How to Take Account of the Value of Not Selecting an Alternative

This note demonstrates that portfolios where *not* selecting a project has a non-zero value can be analyzed by subtracting the value of not selecting each project from the value of selecting it.

Let $a_i, i = 1,2\ldots, N$ be the set of N possible projects to include in a portfolio, where v_i is the value gained by including a_i in the portfolio, and v_i^0 is the value gained by not including a_i in the portfolio. Let I_i be the binary variable that indicates whether or not a_i is included in the portfolio, where $I_i = 1$ means the project is included, and $I_i = 0$ means it is not included.

This portfolio decision can be analyzed by considering not including a_i in the portfolio to be an project just as including a_i is a project. Because a_i cannot both be included and not included in the portfolio at the same time, and you must include one of these two possibilities in any feasible portfolio, therefore it must be true that the binary variables for the possibilities of including and not including a_i always sum to one. Hence, assuming that values add, the total value for any possible portfolio is

$$V = \sum_{i=1}^{N} [v_i I_i + v_i^0 (1 - I_i)] = \sum_{i=1}^{N} (v_i - v_i^0) I_i + \sum_{i=1}^{N} v_i^0$$

However, the term

$$\sum_{i=1}^{N} v_i^0$$

is equal to a constant, and therefore it can be dropped from the value function without changing the ranking of different possible portfolios. Thus, using

$$V = \sum_{i=1}^{N} (v_i - v_i^0) I_i$$

as the value function for the portfolio will give the correct ranking of different portfolios.

This shows that subtracting the value v_i^0 of not selecting a project from the value v_i of selecting the project in the value function for a portfolio will give the correct ranking of portfolios.

If there is a cost associated with not selecting a project, then that cost needs to be taken into account. Suppose that the cost of selecting a_i is c_i and the cost of not selecting a_i is c_i^0, and the total available budget is C_T. Then the cost constraint is

$$\sum_{i=1}^{N} [c_i I_i + (1 - I_i) c_i^0] \le C_T$$

Rearranging terms, this can be written as

$$\sum_{i=1}^{N} (c_i - c_i^0) I_i \le C_T - \sum_{i=1}^{N} c_i^0$$

Therefore, a correct analysis procedure is to change the constraint by subtracting the cost c_i^0 of not selecting an alternative from the cost c_i of selecting the alternative in each term, and also subtracting the total cost $\sum_{i=1}^{N} c_i^0$ of not selecting all of the alternatives from the total available budget C_T.