Clearinghouse Default Funds*

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June 1, 2018

Abstract

A prominent post financial crisis reform to reduce counterparty risk in over-the-counter markets is the adoption of clearinghouses. Current standards require clearing members to contribute to a loss-mutualizing default fund so as to cover the liquidation costs imposed by the default of two members, the "Cover II" rule. We show that such an arrangement is intrinsically vulnerable: although the default funds allow members to share risk ex-post, an inherent externality induces members to take excessive risk ex-ante. We solve for a default fund level that trades off ex-post risk-sharing with ex-ante risk-shifting, thus providing regulators an optimal cover rule for default fund collection.

Keywords: Central counterparties (CCPs), default funds, loss mutualization, externality, risk-taking

JEL: G20, G23, G28, D61, D82

*We are grateful to Bruno Biais, Liyan Yang, Haoxiang Zhu, and Yesha Yadav for constructive comments. We thank the participants of the participants of the 2018 Stern Microstructure Meeting. Contact information, Agostino: ac3827@columbia.edu; Wang: Jessie.Jiaxu.Wang@asu.edu; Zhang: hz2244@columbia.edu

1 Introduction

Central clearing is a key element in authorities' agenda for reforming over-the-counter (OTC) derivatives markets and mitigate systemic risk. Through a process called novation, central clearing replaces the contractual obligations between the trading parties with two equivalent positions between each of the trading parties and the clearinghouse. If one party defaults, its counterparty is insulated from losses because the contractual position is now with the clearinghouse. We refer to Pirrong (2011) for a comprehensive review of clearinghouse functioning mechanisms. The mandate for central clearing applies to all standardized OTC derivatives contracts both in the U.S. and in Europe. The class of products being centrally cleared has been rising steadily. The clearing rate for interest rate swaps has reached 87% in the US, while the comparable figure for the EU as a whole is 62% (CFTC, 2017). The share of outstanding credit default swaps (CDS) cleared increased to 55% at end December 2017 (BIS (2017)).

There is, however, still considerable debate over the determination of clearinghouse arrangements (see, e.g., Dudley, 2014, and *The Financial Times*, 2018). One the key requirements is that members contribute to a loss-mutualizing default fund. The current prevailing requirement is that members jointly post default funds sufficient to cover the liquidation costs of two defaulting members, the "Cover II" rule. More specifically, the European Union law imposes a "Cover II" model, while U.S. rules include both a "Cover I" as well as a "Cover II" option.[1] Using clearinghouse default resources data, Armakola and Laurent (2017) show that the Cover II rule is adopted by major derivative clearinghouses including ICE Clear Credit, CME Clearing, ICE Clear, and LCH.Clearnet. The default loss of an institution that exceeds its initial margins and its default fund contribution are absorbed first by the CCP equity

[1] In the U.S., clearinghouses must abide by a Cover I system at a minimum, but international and systemic U.S. clearinghouses have to comply with a Cover II requirement. We refer to `https://www.law.cornell.edu/uscode/text/7/7a-1`, see, Financial Resources therein, for the Cover I requirement. See `https://www.law.cornell.edu/cfr/text/17/39.33` for the Cover II requirement. CPSS-IOSCO regulatory guidelines require CCPs to maintain a default fund large enough to cover the default of two members, in extreme yet plausible market scenarios. The recent European Market Infrastructure Regulation requires each CCP to cover the default of the clearing member to which it has the largest exposure, or of the second and third largest clearing members if the sum of their exposures is larger.

capital and then by the default fund contributions of the surviving members on a pro-rata basis.[2]

In this paper, we study the optimal sizing of default fund contributions. First, we show that the loss-mutualization arrangement by the CCPs is intrinsically vulnerable: While the default funds allow members to share risk ex post, an inherent externality induces members to take excessive risk ex-ante. Second, we show that the excessive risk-taking behavior can be mitigated by regulating the amount of the default fund. We solve for a default fund level that trades off ex post risk-sharing with ex-ante risk-shifting, thus providing regulators an optimal cover rule for default fund collection. A novel result is that as the number of clearing members increases, the optimal default fund should cover the default costs of a fixed fraction of the members rather than a fixed number of clearing members, as is done, for instance, in the current "Cover II" rule.

To study the seizing of clearinghouse default funds, we develop a game-theoretical model. In the model, a regulator chooses the size of the default fund contribution to maximize social welfare, defined as the aggregate value of clearing members and the CCP. Given the required size of default fund, clearing members decide on the riskiness of the undertaken investments. Specifically, each clearing member maximizes its expected total profits, taking into account the expected costs including those to absorb losses due to other members' defaults.

Using the model, we study the efficiency of the loss-mutualization arrangement and the "Cover II" rule. Loss-mutualization redistributes counterparty risk among all members. However, this risk-sharing benefit comes with a flip side. Sharing the common pool of default funds creates dependency among members. When members can choose to take excessive risk to earn potentially higher returns, a negative externality arises. The degree of the inefficiency directly links to the size of the default fund contribution: while a higher default fund segregation comes with high funding costs, it mitigates the externalities by incentivizing members to take safe

[2]There is not yet a universally agreed upon loss allocation rule. ICE Clear Credit, the leading US CDS clearinghouse adopts a pro-rata basis for cleared futures and options contracts (see ICE, 2016). Similarly, for cleared CDS, it allocates losses among members on a pro-rata basis corresponding to the uncollateralized stress losses of each member.

investments. As such, a regulator faces a tradeoff in collecting default fund contributions between preventing members from excessive risk-taking and not imposing a too high funding cost. We study this tradeoff and identify the default fund size that emerges in equilibrium. To the best of our knowledge, our study is the first to formalize this mechanism in a central clearing setting.

The novelty of our paper is to analytically derive an optimal default fund requirement. Our model predicts that, as the number of clearing members increases, the optimal default fund converges to a fixed fraction of the default cost, provided that the opportunity cost of default fund is not too high. In this case, members choose safe investments in equilibrium and run away from the negative externalities they impose on each other. To the extent that the opportunity cost of default fund is associated with the cost of funding the collateral position, our model predicts that default fund levels should be higher than those prescribed by the Cover II rule in a low funding cost environment. This finding is in line with Murphy and Nahai-Williamson (2014) who argues that the Cover II standard is far from prudent.[3] If instead the cost of funding is high, requiring a level of default fund that is incentive compatible for members would be too costly. In this case, members engage in risk-taking, leaving the clearinghouse in a vulnerable position. These predictions are also consistent with legal studies, e.g., Yadav (2013), who argues that clearing members face risk-taking incentives that can encourage them to pursue risky payoffs at the expense of the clearinghouse.

The solution to the symmetric equilibrium risk-taking problem has a nontrivial methodological contribution. Our solution concept requires an extension of combinatorial techniques used in standard binomial problems, because the optimal risk-taking by other members introduces a mixing parameter in the binomial distribution. The proposed methodology opens the door to solving equilibrium problems in the presence of domino effects triggered by the action of participating agents.

Our main contribution is to develop a tractable model that delivers explicit cover-K re-

[3]The study argues that higher levels of financial resources may be needed to ensure clearing house robustness, "Perhaps a simple backstop to cover 2 could be considered, such as demanding that the default fund in addition meets the requirement that it is larger than some fixed percentage of the 'cover all' requirement."

quirements, accounting for the main economic forces at play—i.e., default costs, opportunity costs of posting collateral, and the risk-return trade-offs of the investments. These rules can be readily employed by clearinghouse supervising authorities to perform macro-stress testing.

Our paper adds to the growing literature on central clearing and its impact on counterparty risk and financial stability. Central clearing improves market conditions from OTC trading in several dimensions.[4] Acharya and Bisin (2014) show that central clearing can correct an inefficiency arising from the lack of portfolio transparency in OTC markets. Zawadowski (2013) argues that a clearinghouse can improve efficiency in interconnected markets as it effectively forces banks to contribute ex-ante to bail-out defaulting counterparties. Biais, Heider, and Hoerova (2012, 2016) show that CCPs can mutualize counterparty risk by appropriately setting margins. Other studies highlight various aspects of inefficiency in central clearing. Duffie and Zhu (2011) shows that central clearing can increase counterparty exposures if the clearing process is fragmented across multiple CCPs. Antinolfi, Carapella, and Carli (2016) illustrate an inefficiency with CCPs as loss-mutualization may weaken the incentives to acquire and reveal information about counterparty risk. Koeppl and Monnet (2010); Koeppl (2012) highlight a type of inefficiency related to market liquidity. Pirrong (2014) argues that central clearing reforms may redistribute risk rather than reduce risk. Different from these studies, we analyze the symmetric equilibrium of risk-taking and highlight a different form of inefficiency due to the loss mutualization mechanism from the default fund contribution.

A closely related paper to ours is Stephens and Thompson (2014), who consider a model economy with good and bad insurers and show that, in equilibrium, buyers choose to contract with bad insurers, resulting in higher counterparty risk relative traditional insurance markets composed of risk-averse insured parties. In an extension with CCPs, they show that the default fund arrangement can make the market riskier as risky insurers crowd out safe insurers. As a remedy to this situation, they propose that the default fund should be conditioned on the counterparty quality. Consistently with their study, we formalize the CCP loss mutualization arrangement and show how protection sellers would engage in risk-taking. Complementary

[4] Loon and Zhong (2014) and Bernstein, Hughson, and Weidenmier (2014) both find evidence consistent with central clearing reducing counterparty risk, using CDS spreads data and historical data, respectively.

to their proposed solution, we show that when the insurers' risk-taking is not contractible, simply regulating the level of default fund contribution is effective to alleviate the inefficiency.

A branch of literature has studied methods for stress testing CCPs and examine whether the Cover-II requirement is sufficient to withstand default losses. Using simulations, Paddrik and Young (2017) show that two members' simultaneous failure will result in insufficient funds at the CCP due to network spillover effects. Heath, Kelly, and Manning (2015); Heath et al. (2016) demonstrate that, owing to the the concentration of risk in CCPs, prefunded financial resources may not be able to absorb losses resulting from liquidity stress. Campbell and Ivanov (2016) highlight that crowded trades may lead to large simultaneous losses at several clearing members. Overall, these quantitative studies conclude that the Cover-II standard is inadequate because it does not account for compounding risk. Our study supports this conclusion and provides a theoretical model to quantify the size of default fund contributions that takes into account negative externalities among members.

Finally, our paper belongs to the nascent literature on default fund requirements. The distinguishing feature of our study, relative to existing literature, is that it is the first to investigate the role of risk-taking incentives under the loss mutualizing default fund arrangement. Existing studies follow exogenously specified risk measures to determine default fund requirements; in our approach, the default fund is endogenously determined by trading-off marginal opportunity costs of collateral resources and risk-taking costs. Menkveld (2017) analyzes systemic liquidation within a crowded trades setting and sets the default fund as the minimum level of funds needed to cover default losses in extreme yet plausible conditions. Ghamami and Glasserman (2017) provide a calibration framework and show that lower default fund requirements reduce the cost of clearing but make CCPs less resilient. Other studies focus on default fund design. Amini, Filipović, and Minca (2015) analyze systemic risk under central clearing in an Eisenberg-Noe clearing network, and propose an alternative structure for a default fund to reduce liquidation costs. In contrast, we solve for the optimal default fund in a symmetric equilibrium setting, taking into account members' incentives. Capponi, Cheng, and Sethuraman (2017) analyze the optimal balance of equity and default fund requirements from the clearinghouse point of view. In their model, the mass of safe and risky members is exogenously specified. Our model differs by endogenizing members' choices of risk and focusing on the socially optimal cover rule.

The paper proceeds as follows. Section 2 introduces the baseline model with binary risk and demonstrates members' incentives for excess risk-taking. Section 3 analyzes the game between the regulator and the clearing members; we demonstrate that although members' risk-taking is unobservable, it can be supervised when the regulator strategically chooses the default fund contribution. Section 4 generalizes the environment to the case of continuous risk choice and compares the social benefit and cost of increasing the size of the default fund. Section 5 concludes. Proofs of technical results are in the Appendix.

2 Baseline Model

In this section we introduce our baseline model in which clearing members have a binary choice of risk. By comparing the (first-best) risk level that maximizes social welfare with the one that maximizes individual members' profit, we demonstrate that members have an incentive to take excessive risk due to an inherent externality associated with loss mutualization.

2.1 The Environment

There are two dates, $t = 0, 1$, N risk-neutral protection sellers that are clearing members of the CCP, N risk-averse protection buyers, and a regulator.

At $t = 0$, buyers and sellers enter into insurance contracts. The price of the contract is normalized to 1, and is paid by the buyer to the seller at $t = 0$. The contract specifies that a payment δ from the seller to the buyer is made at $t = 1$ upon the occurrence of a credit event. Throughout the paper, we assume that the credit event occurs with 100% probability.[5]

[5]In practice, the credit event is stochastic. While one could assign a probability to the occurrence of a credit event, such an addition would complicate the setup without leading to novel economic insight. To make the insurance contracts meaningful, we implicitly assume that the buyers are not able to self-insure or have access to alternative assets that delivers a future payment upon the credit event.

Sellers choose the riskiness of the investments, i.e., whether to invest in a risky (r) or less risky projects which we refer to as safe (s) project. The project types model engagement in risky investments, choice of weak trading counterparties before novation, failure to exert effort in risk management (see, e.g., Biais, Heider, and Hoerova, 2016), or insufficient hedging of counterparty exposure. Such choices are unobservable and capture the risk-return tradeoff faced by the sellers. The project has i.i.d. payoffs. At time $t = 1$, payoffs are realized and payments are allocated. With probability $q \in (0, 1)$, the investment fails and the seller obtains zero payoff. With probability $1 - q$, the investment is successful and delivers a payoff R. Denote the expected return from investment as $\mu = (1 - q)R$. A risky investment has a higher chance of default and a higher payoff when successful. The following assumption formalizes the risk-return trade-off between the risky and safe investments.[6]

Assumption 1B *A member who chooses the risky project has a higher payoff if the investment is successful, but a higher default probability than a member who chooses the safe project—i.e.,*

$$0 < R_s < R_r, \quad 0 < q_s < q_r. \tag{1}$$

The expected return of a risky project is lower than that of a safe project

$$0 < \mu_s - \mu_r < \delta(q_r - q_s). \tag{2}$$

Because the safe project always yields a higher expected return than the risky project, and also has a lower default probability, then it is the choice that maximizes the aggregate values of all agents in the economy (clearing members and the CCP) given by[7]

$$\{a^*\} = \arg\max_{a_i^*} \mathbb{E}\left[\sum_i \left((1+f)R_{a_i}\mathbf{1}_{i \text{ survives}}\right)\right]. \tag{3}$$

Hence, the risk profile $a_i^* = l$ represents the optimal benchmark.

[6] We label the assumptions in this Section with letter "B" to denote for the baseline model. Similar assumptions that directly extend to the continuous case in Section 4 share the same assumption number but have a letter "C" instead.

[7] The payments to the buyers are always delivered so the riskiness of investment does not affect the buyers' utility. The premiums charged from the protection buyers by sellers are wealth transfers between agents in the economy and are proportional to the size of the investment of sellers, $1 + f$.

Our trade-off between risky and safe projects exhibit similarities with the model proposed by Thompson (2010) to study the effect of counterparty risk on credit default swap contracts. In his model, the bank sheds credit risk from two types of loans, a safe and a risky type. Both loans have the same return if they succeed and yield zero if the fail, but the safe loan has a higher probability of succeeding than the risky loan.

Protection buyers are identical and risk averse with mean-variance preferences, i.e., $U(C) = E[C] - \gamma Var[C]$, where $\gamma > 0$ is the mean-variance risk aversion coefficient.

2.2 Bilateral Trading

We describe the utilities of the market participants in the case that trading is not guaranteed by a CCP.

Protection Sellers. If the investment is successful, the protection seller pays the buyer an amount δ and keeps the residual amount $R_a - \delta$, $a \in \{s, r\}$. However, if the investment is unsuccessful, the seller defaults on the buyer, creating counterparty risk. The expected profits for the seller is

$$V_{noCCP}(a) = (1 - q_a)(R_a - \delta) = \mu_a - (1 - q_a)\delta, \quad a = s, r. \tag{4}$$

From (26) and Assumption 1B, the protection seller would always choose the risky project when he does not trade through a CCP.

Protection Buyers. The buyer gets a zero payoff when the seller defaults, which occurs with probability q_r. Otherwise, the buyer gets the full promised payment δ, with probability $1 - q_r$. The expected utility for the buyer is therefore

$$U_{noCCP}(\mathbf{1}_{\text{seller survives}}\delta) = (1 - q_r)\delta - \gamma q_r(1 - q_r)\delta^2. \tag{5}$$

2.3 Trading via the Clearinghouse

This section analyzes the loss allocation mechanisms in the presence of a CCP and the utilities of protection buyers and sellers.

CCP Loss Allocation. Through central clearing, buyers and sellers establish direct contractual relationships with the CCP, which becomes the buyer to every seller, and the seller to every buyer. The CCP mutualizes payment shortfalls caused by the default of a seller, who fails to make the promised payment δ to the protection buyer. This way, central clearing pools the counterparty risk of sellers. The loss mutualization mechanism requires a positive prefunded default fund $F \in (0, \delta)$ from each seller at $t = 0$. Because the default fund is segregated upfront, each member will incur an opportunity cost of βF, where $r \in (0, 1)$ may be interpreted as the cost of funding the collateral position.

We analyze the economic role of default funds and do not model initial margins which usually serve as the first line of defense against default losses. We view the shortfall as describing the losses that exceed the initial margin requirements, and focus entirely on the risk-shifting incentives triggered by the default fund resources.[8] We consider a default waterfall structure in which the clearinghouse capital has seniority over the default funds of surviving members in absorbing losses.[9] After the fund of a defaulted clearing member is exhausted, the loss mutualization mechanism ensures that the remaining losses are allocated proportionally to the available default funds of the surviving members. As such, the default fund increases a surviving member's chances of incurring a loss when others default, creating externality among members. Consistently with the regulatory CCP risk management framework, referred to as the Principles for Financial Market Infrastructures (see CPMI and IOSCO, 2012), we require that the total default fund of the CCP covers the simultaneous defaults of two clearing members. This is formalized by the following assumption:

Assumption 2B *The aggregate default fund contribution satisfies the* Cover II *requirement,*

[8] Each clearing member has two primary accounts to trade with clearinghouse: a "customer account" if the positions are taken on behalf of their clients, and a "house" account if the positions are proprietary. Initial margins for client trades are posted by the clients themselves, while initial margins for proprietary trades are posted by the members. In the case of ICE Clear Credit, the largest clearinghouse for CDS contracts, house accounts contribute to a significantly higher gross notional compared to customer accounts, \$169 billion versus \$46 billion (see Capponi et al. (2017)). Because of these relative magnitudes, we view the entire set of collateral resources (initial margins and default funds) as posted by the clearing member, and focus on the losses in excess of their initial margins.

[9] A well known example is the Korean CCP KRX. The default of a clearing member in December 2013 generated losses that exceeded the defaulter's collateral. According to the KRX's rules, the remaining losses were allocated first to the default fund contributions of surviving participants.

i.e.,

$$2\delta \leq NF. \tag{6}$$

The *Cover II* rule guarantees that the total default fund contributed by all members, equal to NF, is always sufficient to cover the missed payment of two protection selling members. It also sets a lower bound for F, i.e., $F \geq \frac{2\delta}{N}$. If more than two members default, the total default fund contribution may not be enough to cover the losses. In this case, the remaining shortfalls are covered by the CCP's capital.

Assumption 3B *The CCP contributes with his own capital and incurs an equity loss of* $(\mathcal{N}_d\delta - NF)^+$*, where* $\mathcal{N}_d$ *is the number of defaulting members, i.e.,* $\mathcal{N}_d = \sum_j \mathbf{1}_{j \text{ defaults}}$.

Assumption 3B guarantees that there are always enough resources in the system to honor the payments to the buyers in every possible state of the economy.

Protection Sellers. When trading through the CCP, the protection seller charges an additional premium, f, from the buyer at $t = 0$ for the elimination of counterparty risk. At the same time, the seller needs to post the default fund F to the clearinghouse. The premium f will be invested by the seller, which scales up his investment size by $(f \times 100)\%$. For a given default fund requirement F, and investment choice $a_{-i} = (a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N)$ made by other sellers, seller i decides on the riskiness of his investment to maximize the expected profits at $t = 1$,

$$V_i(a_{-i}) = \max_{a_i \in \{r,s\}} \mathbb{E}\left[\mathbf{1}_{i \text{ survives}}\left[(1+f)R_{a_i} - \delta + F - \min\left(F, \frac{\mathcal{N}_d}{N - \mathcal{N}_d}(\delta - F)\right)\right]\right]. \tag{7}$$

The objective function (7) can be understood by analyzing two possible scenarios:

1. If seller i defaults (with probability q_{a_i}), he obtains zero payoff; his segregated default fund F contributes towards his promised insurance payment δ to the protection buyer.

2. If seller i survives (with probability $1 - q_{a_i}$), he obtains proceeds $(1 + f)R_{a_i}$ from his investment, and part of them are used to make the promised payment δ to the protection buyer. The default fund posted by i is used to absorb the shortfalls of the defaulting

members, and the residual amount, if any, is returned to i; hence, the total profit of surviving member i at $t = 1$ is given by

$$(1 + f)R_{a_i} - \delta + F - \min\left(F, \frac{\mathcal{N}_d}{N - \mathcal{N}_d}(\delta - F)\right) \tag{8}$$

To understand the last term in Eq. (8), we can consider the following cases. If no clearing member defaults, the cost from loss mutualization incurred by i is zero. If $\mathcal{N}_d$ number of members default, the total shortfall due to their defaults is $\mathcal{N}_d\delta$. Each defaulted member will first absorb the losses using his own default fund, and the remaining shortfall will be shared equally by the surviving members. Hence, each member i will be charged, on average, an amount equal to $\frac{\mathcal{N}_d}{N-\mathcal{N}_d}(\delta - F)$ capped at maximum amount F that i has segregated.

By participating in central clearing, the seller's total expected profits include the expected profit accrued at $t = 1$ given by Eq. (7), net of the opportunity cost of the default fund requirement, i.e.,

$$V_{CCP}(F) = -(1 + \beta)F + V_i(a_{-i}; F). \tag{9}$$

where $V_i(a_{-i}; F) \equiv V_i(a_{-i})$ in (7) to emphasize its dependence on the default fund F.

Protection Buyers. Compared to trading with protection sellers bilaterally, buyers are hedged against the risk exposure to their selling counterparty if trading occurs through the CCP, thus deriving risk-sharing gains. The payment obligation of the seller is honored with certainty, hence the expected utility of the buyer is

$$U_{CCP}(\delta) = \delta. \tag{10}$$

Participation in Clearing. Comparing equations (5) and (10), the buyer is willing to pay an additional premium f to trade with a seller through the CCP. The premium is a certainty equivalent that the buyer is willing to give up in exchange for eliminating the counterparty risk. The premium f leaves the buyer indifferent between trading with the seller bilaterally or through the CCP, i.e., it satisfies

$$U_{CCP}(\delta - f) = U_{noCCP}(\mathbf{1}_{\text{seller survives}}\delta) \iff f = q_r\delta + \gamma q_r(1 - q_r)\delta^2. \tag{11}$$

This premium consists of two parts, the expected loss due to the counterparty's default $q_r\delta$, and a risk premium for the variance reduction $\gamma q_r(1-q_r)\delta^2$. The higher the risk-aversion γ, the higher this risk premium.[10] Throughout the paper, we impose the following assumption.

Assumption 4B *Buyers are sufficiently risk averse, i.e., their absolute risk aversion coefficient satisfies*

$$\gamma > \frac{\beta + q_r}{\mu_r q_r (1-q_r)\delta} - \frac{1}{(1-q_r)\delta}. \tag{12}$$

As we demonstrate later in the paper, under this assumption, protection buyers are willing to pay a sufficiently high premium to the sellers so that sellers extract risk-sharing benefits from participating in the CCP. This premium can be a very small amount if $r + q_r$ is small.

The following proposition shows that a seller benefits from participating in central clearing.

Proposition 1 *The expected profit of a seller is higher if he trades with the CCP as opposed to bilaterally with a buyer even if all other sellers still choose to invest in the risky project, i.e. when $a_j = r$ for all $j \neq i$, we have $V_{CCP}(a_{-i}; F) - V_{noCCP}(a_i) > 0$, $\forall F \in (0, \delta)$ and $a_i \in \{r, s\}$.*

By trading through the CCP, a seller obtains the lowest profit when the highest default fund requirement is charged, i.e., when $F = \delta$. In this case, the seller is required to post an amount of collateral that would be sufficient to cover the promised payment to the buyer even in the absence of any loss mutualization mechanism. Since the prefunded position is costly, the seller would be worse off if he is charged such a high default fund requirement. Under Assumption 4B, the premium in Eq. (11) is high enough to incentive him to join the CCP, because the expected profit V_{CCP} is always higher than V_{noCCP}. Hence the added value of a CCP is always positive and the seller is willing to become a member of the CCP.

[10] To better capture the value from risk-sharing through participating in central clearing, we assume that the seller has full bargaining power in the contract.

3 Risk Shifting and Default Fund Requirements

The objective of this section is to analyze the strategic interactions between a regulator, who chooses the default fund level to maximize social welfare, and protection sellers who decide between a risky and a safe project to maximize their individual expected utility. We model these strategic choices using a Stackelberg game, in which the regulator is the leader and the protection sellers are the followers.

Definition 1 *A* Nash equilibrium between clearing members and the regulator *is a set of members' risk profiles* $a^e := (a_1^e, \dots, a_N^e)$ *and a default fund contribution* F^e *set by the regulator, such that:*

1. *Taking the default fund* F^e *and other members' risk profile* a_{-i}^e *as given,* a_i^e *solves the optimization problem of clearing member* i *given in* (7).

2. *Taking as given the risk profile of clearing members* a^e*, the regulator chooses a feasible default fund level* F^e*, satisfying assumptions* (2B)–(3B) *to maximize the aggregate value of the members and the equity of CCP net of the total social costs from unsuccessful investments.*

$$F^e = \arg\max_F \left\{ \sum_i V_i - \mathbb{E}\left[(\mathcal{N}_d\ \delta - NF)^+\right] - N(1+\beta)F \right\}. \qquad (13)$$

In the above equation, V_i *is given by Eq.* (7)*, and* $\mathcal{N}_d = \sum_i \mathbf{1}_{i\ defaults}$ *is the number of defaulted members.*

To solve for the equilibrium of the full game, first we analyze the second stage of the game (see Sections 3.1.1), in which the protection sellers coordinate on the choice of investments given an exogenously specified default fund level F. We study the equilibrium choice of F made by the regulator in Section 3.2.

3.1 Equilibrium Members' Investment Choices

In this section, we solve for the best response of members to a given default fund F. Our objective is to compare the investment choice of the profit-maximizing members in equilibrium with the socially optimal choice. We call *risk-shifting* the situation in which the first-best outcome prescribes that all members choose the safe project but the equilibrium response of members is to instead undertake the risky project.

3.1.1 Second-best: optimal investment

We analyze the equilibrium project choices of protection sellers and the incentives behind their decisions. We start with the definition of a Nash equilibrium between members' project choices.

Definition 2 *Under Assumption 2B, the risk profile* $(a_1, a_2, \ldots, a_N) \in \{r, s\}^N$ *is a Nash equilibrium if for all* i*,* a_i *is member* i*'s strategic investment choice that achieves* $V_i(a_{-i})$ *given in Equation (7). A Nash equilibrium* $(a_1, a_2, \ldots, a_N)$ *is Pareto dominating, if, for any other Nash equilibrium* $(b_1, b_2, \ldots, b_N)$*, it holds that* $V_i(a^{-i}) \geq V_i(b^{-i}), \quad i = 1, 2, \ldots, N,$*, and the above inequality holds strictly for at least one* i*.*

Suppose member seller i survives, and g of the remaining $N-1$ member sellers choose the safe project. Then, for a given default fund, the expected contribution of member i to other members' defaults is given by

$$\mathbb{E}\left[\min\left(F, \frac{\mathcal{N}_d}{N - \mathcal{N}_d}(\delta - F)\right)\middle|\text{member } i \text{ survives}\right] := F - \psi(g; F). \tag{14}$$

The left-hand side of (14) is the expected contribution of member i to other members' defaults, given that i survives. The quantity $F - \psi(g; F)$ on the right-hand side of Eq. (14) is the cost due to loss mutualization; $\psi(g; F)$ is the refund amount member i will get from CCP if he survives. A lower value of $\psi(g; F)$ means that the negative externalities on i caused by the risky project choices of the other members are high. The next proposition summarizes our main results on the investment choice of clearing members.

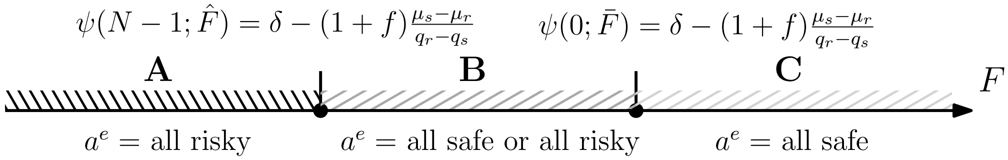


Figure 1. Equilibrium Investment Choices. This figure illustrates how the equilibrium investment choices change as we vary the size F of the default fund. a^e is the equilibrium risk profile from individual members' strategic behavior. In region A, the unique equilibrium differs from the first-best benchmark and features risk-shifting; in region B, we have multiple equilibria; in region C, the unique equilibrium coincides with the first-best.

Proposition 2 *For a given default fund requirement F satisfying Assumption 2B, the equilibrium risk profiles are given by*

$$a^e = \begin{cases} r, \forall i & F < \hat{F} \\ r, \forall i, \text{ or } s, \forall i & \hat{F} \leq F \leq \bar{F} \\ s, \forall i & \bar{F} < F \end{cases} \tag{15}$$

Moreover, $\psi(\cdot;\cdot)$ is a strictly increasing function in both the first and second argument. The parameters $\hat{F}$ and $\bar{F}$ are implicitly defined as[11]

$$\delta - \psi(N-1;\hat{F}) = (1+f)\frac{\mu_s-\mu_r}{q_r-q_s}, \quad \delta - \psi(0;\bar{F}) = (1+f)\frac{\mu_s-\mu_r}{q_r-q_s}. \tag{16}$$

If $\hat{F} \leq F \leq \bar{F}$, the "all safe" equilibrium Pareto dominates the "all risky" equilibrium.

The result in Proposition 2 states that the members' incentives in shifting risk decrease as the required default level of F rises. If the default fund is high enough such that $F > \bar{F}$, then each member decides to run away from the externalities and chooses the safe project. Figure 1 illustrates the equilibrium risk profile as a function of F.

3.1.2 Inefficiency in Investment Choice: risk-shifting

In this section, we show that there exist cases in which the loss mutualization mechanism induces members to take excessive risk *ex ante* due to an inherent externality among members.

[11]If the premium f is such that $(1+f)\frac{\mu_s-\mu_r}{q_r-q_s} \geq \delta$, then both equations in (16) have no solution because the right hand side is always larger when $F \in (0,\delta)$. In this case, "all safe" is the only equilibrium.

Corollary 3 *If $F < \hat{F}$, the first-best benchmark is not an equilibrium, and in fact, "all risky" is the unique inefficient equilibrium. More generally, if $F \leq \bar{F}$, "all risky" is an inefficient equilibrium.*

Under $F < \hat{F}$, if all other members choose the safe project, member i strategically deviates to choose the risky project. Loss mutualization implies that the expected liquidation cost $F - \psi(N-1; F)$ is lower than δ. This wedge shifts the member's incentive from choosing a safe to choosing a risky project, thereby creating a *risk-shifting* problem. Notably, the adoption of CCPs exacerbates the degree of risk-shifting. Without CCP, each seller invests one unit of risky project, whereas with CCP, the risky investment scales up to $1 + f$. By not implementing the right level of default fund, the introduction of a CCP poses a trade-off between risk-sharing and risk-shifting.

In the presence of risk-shifting incentives, it is still possible to achieve the socially optimal risk-taking in equilibrium. This happens when the default fund contribution F is close enough to the default cost such that the externality is restrained.

Corollary 4 *If $\bar{F} < F$, the "safe" equilibrium is the unique equilibrium and is first-best. Moreover, the "safe" equilibrium is the unique Pareto dominant equilibrium if $\hat{F} < F$.*

3.2 Default Fund: a tool to mitigate risk-shifting

In this section, we study the first stage of the game, in which the regulator chooses a default fund that mitigates the inefficiency arising from excessive risk-taking. We show that the regulator can correct the inherent externality by optimally choosing a default fund, balancing the *ex post* risk-sharing benefit and *ex ante* risk-shifting cost.

Corollary 4 indicates that a default fund can potentially correct members' risk-shifting incentives. We solve for the optimal default fund level that satisfies the criterion in Equation (13), taking the functional dependence of a^e on F into account.

Given the assumption of independent defaults, the objective function of the regulator in (13) may be rewritten as follows:

$$\begin{aligned} W(F) &= \mathbb{E}\left[\sum_i V_i - (\mathcal{N}_d\,\delta - NF)^+ - N(1+\beta)F\right] \\ &= \mathbb{E}\left[\sum_i (1+f)R_{a_i} - N\beta F - N\delta\right]. \end{aligned} \tag{17}$$

Thanks to Proposition 2, it suffices to consider either "all safe" or "all risky" profiles. We base our analysis on the equilibrium refinement concept of Pareto dominance: From Proposition 2 and Corollary 4, members all choose the safe project when $\hat{F} \le F$ because it is either the unique equilibrium (region C in Figure 1) or the Pareto dominating equilibrium (in region B). Hence the equilibrium risk profile *switches* from risky to safe projects at the boundary between Regions A and B. To obtain the threshold value of $\hat{F}$ at this boundary, notice that the function $F \mapsto \psi(N-1;F) - (1+f)\frac{\mu_r-\mu_s}{q_r-q_s} - \delta$ is a a strictly increasing function of F by Lemma 9 in Appendix B. Moreover, $\psi(N-1;\delta) = \delta$. Thus if the following condition holds, [12]

$$\psi\left(N-1;\frac{2\delta}{N}\right) < (1+f)\frac{\mu_r-\mu_s}{q_r-q_s} + \delta, \tag{18}$$

there is a unique threshold value $\hat{F}$ such that $\psi(N-1;\hat{F}) = (1+f)\frac{\mu_r-\mu_s}{q_r-q_s} + \delta$.

The function $W(F)$ may be defined piecewisely, with just one discontinuity at $\hat{F}$, as follows:

$$W(F) = N\left(W^s(F)\mathbf{1}_{F\ge\hat{F}} + W^r(F)\mathbf{1}_{F<\hat{F}}\right), \quad W^a(F) = (1+f)\mu_a - \delta - \beta F, \quad a = r, s. \tag{19}$$

Because the 'safe" equilibrium is socially optimal, $W(F)$ exhibits a positive jump as we increase F: the equilibrium switches from "all risky" to "all safe" at $\hat{F}$. The size of the upward jump at $\hat{F}$ is given by

$$W^s(\hat{F}) - W^r(\hat{F}) = (1+f)(\mu_s - \mu_r) > 0. \tag{20}$$

Moreover, $W^r(F)$ and $W^s(F)$ are both strictly decreasing in F. Thus the maximum of $W(F)$ over the set of feasible values for F is attained either at the lower bound $\frac{2}{N}\delta$, or at the switch

[12]When the inequality fails to hold, the aforementioned switch from "all safe" equilibrium to "all risky" equilibrium does not occur. The reason is that the lower bound set by the Cover II rule may already be large enough that choosing risky projects is not attractive for an individual member. In the sequel, we only focus on the (interesting) case that the inequality (18) holds.

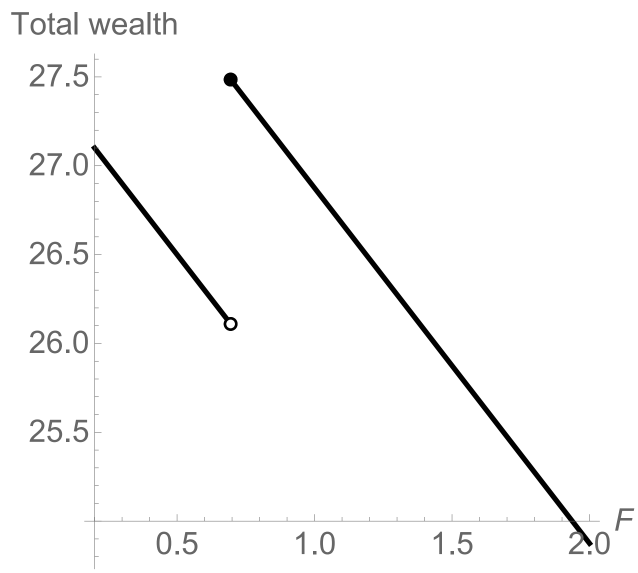


Figure 2. Regulator's Objective Function $W(F)$. This figure plots the total wealth $W(F)$ as a function of F, where F ranges from the lower bound $\frac{2\delta}{N}$ that satisfies the Cover II requirement to the upper bound that equals δ. The graph shows that a default fund given by the Cover II requirement may not be socially optimal. Rather, a higher value of F that corrects the risk-shifting yields the highest total value. Model parameters: Model parameters: $\gamma = 0.14$, $\mu_r = 2.7, \mu_s = 2.755, N = 20, \delta = 2, q_r = 0.1, q_s = 0.05$, and $\beta = 0.1$. The optimal default fund $F^e = \hat{F} = 0.69$, and this alternative default fund rule helps the regulator gain a total wealth of $W(F^e) - W(\frac{2}{N}\delta) = 0.39$ over the default cover-II rule. (From ICE Clear Credit (`https://www.theice.com/clear-credit`), the number of clearing members is between 20 and 30.)

point $\hat{F}$. We summarize this result in the following proposition (see also Figure 2 for an illustration).

Proposition 3 *Suppose the inequality (18) holds. The default fund level that maximizes the regulator's objective function (13), subject to the constraint imposed by the Cover II requirement (6), is given by*

$$F^e = \begin{cases} \hat{F}, & \text{if } W^s(\hat{F}) - W^r(\hat{F}) > \beta\left(\hat{F} - \dfrac{2}{N}\delta\right), \\ \dfrac{2}{N}\delta, & \text{else.} \end{cases} \tag{21}$$

In conclusion, the objective of the regulator is to maximize the total value of the agents in the system, taking members' incentives for risk-shifting into consideration. The choice of default fund yields the following tradeoff. On the one hand, the benefit of having a higher value of F is to correct for risk-shifting: as F increases to $\hat{F}$, we move from region A to region B in Figure 1, where members' risk choices switch from risky to safe. Hence, by mandating a high enough default fund F , the regulator can give the member an incentive to choose risk with a total social value equal to $N\left(W^s(\hat{F}) - W^r(\hat{F})\right)$. On the other hand, increasing F raises the opportunity cost of each member from $\beta\frac{2}{N}\delta$ to $\beta\hat{F}$. If the benefit exceeds the cost, it is optimal to set the default fund at $\hat{F}$, a higher value than required by the Cover II rule; otherwise, the regulator will find it optimal to use the Cover II rule.

3.3 The optimal covering number

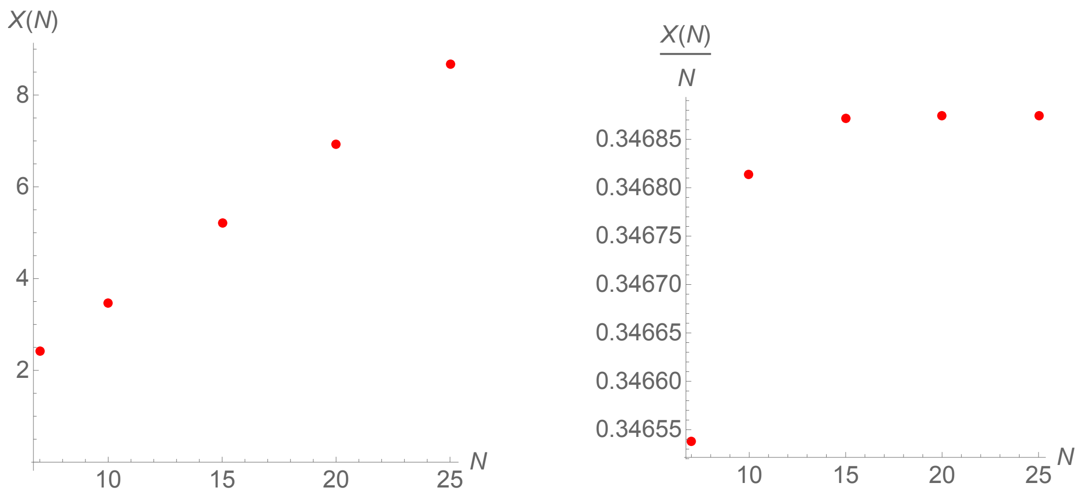
The previous sections have shown that the Cover II rule is not necessarily socially optimal, when we consider the role of mitigating risk-shifting. As the number of clearing members increases, how many members' defaults should the default funds be ready to cover? We propose a Cover X rule so that the implied default fund achieves the efficiency of $\hat{F}$ in Equation (21).

The *generalized Cover X rule* for any given number N of participating clearing members is

$$x(N) := \frac{NF^e(N)}{\delta}, \tag{22}$$

where $F^e(N)$ is the default fund level that maximizes the regulator's objective given in Proposition 3, when N is the number of members in the CCP. When $x(N) > 2$, our model provides a rationale to charge a default fund more than the current regulatory requirement prescribed by the Cover II rule; see also Figure 3 for an illustration.

Interestingly, the exact Cover X rule depends on the number of members. If $N = 7$, then $x(N)$ is approximately equal to two, so Cover II is able to prevent risk-shifting. However, if, for example, $N = 20$, the optimal requirement becomes Cover 6.9. Under the Cover II requirement, clearing members would engage in excessive risk-taking and deviate from the social optimum, resulting in a higher total cost of default. In contrast, the proposed Cover 6.9 requirement induces members to choose safe projects, thereby effectively mitigating risk-shifting.



(a) $x(N)$ (b) $\frac{x(N)}{N}$

Figure 3. Optimal Covering Number. This figure shows the optimal covering number $x(N)$ given in equation (22) as a function of the number of clearing members N. We use the same parameter settings for project return and default probability as in Figure 2. Left panel: the optimal covering numbers $x(N)$; Right panel: the ratio $\frac{x(N)}{N}$. The graph suggests that the optimal default fund should cover payment shortfalls caused by a 34.7% fraction of members defaults.

While the optimal covering number clearly depends on the number of members N, the ratio $x(N)/N$ shows little variation with respect to N, if N is sufficiently large. The next proposition characterizes the asymptotic behavior of the optimal coverage ratio, as the number of clearing members grows large. Specifically, it shows that the ratio between the optimal default fund level $F^e(N)$ and δ, or equivalently, the optimal proportion of covered members, $x(N)/N$, converges to a constant as the size of the CCP network tends to infinity. If the marginal opportunity cost β is sufficiently low, then this limit is a positive number in $(0, 1)$, meaning that the optimal covering number should be proportional to the size of network N (at least for large N); otherwise, this limit is 0, implying that it is optimal to cover only a small portion of the CCP network.

Hence, as the number of clearing members grows large, the cover rule that the regulator should adopt is simple—rather than covering a fixed number of clearing members as prescribed, for instance, by the Cover II rule, the regulator should cover a fixed fraction of the members. Major U.S. derivative clearinghouses consist of more than 20 members (this is the case, for instance, for the major CDS derivative clearinghouse ICE Clear Credit, and interest rates swaps clearinghouse LCH). Our result then implies that the default fund rule is robust with respect to entry and exit of the members in the clearing business.[13]

Proposition 4 *Suppose the fee f is sufficiently small so that $(1+f)\frac{\mu_r-\mu_s}{q_r-q_s}+\delta>0$. In the large CCP network limit – i.e., as $N\to\infty$ – we have*

$$\frac{F^e(N)}{\delta}=\frac{x(N)}{N}\to\begin{cases} q_s+(1-q_s)\dfrac{1}{\delta}\left((1+f)\dfrac{\mu_r-\mu_s}{q_r-q_s}+\delta\right), \\ \textit{if } \mu_s-\mu_r>\beta\dfrac{(q_s\,\delta+(1-q_s)((1+f)\frac{\mu_r-\mu_s}{q_r-q_s}+\delta))}{1+f}, \\ 0, & \textit{else.}\end{cases}$$

It is immediate from Proposition 4 that if the differential ratio $(1+f)\frac{\mu_s-\mu_r}{q_r-q_s}$ is very close to δ (i.e., the risky project yields only a small advantage over the safe project), then the members are more inclined to switch from risky to safe investments as the default fund level increases. Thus the regulator can use a lower default fund level to prevent the risk-shifting. However, unless the marginal funding cost f is large, the default fund level should be no less than $(q_s\times 100)\%$ of δ.

4 Continuous Choice of Risk-Taking

Having illustrated the simple logic of how a default fund can alleviate risk-taking for two possible choices of risky projects, in the rest of the paper we extend the analysis to the more general case of a continuous choice of risky projects. Such a setup allows us to track the marginal impact of setting a higher default fund contribution on the risk-taking behavior of a member, and to obtain a closed form asymptotic expression (as the number of members grows large) for the equilibrium default fund contribution.

[13] For instance, in May 2014, the Royal Bank of Scotland announced the wind down of its clearing business due to increasing operational costs. This was followed by State Street, BNY Mellon, and more recently Nomura, each of whom shut down part or all of their clearing business.

4.1 The Environment

Member i has a continuous choice of risk-levels. The member invests a fraction $a_i \in [0,1]$ of its resources in the risky project and allocates the remaining fraction $1-a_i$ to a risk-free project. The risk-free project has a guaranteed payoff of $(1+\beta)$ times the invested amount, and can be thought as an investment in risk-free assets (e.g., U.S. Treasury securities). The risky project can be interpreted as a portfolio of loans to risky firms in the corporate sector or of mortgages, exposing the member to volatile returns. The payoff, denoted by $\tilde{R}_i$, is assumed to be a random variable at time 0, and is realized at time 1:

$$\tilde{R}_i = \begin{cases} R, & \text{with probability } 1-a_i \\ 0, & \text{with probability } a_i, \end{cases} \tag{23}$$

R can be viewed as the notional value of the loan/mortgage. We assume $R > 1+\beta > 0$: in the good state, the realized payoff is higher than that of the risk-free project, whereas in the bad state the payoff is lower than the return from the same investment in the risk-free project. Similar to the setup in Holmstrom and Tirole (2001) and Acharya, Shin, and Yorulmazer (2010), the risky technology has diminishing returns to scale with risk-taking; i.e., the probability of a good state decreases with a_i.[14] In particular, the probability of observing the good state is $1-a_i$.

Member i defaults if its realized payoff is smaller than $1+\beta$:

$$\mathbf{1}_{i \text{ defaults}} \Leftrightarrow a_i\tilde{R}_i + (1-a_i)(1+\beta) < 1+\beta \Leftrightarrow \tilde{R}_i = 0. \tag{24}$$

The equation above indicates that the default probability of member i is equal to a_i, i.e., the fraction invested by i in the risky project. Default can always be avoided if member i invests entirely in the risk-free project. Notice that at $a = 0$, the marginal profit of risk-taking is R, whereas the marginal cost is $1+\beta$ (forgone return from the risk-free project). We assume that the realizations $\tilde{R}_i$, $i = 1, \ldots, N$, are independent across members, which implies that defaults are independent.

[14] The assumption of diminishing returns to scale is not essential for our results, but helps to obtain a neat expression for the default probability.

Throughout the section, we make the following assumption on the relation between δ, R, and β.

Assumption 1C *The insurance payment δ earned by the seller and the risky project payoff R when successful satisfy*

$$R > \delta - (1 + \beta) \tag{25}$$

Assumption 1C guarantees that the protection sellers is able to generate a positive expected profit by investing a fraction of their resources into the risky asset.[15]

4.2 Bilateral Trading

Protection Sellers. If the investment is successful, the protection seller pays an amount δ to the protection buyer and keeps the residual amount $aR + (1 - a)(1 + \beta) - \delta$, $a \in [0, 1]$. However, if the investment is not successful, the seller's payoff is given by the proceeds $(1 - a)(1 + \beta)$ from the risk-free project, and the seller defaults on the buyer. The expected profit of the protection seller is thus given by

$$V_{noCCP}(a) = (1-a)(aR-\delta)+(1-a)(1+\beta) = -Ra^2+(R-1-\beta+\delta)a+1+\beta-\delta, \quad a \in [0,1]. \tag{26}$$

The optimal investment choice of the protection seller is then given by

$$a_r^* := \frac{R - 1 - \beta + \delta}{2R}, \tag{27}$$

which is guaranteed to be nonnegative by Assumption 1C.

Protection Buyers. The buyer is exposed to the counterparty risk of the seller, and gets a zero payoff if the seller defaults. Such an outcome occurs with probability a_r^*. If the seller survives, the buyer gets the promised payment δ in full. The expected utility of the buyer therefore takes as similar as in (5) and is given by

$$U_{noCCP}(\mathbf{1}_{\text{seller survives}}\delta) = (1 - a_r^*)\delta - \gamma a_r^*(1 - a_r^*)\delta^2. \tag{28}$$

[15] If Assumption 1C fails to hold, the seller will invest all its resources into the risky project and default with certainty.

4.3 Trading via the Clearinghouse

This section analyzes the utilities of protection buyers and sellers along with the distribution of losses, when trades are executed through the CCP. As for the binary case, the protection buyer is willing to finance the payment of the fee f, which the seller incurs when joining the CCP only if he gets sufficient benefits from counterparty risk reduction. Specifically, the fee at which the buyer is indifferent is

$$f = a_r^* \delta + \gamma a_r^* (1 - a_r^*) \delta^2. \tag{29}$$

By paying such a fee, the protection buyer is guaranteed protection against the default of the seller at $t = 1$. The seller will use this fee to scale up his investment by $(f \times 100)\%$. We henceforth adjust Assumption 4B to this model with a continuous choice of risky taking. Specifically, we make the following

Assumption 2C *Buyers are sufficiently risk averse, i.e., their absolute risk aversion coefficient satisfies*

$$\gamma > \frac{\beta + a_r^*}{\mu_r^* a_r^* (1 - a_r^*) \delta} - \frac{1}{(1 - a_r^*) \delta}, \tag{30}$$

where

$$\mu_r^* := -R(a_r^*)^2 + (R - 1 - \beta) a_r^* + 1 + \beta > 0$$

is the expected return of the seller's investment under the optimal risk choice a_r^*.

Similarly to the binary case, protection buyers whose risk-aversion level exceeds the threshold specified in Eq. (30) are willing to pay a fee large enough to incentivize the sellers to participate in the CCP.

Each clearing member i strategically chooses its risk profile so to maximize its expected payoff. Given risk choices a_{-i} made all other members except i, the expected payoff of member

i is given by

$$V_i(a_{-i}) = \sup_{a_i \in [0,1]} E\left[(1+f)(a_i \tilde{R}_i + (1-a_i)(1+\beta)) - \mathbf{1}_{i \text{ survives}} \left[\delta - F + \min\left(F, \frac{\mathcal{N}_d}{N - \mathcal{N}_d}(\delta - F)\right)\right]\right]. \tag{31}$$

We consider the risk choices that arise in the symmetric equilibrium of the game. If all other members choose the same investment strategy $a_j \equiv a \in [0,1]$ for all j. we have

$$V_i(a_{-i}) = \sup_{a \in [0,1]} [-R(1+f)a^2 + (1+f)(R-\beta-1)a + (1-a)(\phi(a_{-i}; F) - \delta) + (1+f)(1+\beta)], \tag{32}$$

where $\phi(a_{-i}; F) \equiv \psi(0; F)$, and $\psi(0; F)$ is obtained by evaluating the function $\psi(g; F)$ defined in Eq. (A3) choosing $q_h \equiv a_{-i}$ therein.

By participating in central clearing, the seller's total expected profits include the expected profit accrued at $t = 1$ given by Eq. (31), net of the opportunity cost of the default fund requirement, i.e.,

$$V_{CCP}(F) = -(1+\beta)F + V_i(a_{-i}; F). \tag{33}$$

where $V_i(a_{-i}; F) \equiv V_i(a_{-i})$ in (31) to emphasize its dependence on the default fund F.

The following proposition shows that a seller benefits from participating in the central clearing.

Proposition 5 *The expected profit of a seller is higher if he trades with the CCP as opposed to bilaterally with a buyer even if all other sellers still choose to invest* $(a_r^* \times 100)\%$ *of his wealth into the risky project, i.e. when* $a_j = a_r^*$ *for all* $j \neq i$*, we have* $V_{CCP}(a_{-i}; F) - V_{noCCP}(a_i) > 0$, $\forall F \in (0, \delta)$ *and* $a_i \in [0,1]$.

Because of our symmetric configuration of clearing members, we restrict our attention to symmetric equilibria. A *symmetric equilibrium* among members is $a^e \in [0,1]$ such that, member i's best response when all other members choose a^e is also a^e; i.e., there is no unilateral deviation. Taking the first-order condition of the objective function in (32), and then setting

a_{-i} with a_i, we obtain the following result.

Proposition 6 *For a large enough successful return $R > 0$, given a default fund level $F \in [\frac{2\delta}{N}, \delta)$, there exists a unique symmetric response of members; i.e., $a_i = a^e\ \forall i$, which satisfies*

$$\frac{(1+f)(R-\beta-1)-\phi(a^e,F)+\delta}{2(1+f)R} - a^e = 0. \tag{34}$$

Moreover, a^e is a strictly decreasing function of F with $a^e(\delta-) = \frac{R-1-\beta}{2R}$; for $F \neq \frac{l\delta}{N}$, $l = 2, 3, \ldots, N-1$, a^e is an infinitely differentiable function of F and $\frac{da^e}{dF} < 0$; for $F = \frac{l\delta}{N}$, $l = 3, 4, \ldots, N-1$, $\frac{da^e}{dF}(F+) - \frac{da^e}{dF}(F-) < 0$, where $\frac{da^e}{dF}(F+)$ and $\frac{da^e}{dF}(F-)$ are respectively the right and left derivatives of $a^e(F)$ at F.

Risk-shifting. We demonstrate that the risk-shifting pattern shown in the binary case also manifests in the continuous risk-taking setup. As a first-best benchmark, we solve for the socially optimal risky asset investment a_s^* that maximizes the aggregate value of all members net of the CCP's equity loss. Equivalently, a_s^* maximizes the expected payoff of a representative member:

$$a_s^* = \arg\max_a \mathbb{E}[(1+f)(aR\mathbf{1}_{i\,\text{survives}} + (1-a)(1+\beta))]. \tag{35}$$

While a_s^* balances the socially desirable tradeoff between risk and return, members have incentives for risk-shifting under the loss mutualization scheme, as shown by the following proposition:

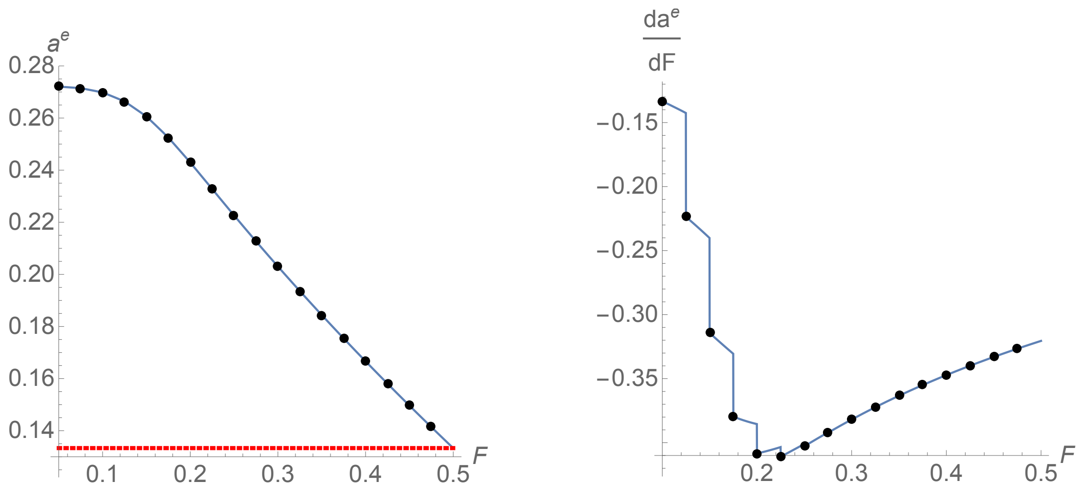
Proposition 7 *The socially optimal investment a_s^* in the risky project satisfies*

$$0 < a_s^* = \frac{R-\beta-1}{2R}. \tag{36}$$

The privately optimal investment choice a^e of the clearing members is given by the solution to Eq. (34) and satisfies

$$a_s^* < a^e < a_r^*. \tag{37}$$

A direct comparison of the privately optimal investment choice a^e (solution to Eq. (34)) and the socially optimal outcome a_s^* (solution to Eq. (36)) immediately reveals the economic



(a) $a^e(F)$ (b) $\frac{da^e}{dF}(F)$

Figure 4. Members' Strategic Response to the Default Fund Level. Panel 4a plots the symmetric strategic response a^e as a function of F (blue solid line). a^e is strictly decreasing in F. The socially optimal invested fraction in the risky project is shown in the red dashed line and is equal to $a_s^* = 0.13$. Panel 4b plots the derivative of a^e with respect to F. There are downward jumps at the kinks $\frac{l\delta}{N}$ for $l = 2, 3, 4, \ldots, N-1$. Kinks are indicated by black dots. The parameters used are: $\gamma = 0.95, R = 1.5, \beta = 0.1, N = 20$, and $\delta = 0.2$. (From ICE Clear Credit (`https://www.theice.com/clear-credit`), the number of clearing members is between 20 and 30.)

mechanism. As $\phi(a; F) < \delta$ (refund from the CCP is always less than F), members strategically chooses higher risk than what is socially optimal. Figure 4 plots $a^e(F)$, a_s^*, and $\frac{da^e}{dF}$: the decreasing pattern of $a^e(F)$ is clearly seen from the figure.

4.4 Equilibrium between Clearing Members and the Regulator

In this section, we show that a default fund higher than the Cover II requirement can be used to regulate members' risk-taking. As in Section 3.2, the regulator selects the default fund level F, which maximizes the social value of the system (including all members and the CCP), anticipating the risk-taking activities chosen by the members in response to his choice. The members' response has been characterized in Proposition 6. Next, we describe the game theoretical setting:

Definition 5 *For a given number N of clearing members, a symmetric* Nash equilibrium between all clearing members and the CCP *is the set of members' risk profiles $\{a^e\}_{i=1}^n$, and* *the default fund contribution F^e set by the CCP such that,*

1. *Taking the default fund F^e and other members' risk profile a^e_{-i} as given, a^e_i solves the optimization problem of clearing member i given in (31).*

2. *Taking as given the risk profile of clearing members a^e, the regulator chooses a feasible default fund level F^e*[16] *to maximize the total value of the system $W(F)$:*

$$W(F) = \sum_i V_i - \mathbb{E}\left[(\mathcal{N}_d\delta - NF)^+\right] - N(1+\beta)F = N(B(F) - \beta F - \delta), \tag{38}$$

$$F^e = \arg\max_F W(F) = \arg\max_F (B(F) - \beta F), \tag{39}$$

where V_i is the payoff of member i given by (31), $\mathcal{N}_d$ is the number of defaults, and $B(F) = (1+f)[-R(a^e(F))^2 + (R-\beta-1)a^e(F) + 1 + \beta]$ is the part of the representative member's value which does not account for the opportunity cost of default fund F.

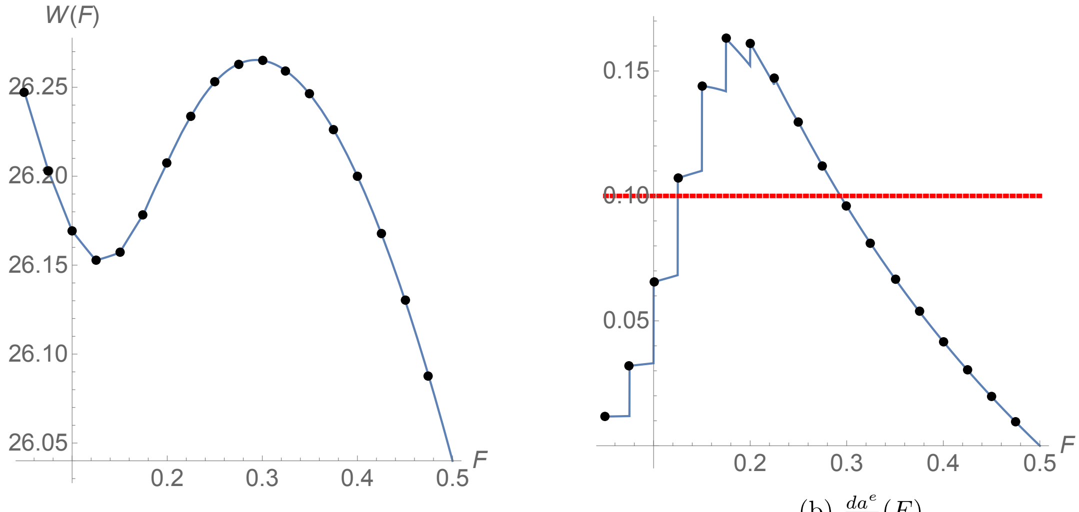
To solve for the tradeoff in choosing the default fund, we analyze the differential properties of the various components of the objective function. First, recall from Proposition 6 that $a^e(F)$ is continuously differentiable in F except over the set of kinks $\{\frac{l\delta}{N}; l = 2, \ldots, N-1\}$. Thus the same property holds for $B(F)$. To see how $B(F)$ changes with F, consider the quadratic function $U(a) = (1+f)[-Ra^2 + (R-\beta-1)a + 1 + \beta]$, which achieves its maximum at $a = a^*_s$. An application of the chain rule shows that the marginal benefit of increasing F is given by

$$B'(F) = \frac{\partial U}{\partial a}\frac{\partial a^e}{\partial F} = -2(1+f)R\left(a^e(F) - a^*_s\right)\frac{da^e}{dF}, \quad \forall F \neq \frac{l\delta}{N}, l = 2, \ldots, N-1. \tag{40}$$

Recall from propositions 6 and 7 that members exhibit risk-shifting ($a^e(F) > a^*_s$), and that a higher default fund can mitigate risk-shifting ($\frac{da^e}{dF} < 0$, $\forall F \neq \frac{l\delta}{N}$, $l = 2, \ldots, N-1$). Therefore, $B(F)$ increases with F; that is, a higher default fund increases value $B(F)$ by steering members' risk-taking closer to the socially optimal level.[17]

[16] We recall that feasible means that the Assumptions 2B-3B are satisfied.

[17] Formally, Equation (40) implies that the marginal benefit of increasing F is positive; i.e., $B'(F) > 0$, $\forall F \in [\frac{2\delta}{N}, \delta)\backslash\{\frac{l\delta}{N}; l = 2, \ldots, N-1\}$. Moreover, as F crosses a kink $\frac{l\delta}{N}$ from below, Proposition 7 indicates that $B'(\frac{l\delta}{N}+) - B'(\frac{l\delta}{N}-) = -2(1+f)R(a^e(\frac{l\delta}{N}) - a^*_s)[\frac{da^e}{dF}(\frac{l\delta}{N}+) - \frac{da^e}{dF}(\frac{l\delta}{N}-)] > 0$. Hence, at each kink value of F, the marginal benefit of increasing F increases with a positive jump. Taken together, $B(F)$ increases with F.



(a) $a^e(F)$

(b) $\frac{da^e}{dF}(F)$

Figure 5. The Social Value Function and the Marginal Value. Panel (a) plots the total wealth $W(F)$. Panel (b) plots the marginal values: the marginal benefit $B'(F)$ (in blue), and the marginal cost β (in red). The parameters used are: $\gamma = 0.95, R = 1.5, \beta = 0.1, N = 20$, and $\delta = 0.2$. The optimal default fund is $F^e \approx 0.26$.

Proposition 8 *The equilibrium default fund F^e set by the regulator is either the Cover II level $\frac{2\delta}{N}$ or a solution to the first-order condition of Equation (39). Formally, $F^e \in \{\bar{F}^e, \frac{2\delta}{N}\}$ where $\bar{F}^e$ satisfies*

$$B'(\bar{F}^e) = \beta, \quad B''(\bar{F}^e) \leq 0, \quad \bar{F}^e \neq \frac{l\delta}{N}, l = 2, \ldots, N. \tag{41}$$

Proposition 8 offers a simple algorithm for the regulator to pin down the equilibrium (constrained optimal) default fund. To maximize the social objective value function (39), the regulator balances the marginal benefit $B'(\bar{F}^e)$ and marginal cost of capital β. While a higher default fund increases the social value by mitigating risk-shifting, it is also costly because the fund needs to be segregated. If the marginal benefit and marginal cost cross paths at a fund value higher than the Cover II lower bound, and the condition $B''(\bar{F}^e) \leq 0$ holds, then $\bar{F}^e$ is the constrained optimal level. In other words, if the marginal value of increasing F is higher than the opportunity cost at the lower bound $\frac{2\delta}{N}$ ($B'(\frac{2\delta}{N}) > 0$), then $\frac{2\delta}{N} < F^e < \delta$. That is, the optimal default fund level exceeds the critical level for the Cover II requirement. Figure 5 considers a scenario in which the Cover II rule is not socially optimal. As the default fund level increases, the social value function increases and peaks at $F^e = 0.26$. This value corresponds to a Cover 11.7 rule instead for a 20-member clearing arrangement.

An Example with Three Members We consider a simplified setting consisting of three clearing members, in which closed-form expressions for $a^e(F)$ and F^e can be obtained. For a given choice of F that satisfies the Cover II requirement, the equilibrium investment in the risky asset is given by

$$a^e(F) = \frac{(1+f)R}{\delta - F} - \frac{1}{2} - \sqrt{\left(\frac{(1+f)R}{\delta - F} - \frac{1}{2}\right)^2 - \frac{(1+f)(R-\beta-1) - F + \delta}{\delta - F}}, \quad F \in [\frac{2\delta}{3}, \delta)$$

with limit $a^e(\delta-) = a_s^*$ and obtained by solving Equation (34). It is immediate to see that $a^e(F)$ is strictly decreasing in F. We can explicitly compute the unique equilibrium default fund F^e. In particular, let $h(a) = a - a_s^* - \beta\frac{1-a^2+a_s^*(1+2a)}{(1+a+a^2)^2}$, which is a strictly increasing function over $[0,1]$, and satisfies $h(a_s^*) < 0$. Then

$$F^e = \begin{cases} \frac{2\delta}{3}, & \text{if } h(a^e(\frac{2\delta}{3})) \leq 0, \\ \delta - 2(1+f)R\frac{a_0 - a_s^*}{1+a_0+a_0^2}, & \text{if } h(a^e(\frac{2\delta}{3})) > 0. \end{cases} \tag{42}$$

where a_0 is the unique root to $h(a_0) = 0$ over the interval $(a_s^*, a^e(\frac{2\delta}{3}))$. This example serves to illustrate that even in a simple central clearing setup consisting of three members, neither Cover II nor Cover III may be the optimal default fund allocation. In fact, $2\delta/3 < F^e < \delta$, and thus an intermediary level for the default fund level might be optimal.

Analogous to the binary case, we can show that the ratio between the optimal default fund level $F^e(N)$ and the insurance payment δ, or equivalently, the optimal proportion of covered members, $x(N)/N$, also converges to a constant as the number of members N grows large. This limit is a positive number in $(0,1)$ when the marginal opportunity cost β is sufficiently low. In particular, paralleling the result in Proposition 4, we have the following asymptotic result.

Proposition 9 *In the large CCP network limit – i.e., as $N \to \infty$ – the unique symmetric* *response of members, $a_i = a^e$ is given by*

$$a^e(F) \to \begin{cases} a_s^* + \dfrac{\delta}{2(1+f)R}, & \text{if } \dfrac{F}{\delta} \leq a_s^* + \dfrac{\delta}{2(1+f)R}, \\ \dfrac{(1+a_s^*) - \sqrt{(1-a_s^*)^2 - \frac{\delta - F}{(1+f)R}}}{2}, & \text{if } \dfrac{F}{\delta} > a_s^* + \dfrac{\delta}{2(1+f)R}. \end{cases} \tag{43}$$

Letting $F(a) = \delta - 2(1+f)(a-a_s^*)(1-a)R$ be the inverse of line two in (43), $a_0^* := \frac{(R+\beta)a_s^*+\beta}{R+2\beta}$, and $\hat{F}_\infty := F(a_0^*)$. Then we have

$$\frac{F^e(N)}{\delta} = \frac{x(N)}{N} \to \begin{cases} \dfrac{\hat{F}_\infty}{\delta}, & \text{if } B(\hat{F}_\infty) - \beta \hat{F}_\infty > B(0) \text{ and } a_0^* < a_s^* + \dfrac{\delta}{2(1+f)R}, \\ 0, & \text{else}, \end{cases} \tag{44}$$

where $B(\cdot)$ is defined in Definition 5.

From Proposition 9, we see that for a CCP consisting of many members, the highest risk choice a^e converges to $a_s^* + \frac{\delta}{2(1+f)R}$, which is still lower than the risk choice without CCP a_r^*, thanks to the fee paid by the buyer. When members take this level of risk, then the optimal response of the regulator is to set the default fund level to zero because the members' risk choice is independent of F. If the marginal opportunity cost of default fund is low, the regulator may attain a higher per member social welfare by charging a larger default fund requirement $\hat{F}_\infty$ to members, that in turn incentivize them to take a lower level risk level a_0^*.

5 Conclusion and Policy Implications

The optimal determination of clearinghouse members' default fund requirement has been the subject of extensive regulatory debate. Current regulatory requirements prescribe that default fund contributions should guarantee the robustness of a clearinghouse in the event that its two largest clearing members default. There is, however, no economic analysis on conditions under which this rule is socially optimal, or of alternative rules that are welfare improving. Our paper fills this important gap and introduces a parsimonious model to study the main economic incentives behind the determination of the default fund requirements. While default funds allow members to effectively share counterparty risk *ex post*, we highlight a novel mechanism related to loss mutualization that induces members to take excessive risk *ex ante* owing to an inherent externality among them. Our analysis shows that the CCPs can mitigate the inefficiency generated by members' excessive risk-taking activities through an optimal choice of the default fund level. Such a choice balances the ex post risk-sharing and the ex ante risk-taking of members.

Our analysis shows that if the clearinghouse consists of a sufficiently large number of clearing members, then the optimal cover rule should fulfill the shortfalls of a constant fraction of members. This finding contrasts with the currently imposed Cover II requirements, whose optimality is supported by our analysis only if the marginal opportunity costs of collateral posting are high.

Our results have important policy implications. They point towards simple rules that guarantees the coverage of the costs generated by the default of a proportion of clearing members. This simple covering requirement is robust to the size of the participating member base. The optimal proportion depends, in an explicit way, on the relation between the premium earned by the member who undertakes high-risk projects and the costs incurred at default. These parameters can be accessed by clearinghouses and their supervisory authorities who typically have detailed information on the risk profile of their members. Owing to its simplicity, the proposed rule can also serve as a benchmark against more complex rules based on simulated scenarios for stress testing.

Our analysis indicates that default funds are negatively correlated with the marginal opportunity cost of default funds. To the extent that these are associated with the costs of funding capital, this implies that large amounts of capital may be tied up with the clearinghouse in the current low interest-rate environment. This would reduce economic opportunities, and thus regulatory policies limiting the increase in default funds when the marginal funding costs are low may be socially desirable. However, our analysis also suggests that increasing costs of funding capital provides clearinghouses with the incentive to reduce default funds, until achieving the Cover II lower bound requirement. This may have serious systemic risk implications if such a period of increasing funding costs is followed by high market stress, given that the clearinghouse may not be vulnerable and not sufficiently capitalized under this scenario.

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A Proof of Proposition 1

From Assumption 4B and condition (11), we have

$$\mu_r f > (q_r + \beta)\delta.$$

When trading through the CCP where all members excepts i are still choosing the high-risk project, member i's expected profit satisfies (using (7), (9) and (14))

$$V_{CCP}(F) \geq -(1+\beta)F + (1-q_r)(\psi(0;F) - \delta) + (1+f)\mu_r,$$

where the inequality is because we considered the case in which member i chooses the high-risk project without the optimization of a in (7). From Lemma 6 below, the seller obtains the lowest profits when facing the highest default fund requirement, i.e., when $F = \delta$. In this case, the seller is required to post a level of prefunded position that would be enough to cover the promised payment to the buyer even without any loss mutualization mechanism. Since prefunded position is costly, the seller would be worse off with such a high default fund requirement. The value of a seller when $F = \delta$ satisifes (see (7))

$$V_{CCP}(\delta) \geq (1+f)\mu_r - (1+\beta)\delta.$$

Hence the expected profit of member i when trading through CCP satisfies

$$V_{CCP}(F) \geq V_{CCP}(\delta) \geq \mu_r[q_r\delta + \gamma q_r(1-q_r)\delta^2] - (1+\beta)\delta + \mu_r > (\beta + q_r)\delta - (1+\beta)\delta + \mu_r = V_{noCCP}.$$

Lemma 6 *Denoting $h(F) = -(1+\beta)F + (1-q_r)\psi(0;F)$, then $h(F)$ is strictly decreasing in F over $[0,\delta]$.*

Proof. We begin by noticing that $h(F)$ can be written as

$$h(F) = -(\beta + q_r)F - (1-q_r)(F - \psi(0;F)). \tag{A1}$$

Recall from (14) that $F - \psi(0;F) = \mathbb{E}[\min(F, (\delta - F)\frac{N-\mathcal{N}_s}{\mathcal{N}_s})|\mathcal{N}_s \geq 1]$, and $\min(F, (\delta - F)\frac{N-\mathcal{N}_s}{\mathcal{N}_s})$ is almost surely concave in F, we can thus conclude that $F - \psi(0;F)$ is concave as well. As a result, from (A1) we know that $h(F)$ is convex over $[0,\delta]$. To complete the proof of the claim, it suffices to show that $h'(\delta-) < 0$.

To that end, consider $F \in [\frac{N-1}{N}\delta, \delta)$. Then we know that $\min(F, (\delta - F)\frac{N-\mathcal{N}_s}{\mathcal{N}_s}) = (\delta - F)(\frac{N}{\mathcal{N}_s} - 1)$. So these F, using (14) we have

$$F - \psi(0;F) = (\delta - F)\mathbb{E}\left[\frac{N}{\mathcal{N}_s} - 1 \middle| \mathcal{N}_s \geq 1\right] \quad \Rightarrow \quad \psi(0;F) = \delta + (F - \delta)\mathbb{E}\left[\frac{N}{\mathcal{N}_s} \middle| \mathcal{N}_s \geq 1\right].$$

Because $\mathcal{N}_s$ follows a binomial distribution with parameter $(N, 1-q_r)$, we know that $(\mathcal{N}_s - 1|\mathcal{N}_s \geq 1)$ follows a binomial distribution with parameter $(N-1, 1-q_r)$. Hence,

$$\begin{aligned}
\psi(0;F) =&\delta + (F-\delta)\mathbb{E}\left[\frac{N}{\mathcal{N}_s}\middle|\mathcal{N}_s \geq 1\right] \\
=&\delta + (F-\delta)\sum_{k=0}^{N-1}\frac{N}{1+k}\binom{N-1}{k}(1-q_r)^k q_r^{N-1-k} \\
=&\delta + (F-\delta)\frac{1}{1-q_r}\sum_{k=0}^{N-1}\binom{N}{k+1}(1-q_r)^{k+1}q_r^{N-1-k} \\
=&\delta + (F-\delta)\frac{1}{1-q_r}\sum_{m=1}^{N}\binom{N}{m}(1-q_r)^m q_r^{N-m} \\
=&\delta + (F-\delta)\frac{1-q_r^N}{1-q_r}.
\end{aligned} \tag{A2}$$

It follows from (A1) that

$$h'(\delta-) = -(1+\beta) + (1-q_r)\frac{d}{dF}\bigg|_{F\uparrow\delta}\psi(0;F) = -(1+\beta) + (1-q_r)\frac{1-q_r^N}{1-q_r} = -(\beta + q_r^N) < 0.$$

Therefore $h(F)$ is strictly decreasing in F over $[0,\delta]$. ■

B Proof of Proposition 2

We first present some technical lemmas to fix preliminary results and notations.

Lemma 7 *Suppose member i is alive, and g of the remaining $N-1$ members choose the low risk project. Then, for any given $F \in [\frac{2\delta}{N}, \delta]$, we have that*

$$\psi(g;F) := \sum_{k=N-1-\lfloor\frac{NF}{\delta}\rfloor}^{N-1} f_g(k)\left(\delta - \frac{N(\delta-F)}{k+1}\right). \tag{A3}$$

Here $\lfloor\cdot\rfloor$ denotes the floor function (giving the greatest integer less than or equal to the argument), and

$$f_g(k) := \sum_{m=0}^{k}\binom{g}{m}(1-q_s)^m q_s^{g-m} \times \binom{N-1-g}{k-m}(1-q_r)^{k-m}q_r^{N-1-g-(k-m)}$$

are positive constant.

Proof. Suppose that the default fund F is such that

$$\frac{l\delta}{N} \leq F < \frac{(l+1)\delta}{N} \text{ or equivalently } 1 - \frac{1+l}{N} < 1 - \frac{F}{\delta} \leq 1 - \frac{l}{N}, \tag{A4}$$

for some integer $l = 2, 3, \ldots, N-1$. Then member i's contribution to other members' default when himself does not default is given by

$$\begin{aligned}\min\left(F, \frac{N-\mathcal{N}_s}{\mathcal{N}_s}(\delta - F)\right) &= F\min\left(1, (\frac{N}{\mathcal{N}_s}-1)(\frac{\delta}{F}-1)\right)\\ &= (\delta - F)\left(\frac{N}{\mathcal{N}_s}-1\right)1_{\mathcal{N}_s \geq (1-\frac{F}{\delta})N} + F1_{\mathcal{N}_s<(1-\frac{F}{\delta})N}\end{aligned}$$

For F in the range (A4), we have

$$N-(l+1) < (1-\frac{F}{\delta})N \leq N-l$$

Thus $\mathcal{N}_s \geq (1-\frac{F}{\delta})N$ if and only if $\mathcal{N}_s \geq n-l$. In other words, among the remaining $N-1$ members, if there are less than or equal to l defaults, member i will pay less than F. But if there are l or more defaults and $F = \frac{l\delta}{N}$ then member i's default fund will be exhausted completely.

Suppose all members except member i choose the low risk project, then if member i survives, his expected contribution is

$$\begin{aligned}&(\delta - F)\mathbb{E}\left[\left(\frac{N}{\mathcal{N}_s}-1\right)1_{\mathcal{N}_s\geq N-l}\middle|\text{member } i \text{ survives}\right] + F\cdot\ Pr(\mathcal{N}_s < N-l|\text{member } i \text{ survives})\\ =&(\delta - F)\sum_{k=N-(l+1)}^{N-1}\binom{N-1}{k}\frac{N-1-k}{k+1}(1-q_s)^k q_s^{N-1-k} + F\sum_{k=0}^{N-(l+2)}\binom{N-1}{k}(1-q_s)^k q_s^{N-1-k}\\ =&(\delta - F)\sum_{k=N-(l+1)}^{N-2}\binom{N-1}{k+1}(1-q_s)^k q_s^{N-1-k} + F\sum_{k=0}^{N-(l+2)}\binom{N-1}{k}(1-q_s)^k q_s^{N-1-k}. \quad \text{(A5)}\end{aligned}$$

Likewise, if all members except member i choose the high risk project, then if member i survives, his expected contribution is

$$\begin{aligned}&(\delta - F)\mathbb{E}\left[\left(\frac{N}{\mathcal{N}_s}-1\right)1_{\mathcal{N}_s\geq N-l}\middle|\text{member } i \text{ survives}\right] + F\cdot\ Pr(\mathcal{N}_s < N-l|\text{member } i \text{ survives})\\ =&(\delta - F)\sum_{k=N-(l+1)}^{N-2}\binom{N-1}{k+1}(1-q_r)^k q_r^{N-1-k} + F\sum_{k=0}^{N-(l+2)}\binom{N-1}{k}(1-q_r)^k q_r^{N-1-k}. \quad \text{(A6)}\end{aligned}$$

In general, if there are g members among the remaining $N-1$ choosing the low risk project, for $g = 0, 1, \ldots, N-1$, then the number of surviving ones among these $N-1$ members, $\mathcal{N}_s - 1$, is the sum of the number of the survived ones choosing the low risk project and that of the survived choosing the high risk project. Specifically, the probability that there are k survived ones is given by

$$f_g(k) := \sum_{m=0}^{k}\binom{g}{m}(1-q_s)^m q_s^{g-m} \times \binom{N-1-g}{k-m}(1-q_r)^{k-m} q_r^{N-1-g-(k-m)}.$$

It follows that, if member i survives, his expected contribution is

$$
\begin{aligned}
&(\delta - F)\mathbb{E}\left[\left(\frac{N}{\mathcal{N}_s} - 1\right)1_{\mathcal{N}_s \geq N-l}\middle|\text{member } i \text{ survives}\right] + F \cdot Pr(\mathcal{N}_s < N - l|\text{member } i \text{ survives}) \\
=&(\delta - F)\sum_{k=N-(l+1)}^{N-1} f_g(k)\frac{N-1-k}{1+k} + F\sum_{k=0}^{N-(l+2)} f_g(k) \\
=&\sum_{k=N-(l+1)}^{N-1} f_g(k)\left(\frac{N(\delta - F)}{k+1} - \delta\right) + F, \qquad \text{(A7)}
\end{aligned}
$$

where the last line comes from the total probability $\sum_{k=0}^{N-1} f_g(k) = 1$. ■

Lemma 8 *For any given $F \in [\frac{2\delta}{N}, \delta)$, the function $\psi(g; F)$ is strictly decreasing in g, i.e.*

$$0 < \psi(0; F) < \psi(1; F) < \ldots < \psi(N-1; F) < F < \delta.$$

Proof. Suppose member i survives, and $\mathcal{N}_s - 1$ is the number of survivals except member i. Given how we define a default event, we have

$$\mathcal{N}_s - 1 = \sum_{j \neq i} 1_{\text{member } j \text{ defaults}} \qquad \text{(A8)}$$

By Lemma 7, we only need to show that

$$g :\mapsto \mathbb{E}\left[\min\left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s}(\delta - F)\right)\right] \equiv F - \psi(g; F) \qquad \text{(A9)}$$

is strictly decreasing in g for $g = 0, 1, 2, \ldots, n-1$, where g is the number of members other than member i, who chooses the low risk project. Because the expression $\psi(g; F)$ only depends on the default probabilities q_r, q_s, not how defaults occur, we can choose a probability model for the defaults that is convenient to our analysis. More precisely, suppose for each member i, there is an independent random variable ϵ_i, with a uniform distribution on $(0, 1)$, such that, if this member has chosen the low risk project, he will default at time 1 if and only if $\epsilon_i < q_s$; on the other hand, if this member has chosen the high risk project, then he will default at time 1 if and only if $\epsilon_i < q_r$. Because the event $\{\epsilon_i < q_s\}$ implies $\{\epsilon_i < q_r\}$, we see that, in this probability model of defaults, increasing g, the number of of remaining members (other than member i) choosing the low risk project, always makes $\mathcal{N}_s$ non-increasing, and can makes $\mathcal{N}_s$ strictly increasing with a positive probability (equal to $q_r - q_s$ in this particular specification of default).

Similarly, there is a positive chance that $\mathcal{N}_s$ may decrease from 1 to N as g increases from 0 to $N-1$. As $\mathcal{N}_s$ varies between 1 and N, $\frac{N-\mathcal{N}_s}{\mathcal{N}_s}(\delta - F)$ varies between $(N-1)(\delta - F)$ and 0, hence the random variable $\min\left(F, \frac{N-\mathcal{N}_s}{\mathcal{N}_s}(\delta - F)\right)$ is non-decreasing with g, and there is a positive chance that it is strictly decreasing with g. As a result, we know that the mapping in (A9) is strictly decreasing.

Lastly, because the expected contribution $F - \psi(N-1; F)$ is positive (member i has to contribute as long as there is at least one other member default), we have $\psi(N-1; F) <$

F. Moreover, the expectation contribution $F - \psi(0; F)$ is strictly less than F (member i contributes less than F when there is no default). ■

Lemma 9 *For any fixed $g = 0, 1, \ldots, N-1$, the function $\psi(g; F)$ is piecewise linear and strictly increasing in F, in the interval $[\frac{2}{N}\delta, \delta]$. In particular, $\psi(g; \delta) = \delta$.*

Proof. From (A3), $\psi(g; F)$ is linear and strictly increasing for $F \in (\frac{l}{N}, \frac{l+1}{N})$ with $l = 2, 3, \ldots, N-1$. Moreover, the nonnegative random variable $\min(F, \frac{N-\mathcal{N}_s}{\mathcal{N}_s}(\delta - F))$ is almost surely continuous in F, and is bounded by δ. By the dominated convergence theorem, we know that $\psi(g; F)$ is continuous. Therefore, the function $\psi(g; F)$ is strictly increasing for all $F \in [\frac{2}{N}\delta, \delta]$. The value of $\psi(g; \delta)$ follows directly from (14). ■

To prove Proposition 2, we next analyze different scenarios for a member's investment choice, taken as given the default fund and other members' investment choices. In particular, we characterize conditions such that member i chooses high risk given all possible combinations of risk profile of other members. When we check those conditions for all members, we will see that "all high risk" and "all low risk" strategy are the only possible equilibria.

Suppose that the other members do not choose the same risk level: There are g members who choose low risk projects and the rest $N - 1 - g$ choose high risk projects, where $g \in \{0, \ldots, N-1\}$.

If member i chooses low risk, his expected utility is (by Lemma 7)

$$\begin{aligned}
&\mathbb{E}\left[(1+f)R_i^l - \mathbf{1}_{i \text{ survives}}\left[\delta - F + \min\left(F, (\delta - F)\,\frac{N - \mathcal{N}_s}{\mathcal{N}_s}\right)\middle| i \text{ survives}\right]\right] \\
&= (1+f)\mu_s + (1-q_s)\,(\psi(g; F) - \delta).
\end{aligned}$$

If member i chooses instead high risk, his expected utility is

$$\begin{aligned}
&\mathbb{E}\left[(1+f)R_i^h - \mathbf{1}_{i \text{ survives}}\left[\delta - F + \min\left(F, (\delta - F)\,\frac{N - \mathcal{N}_s}{\mathcal{N}_s}\right)\middle| i \text{ survives}\right]\right] \\
&= (1+f)\mu_r + (1-q_r)\,(\psi(g; F) - \delta).
\end{aligned}$$

Hence, member i chooses high (low, resp.) risky project when g members choose low and $N - 1 - g$ choose high if and only if

$$(1+f)\frac{\mu_s - \mu_r}{q_r - q_s} < (>, \text{resp.})\delta - \psi(g; F). \tag{A10}$$

When (A10) takes an equality, member i is indifferent to choosing high or low risky project. As a consequence,

1. If $(1+f)\frac{\mu_s - \mu_r}{q_r - q_s} < \delta - \psi(N-1; F)$, every member will choose the high risky project, regardless of other members' choice. Hence, the "all high risk" strategy is the unique equilibrium among members.
2. If $(1+f)\frac{\mu_s - \mu_r}{q_r - q_s} > \delta - \psi(0; F)$, every member will choose the low risky project, regardless of other members' choice. Hence, the "all low risk" strategy is the unique equilibrium among members.

3. If $\delta - \psi(0;F) \geq (1+f)\frac{\mu_s-\mu_r}{q_r-q_s} \geq \delta - \psi(N-1;F)$, it is straightforward to verify that both the "all high risk" strategy and the "all low risk" strategy are equilibriums among members. To prove there cannot be any other forms of equilibrium, suppose there is an equilibrium which is consisted of g low and $(N-g)$ high, for some $g = 1, 2, \ldots, N-1$. Then for any member choosing high, he faces g choosing low and $(N-g-1)$ choosing high, so in order for him to stay high as well, it must hold that

$$(1+f)\frac{\mu_s - \mu_r}{q_r - q_s} \leq \delta - \psi(g;F). \tag{A11}$$

Yet, for any member choosing low, he faces $g-1$ low and $N-g$ high, so for this member to stay low, it must holds that

$$(1+f)\frac{\mu_s - \mu_r}{q_r - q_s} \geq \delta - \psi(g-1;F). \tag{A12}$$

However, (A11) and (A12) cannot hold simultaneously because $\psi(g-1;F) < \psi(g;F)$ (see Lemma 8).

Finally, we prove that the all low risk profile is Pareto dominating the all high risk profile when $\delta - \psi(0;F) \geq (1+f)\frac{\mu_s-\mu_r}{q_r-q_s} \geq \delta - \psi(N-1;F)$ holds. Recall that member i's expected utility under the all low risk profile is

$$(1+f)\mu_s - (1-q_s)\left(\delta - \psi(N-1;F)\right).$$

Likewise, his expected utility under the all high risk profile is

$$(1+f)\mu_r - (1-q_r)\left(\delta - \psi(0;F)\right).$$

The difference is then given by

$$\begin{aligned}&[(1+f)\mu_s - (1-q_s)\left(\delta - \psi(N-1;F)\right)] - [(1+f)\mu_r - (1-q_r)\left(\delta - \psi(0;F)\right)]\\ =&(1+f)(\mu_s - \mu_r) + (1-q_s)\,\psi(N-1;F) - (1-q_r)\psi(0;F) - (q_r - q_s)\delta.\end{aligned}$$

When $(1+f)(\mu_s - \mu_r) \geq (q_r - q_s)(\delta - \psi(N-1;F))$, the above expression is bounded from below by $(1-q_r)\left(\psi(N-1;F) - \psi(0;F)\right) > 0$, due to Lemma 8. Therefore, among the two possible equilibriums, the all low risk profile is Pareto dominating.

C Proof of Proposition 5

When trading through the CCP where all members excepts i are still investing $(a_r^* \times 100)\%$ of their wealth into the risky project, member i's expected profit satisfies (using (31), (33) and (14))

$$V_{CCP}(F) \geq -(1+\beta)F + (1-a_r^*)(\psi(0;F) - \delta) + (1+f)\mu_r^*,$$

where $\mu_r^* = -R(a_r^*)^2 + (R-1-\beta)a_r^* + 1 + \beta$, the inequality is because we considered the case in which member i chooses $a_i = a_r^*$ without the optimization of a in (31). The rest of the proof follows that for Proposition 1.

D Proof of Proposition 6

Recall that a symmetric equilibrium under cover-II requirement is given by $a_i^e = a^e$ for all $i = 1, 2, \dots, N$, with a^e being the root to the follow equation:

$$H(a^e; F) = 0, \text{ where } H(a; F) = \frac{(1+f)(R-\beta-1) - \phi(a; F) + \delta}{2(1+f)R} - a.$$

We first show the existence of such a root. To that end, recall that (14) implies

$$\mathbb{E}\left[\min\left(F, \frac{N-\mathcal{N}_s}{\mathcal{N}_s}(\delta - F)\right) \middle| \text{member } i \text{ survives}\right] = F - \phi(a; F),$$

where $\mathcal{N}_s$ is the number of surviving members, each of who has a default probability of a. It follows that when $a = 0$, we have $\mathcal{N}_s = N$ almost surely, so $0 = F - \phi(0; F)$, which implies that

$$\phi(0; F) = F.$$

Therefore we have

$$H(0; F) = \frac{(1+f)(R-\beta-1) - F + \delta}{2(1+f)R} > \frac{R-\beta-1}{2R} > 0. \tag{A13}$$

On the other hand, because $\min(F, \frac{N-\mathcal{N}_s}{\mathcal{N}_s}(\delta - F)) \le F$ holds almost surely, we know that $\phi(a; F) \ge 0$ for all $a \in [0, 1]$ and $\phi(1; F) = 0$. Therefore, we have [check check check!!!]

$$H(1; F) = \frac{(1+f)(R-\beta-1) + \delta}{2(1+f)R} - 1 < \frac{(1+f)(R-\beta-1+\delta)}{2(1+f)R} - 1 < 0, \tag{A14}$$

due to Assumption 1C. Hence, from (A13) and (A14) we conclude that, for any fixed $F \in [\frac{2\delta}{N}, \delta)$, there exists at least one a^e such that $H(a^e; F) = 0$.

To prove the uniqueness of a^e, we demonstrate that, when $R > 0$ is sufficiently large, $H(a; F)$ is strictly decreasing in a over $(0, 1)$ for every F. Indeed, Lemma 8 indicates that $\phi(q_r; F) = \psi(0; F) < \psi(N-1; F) = \phi(q_s; F)$ for $0 < q_s < q_r < 1$, so $\phi(a; F)$ is strictly decreasing in a. Therefore, the monotonicity of $H(a; F)$ in a is completely determined by the strictly increasing function $-\frac{1}{2(1+f)R}\phi(a; F)$ and the strictly decreasing function $-a$. Intuitively, as R becomes large, we have $-\frac{1}{2(1+f)R}\frac{\partial}{\partial a}\phi(a; F) - 1 < 0$, meaning $H(a; F)$ is strictly decreasing in a. In Lemma 10 below we formally bound $|\frac{\partial}{\partial a}\phi(a; F)|$ from above, which effectively gives a sufficient lower bound for R such that $H(a; F)$ is strictly decreasing, which implies the uniqueness of a^e.

To prove the monotonicity of $a^e(F)$ in F, we use lemma 9 to know that, for each fixed $a \in (0, 1)$, $\phi(a; F)$ is strictly increasing for all $F \in [\frac{2\delta}{N}, \delta)$, so $H(a; F)$ is strictly decreasing in F over the same domain. Let $\frac{2\delta}{N} \le F_1 < F_2 < \delta$, we have

$$0 = H(a^e(F_2); F_2) = H(a^e(F_1); F_1) > H(a^e(F_1); F_2).$$

Because $H(q; F_2)$ is strictly decreasing in q when $R > 0$ is sufficiently large, we must have $a^e(F_2) < a^e(F_1)$. This proves the monotonicity of $a^e(F)$ on F.

The infinite differentiability of $a^e(F)$ in F follows from implicit differentiation and Lemma

9, from which we know that $H(a;F)$ is infinitely differentiable in F if $F \neq \frac{l\delta}{N}$ for $l = 2, 3, \ldots, N-1$. This, in conjunction with the monotonicity of $a^e(F)$ in F, implies that $\frac{da^e}{dF}(F) < 0$ for any $F \in (\frac{2\delta}{N}, \delta)$ but $F \neq \frac{l\delta}{N}$, $l = 3, 4, \ldots, N-1$.

Lastly, to prove that $a^e(F)$ is not differentiable at $F = \frac{l\delta}{N}$, and with a greater left derivative, we use implicit differentiation to obtain that, for any $F \in (\frac{l\delta}{N}, \frac{(l+1)\delta}{N})$ with $l = 3, \ldots, N-2$,

$$\frac{da^e}{dF}(F) = -\frac{\frac{\partial H}{\partial F}}{\frac{\partial H}{\partial a}}\bigg|_{(a,F)=(a^e(F),F)} = -\frac{u_l^{(N)}(a^e(F))}{2(1+f)R + \delta \cdot v_l^{(N)\prime}(a^e(F)) - (\delta - F)u_l^{(N)\prime}(a^e(F))},$$

where $v_l^{(N)}$ and $u_l^{(N)}$ are functions defined in the proof of Lemma 10 below. Thus, the right derivative at $F = \frac{l\delta}{N}$ is given by

$$\frac{da^e}{dF}(\frac{l\delta}{N}+) = -\frac{u_l^{(N)}(a^e(\frac{l\delta}{N}))}{2(1+f)R + \delta \cdot v_l^{(N)\prime}(a^e(\frac{l\delta}{N})) - (c + \delta - \frac{l\delta}{N})u_l^{(N)\prime}(a^e(\frac{l\delta}{N}))}. \tag{A15}$$

Similarly, for the left derivative at $F = \frac{l\delta}{N}$ we obtain that

$$\frac{da^e}{dF}(\frac{l\delta}{N}-) = -\frac{u_{l-1}^{(N)}(a^e(\frac{l\delta}{N}))}{2(1+f)R + \delta \cdot v_{l-1}^{(N)\prime}(a^e(\frac{l\delta}{N})) - (\delta - \frac{l\delta}{N})u_{l-1}^{(N)\prime}(a^e(\frac{l\delta}{N}))}. \tag{A16}$$

To compare (A15) with (A16), we first show that the denominators for $\frac{da^e}{dF}(\frac{lc}{N}+)$ and $\frac{da^e}{dF}(\frac{l\delta}{N}-)$ are the same. To that end, we have

$$\begin{aligned}
&\left(\delta \cdot v_l^{(N)\prime}(a^e(\frac{l\delta}{N})) - \delta\frac{N-l}{N}u_l^{(N)\prime}(a^e(\frac{l\delta}{N}))\right) \\
&- \left(\delta \cdot v_{l-1}^{(N)\prime}(a^e(\frac{l\delta}{N})) - \delta\frac{N-l}{N}u_{l-1}^{(N)\prime}(a^e(\frac{l\delta}{N}))\right) \\
=&\delta\left(\binom{N-1}{N-1-l}[(1-a)^{N-1-l}a^l]' - \frac{N-l}{N}\binom{N}{N-l}[(1-a)^{N-l-1}a^l]'\right)\bigg|_{a=a^e(\frac{l\delta}{N})} \\
=&\delta[(1-a)^{N-1-l}a^l]'|_{a=a^e(F)} \cdot \left(\binom{N-1}{l} - \frac{N-l}{N}\binom{N}{l}\right) = 0.
\end{aligned}$$

On the other hand, recall that when $R > 0$ is large enough, both $\frac{da^e}{dF}(\frac{l\delta}{N}+)$ and $\frac{da^e}{dF}(\frac{l\delta}{N}-)$ are negative. Given that $u_{l-1}^{(N)}(a^e(\frac{l\delta}{N})), u_l^{(N)}(a^e(\frac{l\delta}{N})) > 0$, we know from (A15) and (A16) that the common denominator for $\frac{da^e}{dF}(\frac{l\delta}{N}+)$ and $\frac{da^e}{dF}(\frac{l\delta}{N}-)$ must be positive. Thus,

$$\begin{aligned}
&\frac{da^e}{dF}(\frac{l\delta}{N}+) - \frac{da^e}{dF}(\frac{l\delta}{N}-) \\
=&\frac{-[u_l^{(N)}(a^e(\frac{l\delta}{N})) - u_{l-1}^{(N)}(a^e(\frac{l\delta}{N})]}{2(1+f)R + \delta \cdot v_l^{(N)\prime}(a^e(\frac{l\delta}{N})) - \delta\frac{N-1}{N}u_l^{(N)\prime}(a^e(\frac{l\delta}{N}))} \\
=&\frac{-\binom{N}{N-1}(1-a)^{N-l-1}a^l|_{a=a^e(\frac{l\delta}{N})}}{2(1+f)R + \delta \cdot v_l^{(N)\prime}(a^e(\frac{l\delta}{N})) - \delta\frac{N-1}{N}u_l^{(N)\prime}(a^e(\frac{l\delta}{N}))} < 0.
\end{aligned}$$

Lemma 10 *For any $F \in [\frac{2\delta}{N}, \delta)$, we have that*

$$\sup_{a\in[0,1]} \left| \frac{\partial}{\partial a}\phi(a;F) \right| < \infty.$$

Proof.

Without loss of generality, let us suppose $F \in [\frac{l\delta}{N}, \frac{(l+1)\delta}{N})$ for some $l = 2, 3, \ldots, N-1$, so that $\lfloor \frac{NF}{\delta} \rfloor = l$. Using (A3), we have

$$\begin{aligned}\phi(a;F) =& \sum_{k=N-1-l}^{N-1} \binom{N-1}{k} (1-a)^k a^{N-1-k} \left(\delta - \frac{N(\delta - F)}{k+1} \right) \\ =& \delta \cdot v_l^{(N)}(a) - (\delta - F) u_l^{(N)}(a),\end{aligned} \tag{A17}$$

where

$$v_l^{(N)}(a) = \sum_{k=N-1-l}^{N-1} \binom{N-1}{k} (1-a)^k a^{N-1-k}, \quad u_l^{(N)}(a) = \sum_{k=N-l}^{N} \binom{N}{k} (1-a)^{k-1} a^{N-k}.$$

Functions $v_l^{(N)}(a)$ and $u_l^{(N)}(a)$ do not depend on F and δ, and apparently both of them have continuous first order derivative in a over $[0,1]$. This completes the proof. ■

E Proof of Proposition 7

We first study the first best. To that end, recall from (35) that

$$a_s^* = \arg\max[(1+f)(-Ra^2 + (R - \beta - 1 - c)a + (1+\beta))].$$

Using the first order condition, we obtain that $a_s^* = \frac{R-\beta-1-c}{2R}$.

To prove (37), the upper bound follows immediately from (34) and the fact that $\phi(a;F) \geq 0$:

$$a^e = \frac{(1+f)(R-\beta-1) - \phi(a^e;F) + \delta}{2(1+f)R} < a_r^*.$$

To prove the lower bound, recall from Lemma 9 that, for each fixed $a \in (0,1)$, $\phi(a;F)$ is strictly increasing for all $F \in [\frac{2\delta}{N}, \delta)$, so $\phi(a;F) < \delta$ for all $\frac{2\delta}{N} \leq F < \delta$. Comparing q_s and a^e in expressions (36) and (34), we have that

$$a^e = \frac{(1+f)(R-\beta-1) - \phi(a^e;F) + \delta}{2(1+f)R} > \frac{R-\beta-1}{2R} > a_s^*.$$

This completes the proof.

F Proof of Proposition 8

The objective function $B(F) - \beta F$ is differentiable in F off the set of kinks $\{\frac{l\delta}{N}; l = 2, 3, \dots, N-1\}$. Thus, if the equilibrium F^e is not at one of the kinks, it must solves the regulator's tradeoff between the marginal benefit and marginal cost, i.e. the marginal benefit of increase F in mitigating risk-shifting should equal to the marginal cost of opportunity of default fund segregation. However, as F increases over the kink $\frac{l\delta}{N}$ for some $l = 3, 4, \dots, N-1$, the marginal cost β increases continuously, but the marginal benefit $B'(F)$ increases abruptly. Thus, it is never optimal to choose a default fund level F at one of such kinks.

Next we show that if the marginal value of increasing F is higher than the opportunity cost at the lower bound $\frac{2\delta}{N}$ ($B'(\frac{2\delta}{N}) > \beta$), then $\frac{2\delta}{N} < F^e < \delta$. We claim that

$$B'(\frac{2\delta}{N}) > \beta \leftrightarrow \frac{(a^e(\frac{2\delta}{N}) - a_s^*) \cdot u_2^{(N)}(a^e(\frac{2\delta}{N}))}{1 + \frac{\delta}{2(1+f)R}[v_2^{(N),\prime}(a^e(\frac{2\delta}{N})) - \frac{N-2}{N} u_2^{(N),\prime}(a^e(\frac{2\delta}{N}))]} > \beta. \tag{A18}$$

where $v_2^{(N)}(a) = \sum_{k=N-3}^{N-1} \binom{N-1}{k}(1-a)^k a^{N-1-k}$, $u_2^{(N)}(a) = \sum_{k=N-2}^{N} \binom{N}{k}(1-a)^{k-1} a^{N-k}$. To see (A18), notice that for any $F \neq \frac{l\delta}{N}$,

$$B'(F) = (1+f)[-2Ra^e(F) + R - \beta - 1]\frac{da^e}{dF}(F) = -2(1+f)R(a^e(F) - a_s^*)\frac{da^e}{dF}(F). \tag{A19}$$

Moreover, using implicit differentiation we have

$$-\frac{da^e}{dF}(F) = \frac{\frac{\partial \phi}{\partial F}}{2(1+f)R + \frac{\partial \phi}{\partial a}} = \frac{u_2^{(N)}(a^e(F))}{2(1+f)R + \delta[v_2^{(N)}(a^e(F)) - \frac{N-2}{N} u_2^{(N)}(a^e(F))]}. \tag{A20}$$

From (A19) and (A20) we have

$$B'(\frac{2\delta}{N}+) = \frac{u_2^{(N)}(a^e(\frac{2\delta}{N}))}{1 + \frac{\delta}{2(1+f)R}[v_2^{(N)}(a^e(\frac{2\delta}{N})) - \frac{N-2}{N} u_2^{(N)}(a^e(\frac{2\delta}{N}))]}. \tag{A21}$$

Therefore, the condition in (A18) is equivalent to $B'(\frac{2\delta}{N}+) - \beta > 0$, so the objective function $B(F) - \beta F$ is locally increasing in a small right neighborhood of $\frac{2\delta}{N}$. Similarly, because $a^e(\delta-) = a_s^*$ (see Proposition 7), we know that $B'(\delta-) - \beta = -\beta < 0$. That is, the objective function $B(F) - \beta F$ is locally decreasing in a small left neighborhood of c. It follows that the maximum $F^e < \delta$.

G Proof of Equation (42)

When $N = 3$, the first order equation is a quadratic equation

$$H(a; F) = \frac{(1+f)(R - \beta - 1) + (\delta - F)(1 + a + a^2)}{2(1+f)R} - a = 0. \tag{A22}$$

Its solutions are given by

$$a_\pm = \frac{(1+f)R}{\delta - F} - \frac{1}{2} \pm \sqrt{\left(\frac{(1+f)R}{\delta - F} - \frac{1}{2}\right)^2 - \frac{(1+f)(R-\beta-1) - F + \delta}{\delta - F}}.$$

When $F \in [\frac{2}{3}\delta, \delta)$, we have $\delta - F \le \frac{\delta}{3}$, from which we deduce that $H(1,F) = -(1+f)(R+1+\beta) + 3(\delta - F) < -(R+1+\beta) + \delta < 0$, due to Assumption 1C. From the fact that $H(0,F) > 0$ we know that $0 < a_- < 1 < a_+$. Thus, we have to select a_- as $a^e(F)$ because we knew that $a^e(F) < a_r^* < 1$.

To demonstrate the monotonicity of $a^e(F)$, we notice that, for any $\frac{2\delta}{3} \le F < \delta$ and $0 \le a \le 1$,

$$\begin{aligned}
\frac{\partial H}{\partial a} &= \frac{(\delta - F)(1+2a)}{2(1+f)R} - 1 \le \frac{\frac{\delta}{3}(1+2)}{2(1+f)R} - 1 < \frac{\delta}{2R} - 1 < \frac{R+1+\beta-2R}{2R} = -a_s^* < 0, \\
\frac{\partial H}{\partial F} &= -\frac{1}{2(1+f)R}(1+a+a^2) < 0.
\end{aligned}$$

By the implicit differentiation theorem, we know that $a^e(F)$ is strictly decreasing and differentiable in F.

Now that we have established that the mapping $F :\mapsto a^e(F)$ is one-to-one and decreasing, to obtain the equilibrium F^e, we consider a change of variable: $a^e :\mapsto F(a^e)$, namely, the inverse of $a^e(F)$:

$$F(a) = \delta - 2(1+f)R\frac{a - a_s^*}{1+a+a^2}.$$

Then we know that $F^e = F(\hat{a})$, where

$$\begin{aligned}
\hat{a} &= \operatorname*{arg\,max}_{a \in (a_s^*, a^e(\frac{2\delta}{3})]} [(1+f)(-Ra^2 + (R-\beta-1)a + 1 + \beta) - \delta - \beta F(a)] \\
&= \operatorname*{arg\,max}_{a \in (a_s^*, a^e(\frac{2\delta}{3})]} [(1+f)(-Ra^2 + (R-\beta-1)a + 1 + \beta) - \delta - \beta F(a)]. \qquad \text{(A23)}
\end{aligned}$$

To fix $\hat{a}$, we calculate the derivative of the objective function of (A23) with respect to a:

$$(1+f)[-2Ra + R - \beta - 1] - 2\beta(1+f)R\frac{1 - a^2 + a_s^*(1+2a)}{(1+a+a^2)^2} = -2(1+f)Rh(a). \qquad \text{(A24)}$$

Moreover, notice that

$$h'(a) = 1 + \beta\frac{1 - a^3 + 2a + (2a + 3a^2)a_s^*}{(1+a+a^2)^3} > 1, \forall a \in [0,1]. \qquad \text{(A25)}$$

Therefore, the first order condition equation $h(a) = 0$ can have at most one root. On the other hand, one clearly has $h(a_s^*) < 0$, so if $h(a^e(\frac{2\delta}{3})) \le 0$, then the objective function of (A23) is strictly increasing in a over $[a_s^*, a^e(\frac{2\delta}{3})]$. It follows that $\hat{a} = a^e(\frac{2\delta}{3})$ and $F^e = F(a^e(\frac{2\delta}{3})) = \frac{2\delta}{3}$. On the other hand, if $h(a^e(\frac{2\delta}{3})) > 0$, then there is a unique maximizer $\hat{a} = a_0 \in (a_s^*, a^e(\frac{2\delta}{3}))$, which solves the first order condition equation $h(a_0) = 0$. Hence, the equilibrium default fund $F^e = F(a_0)$.

H The large CCP network limit: proofs of Proposition 4 and Proposition 9

In this section, we derive the limit of $\psi(N-1;F)$ and $\phi(a;F)$ as $N\to\infty$, which will imply the limit of $\hat{F}$ and $a^e(F)$ as the number of members in the CCP network grows without bound. To that end, we recall (A17) that,

$$\phi(a;F) = \delta\, v_l^{(N)}(a) - (\delta - F)\, u_l^{(N)}(a),$$

where $\delta = c+\delta$, $l = \lfloor \frac{NF}{\delta} \rfloor$, and functions $v_l^{(N)}$ and $u_l^{(N)}$ have the following representation: suppose X follows a Binomial distribution with parameter $(N-1,a)$ and Y follows a Binomial distribution with parameter (N,a), then

$$v_l^{(N)}(a) = \mathbb{P}(X\le l) = \mathbb{P}(\sqrt{N-1}(\frac{X}{N-1}-a) \le \sqrt{N-1}(\frac{l}{N-1}-a)), \tag{A26}$$

$$u_l^{(N)}(a) = \frac{\mathbb{P}(Y\le l)}{1-a} = \mathbb{P}(\sqrt{N}(\frac{Y}{N}-a) \le \sqrt{N}(\frac{l}{N}-a)). \tag{A27}$$

By the central limit theorem, both $\sqrt{N-1}(\frac{X}{N-1}-a)$ and $\sqrt{N}(\frac{Y}{N}-a)$ converges in distribution to a normal distribution with mean 0 and variance $a(1-a)$. On the other hand, from $\frac{NF-\delta}{\delta} < \lfloor \frac{NF}{\delta} \rfloor \le \frac{NF}{\delta}$ we know that,

$$\lim_{N\to\infty} \sqrt{N-1}(\frac{l}{N-1}-a) = \lim_{N\to\infty} \sqrt{N}(\frac{l}{N}-a) = \begin{cases} \infty, & \text{if } \frac{F}{\delta} > a, \\ 0, & \text{if } \frac{F}{\delta} = a, \\ -\infty, & \text{if } \frac{F}{\delta} < a \end{cases}.$$

It follows that

$$\lim_{N\to\infty} v_l^{(N)}(a) = 1_{\{\frac{F}{\delta}\ge a\}} + \frac{1}{2}1_{\{\frac{F}{\delta}=a\}}, \quad \lim_{N\to\infty} u_l^{(N)}(a) = \frac{1_{\{\frac{F}{\delta}\ge a\}} + \frac{1}{2}1_{\{\frac{F}{\delta}=a\}}}{1-a},$$

and

$$\lim_{N\to\infty} \phi(a;F) = \delta\cdot 1_{\{\frac{F}{\delta}>a\}}\left(1-\frac{1-\frac{F}{\delta}}{1-a}\right).$$

Returning to the definition of $\phi(a;F)$, we know that

$$\lim_{N\to\infty} \psi(N-1;F) = \delta\cdot 1_{\{\frac{F}{\delta}>q_s\}}\left(1-\frac{1-\frac{F}{\delta}}{1-q_s}\right).$$

Therefore, in the binary case, the switching point $\hat{F}$, which is defined as the unique root to $\psi(N-1;\hat{F}) = \frac{\mu_r-\mu_s}{q_r-q_s}+\delta$, converges to the solution to the equation:

$$\delta\cdot 1_{\{\frac{F}{\delta}>q_s\}}\left(1-\frac{1-\frac{F}{\delta}}{1-q_s}\right) = (1+f)\frac{\mu_r-\mu_s}{q_r-q_s}+\delta.$$

We notice that, as the number of members $N \to \infty$, the cover-II requirement for F, now becomes $F > 0$. Hence, if $\frac{\mu_r - \mu_s}{q_r - q_s} \le \delta$, then we obtain that

$$\hat{F} = q_s\,\delta + (1 - q_s)\Big((1+f)\frac{\mu_r - \mu_s}{q_r - q_s} + \delta\Big).$$

Hence, by (21) we know that, as $N \to \infty$,

$$\frac{x^e(N)}{N} = \frac{F^e(N)}{\delta} \to \begin{cases} \dfrac{\hat{F}}{\delta}, & \text{if } \mu_s - \mu_r > \beta\dfrac{(q_s\,\delta + (1-q_s)((1+f)\frac{\mu_r-\mu_s}{q_r-q_s} + \delta))}{1+f}, \\ 0, & \text{else.} \end{cases} \tag{A28}$$

This proves Proposition 4.

In the continuous case, recall that the optimal risk preference $a^e(F)$ and F satisfies the first order condition equation:

$$a = \frac{(1+f)(R-\beta-1) - \phi(a;F) + \delta}{2(1+f)R}.$$

As $N \to \infty$, we notice that the above equation converges to

$$a = \begin{cases} a_s^* + \dfrac{\delta}{2(1+f)R}, & \text{if } \dfrac{F}{\delta} \le a, \\ a_s^* + \dfrac{\delta - F}{2(1+f)R(1-a)}, & \text{if } \dfrac{F}{\delta} > a. \end{cases}$$

In other words,

$$a^e(F) = \begin{cases} a_s^* + \dfrac{\delta}{2(1+f)R}, & \text{if } \dfrac{F}{\delta} \le a_s^* + \dfrac{\delta}{2(1+f)R}, \\ \dfrac{(1+a_s^*) - \sqrt{(1-a_s^*)^2 - \frac{\delta - F}{(1+f)R}}}{2}, & \text{if } \dfrac{F}{\delta} > a_s^* + \dfrac{\delta}{2(1+f)R}, \end{cases} \tag{A29}$$

which is a continuous function that is strictly decreasing over $(a_s^*\delta + \frac{\delta^2}{2(1+f)R}, \delta)$, with limits $a^e(\delta-) = \frac{R-1-\beta}{2R} > a_s^*$. From (39) we know that, as $N \to \infty$, we have

$$\frac{x^e(N)}{N} = \frac{F^e(N)}{\delta} \to \frac{1}{\delta}\arg\max_F \big((1+f)(-R(a^e(F))^2 + (R-\beta-1)a^e(F) + 1 + \beta) - \beta F\big). \tag{A30}$$

To find the maximizer for the right hand side of (A30), we first notice that

$$\arg\max_{0 < F \le a_s^*\delta + \frac{\delta^2}{2(1+f)R}} \big((1+f)(-R(a^e(F))^2 + (R-\beta-1)a^e(F) + 1 + \beta) - \beta F\big) = 0,$$

with maximum

$$
\begin{aligned}
&(1+f)\bigg[-R\Big(a_s^*+\frac{\delta}{2(1+f)R}\Big)^2+(R-\beta-1)\Big(a_s^*+\frac{\delta}{2(1+f)R}\Big)+1+\beta\bigg]\\
=&(1+f)\bigg[R(a_s^*)^2-R\Big(\frac{\delta}{2(1+f)R}\Big)^2+1+\beta\bigg].
\end{aligned}
$$

On the other hand, the range of the objective function in (A30) for $F \in (a_s^*\delta + \frac{\delta^2}{2(1+f)R}, \delta)$ is the same as that of

$$
G(a) := (1+f)(-Ra^2+(R-\beta-1)a+\beta+1)-\beta F(a), \quad a \in (a_s^*, a_s^*+\frac{\delta}{2(1+f)R}),
$$

where $F(a)$ is the inverse of $a^e(F)$ in (A29) for $F \in (a_s^*\delta + \frac{\delta^2}{2(1+f)R}, \delta)$, given by

$$
F(a) = \delta - 2(1+f)(a-a_s^*)(1-a)R. \tag{A31}
$$

From

$$
G'(a) = 2(1+f)[\beta(1-a)-(R+\beta)(a-a_s^*))], \tag{A32}
$$

we know that $G'(a)$ is strictly decreasing so $G(a)$ is concave. Moreover, since $G'(a_s^*)>0$, we know that the maximum of $G(a)$ over $(a_s^*, a_s^*+\frac{\delta}{2(1+f)R})$ is either $a_s^*+\frac{\delta}{2(1+f)R}$ or the global maximizer of $G(a)$ if the maximizer is insider the interval under consideration. To proceed, we first determine the global maximizer by solving $G'(a)=0$, and obtain $a_0^* = \frac{\beta+(R+\beta)a_s^*}{R+2\beta}$. Thus,

$$
\sup_{a\in(a_s^*, a_s^*+\frac{\delta}{2(1+f)R})} G(a) = \begin{cases} G(a_0^*), & \text{if } a_0^* < a_s^*+\dfrac{\delta}{2(1+f)R},\\ G(a_s^*+\dfrac{\delta}{2(1+f)R}), & \text{else.}\end{cases}
$$

Thus, we know that

$$
\begin{aligned}
&\arg\max_F \big((1+f)(-R(a^e(F))^2+(R-\beta-1)a^e(F)+1+\beta)-\delta-\beta F\big)\\
&=\begin{cases} F(a_0^*), & \text{if } G(a_0^*) > (1+f)\bigg[R(a_s^*)^2-R\Big(\dfrac{\delta}{2(1+f)R}\Big)^2+1+\beta\bigg] \text{ and}\\ & a_0^* < a_s^*+\dfrac{\delta}{2(1+f)R},\\ 0, & \text{else.}\end{cases}
\end{aligned}
\tag{A33}
$$

This proves Proposition 9. Finally, since $a_0^*-a_s^*\to 0$ as $\beta\to 0$, it is easily seen that we are in the first case of (A33) if $\beta>0$ is sufficiently small.