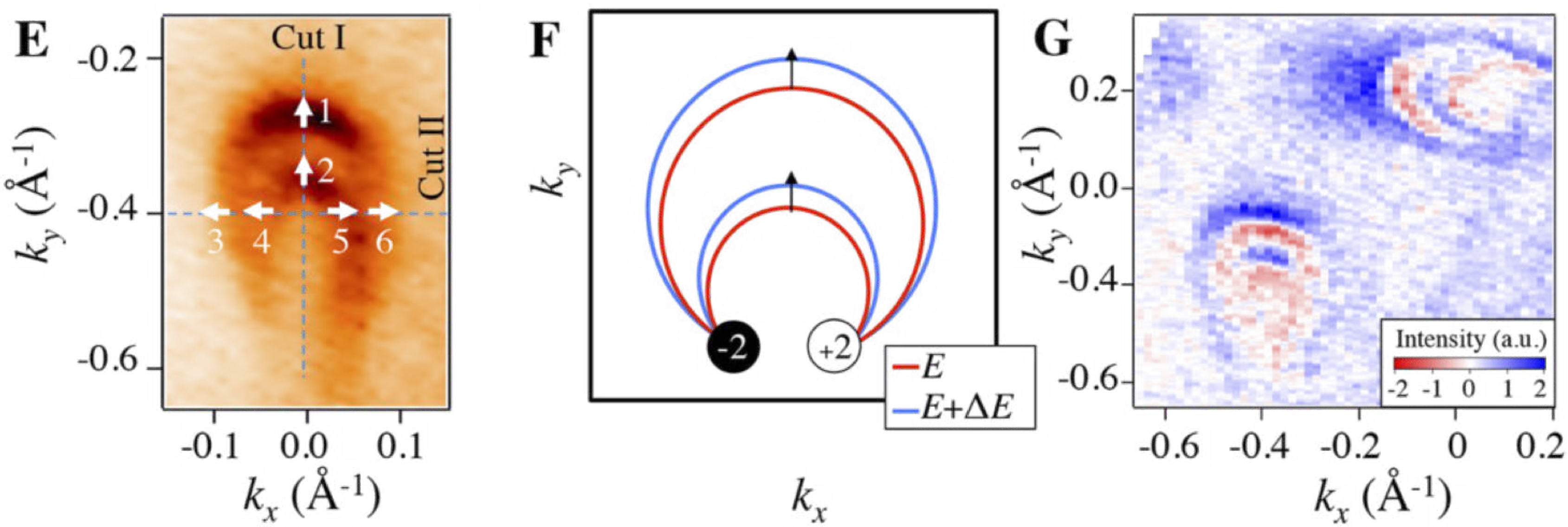
$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5 + b_0\gamma^0\gamma^5\right]\psi$$

Dirac Weyl



WEYL MATERIALS

- TaAs (tantalum arsenide) [S.-Y. Xu et al., Science 349, 613 (2015)] [B. Q. Lv et al., Phys. Rev. X5, 031013 (2015)]
- NbAs (niobium arsenide) [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
- TaP (tantalum phosphide) [S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
- NbP (niobium phosphide) [I. Belopolski et al. arXiv:1509.07465]
- WTe_2 (tungsten telluride) [F. Y. Bruno et al., Phys. Rev. B 94, 121112 (2016)]



LOW-ENERGY DIRAC FERMIONS

- The Hamiltonian for massless Dirac fermions is given by

$$H_D = \begin{pmatrix} v_F(\vec{k}\cdot\vec{\sigma}) & 0 \\ 0 & -v_F(\vec{k}\cdot\vec{\sigma}) \end{pmatrix}$$

- This can we viewed as a combination of two Weyl fermions

$$H_\lambda = \lambda v_F(\vec{k}\cdot\vec{\sigma})$$

where $\lambda = \pm 1$ is a chirality

The Weyl energy eigenstates are given by

$$\psi_k^\lambda = \frac{1}{\sqrt{2\epsilon_k}k_\perp}\begin{pmatrix} \sqrt{\epsilon_k + \lambda k_z}\,k_- \\ \lambda\sqrt{\epsilon_k - \lambda k_z}\,k_\perp \end{pmatrix}$$

They described particles of energy $\epsilon_k = v_F\sqrt{k_x^2 + k_y^2 + k_z^2}$

The mapping $k \to \psi_k^\lambda$ has a nontrivial topology

Berry connection & curvature

- Consider evolution from $\psi_{\boldsymbol{k}}$ to $\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}$:

$$\langle \psi_{\boldsymbol{k}} | \psi_{\boldsymbol{k}+\delta\boldsymbol{k}} \rangle \approx 1 + \delta\boldsymbol{k} \cdot \langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle \approx e^{i\boldsymbol{a}_{\boldsymbol{k}}\cdot\delta\boldsymbol{k}}$$

where $\boldsymbol{a}_{\boldsymbol{k}} = -i\langle \psi_{\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi_{\boldsymbol{k}} \rangle$ is the Berry connection

- The Berry curvature is defined as follows:

$$\boldsymbol{\Omega}_k = \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}}$$

- Not the similarity with gauge fields, but $\boldsymbol{a}_{\boldsymbol{k}}$ and $\boldsymbol{\Omega}_k$ are defined in the momentum space
- It is convenient to define the Chern number (flux of $\boldsymbol{\Omega}_k$)

$$C = \frac{1}{2\pi} \oint \boldsymbol{\Omega}_k \cdot d\boldsymbol{S}_{\boldsymbol{k}}$$

- A nonzero (integer) Chern number indicates a nonzero (topological) charge inside the k-volume surrounded by the closed surface (Gauss's law)

Gauge theory vs. Berry effects

Gauge theory	Berry effects
Local at **coordinate** space	Local at **momentum** space
Gauge field $\vec{\boldsymbol{A}}$	Berry connection $\vec{\boldsymbol{a}}$
Magnetic field $\vec{\boldsymbol{B}} = \vec{\boldsymbol{\nabla}}_{\mathbf{r}} \times \vec{\boldsymbol{A}}$	Berry curvature $\vec{\boldsymbol{\Omega}} = \vec{\boldsymbol{\nabla}}_{\mathbf{k}} \times \vec{\mathbf{a}}$
Aharonov-Bohm phase $\oint d\vec{\boldsymbol{r}}\, \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})$	Berry phase $\oint d\vec{\mathbf{k}}\, \vec{\mathbf{a}}(\vec{\mathbf{k}})$
Magnetic charge (Dirac monopole) $\int d\,\vec{\boldsymbol{r}}\,(\vec{\boldsymbol{\nabla}}_{\mathbf{r}} \cdot \vec{\boldsymbol{B}}) = const$	Berry monopole $\int d\,\vec{\mathbf{k}}\,(\vec{\boldsymbol{\nabla}}_{\mathbf{k}} \cdot \vec{\boldsymbol{\Omega}}) = const$

Berry curvature for Weyl fermions

- In the case of Weyl fermions,

$$\boldsymbol{\Omega}_k = \lambda \frac{\vec{k}}{2k^3}$$

(Note: this looks like a field of a monopole at $\vec{k} = 0$)

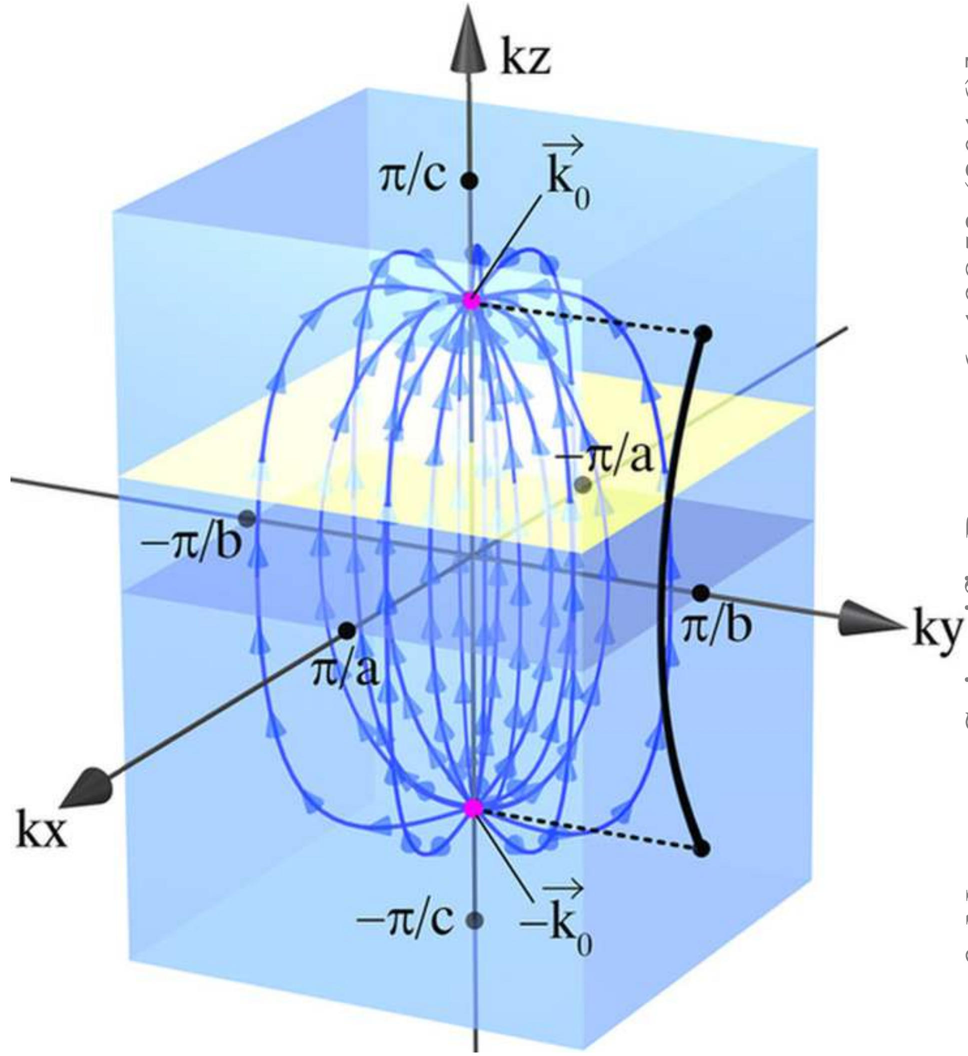
- Let us calculate the total flux of $\boldsymbol{\Omega}_k$-field through the spherical surface of radius K with the center at $\vec{k} = 0$

$$C = \frac{1}{2\pi} \oint \lambda \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda = \pm 1$$

- Thus, the electronic structure of massless Weyl fermions is characterized by a topological monopole at $\vec{k} = 0$
- Is the Berry monopole just a mathematical curiosity?
- Are there any observable consequences?

WEYL FERMIONS ON A LATTICE

- In solid state physics, the momentum space (Brillouin zone) is compact
- A closed surface around a single Weyl node is also a closed surface (of opposite orientation) around a the rest of the Brillouin zone
- Flipping surface orientation changes the sign of the flux
- There must be an opposite charge somewhere in the rest of the zone



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

- Thus, Weyl fermions come in pairs of opposite chirality

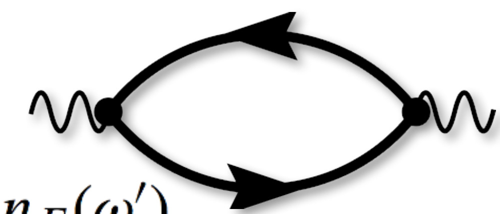
[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]

MAGNETO-CONDUCTIVITY

- Magneto-transport may reveal signature features of Dirac/Weyl materials [Nielsen & Ninomiya, PLB **130**, 389 (1983)],[Aji, PRB **85**, 241101 (2012)], [Son & Spivak, PRB **88**, 104412 (2013)], [Gorbar, Miransky & Shovkovy PRB **89**, 085126 (2014)]
- The conductivity tensor in the Kubo's linear response theory

$$\sigma_{ij} = \lim_{\Omega\to 0} \frac{\mathrm{Im}\,\Pi_{ij}(\Omega + i0; \mathbf{0})}{\Omega}$$

where the polarization function



$$\Pi_{ij}(\Omega + i0; \mathbf{0}) = e^2 v_F^2 \int d\omega \int d\omega' \frac{n_F(\omega) - n_F(\omega')}{\omega - \omega' - \Omega - i0} \times \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathrm{tr}[\gamma^i A(\omega; \mathbf{k}) \gamma^j A(\omega'; \mathbf{k})]$$

is given in terms of the spectral function, obtained from the fermion Green's function in the Landau-level representation

$$A(\omega; \mathbf{k}) = \frac{1}{2\pi i} [\bar{G}_{\mu=0}(\omega - i0; \mathbf{k}) - \bar{G}_{\mu=0}(\omega + i0; \mathbf{k})]$$

RESULTS FOR σ_{12} CONDUCTIVITY

- Topological contribution to the anomalous Hall conductivity

$$\sigma_{12} = -\frac{e^2 s_\perp}{4\pi^2}\sum_n \alpha_n \int dk_3 \frac{\sinh\frac{\mu}{T}}{\cosh\frac{E_n}{T}+\cosh\frac{\mu}{T}} - \frac{e^2}{8\pi^2}\sum_{\chi=\pm}\chi\int dk_3 \frac{\sinh\frac{v_F(k_3-\chi b)}{T}}{\cosh\frac{v_F(k_3-\chi b)}{T}+\cosh\frac{\mu}{T}}$$

- Interestingly, only the LLL contributes. The result is

$$\sigma_{12,\mathrm{anom}} = -\frac{e^2}{8\pi^2 v_F} T \ln \frac{\cosh\frac{v_F(k_3-b)}{T}+\cosh\frac{\mu}{T}}{\cosh\frac{v_F(k_3+b)}{T}+\cosh\frac{\mu}{T}}\Bigg|_{k_3=-\infty}^{k_3=\infty} = \frac{e^2 b}{2\pi^2}$$

(Notice the subtlety in extracting the final result via regularization)

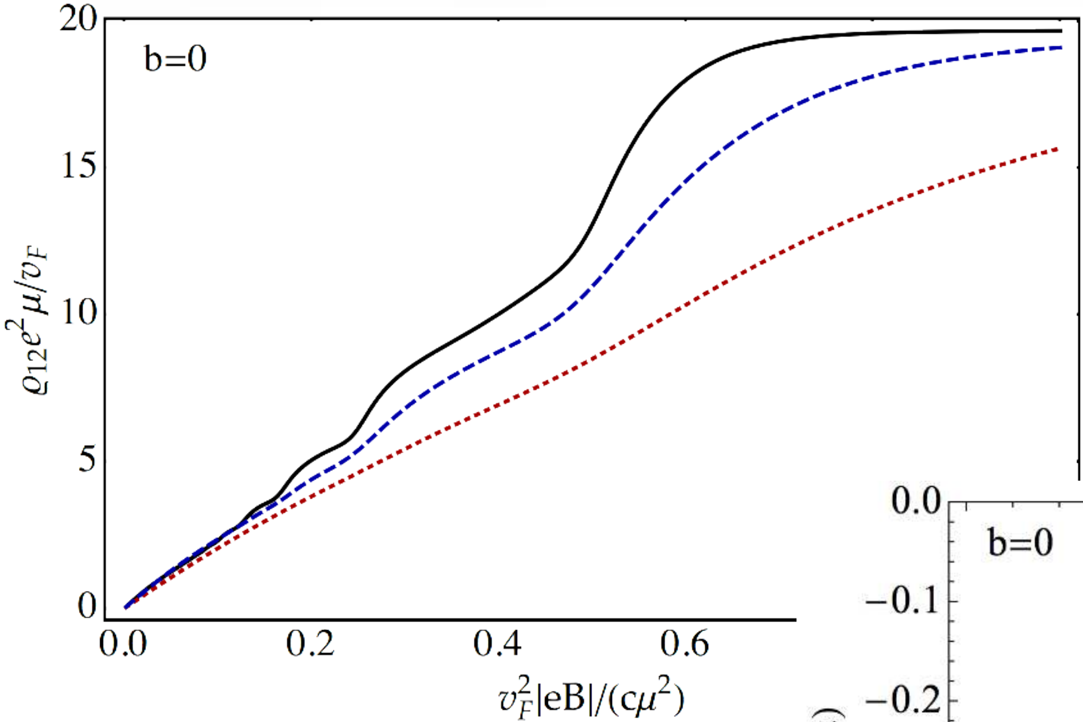
- This topological result implies that there is the current

$$\vec{J} = \frac{e^2}{2\pi^2}(\vec{b}\times\vec{E})$$

where $\vec{b}$ is the chiral shift parameter that determines the momentum space separation between the Weyl nodes, $\Delta\vec{k} = 2\vec{b}$

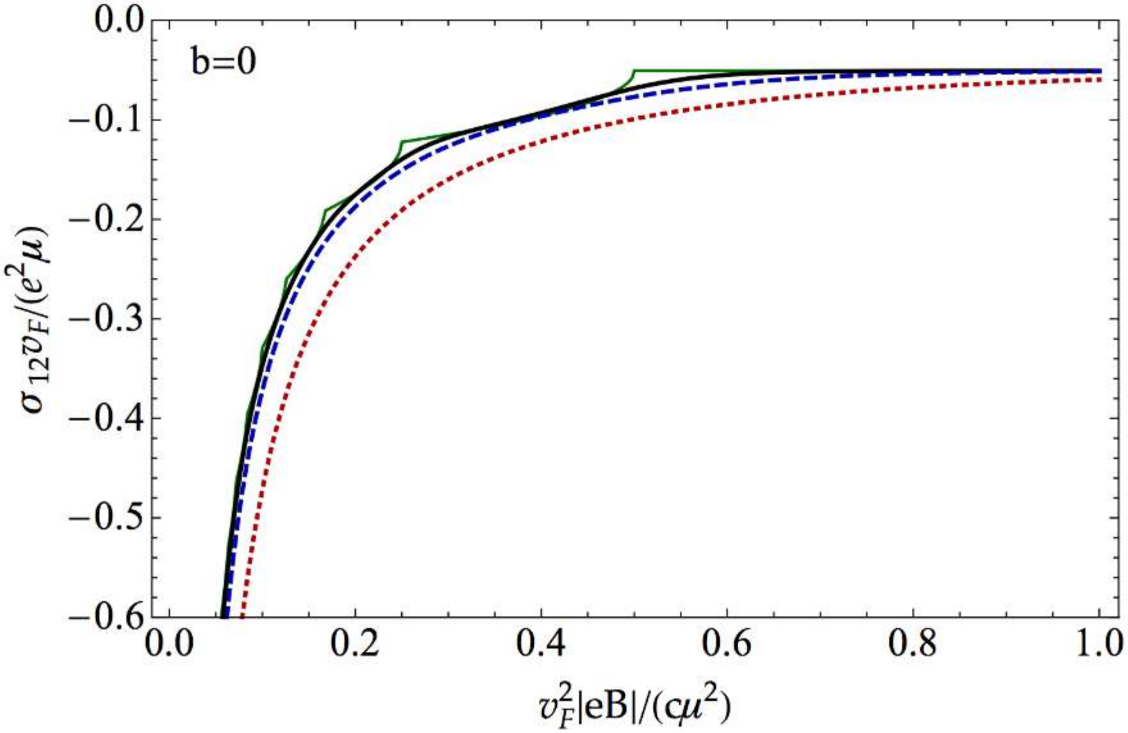
[Burkov & Balents, PRL **107**, 127205 (2011)], [Grushin, PRD 86, 045001 (2012)], [Goswami & Tewari, PRB 88, 245107 (2013)]

MODEL CONDUCTIVITY σ_{12} AT $\mu \neq 0$



$$\rho_{11} = \rho_{22} = \frac{\sigma_{11}}{\sigma_{11}^2 + \sigma_{12}^2}$$

$$\rho_{12} = -\rho_{21} = -\frac{\sigma_{12}}{\sigma_{11}^2 + \sigma_{12}^2}$$



$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\text{anom}}$$

where $\sigma_{12,\text{anom}} = \dfrac{e^2 b}{2\pi^2}$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B 89 (2014) 085126]

LONGITUDINAL CONDUCTIVITY

- Theoretically, it was predicted that the topological nature of the LLL should lead to large negative magnetoresistance [Nielsen & Ninomiya, PLB 130, 390 (1983)], [Son & Spivak, PRB 88, 104412 (2013)]
- From model calculations using Landau-level representation

$$\sigma_{33} = \sigma_{33}^{(LLL)} + \sigma_{33}^{(HLL)}, \quad \text{where} \quad \sigma_{33}^{(LLL)} = \frac{e^2 v_F |eB|}{4\pi^2 c \Gamma_0}$$

- Notice the temperature independent LLL contribution

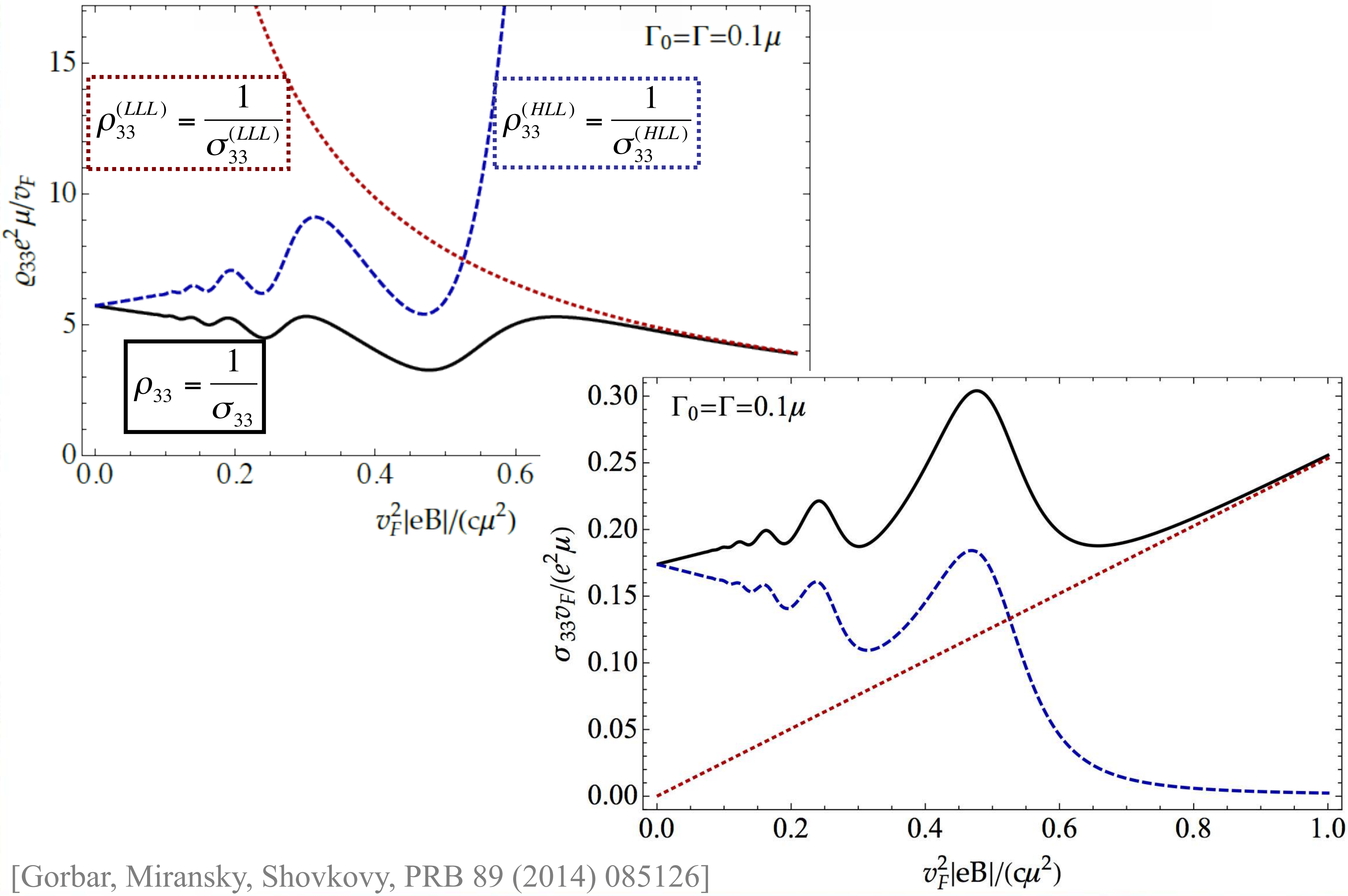
$$\sigma_{33}^{(\mathrm{LLL})} = \frac{e^2 v_F^2}{2^4 \pi^3 l^2 T} \sum_{\chi} \int \frac{d\omega\, dk_3}{\cosh^2 \frac{\omega-\mu}{2T}} \frac{\Gamma_0^2}{[[\omega + s_\perp \chi v_F (k_3 - \chi b)]^2 + \Gamma_0^2]^2}$$

$$= \frac{e^2 v_F}{4\pi^2 l^2 \Gamma_0} = \frac{e^2 v_F |eB|}{4\pi^2 c \Gamma_0}$$

(here Γ_0 is the LLL quasiparticle width, or inverse scattering time)

- The LLL contribution is unlike higher Landau-levels contributions which decrease with the increasing magnetic field

LONGITUDINAL CONDUCTIVITYIN

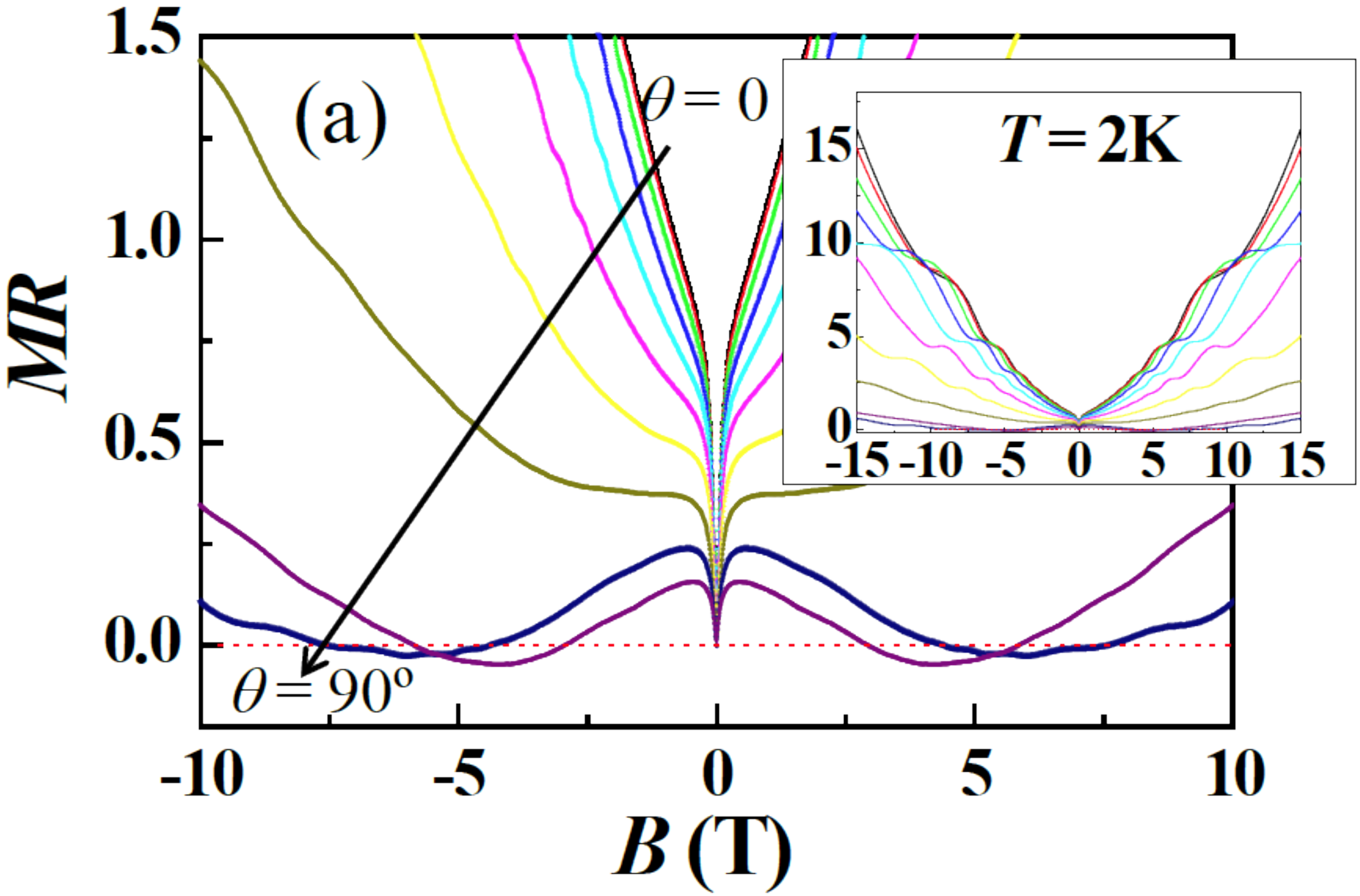


[Gorbar, Miransky, Shovkovy, PRB 89 (2014) 085126]

NEGATIVE MAGNETORESISTANCE

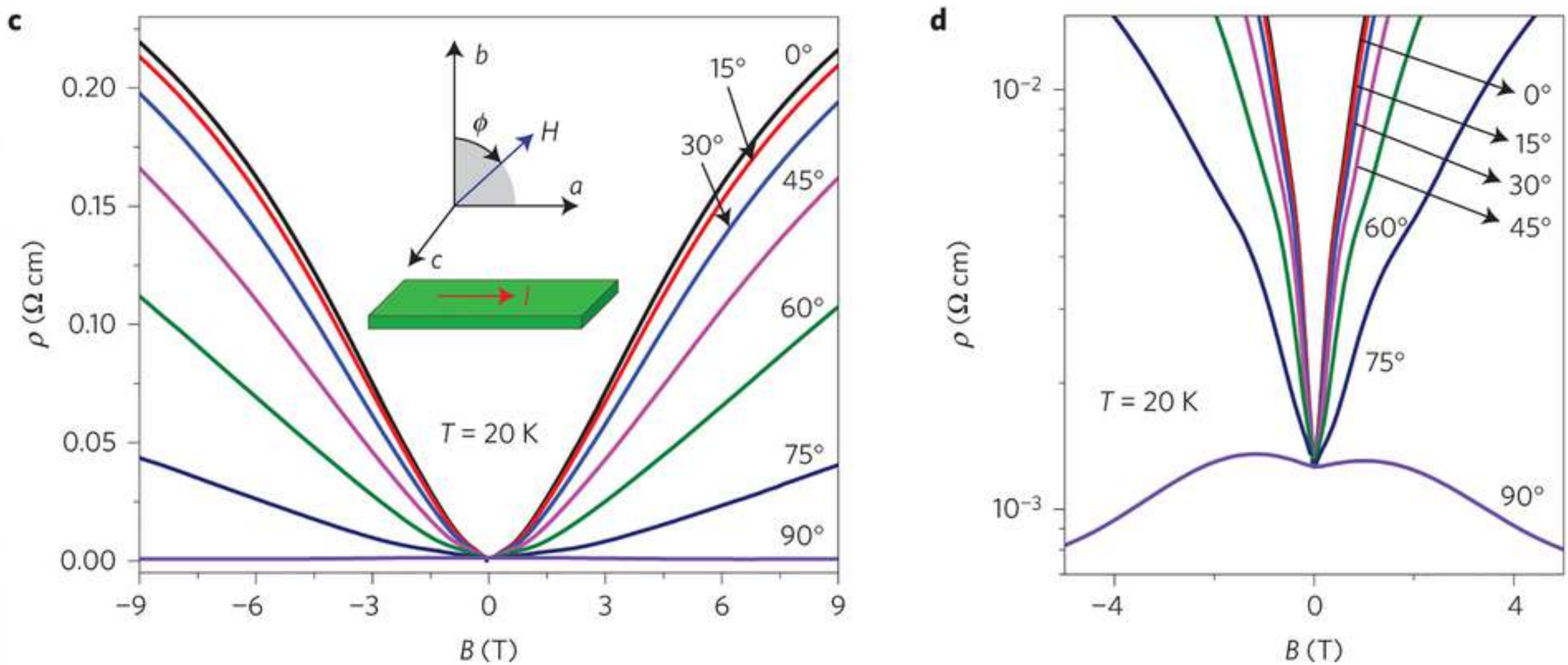
- Experimental confirmation? [Kim, et al., PRL 111, 246603 (2014)]

$Bi_{1-x}Sb_x$ alloy with $x \approx 0.04$



Magneto-transport in $ZrTe_5$

- Positive magnetoresistance for currents perpendicular to magnetic field ($\theta = 0°$)
- Negative magnetoresistance for currents parallel to magnetic field ($\theta = 90°$)



[Li et al., Nature Physics **12**, 550 (2016)]

SIMILAR RESULTS IN OTHER MATERIALS

- Magnetotransport was also studied in other materials, including
  - Na3Bi, Cd3As2 (Dirac materials)

    [Xiong et al., Science **350**, 413 (2015)], [Li et al., Nat. Commun. **6**, 10137 (2015)], [Li et al., Nat. Commun. **7**, 10301 (2016)], …
  - TaAs, NbAs, NbP, TaP (Weyl materials)

    [Huang et al., PRX **5**, 031023 (2015)], [Zhang et al., Nat. Commun. **7**, 10735 (2016)], [Arnold et al., Nat. Commun. **7**, 11615 (2016)], …
- There is no ambuguity that the large negative magneto-resistance is observed in Dirac/Weyl materials
- There is a consensus that it is related to anomalous features of Dirac/Weyl fermions
- However, identifying other signature properties of Dirac/ Weyl materials would be extremely valuable

CHIRAL EFFECTS IN WEYL MATERIALS

- Any qualitative properties of Weyl materials directly sensitive to b_0 and $\vec{b}$?
- Some proposals in the literature:
  - Anomalous Hall effect
  - Anomalous Alfven waves
  - Strain/torsion induced CME
  - Strain/torsion induced quantum oscillations
  - Strain/torsion dependent resistance
  - etc.
- Spectrum of chiral (pseudo-)magnetic plasmons

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Strain in Weyl materials

- Strains affect low-energy quasiparticles in Weyl materials [Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]

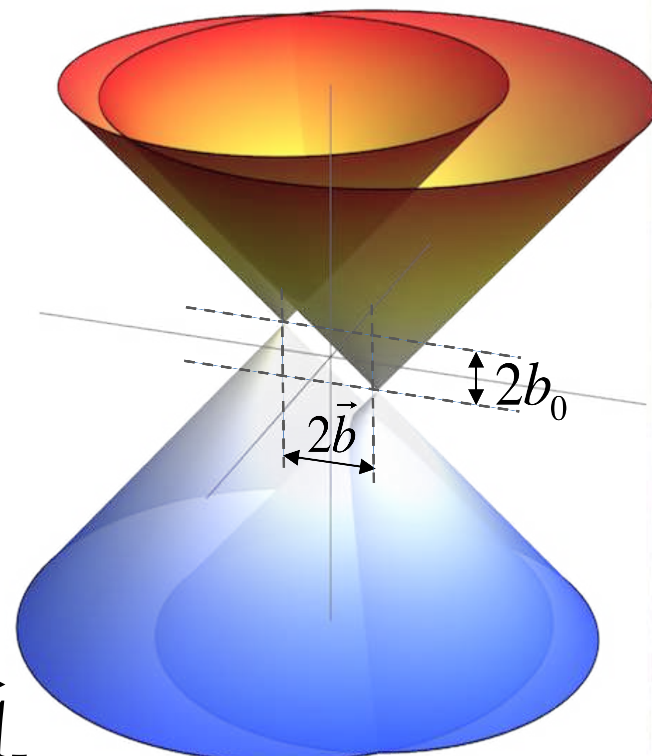
$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\,\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

where the components of the chiral gauge fields are

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$

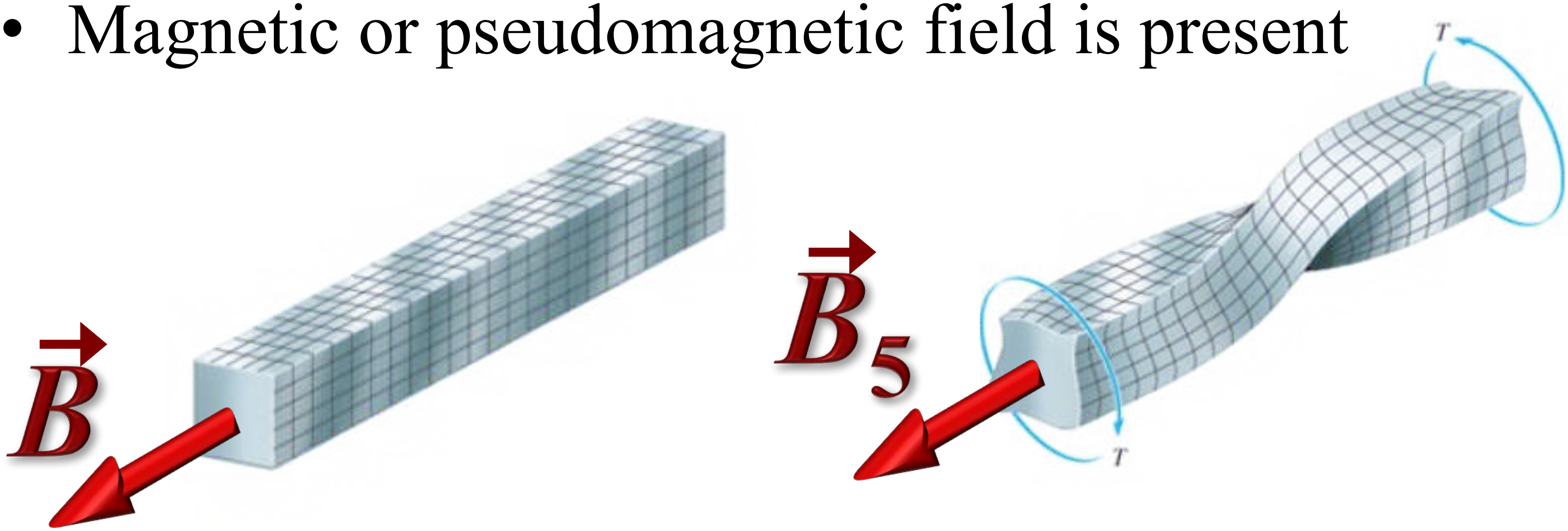


The associated pseudo-EM fields are

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$

GENERAL QUESTION

- What are the properties of plasmons in magnetized chiral material with $b_0 \neq 0$ and $\vec{b} \neq 0$?
- Chiral matter ($\mu_R \neq \mu_L$)
  - This is the case in equilibrium when $b_0 \neq 0$ $(\mu_5 = -e\, b_0)$
- Magnetic or pseudomagnetic field is present



- In general, $\mathbf{E}_\lambda = \mathbf{E} + \lambda \mathbf{E}_5$ and $\mathbf{B}_\lambda = \mathbf{B} + \lambda \mathbf{B}_5$

Chiral kinetic theory

- Kinetic equation:

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\nabla_{\mathbf{p}}f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$

$$+\frac{\left[\mathbf{v}+e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda)+\frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\nabla_{\mathbf{r}}f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\nabla_{\mathbf{r}}\epsilon_{\mathbf{p}}$, $\mathbf{v} = \nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}$,

$$\epsilon_{\mathbf{p}} = v_F p\left[1-\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

CURRENT AND CHIRAL ANOMALY

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda)\right]f_\lambda$$

$$+e\boldsymbol{\nabla}\times\int\frac{d^3p}{(2\pi\hbar)^3}f_\lambda\epsilon_{\mathbf{p}}\boldsymbol{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B})+(\mathbf{E}_5\cdot\mathbf{B}_5)\right]$$ ✔

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B}_5)+(\mathbf{E}_5\cdot\mathbf{B})\right]$$ ✘

CONSISTENT DEFINITION OF CURRENT

- Additional Bardeen-Zumino term is needed,

$$\delta j^\mu = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda}A^5_\nu F_{\rho\lambda}$$

- In components,

$$\delta\rho = \frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{A}^5\cdot\mathbf{B})$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}A^5_0\mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c}(\mathbf{A}^5\times\mathbf{E})$$

- Its role and implications:
  - Electric charge is conserved locally ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)

COLLECTIVE MODES

We search for plane-wave solutions with

$$\mathbf{E}' = \mathbf{E}e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}, \ \ \mathbf{B}' = \mathbf{B}e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

and the distribution function $f_\lambda = f_\lambda^{(\mathrm{eq})} + \delta f_\lambda$,
where $\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$

The polarization vector & susceptibility tensor:

$$P^m = i\frac{J'^m}{\omega} = \chi^{mn} E'^n$$

The plasmon dispersion relations follow from

$$\det\left[\left(\omega^2 - c^2k^2\right)\delta^{mn} + c^2 k^m k^n + 4\pi\omega^2\chi^{mn}\right] = 0$$

CHIRAL MAGNETIC PLASMONS

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

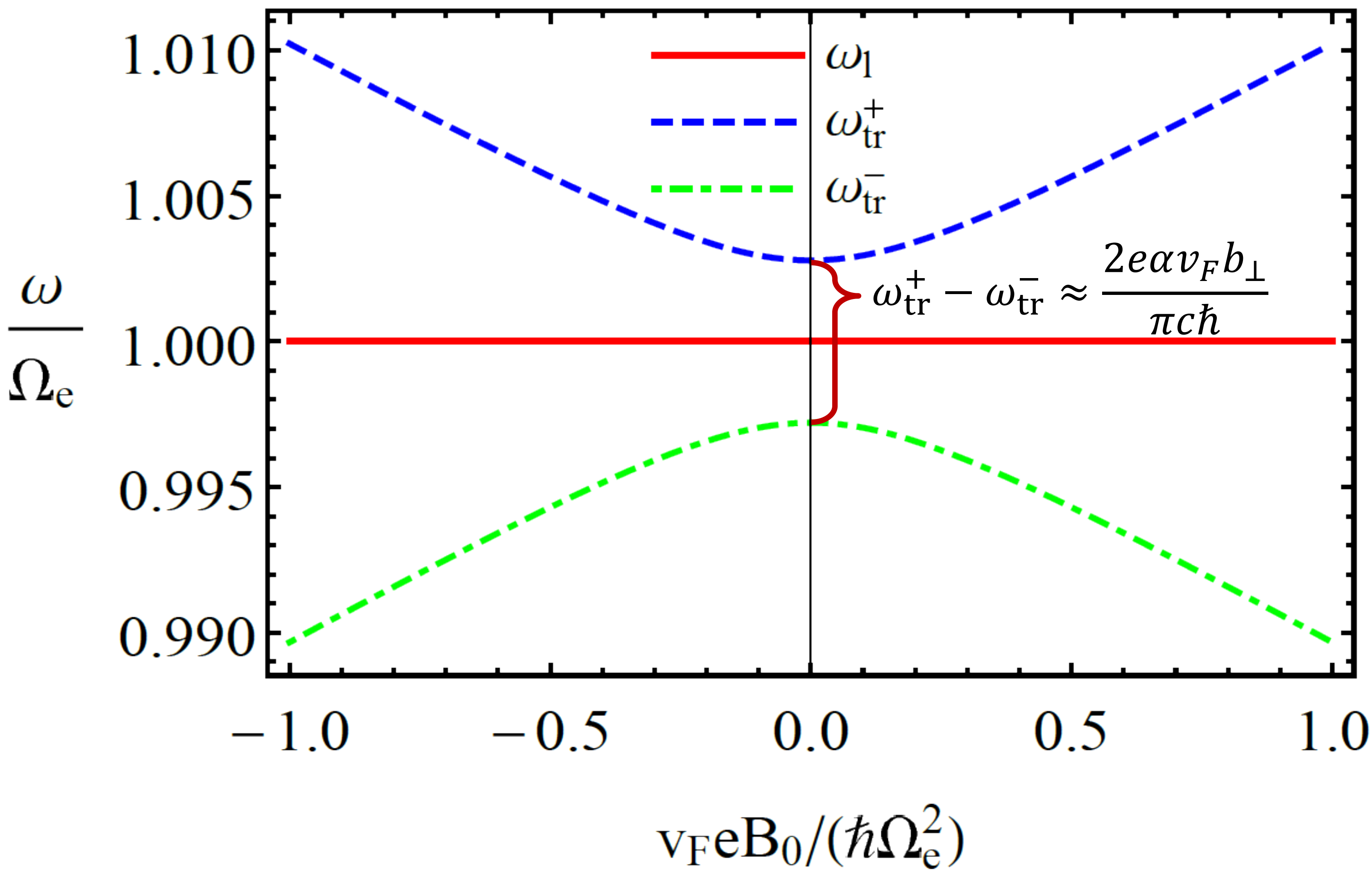
and

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\left\{9\hbar^2 b_\perp^2 + \left[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_{\|} - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\right]^2\right\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

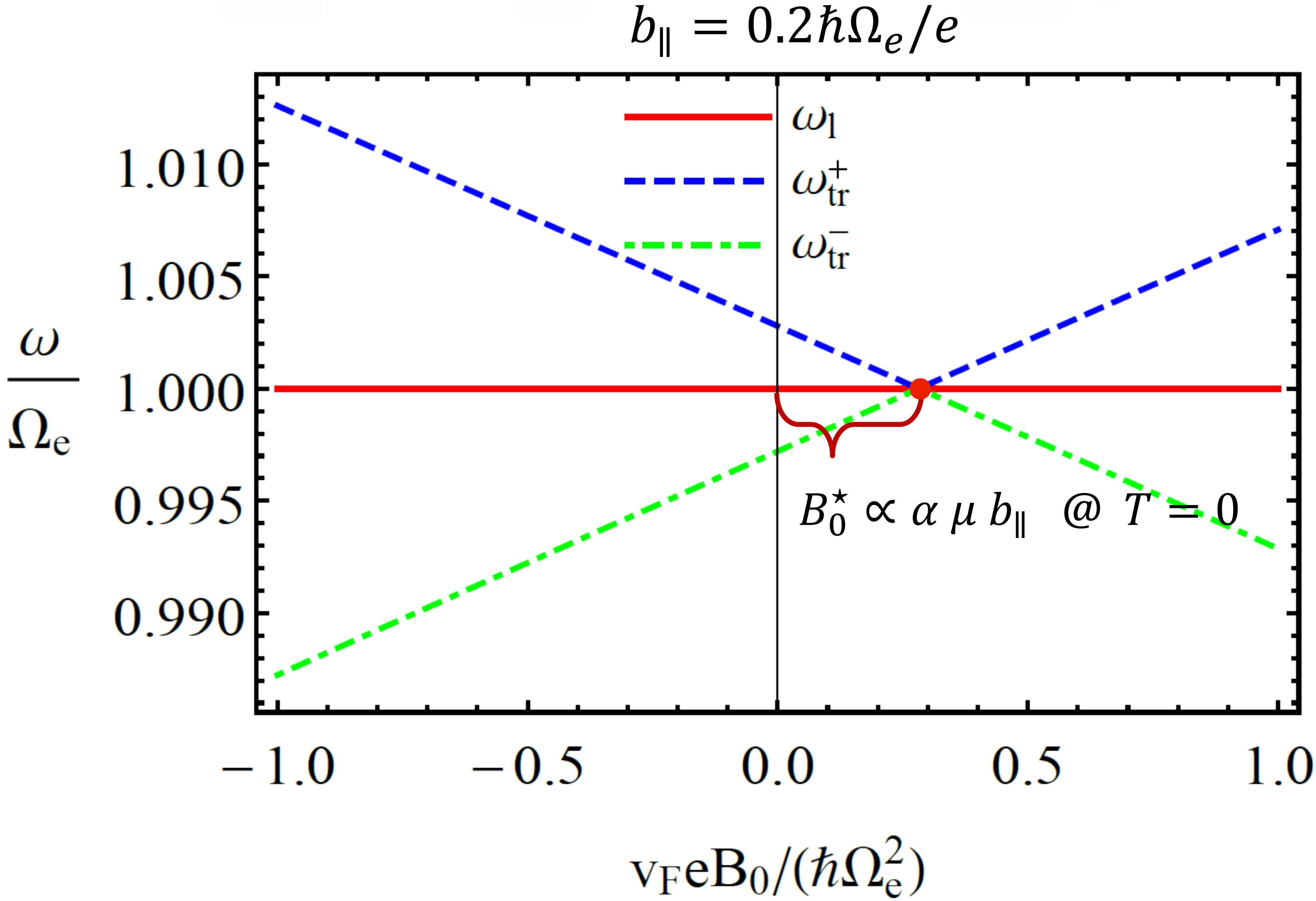
PLASMON FREQUENCIES, $\vec{B} \perp \vec{b}$

$b_\perp = 0.2\hbar\Omega_e/e$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

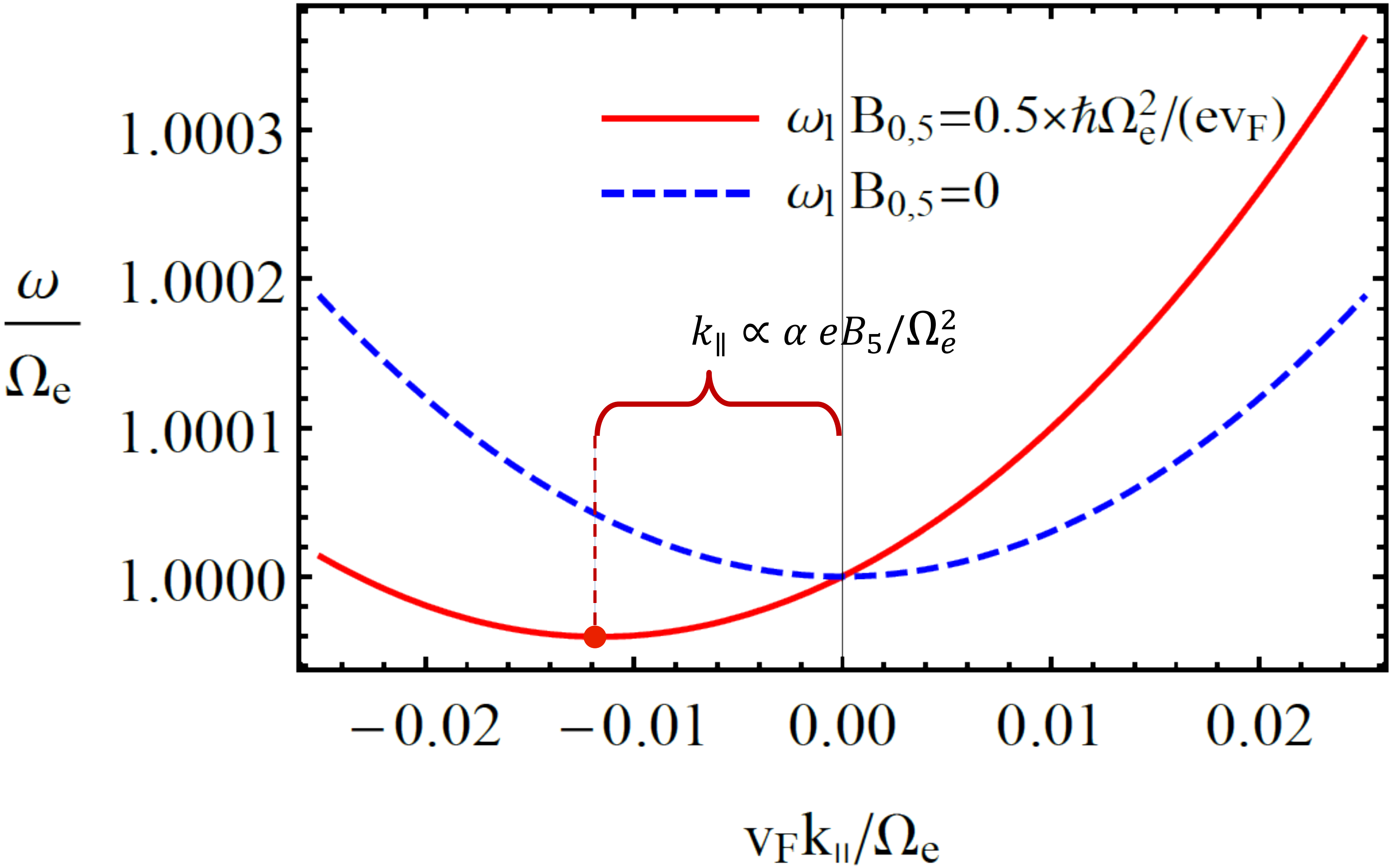
PLASMON FREQUENCIES, $\vec{B} \parallel \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmons with $\vec{k} \neq 0$, $\vec{k} \parallel \vec{B}_5$

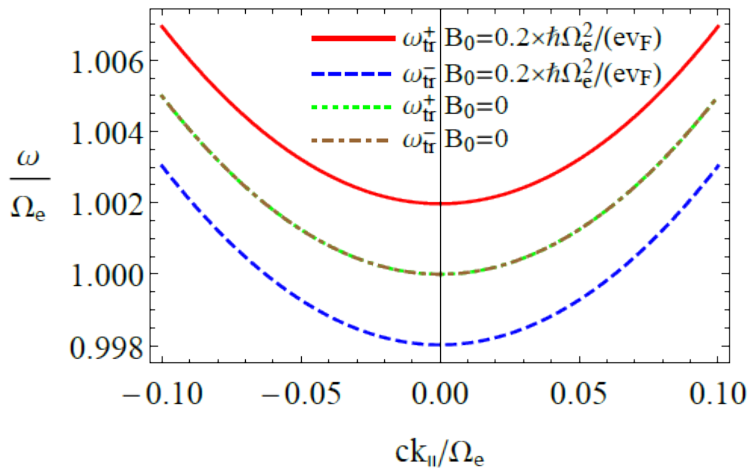
- The longitudinal mode is sensitive to $\vec{B}_5$



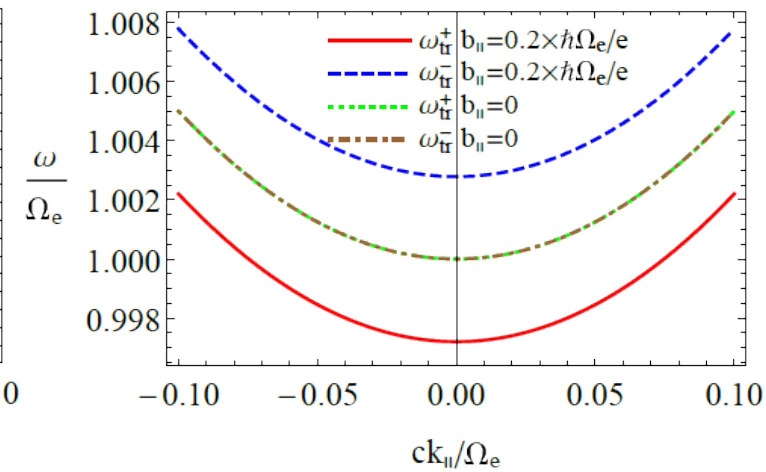
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

PLASMONS WITH $\vec{k} \neq 0, \vec{k} \parallel \vec{B}, \vec{B}_5$

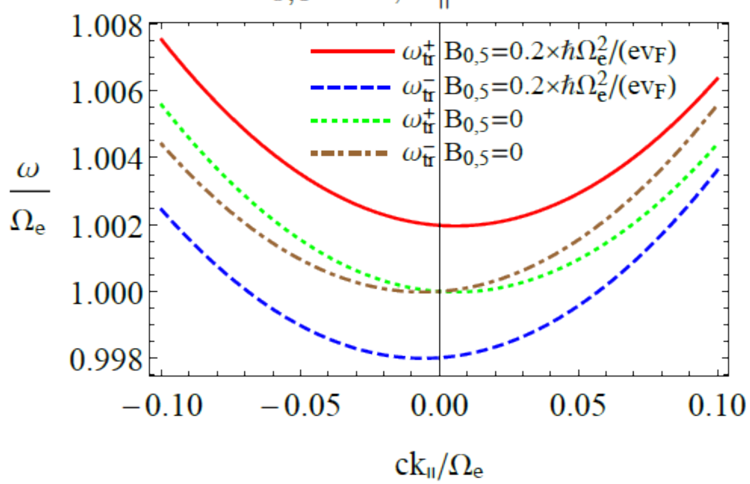
The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$, or (iii) $b_\parallel \neq 0$, or (iv) $b_\perp \neq 0$



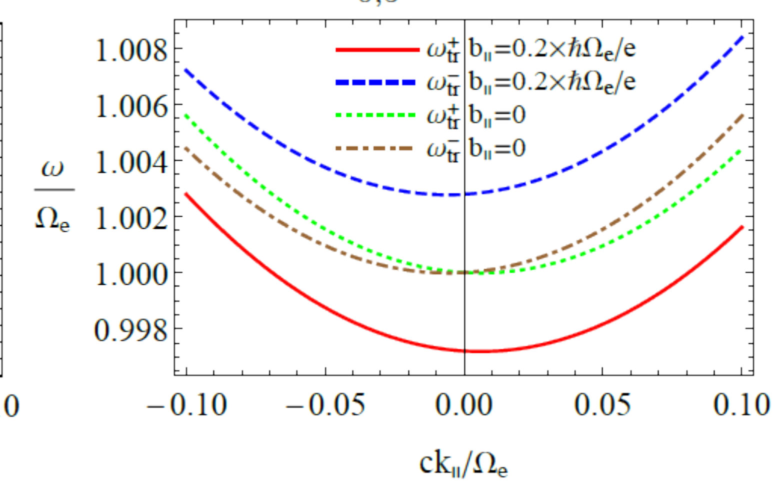
(a) $\mu = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ \mu_5 = 0,\ B_{0,5} = 0,\ b_\parallel = 0$



(c) $\mu = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ \mu_5 = 0,\ B_0 = 0,\ B_{0,5} = 0$



(e) $\mu = 0,\ \mu_5 = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ B_0 = 0,\ b_\parallel = 0$



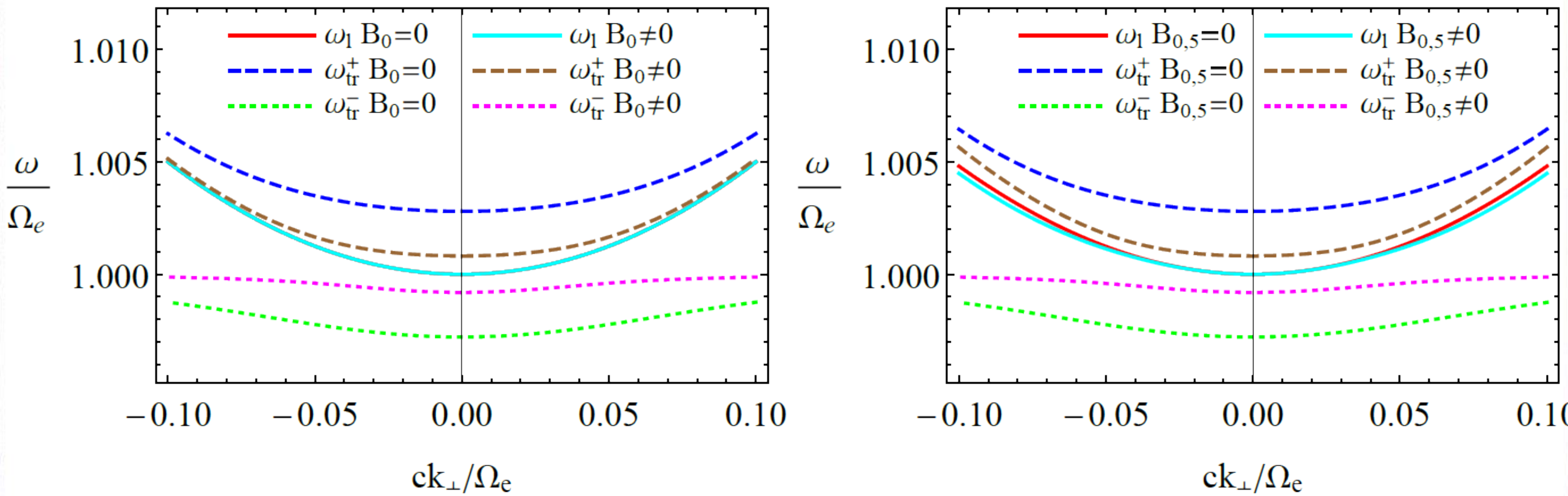
(f) $\mu = 0,\ \mu_5 = \sqrt{3\pi/(4\alpha)}\hbar\Omega_e,\ B_0 = 0,\ B_{0,5} = 0$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Plasmons with $\vec{k} \neq 0, \vec{k} \perp \vec{B}, \vec{B}_5$

The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$

$$b_{\parallel} = 0.2\hbar\Omega_e/e$$

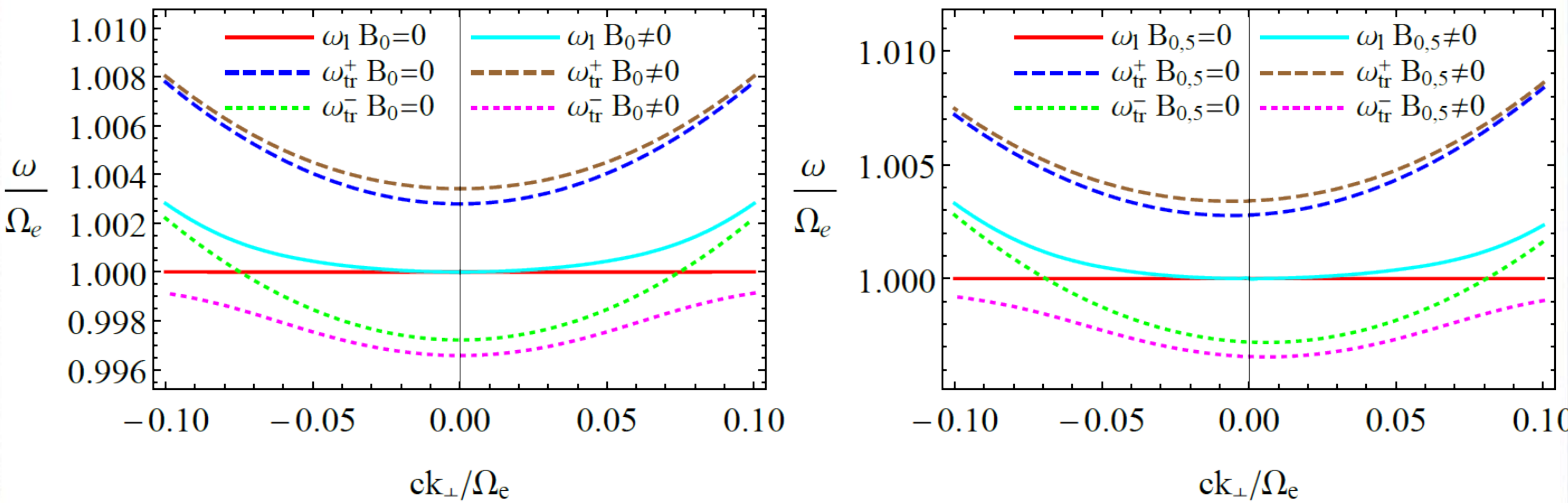


[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

PLASMONS WITH $\vec{k} \neq 0, \vec{k} \perp \vec{B}, \vec{B}_5$

The transverse modes split (in different ways) when (i) $\vec{B} \neq 0$ & $\mu \neq 0$, or (ii) $\vec{B}_5 \neq 0$ & $\mu_5 \neq 0$

$$b_\perp = 0.2\hbar\Omega_e/e$$

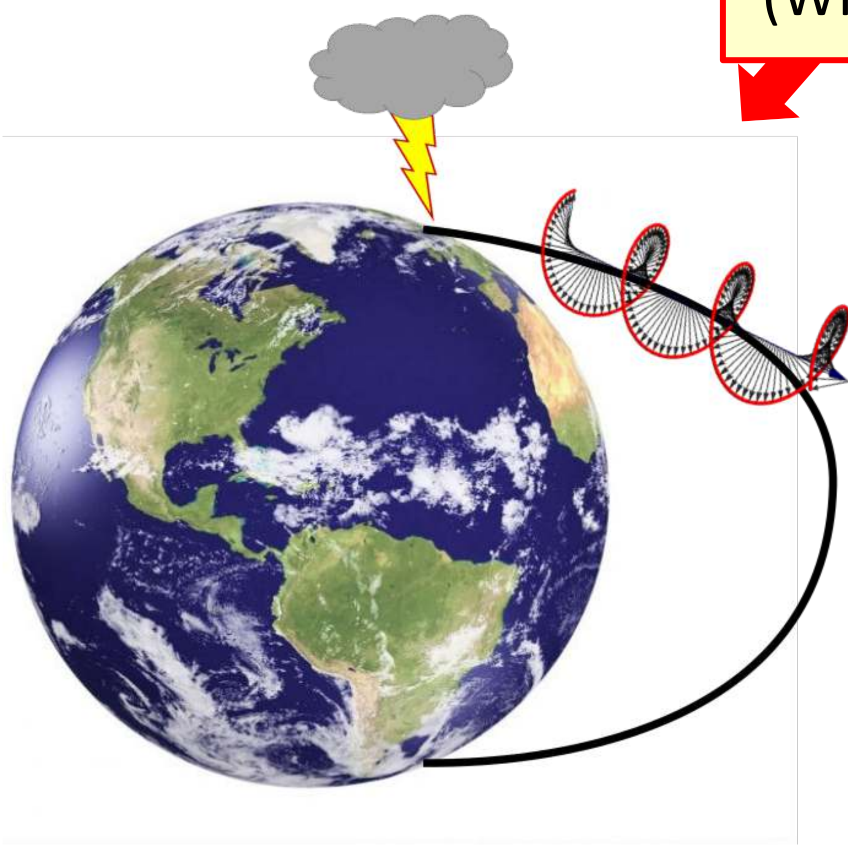


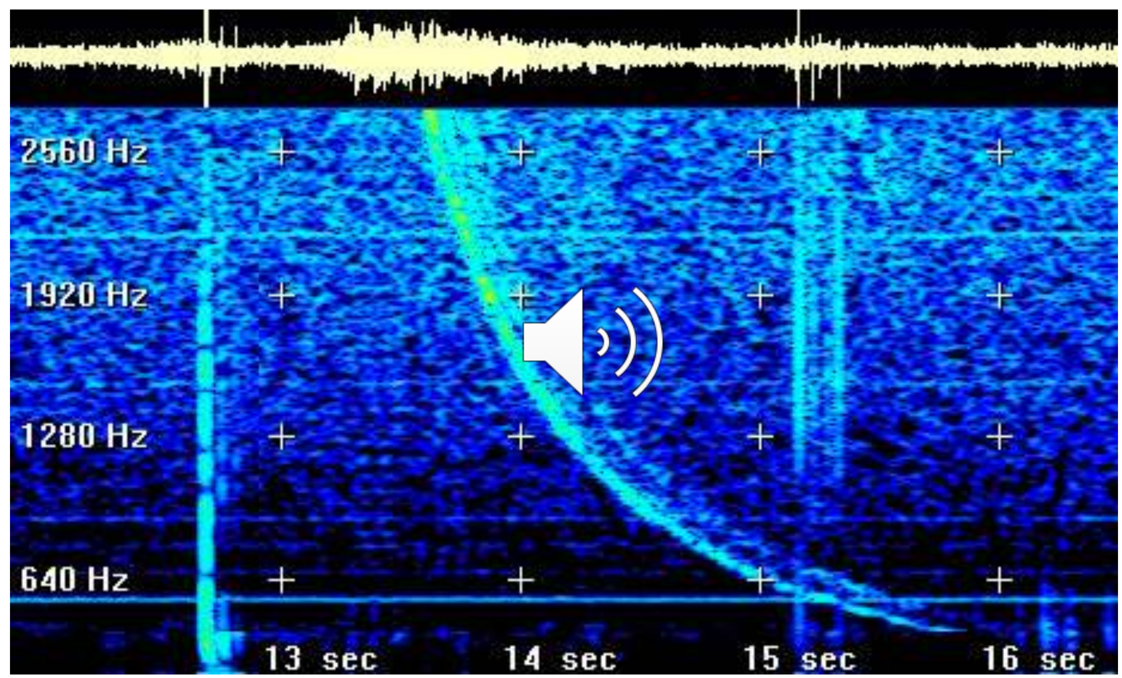
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

(PSEUDO-)MAGNETIC HELICONS

- Usual helicons are transverse low-energy gapless excitations propagating along the background magnetic field $\vec{B}_0$ in uncompensated media (e.g., metals with different electron and hole densities)

Helicon in the ionosphere (Whistler) and its spectrogram





(PSEUDO-)MAGNETIC HELICON

- Helicon dispersion law at $T \to 0$:

$$\omega_h\Big|_{B_{0,5}\to 0,\mu_5\to 0} \stackrel{b_0\to 0}{=} \frac{eB_0 c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu + 2B_0 e^4 v_F^2 b_\parallel} + O(k^3)$$

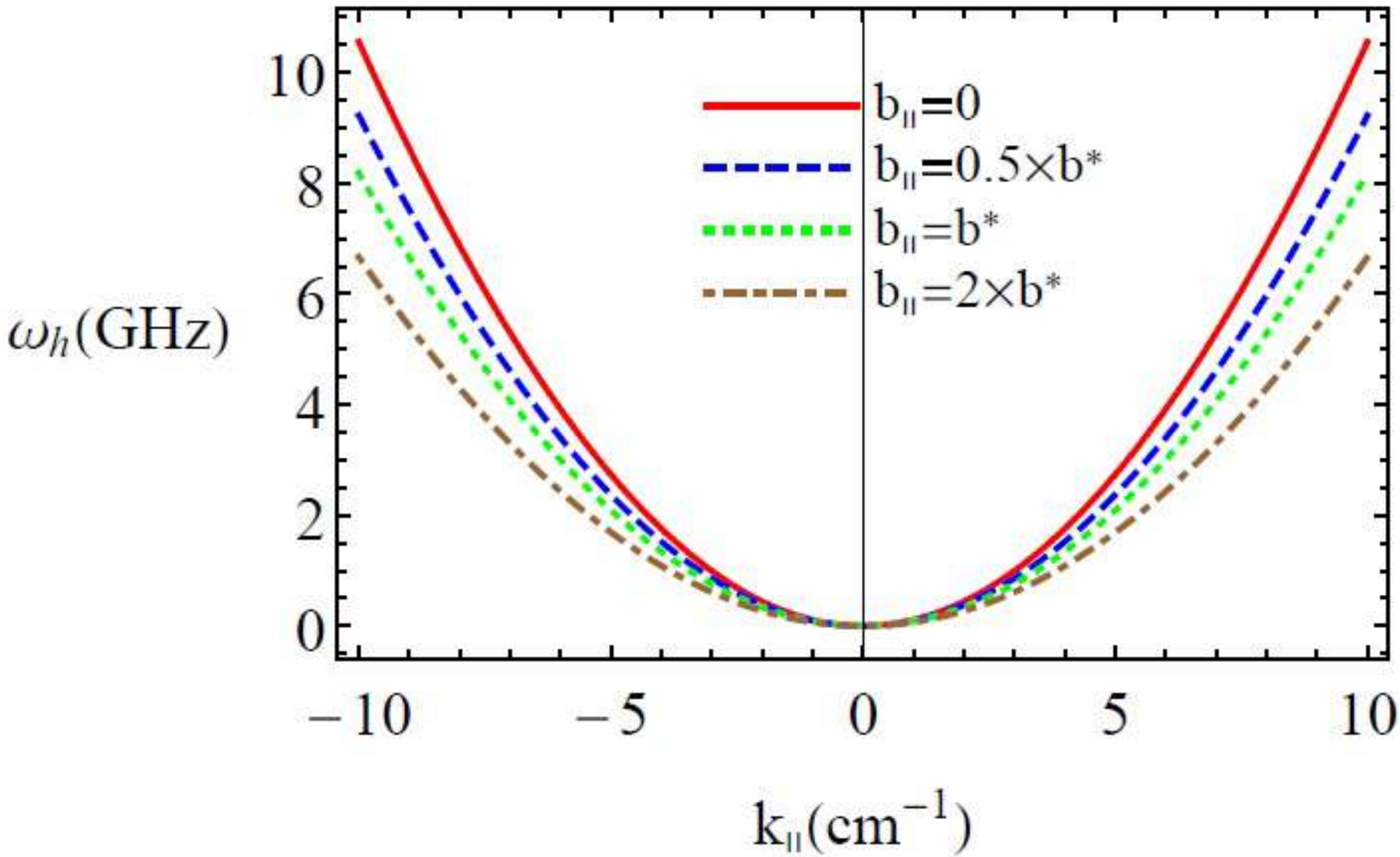
$$\omega_h\Big|_{B_0\to 0,\mu\to 0} \stackrel{b_0\to -\mu_5/e}{=} \frac{eB_{0,5} c^3 \hbar^2 \pi v_F^2 k^2}{\pi \hbar^2 c^2 \Omega_e^2 \mu_5 + 2B_{0,5} e^4 v_F^2 b_\parallel} + O(k^3)$$

- Properties:
  - Gapless electromagnetic wave propagates in metals **without magnetic field**!
  - Chiral shift modifies effective helicon mass
  - In the equilibrium regime $eb_0 = -\mu_5$, the linear in the wave vector term is **absent**

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

HELICONS AT DIFFERENT $b_\parallel$

$eb_0 = -\mu_5, B_{0,5} = 10^{-2}\mathrm{T}, \mu_5 = 5\ \mathrm{meV}, \mu = 0, eb^* = 0.3\pi\hbar v_F/c_3$



HELICONS AT DIFFERENT T

$eb_0 = -\mu_5, B_{0,5} = 10^{-2}\text{T}, b_{\parallel} = 0.5b^*, \mu_5 = 5 \text{ meV}, \mu = 0$