

# Color superconductivity and compact stars

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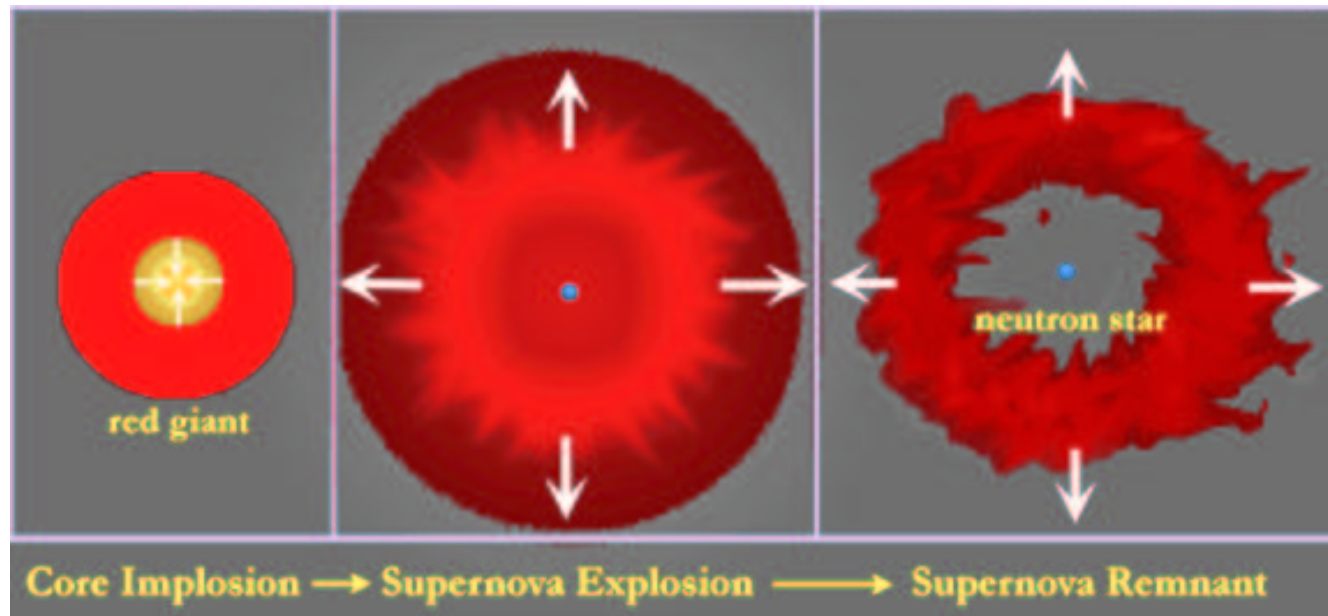
## Collaborators

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- Paul J. Ellis  
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## References

- I.A.S., M. Hanauske & M. Huang, [hep-ph/0303027](#)
- I.A.S. & M. Huang, [hep-ph/0302142](#)
- I.A.S. & P.J. Ellis, [hep-ph/0211049](#), Phys. Rev. C, to appear
- I.A.S. & P.J. Ellis, Phys. Rev. C **66** (2002) 015802

## Birth of compact stars



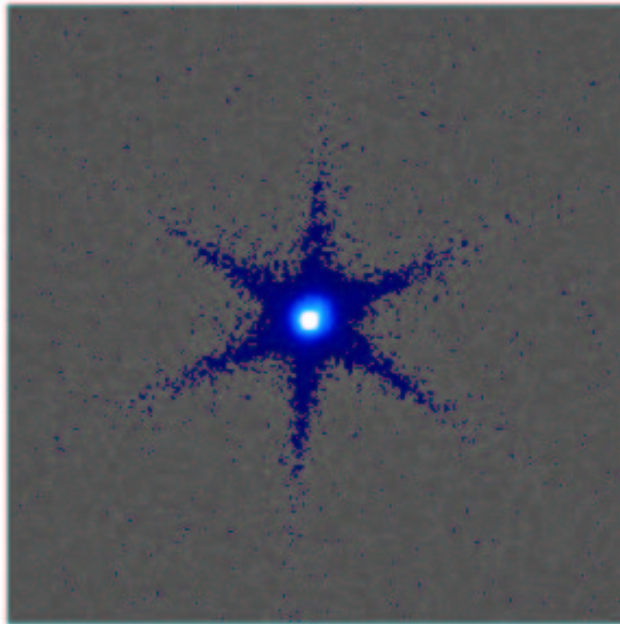
Typical parameters:

Mass	Radius	Temperature	Density
$\sim 1.4 M_{\odot}$	$\sim 10 \text{ km}$	20 - 40 MeV	$\lesssim 10\rho_0$

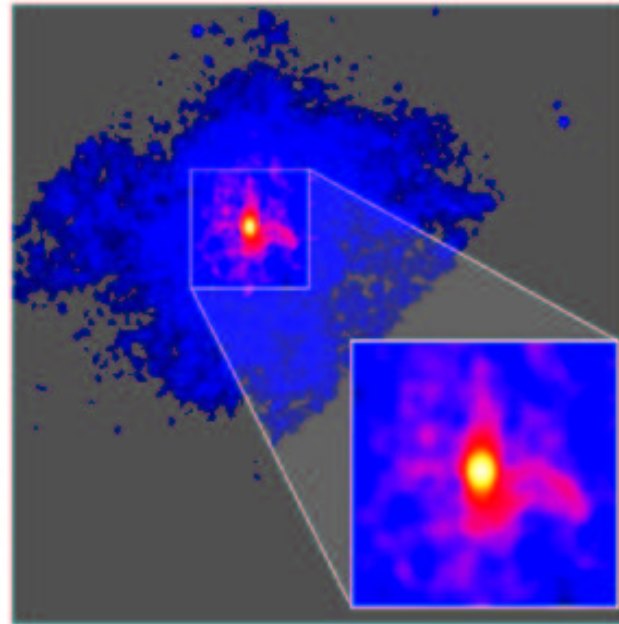
## Quark stars?

CHANDRA X-Ray Observatory (April 10, 2002)

Cosmic X-rays Reveal Evidence For New Form Of Matter



RX J1856.5-3754 [Drake et al.,02]



3C58 [Slane et al.,02]

## Quark stars?

Thursday, April 11, 2002

# The New York Times

“Observations of two stars, one unusually small and the other unusually cold, have led astronomers to think they are seeing evidence of a new form of matter and a new kind of star, one possibly made of elementary particles known as quarks and denser than any cosmic object other than a black hole.”

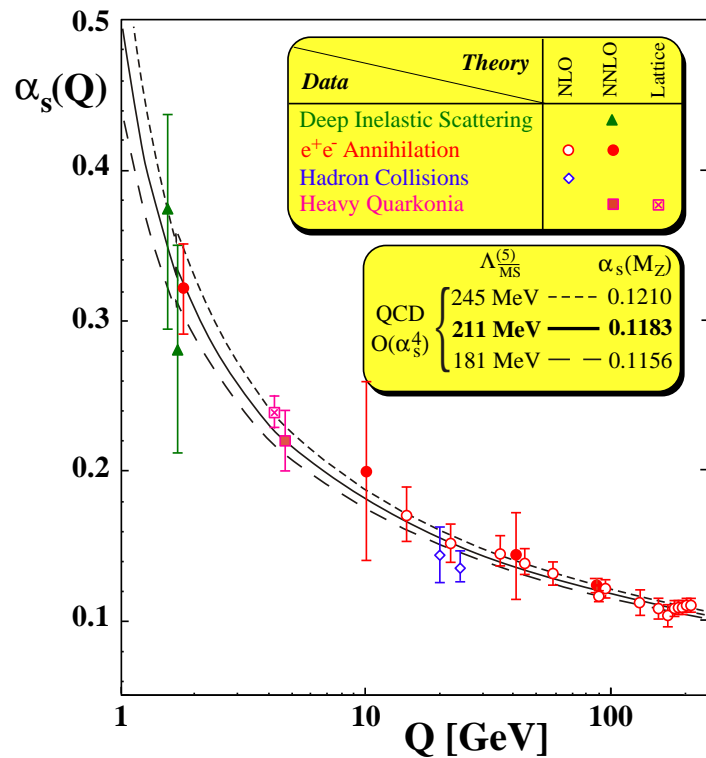
Were quark stars observed? — Perhaps not yet ...

[Walter et al., 02], [Yakovlev et al., 02], [Zane et al., 03]

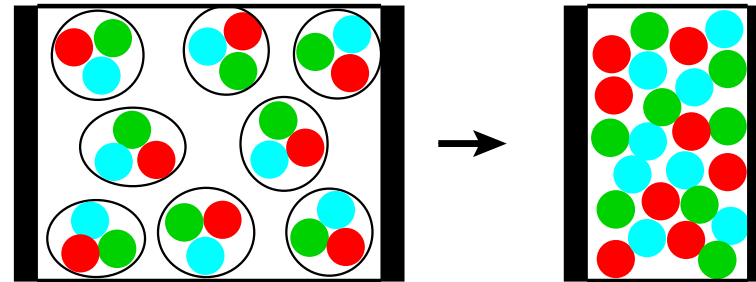
# Weakly interacting quark matter

The property of asymptotic freedom:  $\alpha_s(\mu) \ll 1$  for  $\mu \gg \Lambda_{QCD}$

[Gross & Wilczek,73], [Politzer,73]



- Dense quark matter is weakly interacting [Collins & Perry,75]
- “Squeezing” quark matter



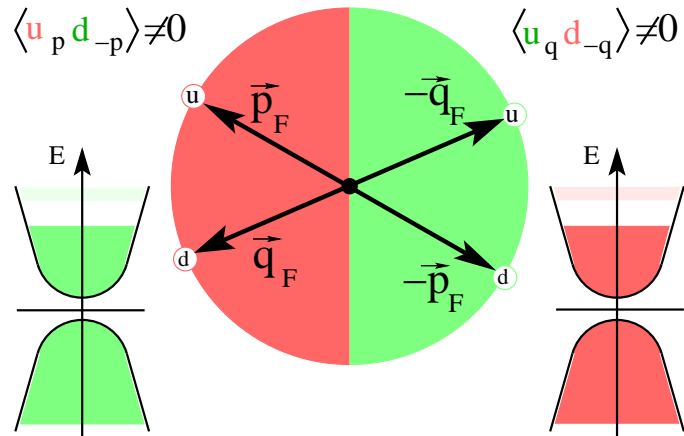
- Realistic densities in compact stars:  $\rho \lesssim 10\rho_0$ , where  $\rho_0 \approx 0.15 \text{ fm}^{-3}$ , (corresponding coupling  $\alpha_s \sim 1$ )

## Basics of color superconductivity

Asymptotic density ( $\mu \gg \Lambda_{QCD}$ ):

- $\alpha_s(\mu) \ll 1$  (weak coupling)
- One-gluon interaction is dominant
- Color  $\bar{3}_a$  channel is **attractive** (!)

$$\begin{array}{c}
 p \quad k \\
 \diagdown \quad / \\
 \text{---} \\
 / \quad \diagdown \\
 -p \quad -k
 \end{array}
 = \bar{3}_a + 6_s$$



- BCS mechanism for quarks leads to color superconductivity
- By using Pauli principle ( $s = 0$ ):

$$N_f = 2 : \varepsilon_{ij} \varepsilon^{3ab} \langle (\psi_a^i)^T C \gamma^5 \psi_b^j \rangle \neq 0$$

$$N_f = 3 : \sum_{I=1}^3 \varepsilon_{ijI} \varepsilon^{abI} \langle (\psi_a^i)^T C \gamma^5 \psi_b^j \rangle \neq 0$$

[Son], [Schafer et al.], [Hong et al.], [Pisarski et al.], [Shovkovy et al.] (1999)

## Properties of 2SC ground state

(up & down quarks only)

- Chiral symmetry  $SU(2)_L \times SU(2)_R$  is intact
- Color symmetry is broken (by Anderson-Higgs mechanism):  
 $SU(3)_c \rightarrow SU(2)_c$ 
  - color Meissner effect (for 5 gluons)
  - low energy  $SU(2)_c$  gluodynamics (decoupled)
- Modified electromagnetic  $U(1)_{\widetilde{em}}$  and modified  $U(1)_{\widetilde{B}}$  survive
  - no electromagnetic Meissner effect
  - no superfluidity
- Approximate  $U(1)_A$  is broken  $\rightarrow$  light pseudo-NG boson
- Parity is preserved



## Signatures of CSC in compact stars

Color superconductivity → **gap** in quasiparticle spectrum

- Thermodynamic properties (equation of state)
  - mass-radius relation [Alford&Reddy,02], [Lugones&Horvath,02]
  - internal star structure [Baldo et al.,02], [Shovkovy et al.,03]
- Transport properties (conductivities, viscosities, mean free paths)
  - cooling rate [Page et al.,02], [Shovkovy&Ellis,02]
  - r-mode instability [Madsen,99]
  - glitches (crystalline phase) [Alford et al.,00]
- Other properties
  - magnetic field generation/penetration [Alford et al.,00]
  - rotational vortices [Iida&Baym,02]

## Neutrality vs. color superconductivity

- The “best” 2SC phase appears when  $n_d \approx n_u$ ,
- but neutral matter appears when  $n_d \approx 2n_u$
- Electrons do not help (!):

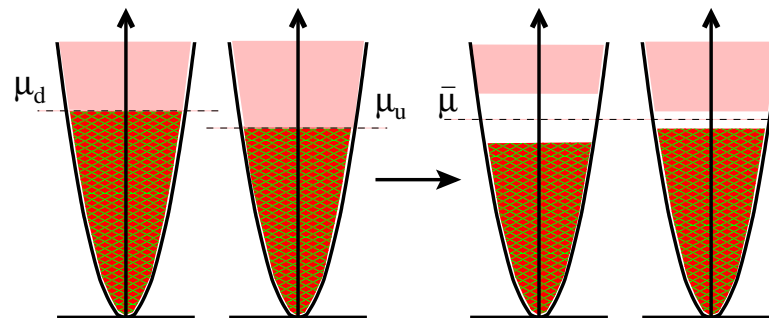
$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u$$

$$\text{Thus, } n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$$

- Cooper pairing with a mismatch between Fermi surfaces of pairing quarks:

$$\mu_d - \mu_u = \mu_e$$

Gaps:  $(\Delta + \mu_e/2)$  and  $(\Delta - \mu_e/2)$



## Neutral quark phases

- Locally neutral phases:
  - Normal quark matter
  - gapless 2SC matter [Shovkovy&Huang,03]
- Globally neutral mixed phases [Glendenning,92], e.g., 2SC+NQ (?)



$$\rho_e^{(MP)} = \chi_B^A \rho_e^{(A)}(\mu, \mu_e) + (1 - \chi_B^A) \rho_e^{(B)}(\mu, \mu_e) = 0$$

where

$$\chi_B^A \equiv \frac{V^{(A)}}{V^{(A)} + V^{(B)}} \text{ is the volume fraction of phase A}$$

## Gibbs construction (2SC+NQ)

- Mechanical equilibrium:

$$P^{(2SC)}(\mu, \mu_e) = P^{(NQ)}(\mu, \mu_e)$$

- Chemical equilibrium:

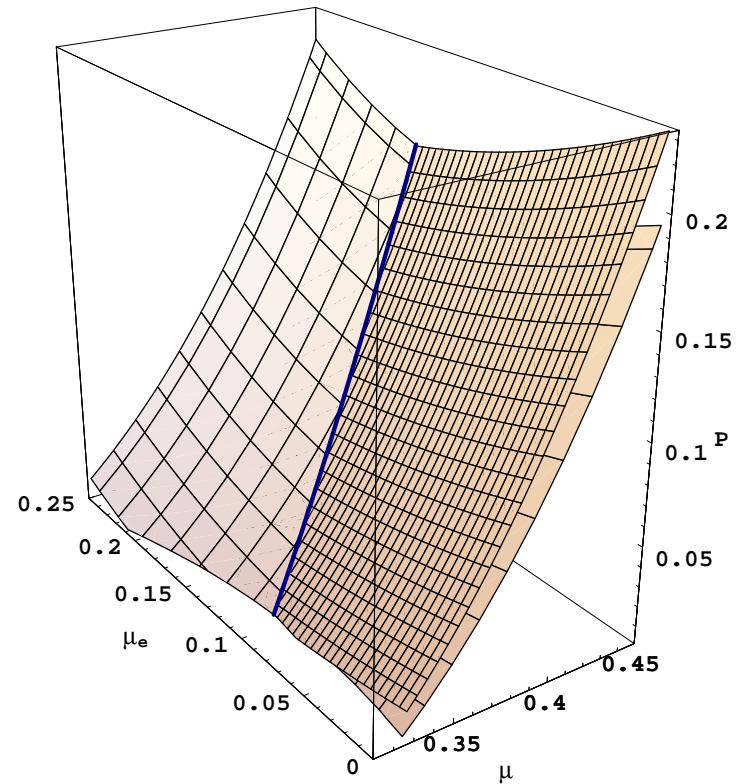
$$\mu = \mu^{(2SC)} = \mu^{(NQ)},$$

$$\mu_e = \mu_e^{(2SC)} = \mu_e^{(NQ)}$$

- From the condition of neutrality

$$\chi_{NQ}^{2SC} = \frac{\rho_e^{(NQ)}}{\rho_e^{(NQ)} - \rho_e^{(2SC)}},$$

$$\text{Energy density: } \varepsilon^{(MP)} = \chi_{NQ}^{2SC} \varepsilon^{(2SC)}(\mu, \mu_e) + (1 - \chi_{NQ}^{2SC}) \varepsilon^{(NQ)}(\mu, \mu_e)$$



## Coulomb forces and surface tension

- Extra surface and Coulomb energies per unit volume  
[Heiselberg et al.,93], [Glendenning & Pei,95]

$$\mathcal{E}_S \simeq C_S^{(\text{geom})} \frac{\sigma}{R}, \quad \mathcal{E}_C \simeq C_C^{(\text{geom})} \left( \rho_e^{(A)} - \rho_e^{(B)} \right)^2 R^2$$

- Minimizing the sum with respect to  $R$ , one gets

$$\mathcal{E}_{C+S} \simeq (8 \text{ MeV fm}^{-3}) \left( \frac{\sigma}{\sigma_0} \frac{\rho_e^{(A)} - \rho_e^{(B)}}{\rho_e^{(0)}} \right)^{2/3}, \quad (\text{“slabs”})$$

where  $\sigma_0 = 50 \text{ MeV fm}^{-2}$  and  $\rho_e^{(0)} = 0.4e \text{ fm}^{-3}$

- Thickness of “slabs”

$$a \simeq (9.4 \text{ fm}) \left( \frac{\sigma}{\sigma_0} \right)^{1/3} \left( \frac{\rho_e^{(0)}}{\rho_e^{(A)} - \rho_e^{(B)}} \right)^{2/3},$$

## Coulomb and surface effects in 2SC+NQ matter

Coulomb effects are easy to estimate, while surface tension is usually **not** well known

There are three possible cases:

- Low surface tension ( $\sigma \lesssim 20 \text{ MeV fm}^{-2}$ ):  
little effect; mixed phase survives (“slabs” with  $a \simeq 10 \text{ fm}$ )
- Intermediate values of surface tension ( $20 \lesssim \sigma \lesssim 50 \text{ MeV fm}^{-2}$ ):  
phase transition occurs at higher densities,  $3\rho_0 \lesssim \rho_B \lesssim 5\rho_0$
- Large values of surface tension ( $\sigma \gtrsim 50 \text{ MeV fm}^{-2}$ ):  
homogeneous phase is more favorable than mixed phase

Similar estimates are valid for hadron-quark mixed phases

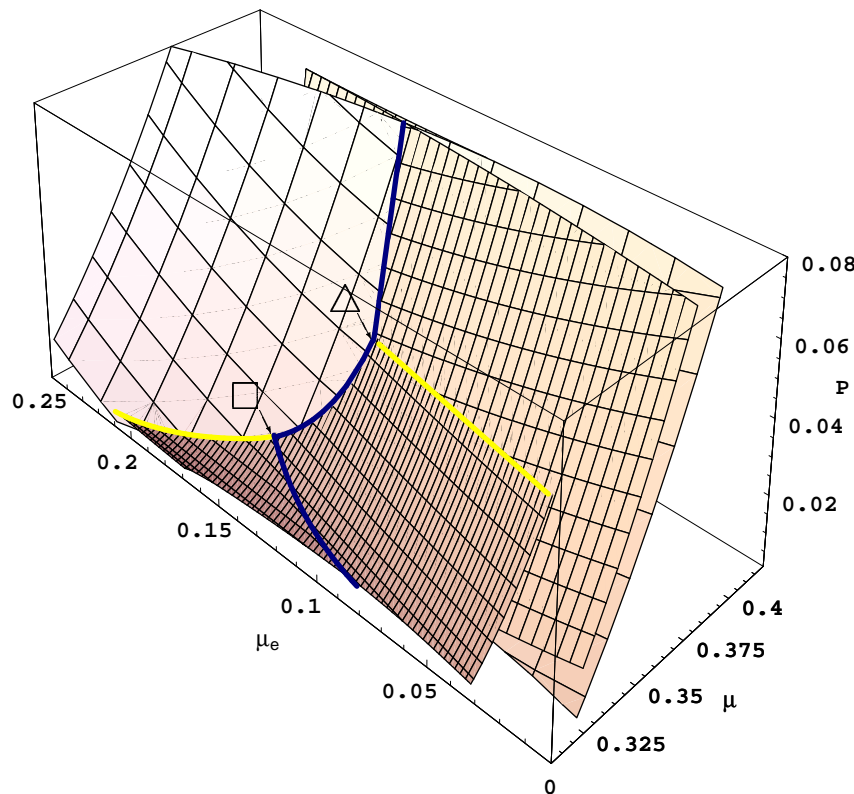
[Heiselberg et al.,01], [Alford et al.,01]

## Hadronic matter

- At low densities  $\sim \rho_0$  quarks are confined
- Some hadronic description is required
- We use hadronic chiral  $SU(3)_L \times SU(3)_R$  model  
[Papazoglou,98], [Papazoglou,99], [Hanuske,00]
  - nonlinear realization of  $SU(3)_L \times SU(3)_R$
  - spontaneous symmetry breaking
  - small explicit symmetry breaking
  - QCD motivated dilaton field is included
- Model describes well hadronic masses, finite nuclei, hypernuclei and neutron star properties

## Hybrid matter

Hadronic phase  $\rightarrow$  Hadron-quark MP  $\rightarrow$  2SC+NQ quark MP

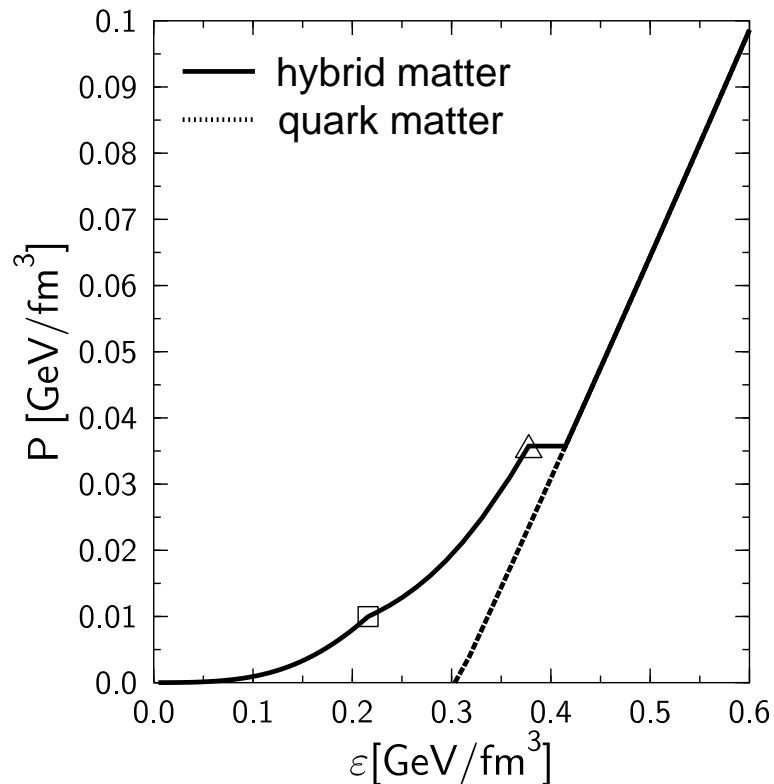


- Star crust matter:  
 $\rho_B \leq 0.08 \text{ fm}^{-3}$  [Baym et al., 71], [Negele & Vautherin, 73]
- Hadronic matter:  
 $0.08 \leq \rho_B \leq 1.49 \text{ fm}^{-3}$
- Hadron-quark mixed phase:  
 $1.49 \leq \rho_B \leq 2.56 \text{ fm}^{-3}$
- 2SC+NQ quark mixed phase:  
 $\rho_B \geq 2.75 \text{ fm}^{-3}$

$\Delta$ -point is a triple point (!)



## Equation of state



Star structure is determined by metric that satisfies Tolman-Oppenheimer-Volkoff equations

**Input:** equation of state  $P(\varepsilon)$

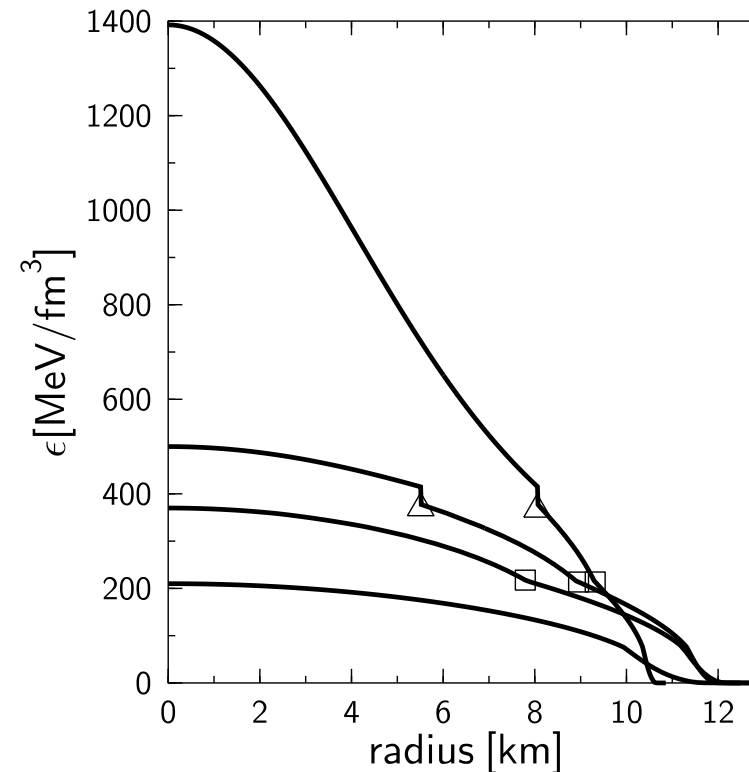
□ – beginning of hadron-quark MP

△ – beginning of 2SC+NQ quark MP

•  $\varepsilon$  and  $\rho_B$  have jumps at the triple point

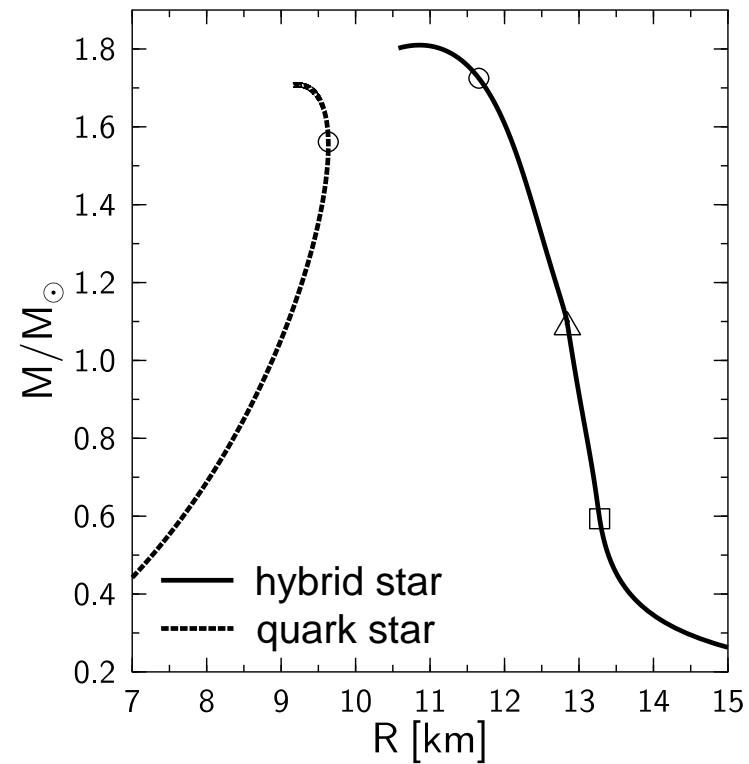
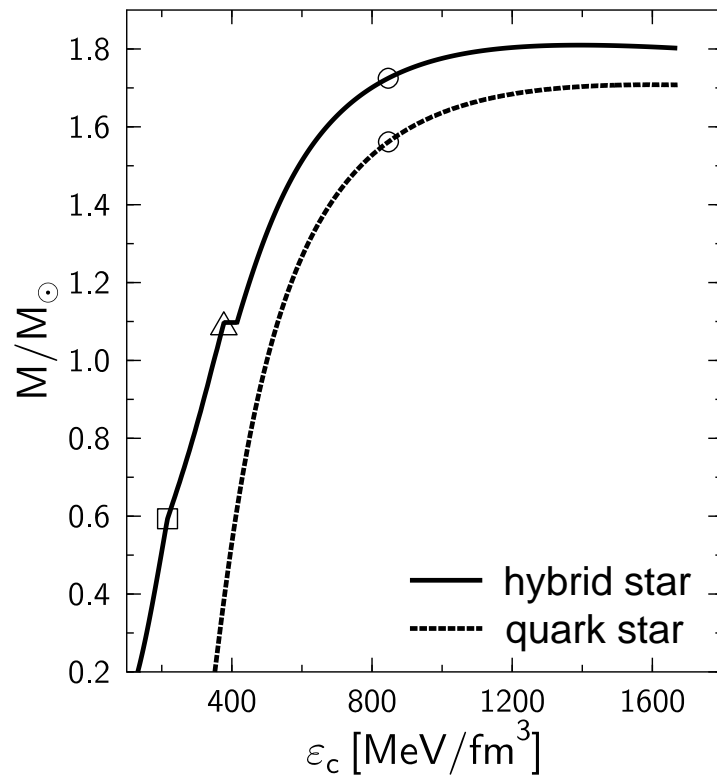
## Compact star structure

- $\epsilon_c = 210 \text{ MeV/fm}^3$  – pure hadronic star;
- $\epsilon_c = 370 \text{ MeV/fm}^3$  – hybrid star without quark core;
- $\epsilon_c = 500 \text{ MeV/fm}^3$  – hybrid star with a quark core;
- $\epsilon_c = 1392 \text{ MeV/fm}^3$  – largest mass star with parameters:  
 $M_{\text{max}} = 1.81 M_{\odot}$   
 $\rho_c = 7.58 \rho_0$   
 $R = 10.86 \text{ km}$



There are no stars with  $378 \leq \epsilon_c \leq 415 \text{ MeV/fm}^3$

## Mass-radius relation



Stars heavier than “ $\odot$ -stars” may have strange matter in their cores

## Summary

- Realistic EoS of nonstrange hybrid baryon matter is constructed
- Charge neutrality and  $\beta$ -equilibrium are taken into account; they play very important role
- Two-flavor color superconducting matter appears naturally as a component of 2SC+NQ mixed phase
- Construction of 2SC+NQ mixed phase is very stable: volume fractions of components change little with changing density
- The first example of a triple point is obtained and studied
- Sharp interface between the two mixed phases is observed; this is smoothed over distances of about 10 fm

## Outlook

- Generalization including strange quarks is needed
- All kinds of hybrid star properties should be studied
  - neutrino emissivity and mean free path
  - cooling rates of mixed phases
  - magnetic properties
- Surface tension effects and screening of Coulomb forces should be addressed
- Studies of color superconductivity in rotating stars are of interest
- Search for signatures of color superconductivity in compact stars should be made systematic