CSE 494/598, Fall 2007 Homework 2
Due on Wednesday, September 12

1. Let $D \in \mathbb{R}^{n\times n}$ be a diagonal matrix whose i-th diagonal entry is d_i. Show that $||D||_p = \max_i |d_i|$, where $1 \leq p \leq \infty$.

2. Let A and B be two positive semi-definite matrices in $\mathbb{R}^{n\times n}$. Prove or disprove:

   (1) $A + B$ is positive semi-definite;

   (2) AB is positive semi-definite;

   (2) B^T is positive semi-definite.

3. Let A and B be two matrices in $\mathbb{R}^{n\times n}$ and let $Q \in \mathbb{R}^{n\times n}$ be an orthogonal matrix. Show that

   (1) $\text{trace}(AB) = \text{trace}(BA)$;

   (2) $||A||_F^2 = \text{trace}(AA^T)$; and

   (2) $||QA||_F = ||A||_F$, where $||A||_F$ denotes the Frobenius norm of the matrix A.

4. Let $A = (a_{ij})$ be any matrix of size m by n. Show that $||A||_1 = \max_j \sum_{i=1}^m |a_{ij}|$.

5. Let A be a matrix of size n by n whose entries are real numbers. Assume that $A^T = -A$. Show that

   (1) $I_n - A$ is nonsingular, where I_n is the identity matrix of size n by n; and

   (2) $(I_n - A)^{-1}(I_n + A)$ is orthogonal.