

Verify the following identities

Here each of the identities is verified. Where possible, how to verify the identity working with either side is shown.

1.
$$\frac{\sin x - \cos x}{\sin x} = 1 - \frac{1}{\tan x}$$

Working with the left side only	Working with the right side only
$\frac{\sin x - \cos x}{\sin x}$ $1 - \cot x$ $1 - \frac{1}{\tan x}$	$1 - \cot x$ $1 - \frac{\cos x}{\sin x}$ $\frac{\sin x - \cos x}{\sin x}$ $\frac{\sin x - \cos x}{\sin x}$

2.
$$\frac{1}{\sec x + \tan x} = \sec x - \tan x$$

Working with the left side only	Working with the right side only
$\frac{1}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$ $\frac{1}{\left(\frac{1 + \sin x}{\cos x}\right)}$ $1 \div \frac{1 + \sin x}{\cos x}$ $1 \times \frac{\cos x}{1 + \sin x}$ $\frac{\cos x}{1 + \sin x}$ $\frac{\cos x}{(1 + \sin x)} \times \frac{1 - \sin x}{1 - \sin x}$ $\frac{\cos x(1 - \sin x)}{1 - \sin^2 x}$ $\frac{\cos^2 x}{\cos x(1 - \sin x)}$ $\frac{1 - \sin x}{\cos x}$ $\frac{1}{\cos x} - \frac{\sin x}{\cos x}$ $\sec x - \tan x$	$\frac{1}{\cos x} - \frac{\sin x}{\cos x}$ $\frac{1 - \sin x}{\cos x}$ $\frac{1 - \sin x}{\cos x} \cdot \frac{1 + \sin x}{1 + \sin x}$ $\frac{1 - \sin^2 x}{\cos x(1 + \sin x)}$ $\frac{\cos^2 x}{\cos x(1 + \sin x)}$ $\frac{\cos^2 x}{\cos x(1 + \sin x)} \cdot \left(\frac{1}{\cos^2 x}\right)$ $\frac{1}{\left(\frac{\cos x(1 + \sin x)}{\cos^2 x}\right)}$ $\frac{1}{\left(\frac{1 + \sin x}{\cos x}\right)}$ $\frac{1}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$ $\frac{1}{\sec x + \tan x}$

$$3. \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

Working with the left side only	Working with the right side only
$\frac{\sin x}{\left(\frac{1}{\sin x}\right)} + \frac{\cos x}{\left(\frac{1}{\cos x}\right)}$ $\left(\frac{\sin x}{1} \div \frac{1}{\sin x}\right) + \left(\frac{\cos x}{1} \div \frac{1}{\cos x}\right)$ $\left(\frac{\sin x}{1} \times \frac{\sin x}{1}\right) + \left(\frac{\cos x}{1} \times \frac{\cos x}{1}\right)$ $\sin^2 x + \cos^2 x$ 1	

$$4. \frac{\csc x}{\tan x + \cot x} = \cos x$$

Working with the left side only	Working with the right side only
$\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)}$ $\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{\sin x \cdot \sin x}{\cos x \cdot \sin x} + \frac{\cos x \cdot \cos x}{\sin x \cdot \cos x}\right)}$ $\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}\right)}$ $\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)}$ $\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{1}{\cos x \sin x}\right)}$ $\frac{1}{\sin x} \div \frac{1}{\cos x \sin x}$ $\frac{1}{\sin x} \times \frac{\cos x \sin x}{1}$ $\frac{\cos x \sin x}{\sin x}$ $\cos x$	

$$5. \frac{\sec x}{\tan x + \cot x} = \sin x$$

Working with the left side only	Working with the right side only
$\frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)}$ $\frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}\right)}$ $\frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}\right)}$ $\frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)}$ $\frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{1}{\sin x \cos x}\right)}$ $\frac{1}{\cos x} \div \frac{1}{\sin x \cos x}$ $\frac{1}{\cos x} \times \frac{\sin x \cos x}{1}$ $\sin x$	

$$6. \frac{\csc x}{\cot x} = \sec x$$

Working with the left side only	Working with the right side only
$\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{\cos x}{\sin x}\right)}$ $\frac{1}{\sin x} \div \frac{\cos x}{\sin x}$ $\frac{1}{\sin x} \times \frac{\sin x}{\cos x}$ $\frac{1}{\cos x}$ $\sec x$	

$$7. \sin^4 x = \frac{1 - \cos^2 x}{\csc^2 x}$$

Working with the left side only	Working with the right side only
$\sin^2 x \sin^2 x$ $(1 - \cos^2 x) \frac{1}{\csc^2 x}$ $\frac{1 - \cos^2 x}{\csc^2 x}$	$\frac{\sin^2 x}{\left(\frac{1}{\sin^2 x}\right)}$ $\sin^2 x \div \frac{1}{\sin^2 x}$ $\sin^2 x \times \frac{\sin^2 x}{1}$ $\sin^4 x$

$$8. \sec x(1 - \sin^2 x) = \cos x$$

Working with the left side only	Working with the right side only
$\frac{1}{\cos x} (\cos^2 x)$ $\frac{\cos^2 x}{\cos x}$ $\cos x$	

$$9. \frac{\cos x}{\cot x} = \sin x$$

Working with the left side only	Working with the right side only
$\frac{\cos x}{\left(\frac{\cos x}{\sin x}\right)}$ $\frac{\cos x}{1} \div \frac{\cos x}{\sin x}$ $\frac{\cos x}{1} \cdot \frac{\sin x}{\cos x}$ $\frac{\sin x}{1}$ $\sin x$	

$$10. \frac{\tan x - \sin x}{\sin^3 x} = \frac{\sec x}{1 + \cos x}$$

Working with the left side only	Working with the right side only
$\frac{\tan x}{\sin^3 x} - \frac{\sin x}{\sin^3 x}$ $\frac{\left(\frac{\sin x}{\cos x}\right)}{\sin^3 x} - \frac{1}{\sin^2 x}$ $\frac{\sin x}{\sin x \cos x} - \frac{1}{\sin^2 x}$ $\frac{\cos x \sin^3 x}{1} - \frac{\sin^2 x}{1}$ $\frac{\cos x \sin^2 x}{1} - \frac{\sin^2 x}{\cos x}$ $\frac{\cos x \sin^2 x}{1 - \cos x} - \frac{\cos x \sin^2 x}{1 - \cos x}$ $\frac{\cos x(1 - \cos^2 x)}{1 - \cos x}$ $\frac{\cos x(1 - \cos x)(1 + \cos x)}{1}$ $\frac{1}{\cos x(1 + \cos x)}$ $\frac{1}{\cos x} \cdot \frac{1}{1 + \cos x}$ $\sec x \cdot \frac{1}{1 + \cos x}$ $\frac{\sec x}{1 + \cos x}$	$\frac{\sec x}{(1 + \cos x)} \times \frac{(1 - \cos x)}{(1 - \cos x)}$ $\frac{\sec x(1 - \cos x)}{1 - \cos^2 x}$ $\frac{1}{\cos x}(1 - \cos x)$ $\frac{\sin^2 x}{\left(\frac{1 - \cos x}{\cos x}\right)}$ $\frac{1 - \cos x}{\cos x \sin^2 x}$ $\frac{1 - \cos x}{\cos x \sin^2 x} \times \frac{\sin x}{\sin x}$ $\frac{\sin x(1 - \cos x)}{\cos x \sin^3 x}$ $\frac{\sin x - \sin x \cos x}{\cos x \sin^3 x}$ $\frac{\sin x}{\cos x \sin^3 x} - \frac{\sin x \cos x}{\cos x \sin^3 x}$ $\frac{\sin x}{\cos x} \cdot \frac{1}{\sin^3 x} - \frac{\sin x}{\sin^3 x}$ $\tan x \cdot \frac{1}{\sin^3 x} - \frac{\sin x}{\sin^3 x}$ $\frac{\tan x}{\sin^3 x} - \frac{\sin x}{\sin^3 x}$ $\frac{\tan x - \sin x}{\sin^3 x}$

$$11. \tan x + \cot x = \frac{1}{\sin x \cdot \cos x}$$

Working with the left side only	Working with the right side only
$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ $\frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$ $\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}$ $\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$ $\frac{1}{\sin x \cos x}$	$\frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x}$ $\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}$ $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ $\tan x + \cot x$

$$12. \frac{\tan x}{\sin x} = \sec x$$

Working with the left side only	Working with the right side only
$\frac{\left(\frac{\sin x}{\cos x}\right)}{\sin x}$ $\frac{\sin x}{\cos x} \div \sin x$ $\frac{\sin x}{\cos x} \times \frac{1}{\sin x}$ $\frac{1}{\cos x}$ $\sec x$	

$$13. 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

Working with the left side only	Working with the right side only
	$\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$ $\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \right) \div \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right)$ $\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \right) \times \frac{\cos^2 x}{\cos^2 x}$ $\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \div \frac{1}{\cos^2 x}$ $\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \frac{\cos^2 x}{1}$ $\cos^2 x - \sin^2 x$ $(1 - \sin^2 x) - \sin^2 x$ $1 - 2\sin^2 x$

$$14. \frac{\sin x}{\cot x} = \sec x - \cos x$$

Working with the left side only	Working with the right side only
$\frac{\sin x}{\left(\frac{\cos x}{\sin x}\right)}$ $\sin x \div \frac{\cos x}{\sin x}$ $\sin x \times \frac{\sin x}{\cos x}$ $\frac{\sin^2 x}{\cos x}$ $\frac{1 - \cos^2 x}{\cos x}$ $\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$ $\sec x - \cos x$	$\frac{1}{\cos x} - \cos x$ $\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$ $\frac{1 - \cos^2 x}{\cos x}$ $\frac{\sin^2 x}{\cos x}$ $\frac{\sin x}{\cos x} \sin x$ $\tan x \sin x$ $\frac{1}{\cot x} \sin x$ $\frac{\sin x}{\cot x}$

$$15. 2\sin^2 x - 1 = \sin^4 x - \cos^4 x$$

Working with the left side only	Working with the right side only
	$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$ $(\sin^2 x - \cos^2 x) \cdot 1$ $\sin^2 x - \cos^2 x$ $\sin^2 x - (1 - \sin^2 x)$ $\sin^2 x - 1 + \sin^2 x$ $2\sin^2 x - 1$

$$16. \frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = 1 + \tan x + \cot x$$

Working with the left side only	Working with the right side only
$\frac{\tan x}{\left(1 - \frac{\cos x}{\sin x}\right)} + \frac{\cot x}{\left(1 - \frac{\sin x}{\cos x}\right)}$ $\frac{\tan x}{\left(\frac{\sin x - \cos x}{\sin x}\right)} + \frac{\cot x}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$ $\frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{\sin x - \cos x}{\sin x}\right)} + \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$ $\left(\frac{\sin x}{\cos x} \div \frac{\sin x - \cos x}{\sin x}\right) + \left(\frac{\cos x}{\sin x} \div \frac{\cos x - \sin x}{\cos x}\right)$ $\left(\frac{\sin x}{\cos x} \times \frac{\sin x}{\sin x - \cos x}\right) + \left(\frac{\cos x}{\sin x} \times \frac{\cos x}{\cos x - \sin x}\right)$ $\frac{\sin^2 x}{\cos x(\sin x - \cos x)} + \frac{\cos^2 x}{-1\sin x(\sin x - \cos x)}$ $\frac{\sin^2 x}{\cos x(\sin x - \cos x)} - \frac{\cos^2 x}{\sin x(\sin x - \cos x)}$ $\left(\frac{\sin^2 x}{\cos x(\sin x - \cos x)} \cdot \frac{\sin x}{\sin x}\right) - \left(\frac{\cos^2 x}{\sin x(\sin x - \cos x)} \cdot \frac{\cos x}{\cos x}\right)$ $\frac{\sin^3 x}{\cos x \sin x(\sin x - \cos x)} - \frac{\cos^3 x}{\sin x \cos x(\sin x - \cos x)}$ $\frac{\sin^3 x - \cos^3 x}{\cos x \sin x(\sin x - \cos x)}$ $\frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\cos x \sin x(\sin x - \cos x)}$ $\frac{(\sin^2 x + \sin x \cos x + \cos^2 x)}{\cos x \sin x}$ $\frac{\sin^2 x}{\cos x \sin x} + \frac{\sin x \cos x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}$ $\frac{\sin x}{\cos x} + 1 + \frac{\cos x}{\sin x}$ $\tan x + 1 + \cot x$	