Circle your answer choice on the exam **AND** fill in the answer sheet below with the letter of the answer that you believe is the correct answer.

Problem Number	Letter of Answer	Problem Number	Letter of Answer
7.	A	12.	D
8.	C	13.	B
9.	C	14.	D
10.	C	15.	C
11.	A	16.	C

For Part I – Free Response. **Show all work (as described on page 1).**

1. **[8 pts]** Use linear approximation to approximate the value of $\sqrt{139}$ to four decimal places.

$\sqrt{139}$ $\quad x = 139 \quad a = 144$ $\quad f(x) \approx f(a) + f'(a)(x-a)$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$ or $\frac{1}{2\sqrt{x}}$

$f(x) \approx f(144) + f'(144)(139-144)$

$f(x) \approx \sqrt{144} + \frac{1}{2\sqrt{144}}(-5)$

$f(x) \approx 12 + \frac{1}{24}(-5)$

$f(x) \approx 12 + \frac{-5}{24}$

$\boxed{f(x) \approx 11.7917}$

2. **[9 pts]** A beverage company works out a demand function for its sale of soda and find it to be $q = 3700 - 25p$ where q is the quantity of sodas sold when the price per can, in cents, is p. At what price is the revenue a maximum?

$\frac{dq}{dp} = -25$

$E = -\frac{p}{q} \cdot \frac{dp}{dq}$

$E = -\frac{p}{3700-25p}(-25)$

$E = \frac{25p}{3700-25p}$

$\frac{1}{1} = \frac{25p}{3700-25p}$

$3700 - 25p = 25p$

$+25p \quad +25p$

$\frac{3700}{50} = \frac{50p}{50}$

$\boxed{74 = p}$

3. **[9 pts]** Find the x-values of all points where the function $f(x) = x^{\frac{4}{3}} - x^{\frac{2}{3}}$ has any relative extrema. Find the value(s) of any relative extrema. Write your final answer in the appropriate box on the right. Answers must be exact.

appropriate box on the right. Answers must be exact.

$f(x) = x^{4/3} - x^{2/3}$

$f'(x) = \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{4x^{1/3}}{3} - \frac{2}{3x^{1/3}}$

$3x^{1/3} = 0$

$x^{1/3} = 0$

$x = 0$

when $f'(x) = 0$

$\frac{2}{3x^{1/3}} = \frac{4x^{1/3}}{3}$

$6 = 12x^{2/3}$

$\left(\frac{1}{2}\right)^{3/2} = \left(x^{2/3}\right)^{3/2}$

$\pm\left(\frac{1}{2}\right)^{3/2} = x$

$f\left(\frac{1}{2}\right)^{3/2} = \left(\left(\frac{1}{2}\right)^{3/2}\right)^{4/3} - \left(\left(\frac{1}{2}\right)^{3/2}\right)^{2/3}$

$\left(\frac{1}{2}\right)^2 - \frac{1}{2}$

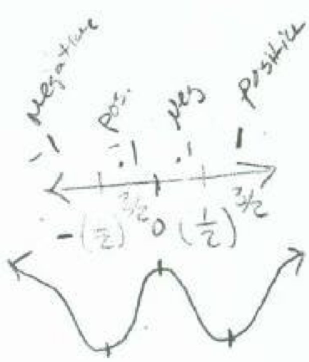
$f\left(\frac{1}{2}\right)^{3/2} = \frac{1}{4} - \frac{1}{2}$

$f\left(\frac{1}{2}\right)^{3/2} = -\frac{1}{4}$

$f\left(-\frac{1}{2}\right)^{3/2} = \left(\left(-\frac{1}{2}\right)^{3/2}\right)^{4/3} - \left(\left(-\frac{1}{2}\right)^{3/2}\right)^{2/3}$

$f\left(-\frac{1}{2}\right)^{3/2} = \left(\frac{1}{2}\right)^2 - \frac{1}{2}$

$f\left(-\frac{1}{2}\right)^{3/2} = -\frac{1}{4}$



Relative Maxima	
x-value	value
0	0

Relative Minima	
x-value	value
$-\left(\frac{1}{2}\right)^{\frac{3}{2}}$	$-\frac{1}{4}$
$\left(\frac{1}{2}\right)^{\frac{3}{2}}$	$-\frac{1}{4}$

4. **[8 pts]** Find dy for $y = 3x^4 - 5x^3 + 2x + 1$, $x = -1$, and $\Delta x = \frac{1}{3}$.

$dy = (12x^3 - 15x^2 + 2)dx$

$dy = (12(-1)^3 - 15(-1)^2 + 2)(1/3)$

$dy = (-25)(1/3)$

$\boxed{dy = \frac{-25}{3}}$

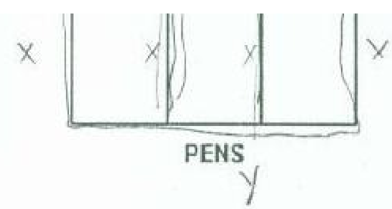
5. **[8 pts]** Evaluate the indefinite integral $\int (x^4 + e^{5x})dx$

$\int x^4 + \int e^{5x}$

$\frac{1}{a+1}x^{a+1} + \frac{1}{a}e^{ax} + C$

$\frac{1}{5}x^5 + \frac{1}{5}e^{5x} + C$

6. **[9 pts]** A farmer decides to make three identical pens with 144 feet of fencing. The pens will be next to each other sharing a fence and will be up against a barn. The barn side needs no fence (see picture to the right). What dimensions for the total enclosure (rectangle including all pens) will make the area as large as possible?



$A = xy$ $\quad 144 = 4x + y$ 144 ft

$A = x(144 - 4x)$ $\quad 144 - 4x = y$

$A = 144x - 4x^2$

$A' = 144 - 8x$

$0 = 144 - 8x$

$-144 = -8x$

$\boxed{18 = x}$

$144 = 4(18) + y$

$144 = 72 + y$

$\boxed{72 = y}$

$x = 18$

$y = 72$

For Part II – Multiple Choice: Circle your answer choice on the exam **AND** fill in the answer sheet on the front of page 2 with the letter of the answer that you believe is the correct answer.

7. **[5 pts]** The concentration of a certain drug in the bloodstream x hours after being administered is approximately $C(x)=\dfrac{3x}{11+x^2}$. Use the differential to approximate the change in concentration as x changes from 1 to 1.37.

A. 0.08 B. 0.30 C. 0.11
D. 0.21 E. None of these

$$dC(x)=\frac{3(11+x^2)-3x(2x)}{(11+x^2)^2}dx$$
$$dC(x)=\frac{33+3x^2-6x^2}{(11+x^2)^2}dx$$
$$dC(x)=\frac{33-3x^2}{(11+x^2)^2}dx$$

$$dC(x)=\frac{33-3(1)^2}{(11+(1)^2)^2}(1.37-1)$$
$$dC(x)=.077083333$$

8. **[5 pts]** Find the elasticity of the demand function $q=\sqrt{750-p}$ at the price p = \$580 and state whether the demand is elastic, inelastic or whether it has unit elasticity.

A. $\frac{29}{17}$; inelastic B. 1; unit elasticity C. $\frac{29}{17}$; elastic
D. $\frac{58}{17}$; elastic E. None of these

$$E=-\frac{p}{q}\cdot\frac{dq}{dp}$$
$$\frac{dq}{dp}=\frac{1}{2}(750-p)^{-\frac{1}{2}}(-1)$$
$$E=-\frac{p}{\sqrt{750-p}}\cdot\frac{-1}{2\sqrt{750-p}}$$
$$E=\frac{p}{2(750-p)}$$
$$E=\frac{580}{2(750-580)}$$
$$E=\frac{29}{17}>1 \text{ thus elastic}$$

9. **[5 pts]** Find the open interval(s) where the function $f(x)=\dfrac{1}{x^2+1}$ is increasing.

A. $(0, \infty)$ B. $(1, \infty)$ C. $(-\infty, 0)$ D. $(-\infty, 1)$ E. None of these

$$f'(x) = -1(x^2+1)^{-2}(2x)$$

$$f'(x) = \frac{-2x}{(x^2+1)^2}$$

$$0 = \frac{-2x}{(x^2+1)^2}$$

$$0 = 2x$$

$$x = 0$$

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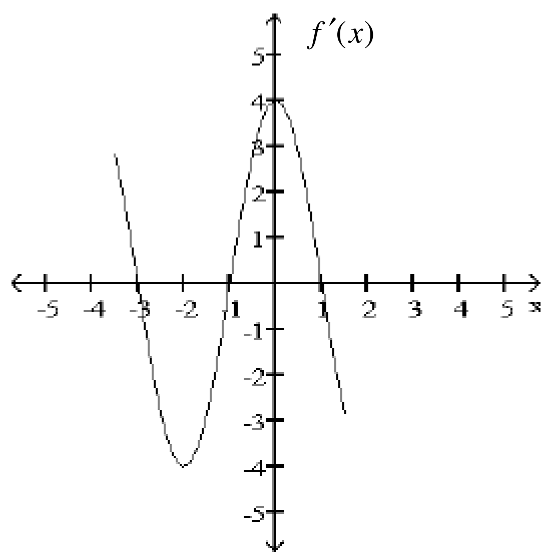
check -1 0 check 1

$$f'(-1) = \frac{-2(-1)}{((-1)^2+1)^2} \qquad f'(1) = \frac{-2(1)}{((1)^2+1)^2}$$

$$f'(-1) = \frac{2}{4} = \frac{1}{2} \qquad f'(1) = -\frac{2}{4} = -\frac{1}{2}$$

positive negative

10. **[5 pts]** Suppose that the graph below is the graph of the derivative of the function $f(x)$ (i.e. this is the graph of $f'(x)$). Find the locations of all extrema and tell whether each extrema is a relative maximum or relative minimum.



A. relative minimum at -3 and 1; relative maximum at -1
B. relative maximum at 0 and relative minimum at -2
C. relative maxima at -3 and 1 and relative minimum at -1
D. no relative extrema
E. None of these

The relative extrema occur when the derivative is equal to 0. This will be at x = -3, x = -1, and x = 1. If the derivative is positive to the left and negative to the right of the zero of the derivative, then the root represents a relative maximum of the function. If the derivative is negative to the left and negative to the right of the zero of the derivative, then the root represents a relative minimum of the function.

11. **[5 pts]** Find the absolute maximum and absolute minimum values of the function $f(x) = x^2 - 8x + 14$ on the interval $[0, 6]$

A. absolute maximum 14, absolute minimum -2
B. absolute maximum 2, absolute minimum -2

C. absolute maximum 14, absolute minimum2
D. absolute maximum -2, no absolute minimum
E. None of these

$f'(x) = 2x - 8$

$0 = 2x - 8$
$8 = 2x$
$4 = x$

Find function values at $x = 4$, $x = 0$, and $x = 6$.

$f(0) = 0^2 - 8(0) + 14 = 14$
$f(4) = 4^2 - 8(4) + 14 = -2$
$f(6) = 6^2 - 8(6) + 14 = 2$

12. **[5 pts]** Evaluate the indefinite integral $\int \frac{3x^4 - 3}{x} dx$

A. $\frac{3}{4}x^4 - 3x^0 + C$ B. $3x^4 - 3\ln|x| + C$ C. $\frac{4}{3}x^4 - 3\ln|x| + C$
D. $\frac{3}{4}x^4 - 3\ln|x| + C$ E. None of these

We have to start by breaking up the integral.

$\int \left(\frac{3x^4}{x} - \frac{3}{x} \right) dx$

$\int \left(3x^3 - \frac{3}{x} \right) dx$

$\int 3x^3 dx - \int \frac{3}{x} dx$

$3\int x^3 dx - 3\int \frac{1}{x} dx$

$3\left(\frac{1}{3+1} \right) x^{3+1} - 33\ln|x| + C$

$\frac{3}{4}x^4 - 3\ln|x| + C$

13. **[5 pts]** Evaluate the indefinite integral $\int 3^x dx$

A. $\frac{3^{x+1}}{x+1} + C$ B. $\frac{3^x}{\ln 3} + C$ C. $\frac{3^x}{3} + C$
D. $3^x(\ln 3) + C$ E. None of these

$$\frac{1}{\ln 3}3^x + C$$

$$\frac{3^x}{\ln 3} + C$$

14. **[5 pts]** Evaluate the indefinite integral $\int (2x^5 - 7x^3 + 4)dx$

A. $6x^6 - \frac{7}{4}x^4 + 4x + C$ B. $6x^6 - \frac{7}{3}x^4 + 4x + C$ C. $\frac{1}{2}x^6 - \frac{7}{3}x^4 + 4x + C$

D. $\frac{1}{3}x^6 - \frac{7}{4}x^4 + 4x + C$ E. None of the above

$$\int (2x^5 - 7x^3 + 4)dx = 2\int x^5 dx - 7\int x^3 dx + 4\int x^0 dx$$

$$2 \cdot \frac{1}{5+1}x^{5+1} - 7 \cdot \frac{1}{3+1}x^{3+1} + 4 \cdot \frac{1}{0+1}x^{0+1} + C$$

$$2 \cdot \frac{1}{6}x^6 - 7 \cdot \frac{1}{4}x^4 + 4 \cdot \frac{1}{1}x^1 + C$$

$$\frac{1}{3}x^6 - \frac{7}{4}x^4 + 4x + C$$

15. **[5 pts]** The function $g(x)$ is the inverse of the function $f(x) = x^2 - x$ where $x > 0$. Find $g'(6)$.

A. $g'(6) = 11$ B. $g'(6) = 5$ C. $g'(6) = \frac{1}{5}$

D. $g'(6) = \frac{1}{11}$ E. None of the above

$g'(y_0) = \frac{1}{f'(x_0)}$ where (x_0, y_0) is a point on $f(x)$

We need to find x that goes with the y-coordinate 6.

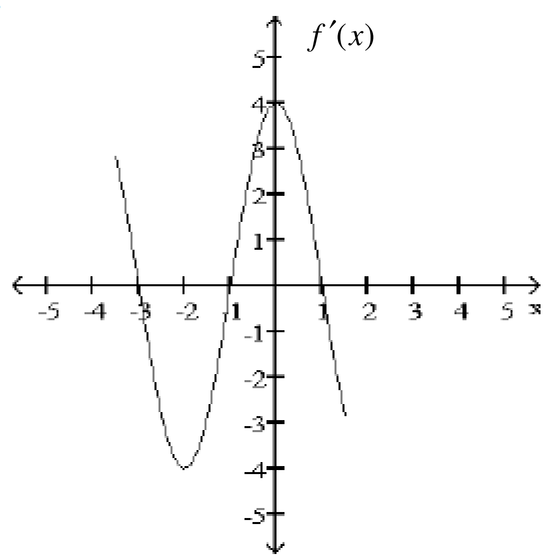
$$6 = x^2 - x$$
$$0 = x^2 - x - 6$$
$$0 = (x-3)(x+2)$$
$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 3 \quad \text{or} \quad x = -2$$

The one that we use is $x = 3$ since according to the problem x is greater than 0.

$$f'(x) = 2x - 1$$
$$g'(6) = \frac{1}{2(3)-1} = \frac{1}{5}$$

16. **[5 pts]** Suppose that the graph to the right is the graph of the derivative of the function $f(x)$ (i.e. this is the graph of $f'(x)$). Find all open interval(s) where the function $f(x)$ is concave downward.

A. $(-\infty, -1)$
B. $(-2, 0)$
C. $(-\infty, -2) \cup (0, \infty)$
D. $f(x)$ is never concave downward
E. None of these



In general, concavity changes (an inflection point exists) where the second derivative is equal to 0 (or does not exist). The second derivative is equal to 0 where the first derivative has a maximum or a minimum. Thus the inflection points for $f(x)$ are at $x = -2$ and at $x = 0$. The next thing that we need to do is check the sign of the second derivative to the left of $x = -2$, between $x = -2$ and $x = 0$, and to the right of $x = 0$. When the second derivative is negative, this indicates that the slope of the tangent lines of the first derivative are negative. Thus the original function is concave down (occurs when the second derivative is negative) when the slope of the tangent lines of the first derivative are negative. Thus the function is concave downward on the interval $(-\infty, -2) \cup (0, \infty)$