A Theory of Collateral Requirements for Central Counterparties

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Motivation

- A key post-crisis reform is the mandatory clearing of standardized OTC derivatives (CDS+IRS).
  - CCPs act as the buyer to every seller and the seller to every buyer
  - CCPs guarantee terms of trades by pooling the counterparty risks
  - risk management of CCPs is crucial for financial stability

- CCPs collect prepaid collateral: initial margin and default fund
  - initial margins are only used to absorb losses of the posting member
  - default funds are shared across members for loss mutualization

- Little is known about the design of collateral requirements for CCPs
  - how to allocate collateral in initial margin and default funds?
  - what are the economic tradeoffs between the two types of collateral?
Bilateral Trading Markets
Centrally Cleared Markets
Collateral and CCP’s Default Waterfall

Members post two types of collateral: initial margin and default fund.

1. Defaulting member’s Initial Margin
2. Default funds
3. Defaulting member’s Default Fund
4. CCP’s equity capital (tiny)
5. Surviving members’ Default Funds (loss mutualization)
6. End-of-Waterfall Resources (Assessments, IM Haircutting, VMGH)
Lack of Global Standards for Collateral Requirements

- Regulation of collateral is still debatable: lack of global standards (Cunliffe, 2018; Duffie, 2019)
  - initial margin is usually set at some Value-at-Risk level
  - default fund is subject to “Cover 2”—total default funds should cover shortfalls of the two largest members (CPSS-IOSCO)
  - large cross-sectional variation in how CCPs allocate collateral

<table>
<thead>
<tr>
<th>Number of CCPs</th>
<th>Asia</th>
<th>Australia</th>
<th>Europe</th>
<th>North America</th>
<th>South America</th>
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</thead>
<tbody>
<tr>
<td>Initial margin</td>
<td>69.2</td>
<td>92.8</td>
<td>74.0</td>
<td>85.2</td>
<td>99.6</td>
</tr>
<tr>
<td>Default fund</td>
<td>18.7</td>
<td>4.5</td>
<td>25.3</td>
<td>13.5</td>
<td>0.2</td>
</tr>
<tr>
<td>CCP capital</td>
<td>12.2</td>
<td>2.7</td>
<td>0.7</td>
<td>1.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Q: How to design collateral requirements for central clearing?
The first framework to jointly solve for initial margin and default funds

- both raise members’ pledgeable income, but are imperfect substitutes
- initial margin is more cost-effective to align members’ incentives
- default fund is less cost-effective for incentive provision ex-ante, but more valuable for CCP’s resilience ex-post

The optimal design of collateral trades off between minimizing

- members’ collateral opportunity cost ⇒ call for more initial margin
- CCP recapitalization cost ⇒ call for more default fund

Offer policy implications for CCP resilience and value of clearing
Literature

- The Role of Collateral in Derivatives Markets
  - Oehmke 2014; Bolton, Oehmke 2015; Biais, Heider, Hoerova 2016, 2019
  - We model the composition of collateral in central clearing

- Loss-mutualization and Insurance
  - Arrow, 1974; Raviv, 1979; Parlour, Plantin, 2008
  - We highlight the distinguishing features of central clearing

- Central Clearing and CCP Regulation
  - Acharya, Bisin 2014; Koeppel, Monnet, Temzelides 2012; Duffie, Zhu 2011; Huang, 2019; Biais, Heider, Hoerova 2012
  - Boissel et al. 2017; Menkveld 2017; Paddrik, Rajan, Young 2019
  - We provide a normative analysis on the design of collateral requirements
Model
Model Setup: agents and assets

• A continuum of protection buyers
  - mean-variance preferences and risk-aversion parameter $\gamma > 0$
  - an aggregate credit shock (prob = $p_c$) of $(-D)$ to endowed payoff
  - seek insurance against credit risk from dealers at price $A$

• $N$ risk-neutral dealers, each selling a CDS and enjoying limited liability
  - fully invests $A$ in a risky asset
  - makes risk-management choice $a = \{s$ (to hedge), $r$ (not to hedge)$\}$
    - default in bad state (prob = $q_a$) with zero payoff
    - survive in good state (prob = $1 - q_a$) with payoff $A \times R_a - p_c D$
  - $0 < q_s < q_r$, $R_s < R_r$
  - choice $a$ is unobservable and non-contratible

• Let $\mu_a = (1 - q_a)R_a$; assume $\mu_s > \mu_r$ so risk management is efficient
Bilateral Trading Market

- $t = 0$: a buyer and a dealer trade CDS; buyer pays a unit price $A_{BT}$

- $t = 1$: i.i.d. payoffs are realized; promise $D$ needs to be fulfilled in the aggregate credit event, which occurs with prob $p_c \in (0, 1)$; a dealer defaults with prob $p_c q_a$
Centrally Cleared Market

- CCP guarantees insurance payment $D$ to buyers with certainty.
- $t = 0$: CCP collects collateral from members: initial margin $I \in [0, D]$, default fund $F \in [0, D - I]$. Members have opportunity cost $\beta(I + F)$.
- $t = 1$: $N_d$ members default; if all collateral is exhausted, CCP invokes end-of-waterfall resources at a cost $(1 + \beta)(N_d(D - I) - NF)^+$. 
- Stylized model of the default waterfall
  - end-of-waterfall resources are sufficient to prevent CCP’s default
  - we do not model “CCP’s skin-in-the-game” (tiny in the waterfall)
  - CCP acts as a social planner to maximize value of all market participants
Centrally Cleared Market: default waterfall

1. Defaulting member’s Initial Margin
   \[ I \in [0, D] \]

2. Defaulting member’s Default Fund
   \[ F \in [0, D - I] \]

3. Surviving members’ Default Funds (loss mutualization)
   \[ (N - N_d) F \]

4. End-of-Waterfall Resources
   \[ (N_d(D - I) - NF)^+ \]
Pricing of Bilateral and Centrally Cleared CDS

- Assumption 1: buyers have zero bargaining power and are sufficiently risk-averse, $\gamma > \gamma$.
  - allows dealers to collect a high premium for a centrally cleared CDS
  - dealers’ participation constraint is satisfied

- A bilaterally traded CDS reduces buyer’s credit risk to $p_c q a$

$$A_{BT} = p_c D (1 - q_r) (1 + \gamma D (1 - p_c - p_c q_r))$$

- A centrally cleared CDS reduces buyer’s credit risk to 0

$$A_{CCP} = p_c D (1 + \gamma D (1 - p_c)) > A_{BT}$$

- Assumption 2 (technical restrictions):
  - $D > \frac{A_{CCP} (\mu_s - \mu_r)}{(q_r - q_s) p_c}$ ⇒ dealers do not hedge without posting collateral
  - $A_{BT} R_s > D$ ⇒ a dealer is able to pay $D$ in the good state
Dealer’s Expected Profit: bilateral vs. central clearing

- A bilateral dealer chooses not to hedge ⇒ risk-shifting

\[ V_{BT} = \max_{a \in \{s, r\}} (1 - q_a)(A_{BT}R_a - p_cD) = A_{BT}\mu_r - (1 - q_r)p_cD. \]

- The risk-management decision of a clearing member depends on the collateral requirements

\[ V(a_i, a_{-i}; I; F) = -(1 + \beta)(I + F) + (1 - p_c)(I + F) + \]

\[ (1 - q_{a_i}) \left( A_{CCP}R_{a_i} - p_cD + p_cI + p_cE^a \left[ \left( F - \frac{N_d(D - I - F)}{N - N_d} \right)^+ \right] \right) \]

expected refund of collateral ⇒ increase in pledgeable income
The First-Best: no collateral is needed

In the first-best benchmark, the CCP, acting as a social planner, chooses \((a; I; F)\) to maximize the value of all market participants

\[
\max_{(a; I; F)} \left\{ \sum_i V(a; I; F) - (1 + \beta)p_c \mathbb{E}^a \left[ (N_d(D - I) - NF)^+ \right] \right\}.
\]

Proposition 1: The first-best risk-management choice is \(a^F_B i = s\), \(\forall i\); the first-best collateral is \(I^{FB} = F^{FB} = 0\).

Costly collateral is collected only as an incentive device.
The Second-Best: incentive-constrained optimal collateral

- Collateral \((I, F)\) satisfy *incentive compatibility* if
  \[
  V(a = s; I; F) \geq V(a = r; I; F);
  \]  
  (IC)

- Collateral \((I, F)\) satisfy *individual rationality* if
  \[
  V(s; I; F) \geq V_{BT};
  \]  
  (IR)

- Given \(a(I; F)\), the CCP (social planner) chooses optimal collateral \((I^{SB}, F^{SB})\) to maximize the value of all market participants
  \[
  \max_{(I,F)} \left\{ \sum_i V_i(a(I; F); I; F) - (1 + \beta) p_c E^a \left[ (N_d(D - I) - NF)^+ \right] \right\}
  \]
  subject to IC and IR.
Members’ Risk-shifting Incentives and Collateral

Proposition 2: Members’ risk-management decisions depend on collateral $I$ and $F$.

- posting collateral increases pledgeable income, alleviates risk-shifting
- total refundable collateral needs to be $D - \frac{A_{CCP}(\mu_s - \mu_r)}{(q_r - q_s)p_c}$
- the IC curve satisfies $\partial \hat{F}(I) / \partial I < -1$

$\Rightarrow$ when initial margin decreases by 1, default fund increases by $1+$.
$\Rightarrow$ initial margin is more cost-effective in aligning members’ incentives.
Proposition 3: The incentive-constrained optimal collateral is

\[ I_{SB} = D - \frac{A_{CCP}(\mu_s - \mu_r)}{(q_r - q_s)p_c} \]

and \( F_{SB} = 0 \) when the marginal collateral opportunity cost and CCP’s recapitalization cost are both equal to \( \beta \).

- CCP recapitalization is financed ex-post, thus is cheaper \((\beta > p_c/\beta)\)
- minimizing members’ collateral posting is the primary goal

⇒ initial margin is preferred.
Optimal Collateral Requirements: extreme market events

Prefunded Collateral with cost $\beta$

- Defaulting member’s Initial Margin
  \[ I \in [0, D] \]

- Default funds

  - Defaulting member’s Default Fund
    \[ F \in [0, D - I] \]

  - Surviving members’ Default Funds (loss mutualization)
    \[ (N - N_d)F \]

- Not prefunded, with cost $\alpha$

  - End-of-Waterfall Resources
    \[ (N_d(D - I) - NF)^+ \]
Proposition 4: The incentive-constrained optimal collateral is \( I^{SB} = 0 \) and
\[
F^{SB} = \left( \frac{1-v_l^{(N)}(q_s)}{u_l^{(N)}(q_s)} + 1 \right) D - \frac{A_{CCP}(\mu_s-\mu_r)}{(q_r-q_s)p_c u_l^{(N)}(q_s)} \] when CCP recapitalization cost is higher than collateral cost, \( \beta < p_c \alpha \).

- reducing recapitalization costs via mutualization is the primary goal
  ⇒ default fund is preferred.
  ⇒ offers a rationale for collecting default funds.
Optimal Cover Rule for Default Funds: “Cover x%”

Proposition 5: Default fund follows a “Cover x%” rule; as $N \to \infty$,

$$x(I^{SB}; N) \to 1 - \frac{(1 - q_s)(\mu_s - \mu_r)D (1 + \gamma D (1 - p_c))}{(q_r - q_s)(D - I^{SB})}$$

- cover number increases with $N$; Cover x% has little variation with $N$

$\Rightarrow$ implications: cover a fixed fraction rather than a fixed number.

$\Rightarrow$ the rule is robust to entry and exit of members.
In systemic events, i.e., when multiple members default, CCP might face increasing marginal costs to raise end-of-waterfall resources:

$$\alpha \left( (N_d(D - I) - NF)^+ \right)^2$$

- the trade-off between initial margin and default fund is robust.

$$\Rightarrow$$ nonlinearity allows to pin down interior levels of collateral.
Robustness 2: heterogeneity in size

CCPs’ exposures tend to concentrate on a few large clearing members. Suppose $i$ is $K$ times ($K > 1$) the size of others: $KD, KACCP Ra$

- economic forces from the baseline model persist
- required collateral is disproportionately lower for the big member
  ⇒ it is easier for the big member (who acts as an internally coordinated group of $K$ small members) to internalize the externalities
Policy Implications: collateral design and CCP resilience

- Our results inform the optimal design of collateral requirements
  - CCPs are required to quarterly disclose their default waterfalls
  ⇒ it is feasible to closely monitor CCP collateral requirements

- CCP having to recapitalize is a warning sign against its resilience
  - high recapitalization cost $\alpha$ may be viewed as a strong regulatory weight on reducing systemic distress
  ⇒ the expected loss at the CCP under the optimal collateral requirements $(I^{SB}, F^{SB})$ converges to 0.

- Our results show the value of central clearing under optimal collateral
  - members prefer to join CCP than trade bilaterally
Policy Implications: the value of central clearing

What if trades in bilateral markets are fully collateralized?
⇒ Dealers choose to hedge, and impose zero counterparty risk to buyers
⇒ But full collateralization is too costly ⇒ members prefer CCP

\[ I = D, \quad F = 0 \]

\[ F \in [0, D - I] \]

\[ \hat{F}(I) \]

(0, 0) \quad \hat{F}(I) \quad F \in [0, D - I]
Policy Implications: the value of central clearing

What if using incentive-compatible margin in bilateral markets?
⇒ Dealers choose to hedge, and impose counterparty risk $p_c q_s$ to buyers.
⇒ But collateral is higher than in central clearing ⇒ members prefer CCP

$I = D - \frac{A_{BT}(\mu_s - \mu_r)}{(q_r - q_s)p_c}
\hat{F}(I)$

$F \in [0, D - I]$
Empirical Implications

1. CCP collects more collateral when the CDS reference entity has higher credit risk; Members with more effective risk management post less collateral.

2. The fraction of initial margin increases with the collateral opportunity cost, decreases with measures of systemic distress.

3. Total default funds increase with number of members. Larger members post more collateral, but disproportionately lower default fund.
Conclusions

• This paper develops the first theoretical framework of collateral requirements for CCPs.

• We study the trade-off between initial margin and default fund.
  - initial margin is more cost-effective to align incentives;
  - default fund allows for members’ loss-sharing ex-post, but is less valuable for incentive provision ex-ante.

• We solve for the incentive-constrained optimal collateral
  - use initial margin during normal times, use default fund in extreme market events when CCP recapitalization is very costly
  - offer a rationale for collecting default funds and a “Cover x%” rule
  - insights generalize to settings with heterogeneous members.