

CSE 494/598, Fall 2007 Homework 3
Due on Wednesday, September 26

1. Let $x = (0, 12, 3, 4)^T$, $y = (13, 0, 0, 0)^T$, $z = (0, 13, 0, 0)^T$.

- (i) Compute the Householder matrix P that transforms x to z as $Px = z$.
- (ii) Compute three Givens (rotation) matrices G_1, G_2, G_3 that transform x to y as $G_3G_2G_1x = y$.

2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 8 \\ 4 & 7 & 9 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

- (i) Apply the Gram-Schmidt procedure to compute the QR decomposition of A .
- (ii) Compute $x \in \mathbb{R}^3$ that solves the following least squares problem: $\min_x \|Ax - b\|_2$.

3. Let $A \in \mathbb{R}^{m \times n}$ be a matrix of rank r . Prove that $\|A\|_F \leq \sqrt{r}\|A\|_2$. Here $\|A\|_F$ denotes the Frobenius norm of A .

4. Prove that for $A \in \mathbb{R}^{m \times n}$, the following equality holds:

$$\sigma_{\max}(A) = \max_{y \in \mathbb{R}^m, x \in \mathbb{R}^n} \frac{y^T Ax}{\|x\|_2 \|y\|_2}.$$

5. Let $A \in \mathbb{R}^{m \times n}$ be a matrix of rank n . Prove that $\|A(A^T A)^{-1} A^T\|_2 = 1$.