Implications of sediment-flux-dependent river incision models for landscape evolution

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Received 7 November 2000; revised 17 September 2001; accepted 27 September 2001; published 22 February 2002

[1] Developing a quantitative understanding of the factors that control the rate of river incision into bedrock is critical to studies of landscape evolution and the linkages between climate, erosion, and tectonics. Current models of long-term river network incision differ significantly in their treatment of the role of sediment flux. We analyze the implications of various sediment-flux-dependent incision models for large-scale topography, in an attempt (1) to identify quantifiable and diagnostic differences between models that could be detected from topographic data or from the transient responses of perturbed systems and (2) to explain the apparent ubiquity of mixed bedrock-alluvial channels in active orogens. Although certain forms of the various models can be discarded as inconsistent with morphological data, we find that the relative intrinsic concavity indices of detachment- and transport-limited systems (defined herein) largely dictate whether the various models can be tied to distinctive steady state morphologies. Preliminary data suggest that no such diagnostic differences may exist, and other methods must be developed to test models. Accordingly, we develop and explore differences in the scaling behavior of topographic relief and the extent of detachment- versus transport-limited channels as a function of rock uplift rate that may allow discrimination among various models. Further, we explore potentially diagnostic differences in the rates and patterns of transient channel response to changes in rock uplift rate. In addition to general differences between detachment- and transport-limited systems our analysis identifies an interesting hysteresis in landscape evolution: "hybrid" channels at the threshold between detachment- and transport-limited conditions are expected to act as detachment-limited systems in response to an increase in rock uplift rate (or base level fall) and as transport-limited systems in response to a decrease in rock uplift rate, especially during postorogenic topographic decline. The analyses presented set the stage for field studies designed to test quantitatively the various river incision models that have been proposed. *INDEX TERMS:* 1815 Hydrology: Erosion and sedimentation; 1824 Hydrology: Geomorphology (1625); 8107 Tectonophysics: Continental neotectonics; *KEYWORDS:* Tectonic geomorphology, Erosion, Sediment-flux, Bedrock channels, Relief

1. Motivation

[2] Spurred by the recognition that river incision into bedrock importantly influences the style and tempo of landscape evolution in mountainous regions, the morphology of the resulting landscapes, and orogenic evolution in general, much research over the past decade has been devoted to understanding this process [e.g., *Seidl and Dietrich*, 1992; *Howard et al.*, 1994; *Wohl et al.*, 1994; *Tinkler and Wohl*, 1998; *Whipple et al.*, 2000a]. Bedrock channels set much of the relief structure of unglaciated mountainous landscapes and convey signals of tectonic and climatic change across landscapes, thus setting, to first order, landscape response time [*Whipple and Tucker*, 1999; *Whipple*, 2001]. Bedrock channel incision rate fundamentally determines landscape denudation rate because it is the lower boundary condition for all hillslopes. The controls on bedrock channel incision rate, further, largely dictate the relationships among substrate lithology, climatic conditions, tectonic setting, and topographic relief [*Howard*, 1994; *Howard et al.*, 1994; *Tucker and Slingerland*, 1996; *Tucker and Bras*, 1998; *Whipple et al.*, 1999; *Whipple and Tucker*, 1999; *Tucker and Whipple*, 2002]. Although these connections have been recognized to some degree since at least the time of *Gilbert* [1877], the recent focus on detachment-limited river incision has been a significant departure from previous emphasis on transport-limited processes, often in the context of gravel bedded rivers [e.g., *Mackin*, 1948]. However, over the last few years, researchers have begun questioning the detachment-limited paradigm.

[3] There is a dual demand placed on "bedrock" rivers in a steady state landscape, where "steady state" refers to a condition of temporally invariant topography (in a statistical sense), achieved through a long-term balance between rock uplift and erosion. First, rivers must erode rock from the channel bed at a rate equal to the rock uplift rate (measured relative to a fixed base level). Second, rivers must transport all the sediment supplied to them from upstream. Thus, depending on circumstances, it is plausible that although actively incising through bedrock, the gradient of a given river could be set primarily by the need to transport the sediment load: a transport-limited bedrock channel [*Howard*, 1980; *Howard and Kerby*, 1983; *Tucker and Slingerland*, 1994, 1996; *Sklar and Dietrich*, 1998]. Moreover, researchers are increasingly recognizing the potentially important, some argue dominant, role of sedi-

0148-0227/02/2000JB000044$09.00

ment flux in determining river incision rates into bedrock [*Beaumont et al.*, 1992; *Sklar and Dietrich*, 1997; *Slingerland et al.*, 1997; *Howard*, 1998; *Sklar and Dietrich*, 1998; *Slingerland et al.*, 1998]. The implied coupling of bedrock incision with sediment supply and transport speaks to an interesting, complex interplay between hillslopes and channels [e.g., *Howard*, 1998; *Sklar and Dietrich*, 1998; *Hovius et al.*, 2000] and draws into question the simple distinction between bedrock and ''alluvial'' systems that is often made. Similarly, a number of researchers have noted that ''mixed bedrock-alluvial'' channels, characterized by frequent outcrops of bedrock but largely blanketed by a thin layer of alluvium or a lag of locally derived boulders, are surprisingly common [*Miller*, 1991; *Seidl and Dietrich*, 1992; *Wohl*, 1992, 1993; *Howard et al.*, 1994; *Howard*, 1998; *Snyder et al.*, 2000]. Clean, bedrock-floored channels are relatively rare. The question of how (or whether) to draw the distinction between detachment- and transport-limited systems is particularly important in the case of these mixed bedrock-alluvial channels and plays into all the aforementioned linkages among climate, erosion, and tectonics.

2. Channel Classification and Definitions

[4] Channels can be classified either on the basis of observable bed morphology or on the basis of their dynamics. *Howard* [1980, 1987, 1998] and *Howard et al.* [1994] have defined five channel types primarily on the basis of bed morphology: (1) live bed sand alluvial; (2) live bed gravel alluvial; (3) threshold gravel alluvial; (4) mixed bedrock-alluvial; and (5) bedrock. Alluvial channels are characterized by a coherent blanket of transportable sediment on both the bed and banks. This blanket of sediment may be a thin cover over bedrock, so long as it is coherent in both space and time. Among alluvial channel types, live bed conditions imply that the channel gradient is set primarily by sediment flux, whereas threshold conditions imply that the channel gradient is set primarily by the critical shear stress for the initiation of motion (a dynamic criterion, but one in principle recognizable on morphological grounds). Bedrock channels are characterized by frequent exposures of bedrock in the bed and banks and a lack of a coherent blanket of sediment, even at low flow. Mixed bedrock-alluvial channels either have alternating bedrock and alluvial segments or are bedrock channels with a thin and patchy alluvial cover (at low flow).

[5] A similar classification scheme can readily be defined on the basis of what factors dictate the rate of channel incision. The rate of incision in transport-limited systems is by definition set by the downstream divergence of sediment transport capacity. In other words, if and only if the local transport capacity of the stream is greater than the sediment load supplied from upstream, the stream will incise. The stream may incise into bedrock and still be transport-limited, so long as it is the divergence of the transport capacity that limits the rate of incision [*Howard*, 1980; *Howard and Kerby*, 1983]. Conversely, the rate of incision in detachment-limited systems is by definition determined by the stream's ability to erode the bed, usually by a combination of abrasion and plucking [e.g., *Hancock et al.*, 1998; *Whipple et al.*, 2000a].

[6] The different controls on channel incision rate in transport- and detachment-limited systems mean that channel gradient under steady state conditions (e.g., erosion balancing baselevel fall) is also set by different factors, with important implications for the dynamics of channel profile evolution. The gradient of a transport-limited channel is by definition adjusted to just transport the sediment load (including both sediment delivered from upstream and derived from local bed erosion) [e.g., *Willgoose et al.*, 1991; *Tucker and Bras*, 1998]. Thus long-term sediment carrying capacity and sediment supply are by definition in balance in a transport-limited river at steady state. Note that as pointed out by *Howard* [1980] and *Howard and Kerby* [1983] on theoretical and empirical grounds, respectively, transport-limited channels may actively incise into bedrock at a limited rate. Bedrock erosion in such a case presumably occurs when scour penetrates to the base of a thin alluvial blanket during floods [e.g., *Howard*, 1998].

[7] For a detachment-limited channel the steady state gradient is set by a combination of the river's ability to erode the bed and the prevailing rate of rock uplift or base level fall [e.g., *Howard*, 1980; *Whipple and Tucker*, 1999]. An important corollary is that the long-term sediment carrying capacity of a detachment-limited river greatly exceeds the sediment supply. An intermediate or ''hybrid'' channel type can be defined as one where both the transport capacity and the efficiency of bedrock detachment importantly influence channel gradient.

[8] In principle, under constant environmental conditions, there should be a simple correspondence between the two classification schemes outlined above. For example, live bed alluvial channels are also transport-limited channels. However, channel morphology at any one time is not necessarily a reliable indicator of long-term behavior because bed morphology can change quickly in response to climate change, human activity, or episodic input of sediment from tributaries and hillslopes [e.g., *Benda and Dunne*, 1997; *Hovius et al.*, 2000]. Because we are interested in long-term behavior, our analysis uses the behavioral channel classification (detachment-limited, transport-limited, and hybrid) rather than a morphologic classification [e.g., *Howard et al.*, 1994]. We only discuss channel bed morphology when describing field evidence for transport-limited conditions in a suite of field sites and emphasize the uncertainty involved in such interpretations.

3. Approach and Scope

[9] This paper addresses, in a general way, issues raised independently by *Sklar and Dietrich* [1997, 1998], *Slingerland et al.* [1997, 1998], and *Howard* [1998] regarding the potentially important role of sediment flux in the bedrock channel incision problem. The approach is theoretical, though guided by field observations. The focus is on long-term interactions between rock uplift and channel incision at the drainage basin scale (a few to hundreds of square kilometers). Only strictly fluvial erosion processes are considered. Our treatment of sediment flux and transport-limited channel gradients emphasizes conditions in live bed gravel rivers. The dynamics of systems involving transitions between bedrock and live bed sand channel types may be different. Analysis of the role of storms and short-term climatic fluctuations and the associated variability in sediment flux is intentionally omitted, although we recognize them as potentially important. Although most of the analysis focuses on steady state landscape form, we also briefly consider transient responses of hybrid channels to step changes in uplift rate.

[10] The intention here is to explore the controls on the occurrence, form, and large-scale dynamics of hybrid channels in order to develop testable hypotheses to guide further directed field, laboratory, and topographic (digital elevation model) investigation. We do not test the relative merits of the various river incision models considered but rather attempt to establish, in a systematic way, how the models differ and therefore how they may be tested.

[11] First, using standard formulations of detachment- and transport-limited river incision, we explore the controls on the transition between the two conditions at steady state, following earlier work on this problem by *Howard* [1980, 1987] and *Howard and Kerby* [1983]. Next, we explore the steady state topographic consequences of various formulations of the coupling between sediment flux and bedrock incision rate. The formulations used here are based on the models due to (1) *Beaumont et al.* [1992] and (2) *Sklar and Dietrich* [1998], though we generalize both models and employ only a simplified abstraction of the Sklar and Dietrich model. Critical parameters and testable topographic predictions of each model are emphasized and compared to the familiar detachment-limited stream power model [*Howard et al.*, 1994; *Whipple and Tucker*, 1999; *Tucker and Whipple*, 2002]. We intentionally

omit from the analysis some potentially important internal feedbacks, such as possible interdependencies among erosion rate, channel width, grain-size distribution of sediment delivered to channels, and rates of downstream fining due to selective transport and abrasion. Some field evidence relevant to one critical parameter identified in our analysis (the intrinsic concavity index of transport-limited channels) is briefly examined. Finally, a two-dimensional (2-D) landscape evolution model is used to explore the transient dynamics of hybrid channels that exhibit transitions between detachment- and transport-limited behavior in response to changing rock uplift rates.

4. Theoretical Background

[12] Detachment-limited bedrock incision is often modeled as a power function of unit stream power (or shear stress). The familiar stream power river incision model can be written as [*Howard and Kerby*, 1983; *Howard et al.*, 1994; *Whipple and Tucker*, 1999]:

$$E = Kf(q_s)A^m S^n, \tag{1}$$

where E denotes vertical incision rate, Adenotes upstream drainage area (a proxy for discharge), Sdenotes local channel gradient, K is a dimensional erosion efficiency factor, and m and $n$are positive constants that depend on basin hydrology, hydraulic geometry, and erosion process. In principle, (1) should include an erosion threshold. Although a threshold term will be important for some problems [e.g., *Howard*, 1994; *Rinaldo et al.*, 1995; *Howard*, 1997; *Tucker and Slingerland*, 1997; *Tucker and Bras*, 2000; *Snyder*, 2001], we omit it here in the interest of obtaining analytical solutions. Following *Whipple and Tucker* [1999], the $f(q_s)$ term represents a variety of plausible models for the dependence of river incision rate on sediment flux. Several generic models for the $f(q_s)$ term will be explored below. The function $f(q_s)$ is assumed to equal unity in the standard detachment-limited stream power model, which is by definition applicable only where sediment transport capacity (Q_c) exceeds sediment flux (Q_s). Where sediment flux equals or exceeds transport capacity ($Q_s/Q_c \geq 1$) the system is by definition transport-limited (and further is depositional if $Q_s/Q_c > 1$). Note that the standard stream power model (with $f(q_s) = 1$) does not necessarily assume that sediment flux is unimportant; rather it assumes that the influence of sediment flux is sufficiently captured by writing incision rate as a power function of unit stream power (or shear stress). In effect, relative sediment supply is subsumed into the model parameters (K, m, and n).

[13] Combined with a statement of conservation of mass of rock, a channel profile evolution equation based on (1) can be written as

$$dz/dt = U - Kf(q_s)A^m S^n, \tag{2}$$

where dz/dt is the time rate of change of channel bed elevation and U is rock uplift rate defined relative to erosional base level. Equation (2) is a nonlinear kinematic wave equation, with important implications for the style and duration of transient responses to external forcing as discussed by *Humphrey and Heller* [1995], *Weissel and Seidl* [1998], *Whipple and Tucker* [1999], *Royden et al.* [2000], and *Tucker and Whipple* [2002]. At steady state, $dz/dt = 0$ by definition, and a relationship for steady state detachment-limited channel gradient (S_d) is easily derived [*Howard*, 1980]:

$$S_d = (U/K)^{1/n} A^{-\theta_d} \tag{3a}$$

$$\theta_d = m/n, \tag{3b}$$

where we, for convenience, define θ_d as the intrinsic concavity index of detachment-limited systems. Equations (3a) and (3b) show that under conditions of spatially uniform erosion coefficient, rock uplift rate, and erosion process, the concavity of stream profiles is set by drainage basin shape (which determines A as a function of downstream distance) and the intrinsic concavity index ($\theta_d = m/n$). Intrinsic concavity index is thus distinguished from the actual concavity index of the river profile which may be influenced by nonuniform uplift [*Kirby and Whipple*, 2001], nonuniform substrate lithology [*Moglen and Bras*, 1995], or downstream variations in sediment flux [*Sklar and Dietrich*, 1998; *Slingerland et al.*, 1998], as discussed below. We have argued elsewhere on theoretical and empirical grounds that the m/n ratio for detachment-limited fluvial systems is typically ~0.5 [*Whipple and Tucker*, 1999; *Snyder et al.*, 2000; *Kirby and Whipple*, 2001]. The slope exponent (n) has been argued to depend on the dominant erosion process and to vary between ~2/3 and ~5/3 [*Whipple et al.*, 2000a].

[14] Transport-limited incision can be modeled in a similar fashion. In this case, volumetric sediment transport capacity, rather than erosion rate, is written as a power function of unit stream power (or shear stress) [*Willgoose et al.*, 1991; *Tucker and Bras*, 1998]:

$$Q_c = K_t A^{m_t} S^{n_t}, \tag{4}$$

where K_t is a dimensional transport coefficient and m_t and n_t are positive constants. Although an initiation of motion threshold is not explicit in (4), it can be shown that models that include both a threshold of motion and a Parker-type channel width closure rule [*Parker*, 1978] do indeed collapse to the form of (4) (see related derivation by *Paola et al.* [1992]). A one-dimensional transport-limited river profile evolution equation analogous to (2) can be written by noting that incision in these systems is dictated by the downstream divergence of sediment flux:

$$\frac{dz}{dt} = U - \frac{1}{1-\lambda_p}\frac{d}{dx}\frac{1}{W}\left(K_t A^{m_t} S^{n_t}\right), \tag{5}$$

where λ_p is the porosity of sediment and W is the channel width. Note that the transport-limited evolution equation is a nonlinear diffusion equation; the transient responses of detachment- and transport-limited systems to external forcings are quite distinct as will be illustrated briefly below.

[15] A relationship for steady state transport-limited channel gradient can be derived by recognizing that at steady state the channel transport capacity at every point in the basin must equal the sediment flux delivered from upstream and from local erosion of the channel bed [*Willgoose et al.*, 1991; *Tucker and Bras*, 1998]. We assume that it is the supply and transport capacity of the bed load fraction that determines stream gradients in actively incising systems (live bed gravel rivers). Therefore we introduce a new variable (β) that explicitly denotes the fraction of total load delivered to channels as bed load (assumed to be spatially and temporally invariant in the present treatment). At steady state, erosion everywhere balances rock uplift rate such that under conditions of spatially uniform uplift the total sediment flux at a given point along a river must equal the product of upstream drainage area (A) and the rock uplift rate. Thus the steady state bed load sediment flux at all points in the river network is given by

$$Q_s = \beta A U. \tag{6}$$

By setting sediment transport capacity (equation (4)) equal to sediment flux (equation (6)) the steady state transport-limited channel gradient (S_t) is readily found [*Willgoose et al.*, 1991]:

$$S_t = \left(\frac{\beta U}{K_t}\right)^{1/n_t} A^{-\theta_t} \tag{7a}$$

$$\theta_t = (m_t - 1)/n_t, \tag{7b}$$

where analogous to the detachment-limited case, we define θ_t as the intrinsic concavity index of transport-limited systems. As can be seen by a comparison of (3a) and (3b) and (7a) and (7b), detachment-limited and transport-limited systems can have identical steady state forms; only the transient responses of these systems exhibit diagnostic differences [see also *Tucker and Whipple*, 2002].

[16] An extensive literature on sediment transport exists, and parameters of the transport-limited model are more tightly constrained than is the case for the detachment-limited model. However, important uncertainties remain regarding transport of mixtures of grain sizes and the effects of downstream fining (and its causes) [e.g., *Parker et al.*, 1982; *Parker*, 1991; *Ferguson et al.*, 1996; *Sinha and Parker*, 1996; *Seal et al.*, 1997; *Gasparini et al.*, 1999]. *Howard* [1980, 1987] and *Willgoose et al.* [1991] present a derivation using the Einstein-Brown total load equation that yields the estimates $n_t \sim 2$, $m_t \sim 2$, applicable to live bed sand rivers. *Howard and Kerby* [1983] discuss a field case where this is appropriate in the context of the bedrock-alluvial transitions discussed here. However, given our argument that transport of the gravel bed load fraction will predominantly dictate channel gradient in mountain rivers, it seems most appropriate to use estimates of n_t and m_t based on gravel bed load transport relations (noting that therefore sand- and gravel-dominated systems may exhibit important differences). In almost all bed load transport relations, sediment flux scales either with basal shear stress to the 3/2 power or linearly with unit stream power (beyond a critical entrainment threshold in both cases) [e.g., *Bagnold*, 1980; *Gomez and Church*, 1989]. Either model yields $n_t = 1$ as the most appropriate a priori estimate (when shear stress is written in terms of slope and area, the slope exponent is 2/3 the shear stress exponent; see derivation of *Whipple and Tucker* [1999]). Accordingly, we assume $n_t = 1$ throughout. As downstream fining importantly influences the concavity of transport-limited, gravel-bedded river profiles [e.g., *Snow and Slingerland*, 1990; *Sinha and Parker*, 1996; *Gasparini*, 1998], we allow m_t to vary as a crude proxy for this effect, analogous to the parameterization of downstream fining proposed by *Howard* [1980].

5. Transitions From Detachment-Limited to Transport-Limited Conditions

[17] In this section we address the questions of what determines whether a given channel segment is detachment- or transport-limited and under what conditions a downstream transition from one to the other occurs. This analysis is done within the context of the standard stream power river incision model ($f(q_s) = 1$) described above. Although nontrivial models for the $f(q_s)$ term predict a gradual transition between detachment- and transport-limited conditions, the standard model provides a useful end-member case for comparison and facilitates discussion of the more complex models. We define first a nondimensional detachment-transport transition number to distinguish detachment- from transport-limited channel segments. Next, we derive an expression for the critical drainage area (A_{cr}) at which a downstream transition would occur. In addition, we consider the implications of the common perception that detachment-limited channels are favored in rapidly uplifting landscapes.

[18] As noted earlier, detachment-limited channels occur by definition only where sediment transport capacity exceeds sediment supply ($Q_c > Q_s$) and thus only where the stream gradient required to erode the bed at a rate equal to rock uplift exceeds the stream gradient required to transport the sediment load ($S_d > S_t$) [*Howard*, 1980]. Following *Howard* [1980], a dimensionless detachment-transport transition number is defined as the ratio of these two gradients:

$$N_{dt} = \frac{S_d}{S_t} = K^{-1/n}(K_t/\beta)^{1/n_t} U^{(1/n - 1/n_t)} A^{(\theta_t - \theta_d)}, \tag{8}$$

where θ_t and θ_d denote the intrinsic concavity indices of transport- and detachment-limited systems, respectively, as defined above, and m_t, n_t, m, and n are all positive constants. If and only if $N_{dt} > 1$ is the channel detachment limited; otherwise, it is transport limited. Note that for $n_t = 1$ (all cases considered here), N_{dt} is exactly equal to the ratio of sediment transport capacity to sediment flux (where steady state conditions are assumed and Q_c is calculated for the steady state detachment-limited channel gradient (S_d)). We will exploit this fact later in the analysis of hybrid channel systems.

[19] Several important conclusions can be drawn from (8). First, an easily erodible rock substrate (high K) favors transport-limited conditions regardless of the rock uplift rate. Second, only if $n < n_t$ are detachment-limited conditions favored by higher rock uplift rates, as is commonly assumed. This important implication has apparently not been previously recognized and certainly has never been exploited in field tests of the stream power incision model. Third, if the intrinsic concavity index of transport-limited systems ($(m_t - 1)/n_t$) is less than that of detachment-limited systems (m/n), as is commonly argued, then transport-limited conditions are favored at large drainage areas and a downstream transition from detachment- to transport-limited conditions is expected (Figure 1) [*Howard*, 1980, 1987; *Howard et al.*, 1994; *Tucker and Slingerland*, 1996; *Tucker and Slingerland*, 1997; *Tucker et al.*, 2001]. As we are considering only steady state erosional systems, this transition is neither a shift from erosion to deposition nor necessarily a shift from easily distinguished bedrock to alluvial channel types, although *Howard and Kerby* [1983] did successfully recognize this transition based on morphologic criteria in their field area (rapidly eroding artificial badlands). Regardless of how clear the channel bed morphology transition is, plots of channel gradient versus drainage area on a log-log scale will show a kink at the critical drainage area (Figure 1).

[20] Although stochastic variation in sediment supply and flood discharge may be expected to cause lateral shifts in the transition point (Figure 1), potentially blurring the morphologic expression of the transition [see, e.g., *Tucker et al.*, 2001, Figure 8], a long-term average of the critical drainage area (A_{cr}) at which this transition occurs can be found by setting N_{dt} equal to unity and solving for area:

$$A_{cr} = \left(K^{1/n}[K_t/\beta]^{-1/n_t}\right)^{1/(\theta_t - \theta_d)} U^{(1/n_t - 1/n)/(\theta_t - \theta_d)}. \tag{9}$$

As also seen in (8), only if $n < n_t$ will larger portions of a channel network be detachment-limited (i.e., greater A_{cr}) at higher rock uplift rates (assuming $\theta_d > \theta_t$ and therefore that downstream transitions from detachment- to transport-limited channel reaches are expected). If our argument that bed load gravel transport capacity is the important quantity (implying $n_t = 1$) is correct, then this widely held assumption about the occurrence of bedrock channels in actively uplifted landscapes requires that n in the detachment-limited stream power incision model (equation (1)) be restricted to values less than unity.

[21] If θ_t and θ_d are equal, (9) is undefined because there is no downstream transition point; the system is either detachment- or transport-limited for the entire length of the stream, whichever

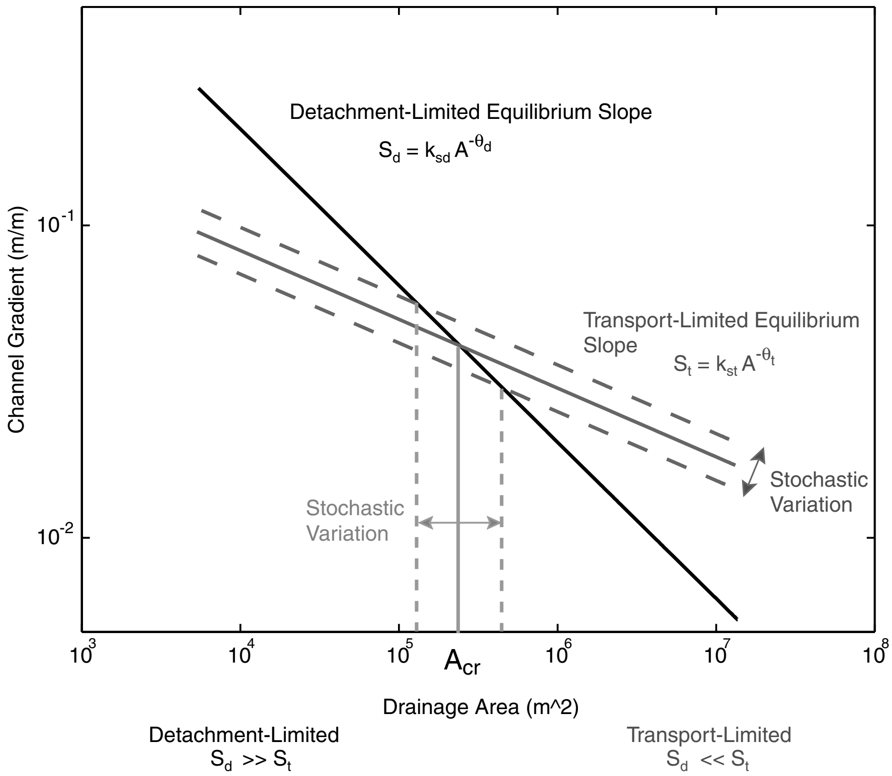


Figure 1. Sketch illustrating the sudden downstream transition from detachment- to transport-limited conditions expected for the standard stream power model ($f(q_s) = 1$ where detachment-limited) when $\theta_t < \theta_d$. Note that the concavity index (θ) determines the slope of the linear relationship between log channel gradient and log drainage area. Steady state channel gradient is the greater of either the detachment- or the transport-limited relation. A_{cr} marks a transition in the rate-limiting process only; channels remain erosional. Dashed lines illustrate schematically how A_{cr} may fluctuate in response to stochastic floods and sediment delivery or climate change.

requires the greater slope under the circumstances. From (8) the entire network will be detachment-limited if

$$K^{-1/n}(K_t/\beta)^{1/n_t}U^{(1/n-1/n_t)} > 1, \tag{10}$$

and transport-limited otherwise. A logical implication is that a change in any of these variables (reflecting climate, hydrology, tectonics, and lithology) could drive a transition from detachment- to transport-limited behavior or vice versa. Transport-limited behavior will generally be favored by more erodible rocks, inefficient transport, and larger gravel fractions, and conversely for detachment-limited behavior. Once again, if and only if n is less than n_t is detachment-limited behavior favored by higher uplift rates.

6. Sediment-Flux Dependent River Incision and Steady State Landscape Form

[22] The standard detachment- and transport-limited stream power river incision models predict an abrupt downstream transition from one regime to the other if their intrinsic concavity indices are not equal [see also *Howard*, 1980]. This transition would be easily recognizable in river profile data as a kink in the scaling of channel gradient with drainage area (Figure 1), although, as noted above, stochastic variability in sediment supply and flood discharge may blur the morphologic transition [*Tucker et al.*, 2001]. However, sediment in transport both provides the tools for abrasion and fracture of rock and, if overly abundant, can protect the bed from erosion [*Sklar and Dietrich*, 1997; *Slingerland et al.*, 1997; *Howard*, 1998; *Sklar and Dietrich*, 1998; *Slingerland et al.*, 1998]; sediment flux seemingly must play an important role in river incision. Nontrivial forms of the sediment-flux-dependent term in equation (1) ($f(q_s) \neq 1$) predict a more gradual transition between end-member detachment- and transport-limited conditions, with important consequences for steady state landscape form. These predictions constitute testable hypotheses. In principle, steady state landscape form (in areas with uniform lithology, climate, and rock uplift rate) can be studied to discriminate between various models of the dependence of river incision rate on sediment flux. However, we will demonstrate below that this only holds if the intrinsic concavity indices of detachment- and transport-limited river systems are sufficiently different.

[23] In this section we develop and explore the implications of two generic models for the $f(q_s)$ term for steady state landscape form. *Slingerland et al.* [1998] presented preliminary work on this problem, taking a somewhat different approach than that used here. The models considered here are motivated by, and generalized from, hypotheses put forward by *Beaumont et al.* [1992] and *Sklar and Dietrich* [1998]. We will refer to the two generalized forms considered here as the "linear decline" and "parabolic" models. In the case of the parabolic model in particular, it is important to keep in mind that it represents a considerable simplification of the *Sklar and Dietrich* [1998] model. We have considered many additional simple, plausible forms, such as a linear rise to a maximum followed by a linear decline, but these are not presented here in the interest of space as the linear and parabolic models capture much of the variability in behavior.

[24] The linear decline model holds that the primary influence of sediment flux is to inhibit erosion. The concept is that incision potential decreases as a larger fraction of stream energy is expended on transport, rather than erosion, processes. Thus detachment potential decreases linearly from a maximum where there is

negligible sediment flux to zero where sediment flux equals transport capacity:

$$f(q_s) = 1 - Q_s/Q_c. \quad (11)$$

The undercapacity model [*Beaumont et al.*, 1992; *Kooi and Beaumont*, 1994] is a special case of the linear decline model as defined above. Substituting (11) and (4) into (1), setting m = 1, n = 1, m_t = 1, n_t = 1, and $K = K_t/L_f$ (where L_f is the bedrock erosion length scale defined by *Beaumont et al.* [1992]), and rearranging yields the undercapacity model exactly:

$$E = \frac{1}{L_f}(Q_c - Q_s). \quad (12)$$

The assumed linearity in area and slope terms in the undercapacity model [*Beaumont et al.*, 1992; *Kooi and Beaumont*, 1994] limits the range of predicted steady state landforms and their dependence on rock uplift rate, climate, and lithologic resistance.

[25] Unlike the linear decline model, the parabolic model holds that sediment flux has a dual role in the erosion process. When sediment flux is low relative to capacity, erosion potential increases with sediment flux because the sediment provides the tools for bedrock abrasion [*Sklar and Dietrich*, 1998] and perhaps also for creating the fractures required for plucking [*Whipple et al.*, 2000a]. However, as in the linear decline model, eventually sediment flux inhibits erosion as sediment swamps the system and begins to protect the bed from impacts by saltating particles. Following *Sklar and Dietrich* [1998], we adopt a parabolic form that reaches a maximum at Q_s/Q_c = 1/2:

$$f(q_s) = 1 - 4(Q_s/Q_c - 1/2)^2. \quad (13)$$

Note that in both cases, $f(q_s)$ varies between 0 and 1, in keeping with the generic form of the stream power incision model (equation (1)) and the postulation by *Whipple and Tucker* [1999] that the sediment flux dependence can be treated in a general way as an efficiency factor in the stream power incision model (Figure 2). One might argue that incision rate should also depend on total sediment flux (as well as the flux-capacity ratio): $f(q_s) = (Q_s/W)\,(1 - Q_s/Q_c)$, for example. The behavior of such models, however, is very similar to that of the generic parabolic $f(q_s)$ model above (13). Therefore we restrict discussion of such models to brief comments on the few important differences between them and the models considered in-depth. As will be developed below, all nontrivial forms of $f(q_s)$ predict steady state stream profiles with concavity indices that both differ significantly from the intrinsic detachment-limited concavity index given by the m/n ratio and vary downstream, broadly consistent with the preliminary simulation results presented by *Sklar and Dietrich* [1998].

[26] Analytical solutions for steady state (dz/dt = 0 in equation (2)) channel gradient as a function of drainage area can be derived for certain special cases by substituting (11) or (13) into (2) and solving for S. In keeping with our assumption that gravel bed load transport dictates the required transport gradient in mountain streams, we restrict our analysis to cases with n_t = 1. With this assumption, analytical solutions can be found for n = 1 and n = 2 for both the linear decline and parabolic models. For clarity, we denote steady state channel gradients as S_{l1}, S_{l2}, S_{p1}, and S_{p2}, depending on the $f(q_s)$ model (l, linear; p, parabolic) and the assumed value of the exponent n.

[27] Steady state channel gradient for the linear decline model where $n_t = n = 1$ (S_{l1}) is given by

$$S_{l1} = \frac{U}{K}A^{-m} + \frac{\beta U}{K_t}A^{1-m_t}. \quad (14)$$

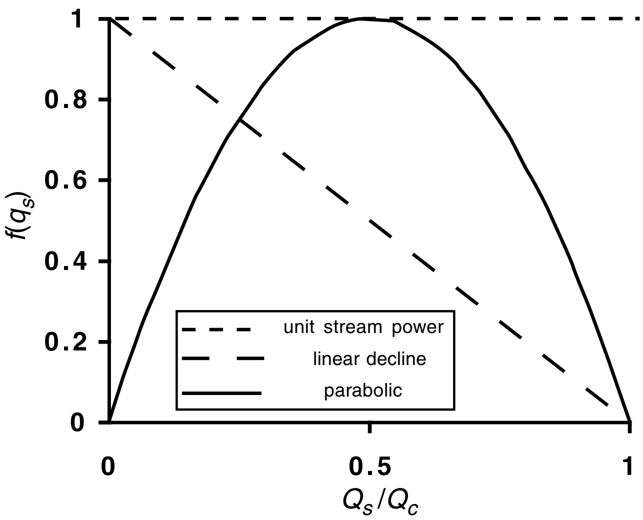


Figure 2. Illustration of postulated models of the $f(q_s)$ term in equation (1).

Interestingly, in this case, steady state channel gradient is a simple linear sum of the detachment-limited and the transport-limited gradients ($S_{l1} = S_d + S_t$). This results in a smooth, gradual downstream transition from effectively detachment-limited to effectively transport-limited conditions if the intrinsic concavity index of the transport-limited system is less than that of the detachment-limited system (Figure 3). In all cases, S_{l1} is linear in uplift rate. Note that in this and all other models with nontrivial $f(q_s)$ terms, end-member detachment- and transport- limited conditions are only asymptotically approached. Again, (14) is exactly equal to the steady state form of the undercapacity river incision model [*Beaumont et al.*, 1992] if and only if $m = m_t = 1$, $n = n_t = 1$, and $K = K_t/L_f$. Unlike the generic detachment- and transport-limited models, the Beaumont undercapacity model does not imply a power law slope-area relationship; rather, by (14), the concavity index decreases systematically downstream from $\theta \sim 1$ to $\theta \sim 0$. To our knowledge, no stream profiles with this form have been observed.

[28] The steady state channel gradient for the linear decline model for the case n_t = 1, n = 2 (S_{l2}) has a similar form:

$$S_{l2} = \frac{\beta U}{2K_t}A^{1-m_t} + \sqrt{\left(\frac{\beta U}{2K_t}A^{1-m_t}\right)^2 + \frac{U}{K}A^{-m}}. \quad (15)$$

Again steady state channel gradient is approximately equal to a sum of the detachment-limited and the transport-limited gradients, and a gradual transition from detachment- to transport-limited conditions is expected (Figure 3). S_{l2} is weakly nonlinear in rock uplift rate (Figure 4).

[29] The parabolic $f(q_s)$ model exhibits considerably more complex behavior. Steady state channel gradient for the parabolic model for the case $n_t = n = 1$ (S_{p1}) is given by

$$S_{p1} = \frac{\beta U}{K_t}A^{1-m_t}\left(1 - \frac{K_t}{4K\beta}A^{m_t-m-1}\right)^{-1}, \quad (16)$$

which is defined for $(K_t/4K\beta)A^{m_t-m-1} < 1$. As expected from the low efficiency of erosion where sediment flux is well under capacity (Figure 2), the parabolic model predicts streams with considerably higher concavity indices in their upper reaches and a strongly curving slope-area relation on log-log plots (Figure 3). Interestingly,

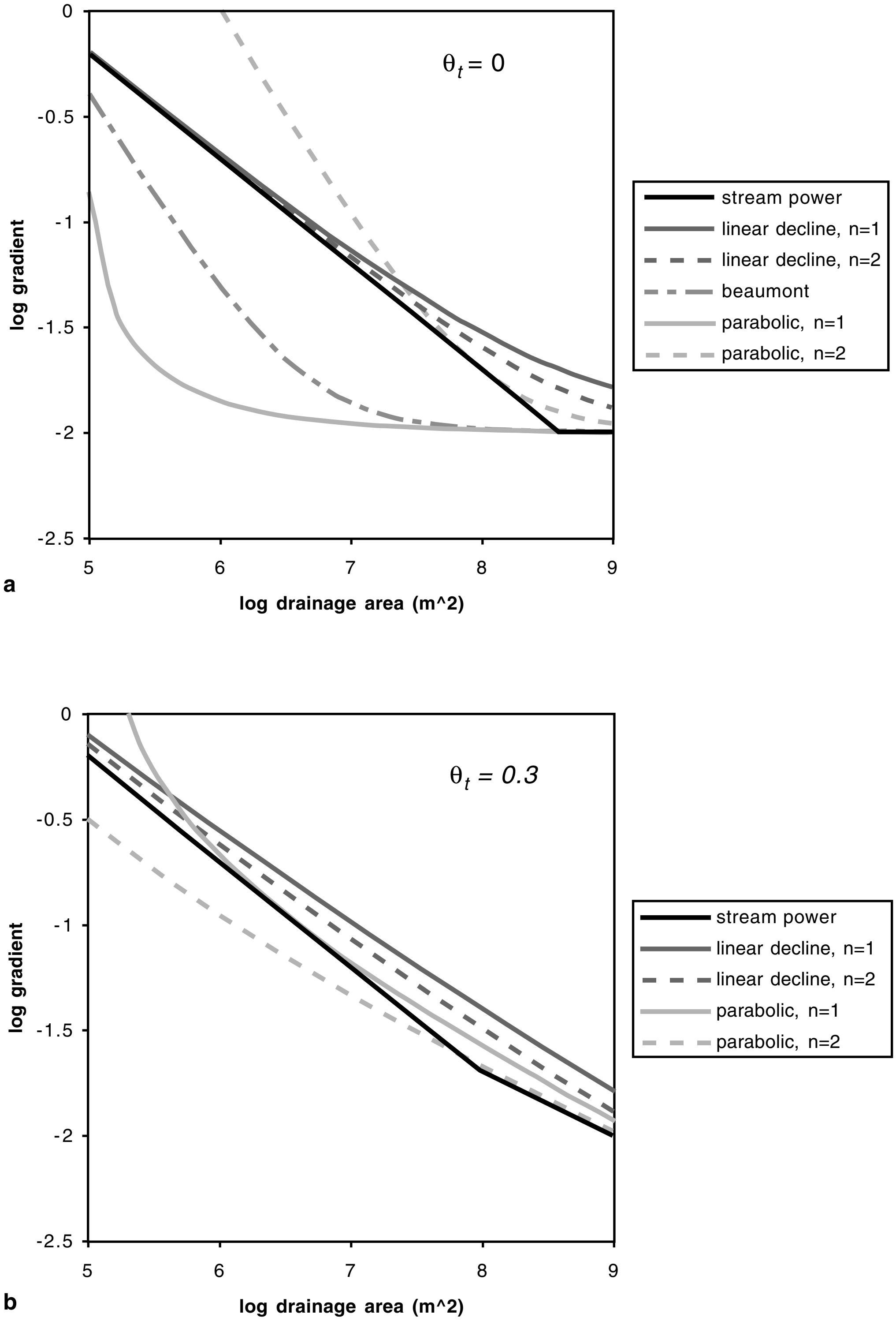


Figure 3. Steady state slope-area relations for various $f(q_s)$ models, with $\theta_d = 0.5$ and $n_t = 1$ (bed load transport) assumed in all cases. The parameter θ_t crudely represents varying degrees of downstream fining. (a) Relations for $\theta_t = 0$ (no downstream fining). (b) Relations for $\theta_t = 0.3$ (significant downstream fining). For $\theta_t = \theta_d$ all curves collapse to linear parallel lines.

because of the high intrinsic concavity index of the Beaumont undercapacity model (m/n = 1) the parabolic model and the Beaumont undercapacity model predict similar steady state forms (Figure 3a). Similar to the linear decline model, as K becomes large or K_t small, the bracketed term in (16) approaches unity and channel gradient asymptotically approaches a transport-limited condition. S_{p1} is linear in rock uplift rate. However, the parabolic model is inherently unstable [*Sklar and Dietrich*, 1998] (albeit only so for n = 1 in our formulation). As K becomes small (e.g., harder rocks) or K_t becomes large, steady state channel gradient approaches

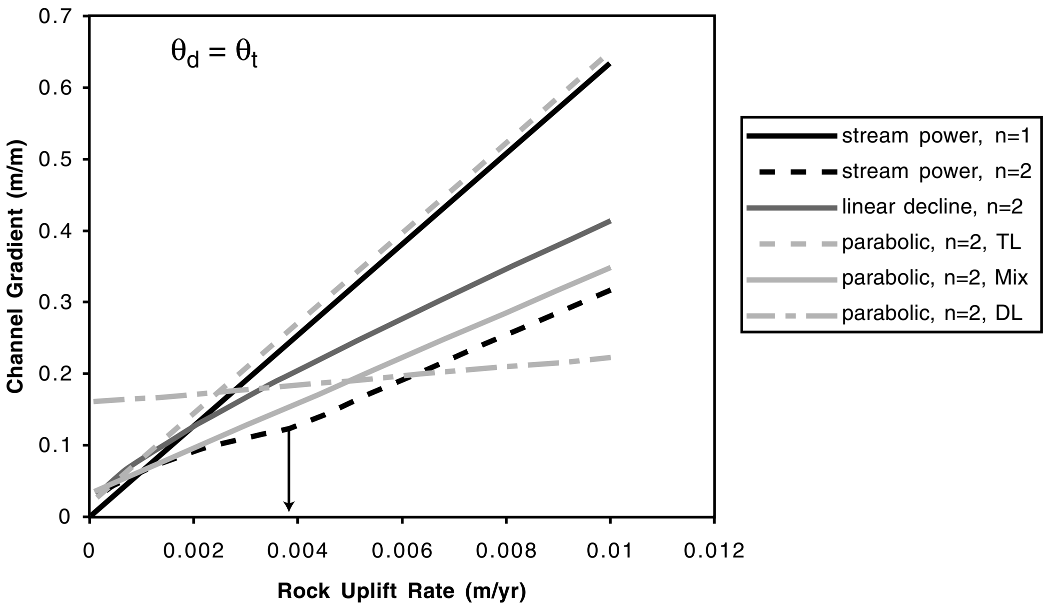


Figure 4. Steady state channel gradient versus rock uplift rate (U) for various $f(q_s)$ models with $\theta_d = \theta_t$. All models with $n = 1$ are linear in uplift and parallel the line for the standard stream power model. For the parabolic $f(q_s)$ with $n = 2$ case, TL indicates transport-limited, DL indicates detachment-limited, and Mix indicates hybrid conditions. A factor of 10 decrease in the K_t/β ratio (only) differentiates the illustrated DL from TL conditions (see Table 1). Note that TL conditions are favored by high rock uplift rates for this model (equations (8) and (17)).

infinity. The physical interpretation is that once Q_s/Q_c drops below 1/2 and $f(q_s)$ begins a slide down the rising limb of the parabola, the system becomes tool starved (Figure 2). A runaway feedback develops in which, because insufficient tools are available to incise at a rate equal to the uplift rate, the channel is forced to steepen, which leads to a further drop in the Q_s/Q_c ratio and further steepening. This implies that there is a very narrow range of conditions over which a channel will shift from essentially transport-limited to an unstable state in which the channel rapidly steepens until other processes, such as plunge pool erosion or debris flow scour, take over [*Howard*, 1998; *Sklar and Dietrich*, 1998; *Stock and Dietrich*, 1998]. Note that for the case $n = n_t$ the Q_s/Q_c ratio (8) is independent of rock uplift rate.

[30] Steady state channel gradient for the parabolic model for the case $n_t = 1$, $n = 2$ (S_{p2}) is given by

$$S_{p2} = \frac{K_t}{4K\beta} A^{m_t - m - 1} + \frac{\beta U}{K_t} A^{1 - m_t}. \tag{17}$$

As for all other models, as K becomes large (e.g., where rocks are highly erodible) or K_t becomes small, the first right-hand term shrinks and the solution converges on the transport-limited gradient ($S_{p2} \approx S_t$). In this limit, steady state channel gradient is linear in rock uplift rate, as for the models considered above. Interestingly, the inherent instability of the case with $n = 1$ (S_{p1}) is gone. A wider range of hybrid and detachment-limited conditions are therefore allowed (i.e., there are stable steady state forms associated with a wide range of rock uplift rates, substrate erodibilities, transport efficiencies, and gravel fractions). The physical explanation is that for $n = 2$, small increases in channel gradient greatly enhance the efficiency of the detachment process such that the runaway steepening seen in the $n = 1$ case does not occur. However, unlike all other models, as K_t becomes large, or K becomes small, and the system shifts toward detachment-limited behavior, the dependence on uplift rate becomes weaker (Figure 4) as the second term on the right-hand side (17) becomes less important. However, increasing the rock uplift rate does shift the system toward transport-limited conditions (see also (8) for the case where $n > n_t$). The finding that the gradient-uplift relation depends on the flux-capacity ratio (Q_s/Q_c) constitutes an important testable prediction of this form of the river incision model.

[31] Figure 3 highlights the differences in steady state longitudinal river profile form predicted by the various models for the coupling of incision rate and sediment flux considered. If the intrinsic concavity index of transport-limited erosional systems is less than that of detachment-limited systems, as is often argued [e.g., *Howard*, 1994; *Howard et al.*, 1994], then there are significant topographic implications of each model that could, in principle, be used to determine which $f(q_s)$ model best captures the behavior of natural systems. Further, the relative magnitude of the difference in intrinsic concavity indices significantly affects the predicted topographic forms. However, although not emphasized above and not shown in Figure 3, if the intrinsic concavity indices are equal ($m/n = (m_t - 1)/n_t$), inspection of (3), (7a), (7b), and (14)–(17) reveals that log-log plots of steady state channel gradient against drainage area (like Figure 3) will be utterly nondiagnostic. That is, there will be a simple power law scaling between channel gradient and drainage area, with the same scaling exponent for all $f(q_s)$ models. There are two exceptions. A first exception can be made for a model of the form $f(q_s) = Q_s/W(1 - Q_s/Q_c)$ (parabolic with a maximum of $Q_c/4W$). This model exhibits behavior that closely mimics that described for the parabolic, $n = 2$ case, except that steady state channels retain a high concavity index ($\theta \sim 1$) at small drainage areas and curving slope-area relationships even if $\theta_t = \theta_d$. A second important exception can be made for the expected influence of abrupt, downstream changes in either rock erodibility (K) or rock uplift rate (U). Only systems near the detachment-limited end-member will exhibit abrupt gradient changes that coincide precisely with the change in lithology or rock uplift rate. Increasingly gradual adjustments in channel gradient may be expected as systems approach transport-limited conditions because for these systems, local gradient is controlled primarily by the integrated sediment flux from upstream and less

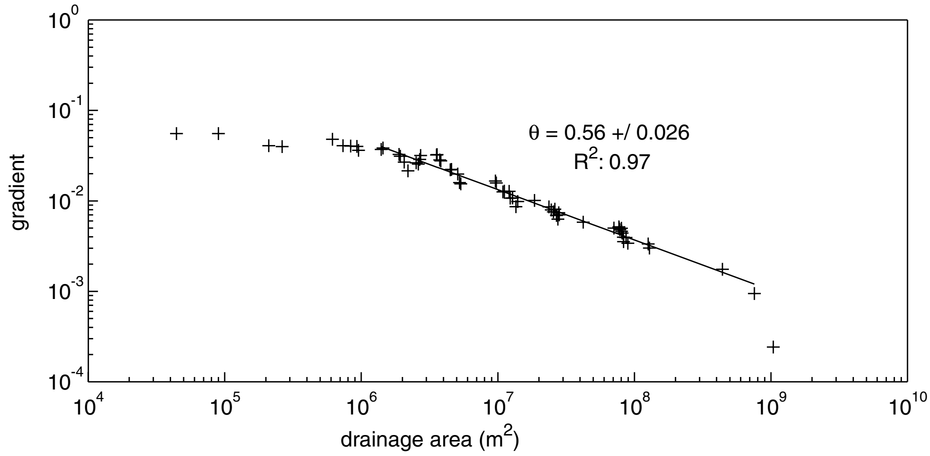


Figure 5. Slope-area data for the middle branch of the North River, Virginia (studied by *Hack* [1957]). Break in scaling at $\sim 10^6$ m^2 may reflect a transition to debris flow-dominated erosion [*Montgomery and Foufoula-Georgiou*, 1993; *Stock and Dietrich*, 1998].

so by local conditions. In all other cases, if $\theta_t = \theta_d$ only study of transient landscape response to abrupt changes in rock uplift rate (U), sediment supply (Q_s/Q_c, β), or climate (K, K_t) holds promise for testing among models of bedrock channel incision [e.g., see *Stock and Montgomery*, 1999; *Whipple et al.*, 2000b].

6.1. Field Evidence for Concavity Index of Transport-Limited Erosional Systems

[32] Although it is commonly argued that transport-limited systems should have lower intrinsic concavity indices than detachment-limited systems, there are several independent lines of evidence which suggest that this may not be true. First, *Gasparini* [1998] show that models of gravel-bedded, transport-limited rivers that incorporate transport, sorting, and downstream fining of gravel-sand mixtures [*Wilcock*, 1998] typically have concavity indices of around 0.4. In their models, only sand-dominated systems have low concavity indices ($\theta \approx 0.2$), consistent with *Howard*'s [1994, 1997] argument that sand-bedded, transport-limited systems should have low concavity indices. Second, *Tarboton et al.* [1991] report stream profile concavity indices for 21 basins, 19 of which range from 0.29 to 0.58 (outliers of 0.25 and 0.85). Both simple theoretical arguments and data suggest this range is typical for detachment-limited systems [e.g., *Slingerland et al.*, 1998; *Whipple et al.*, 1999; *Whipple and Tucker*, 1999; *Snyder et al.*, 2000]. However, no effort was made by Tarboton et al. to distinguish transport-limited from detachment-limited systems. It seems likely that their random sample of drainage basins would include some of both types of system. Indeed, Schoharie Creek, $\theta = 0.43$ [*Tarboton et al.*, 1991], with classic gravel bedded channel morphologies (cobble-boulder surface sizes), is currently incised $\sim$20 m into an alluvial fill, and bedrock is only rarely exposed in either the banks or channel floor. These field observations suggest transport-limited rather than detachment-limited conditions, although it is possible that the gradient of Schoharie Creek is controlled more by a boulder entrainment threshold than by sediment flux (for a discussion of such "threshold" channels see *Howard* [1980]). Third, the detailed field data on the distribution of bedrock and alluviated channel segments in the Olympic Mountains, Washington, presented by *Massong and Montgomery* [2000] clearly imply very similar slope-area scaling for bedrock (whether detachment- or transport-limited is arguable) and alluvial (presumably transport-limited) channel reaches: their data demonstrate that a power law slope-area relationship accurately discriminates slightly steeper bedrock channel reaches from gentler alluviated reaches over 3 orders of magnitude in drainage area. The scaling exponent in their discriminant function (0.42, 0.49, and 0.69 in the three basins studied) is within the range expected for the detachment-limited channel concavity index and implies that the concavity indices of transport-limited systems are similar. Finally, *Hack* [1957] showed that mixed bedrock-alluvial channels in the Appalachian Mountains of Virginia have channel gradients that are adjusted to the size of the gravel and boulder bed material. This finding implies transport-limited channel gradients on rivers with concavity indices similar to those expected for detachment-limited systems (Middle River (three branches):θ = 0.64, 0.59, 0.49; North River (four branches):θ = 0.43, 0.47, 0.56, 0.52) (Figure 5).

[33] Two additional river basins thought to be transport-limited erosional systems merit mention: the Waipaoa River basin on the North Island of New Zealand and the Enza River basin in the Appenines of northern Italy. Both the Waipaoa and the Enza Rivers are known to be rapidly incising through erodible bedrock [*Talling and Sowter*, 1998; *Berryman et al.*, 2000]. Both are gravel bedded with channel gradient adjusted to bed material size [*Gomez et al.*, 2001; *Talling*, 2000]. Both basins are characterized by bedrock strath terraces topped with significant thicknesses of gravel. It seems reasonable to infer transport-limited conditions for both. Slope-area analysis of the Waipaoa River yields a mean estimate of the channel profile concavity index (θ) of 0.55 (0.61, 0.53, 0.49, 0.55, and 0.57 for the five major branches of the Waipaoa). Similarly, the Enza River, downstream of a zone of complex tectonics in the headwaters, yields a best fit concavity index of 0.56.

[34] In summary, the concavity indices of erosional, gravel-bedded, arguably transport-limited rivers are not demonstrably different from the concavity indices of bedrock, and arguably detachment-limited, rivers. Although it is possible either that many or most bedrock channels are in fact transport-limited or that the rivers analyzed above are actually detachment-limited despite evidence to the contrary, the data do suggest that detachment- and transport-limited erosional systems may have similar intrinsic concavity indices. If correct, a direct implication of this finding is that there may be no readily discernable steady state topographic signature of the various models for the coupling of sediment flux and bedrock incision considered above. Given the importance of the relative intrinsic concavity indices of transport- and detachment-limited channels and the

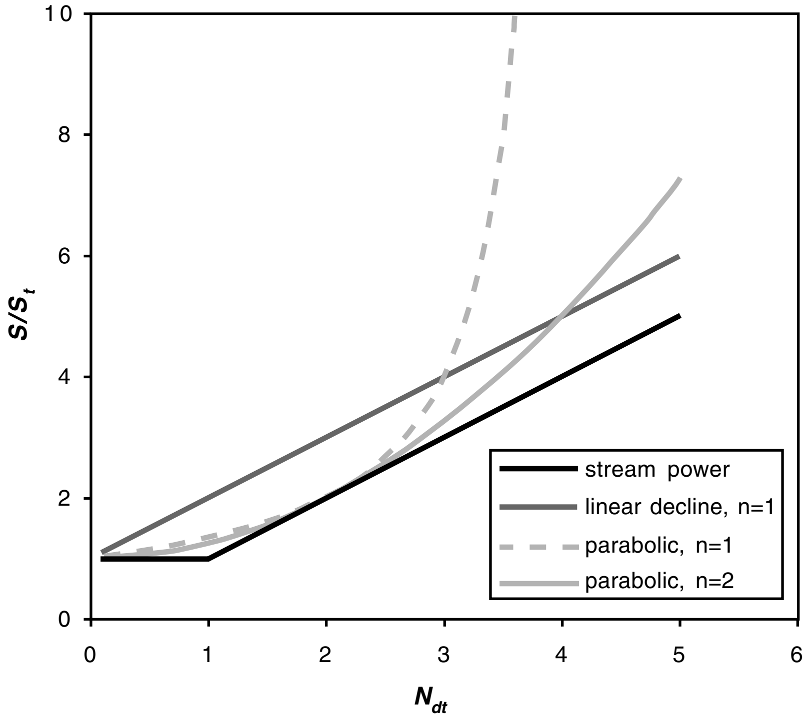


Figure 6. Steady state channel gradient normalized by equivalent transport-limited channel gradient (S/S_t) versus N_{dt} (equations (8) and (18)) for various $f(q_s)$ models. Gradient of models with a parabolic $f(q_s)$ term is equal to the detachment-limited gradient (S_d) for $N_{dt} = 2$ ($Q_s/Q_c = 1/2$), the point at which $f(q_s)$ attains its maximal value of unity. Note the instability of parabolic model with $n = 1$ (channel gradient becomes infinite at $N_{dt} = 4$).

uncertainty in the preliminary analysis above, directed fieldwork on this problem is needed.

6.2. Controls on Steady State Channel Gradient if $\theta_d = \theta_t$

[35] In the special case where $\theta_d = \theta_t$, the relations for steady state channel gradient in (14), (16), and (17) can be simplified to algebraic functions of either one of the equivalent (i.e., all the same parameter values) end-member detachment- or transport-limited cases (S_d or S_t, respectively). This is useful in that it helps clarify some of the behavior implied by the relations in (14)–(17) and because the steady state gradient of hybrid channel systems can be expressed directly in terms of the well-known detachment- or transport-limited models.

[36] For the standard stream power model ($f(q_s) = 1$), detachment-limited channel gradient (S_d) can be expressed as the product of the transport-limited channel gradient (S_t) and N_{dt}, the non-dimensional detachment-transport transition number (see equation (8)). Similarly, where $\theta_d = \theta_t$, N_{dt} (8) simplifies to

$$N_{dt} = K^{-1/n}(K_t/\beta)^{1/n_t}U^{(1/n-1/n_t)}, \tag{18}$$

and the steady state gradient of hybrid channels (equations (14), (16), and (17)) can be written in terms of S_t and N_{dt}:

$$S_{l1} = (1 + N_{dt})S_t; \tag{19}$$

$$S_{p1} = (1 - N_{dt}/4)^{-1}S_t, \qquad N_{dt} < 4; \tag{20}$$

$$S_{p2} = (1 + N_{dt}^2/4)S_t. \tag{21}$$

In all cases, steady state channel gradient can alternatively be expressed in terms of the detachment-limited gradient (S_d) through (8). For the case of the linear decline $f(q_s)$ model with $n_t = 1$ and $n = 2$, an analytical solution in this form has not been found. However, as shown earlier (equations (14) and (15) and Figure 2), its behavior is not markedly different from that given in (19) for the $n = 1$ case.

[37] Some of the diagnostic characteristics of the different $f(q_s)$ models described above are made very clear by examination of (19)–(21) (see Figures 4 and 6). First, the linear combination of S_d and S_t in the linear decline case is obvious from (19). More significantly, the inherent instability of the parabolic model in the $n = 1$ case, and its sensitivity to the relative magnitudes of K, K_t, and β, are clearly highlighted in (18) and (20). From both (20) and (21) it can be shown that for the parabolic model in general, channel gradient equals the detachment-limited gradient (S_d) at $N_{dt} = 2$, as expected. This follows because the parabolic $f(q_s)$ term attains its maximum value of unity at $Q_s/Q_c = 1/2$ (equation (13)) and $Q_s/Q_c = 1/N_{dt}$ if $n_t = 1$, as assumed here (see discussion of (8)). Thus, at $N_{dt} = 2$ the channel is fully in the detachment-limited regime. However, for the parabolic model with $n = 1$, only a very narrow range of parameter space is occupied by stable detachment-limited channels. By (20), channel gradient is predicted to become infinite as N_{dt} approaches 4 (Figure 6). Thus, with only a factor of 2 change in K, K_t, or ß (N_{dt} is independent of U if $n = n_t$; equations (8) and (18)) a channel in the detachment-limited regime could become unstable and steepen until other processes (landsliding, plunge pool erosion, debris flow scour [*Howard*, 1998; *Stock and Dietrich*, 1998]) take over. With $n = 2$ the parabolic model loses this instability but does steepen relative to the transport-limited gradient (S_t) rapidly as N_{dt} increases beyond 2 and the channel becomes increasingly detachment-limited and tool-starved (Figure 6). Both models rapidly converge on transport-limited conditions as N_{dt} becomes small. Note that for $n = 2$ and $n_t = 1$, N_{dt} scales with rock uplift to the $-1/2$ power, confirming that higher uplift rates drive the system toward transport-limited conditions (equations (8) and (18)).

Table 1. Model Parameters

	K	m	K_t	m_t	n_t	β
θ_t = 0 (Figure 3a)						
n = [a]	5.00E-06	0.5	1.00E-02	1	1	0.1
n = 2	2.50E-08	1	1.00E-02	1	1	0.1
Beaumont (n = 1)[b]	2.50E-08	1	1.00E-02	1	1	0.1
θ_t = 0.3 (Figure 3b)						
n = 1	5.00E-06	0.5	2.00E-05	1.3	1	0.1
n = 2[c]	2.50E-08	1	2.00E-05	1	1	0.1
θ_t = 0.5 (Figure 4)						
n = 1	5.00E-06	0.5	1.00E-06	1.5	1	0.1
n = 2	2.50E-08	1	1.00E-06	1.5	1	0.1
parabolic (n = 2, DL)	2.50E-08	1	5.00E-06	1.5	1	0.1
parabolic (n = 2, mixed)	2.50E-08	1	1.00E-06	1.5	1	0.1
parabolic (n = 2, TL)	2.50E-08	1	5.00E-07	1.5	1	0.1
θ_t = 0.5 (Figures 7–10)						
n = 0.66	2.92E-05	0.33	5.00E-06	1.5	1	1
n = 1	5.00E-06	0.5	5.00E-06	1.5	1	1
n = 1.5	3.53E-07	0.75	5.00E-06	1.5	1	1
n = 2	2.50E-08	1	5.00E-06	1.5	1	1

[a] K = 8.5E-05 (read 8.5E-05 as 8.5×10^{-5}) for the parabolic (n = 1) case to avoid the unstable regime ($S \rightarrow$ infinity).
[b] L_f in the *Beaumont et al.* [1992] model is set at 4.0E-05.
[c] K = 1.0E-7 for the parabolic (n = 2) case to offset the slope-area curve for clarity.

Thus some care must be taken when considering the curves plotted in Figure 6. For example, an increase in rock uplift rate results in a decrease in N_{dt}, and therefore a decrease in the S/S_t ratio (equation (21) and Figure 6), not because channel gradient decreases but rather because S_t increases more rapidly with U than does S_{p2} (equation (17) and Figure 4).

7. Transient Response of Threshold Channels

[38] Thus far we have considered only steady state conditions in which river incision rates everywhere balance rock uplift rate. Unfortunately, detachment-limited, transport-limited, and hybrid systems can have identical steady state morphologies if their intrinsic concavity indices are the same or similar, as suggested by the data presented above. However, even in this case the tempo, pattern, and duration of transient responses to external forcings will be different among the various models, with important implications for landscape evolution and the interpretation of landforms in terms of tectonics and climate [*Tucker and Whipple*, 2002]. As the parameter space defined by the range of models discussed above is extensive, we purposefully analyze the transient response of only the simplest case: the standard stream power model in which an abrupt, rather than gradual, transition from detachment- to transport-limited conditions is expected at the point (in space and/or time) that $N_{dt} = 1$ (and therefore $Q_s/Q_c = 1$). The abrupt transition in behavior expected for this simplest model has the advantage of producing a clearly recognizable signal that can be exploited to gain insight into the occurrence and consequences of any transient shifts from detachment- to transport-limited conditions or vice versa. The linear decline $f(q_s)$ models can be anticipated to behave in a broadly similar way but with a gradual transition from one end-member behavior to the other. Similarly, natural stochastic variation in sediment supply and storm discharge can be anticipated to blur the abrupt transitions expected for the simplest model [see *Tucker et al.*, 2001, Figure 8]. The parabolic $f(q_s)$ models can be anticipated to involve somewhat more complex transient responses. Finally, we analyze below only cases in which the intrinsic concavity indices of the two systems are the same ($\theta_d = \theta_t$). We impose this restriction in part because this is the scenario in which only analyses of transient responses can hope to provide useful tests of model success and in part because it removes some of the complication associated with spatial transitions between detachment- and transport-limited conditions.

[39] Unlike simple detachment-limited systems, transient responses of hybrid or transport-limited systems cannot be illustrated with one-dimensional channel profile evolution simulations because local incision rates depend on transient sediment fluxes from all upstream sub-catchments. Accordingly, we use the GOLEM model [*Tucker and Slingerland*, 1994, 1996] to perform two-dimensional simulation experiments. As we are interested here in the evolution of the fluvial channel system at the orogen scale, we do not model the transient response of hillslopes. Every model cell tracks the elevation of the largest stream channel within its bounds and all hillsides are assumed to lower instantaneously at the rate of channel incision.

[40] Parameters used in all simulations are given in Table 1. Values of m and m_t were chosen to give intrinsic concavity indices of 0.5 in all experiments ($\theta_d = \theta_t = 0.5$), and n_t was set to unity in all simulations. All simulation experiments presented below were run in the following manner. All runs began from the same steady state initial condition with $U = 0.001$ m yr^{-1}. At time $t = 0$ (before the first time step) rock uplift rate was changed to U_f and all other parameters held constant. Simulations were run until a new steady state was achieved (i.e., for a time equal to or exceeding the fluvial response time).

[41] Although the focus here is on the behavior of channels at the threshold between detachment- and transport-limited conditions, it is first necessary to establish the main distinguishing characteristics of the transient response of each of these end-member states. Accordingly, a pair of simulations of purely detachment-limited and transport-limited systems were run to briefly highlight the differences between them in terms of the tempo, style, and duration of a transient response. In all subsequent runs, parameters were chosen such that the initial steady state was everywhere precisely at the transition from detachment- to transport-limited conditions, such that the transient response of a threshold, or hybrid, channel system could be explored. This device had the effect of minimizing the run time required to observe the evolution of a landscape through a process transition. Runs completed include cases with $n < 1$, $n = 1$, and $n > 1$, for both increases and decreases in rock uplift rate.

[42] End-member detachment- and transport-limited runs exhibit strikingly different transient responses, even when both initial and final steady state forms are identical (Figure 7 and

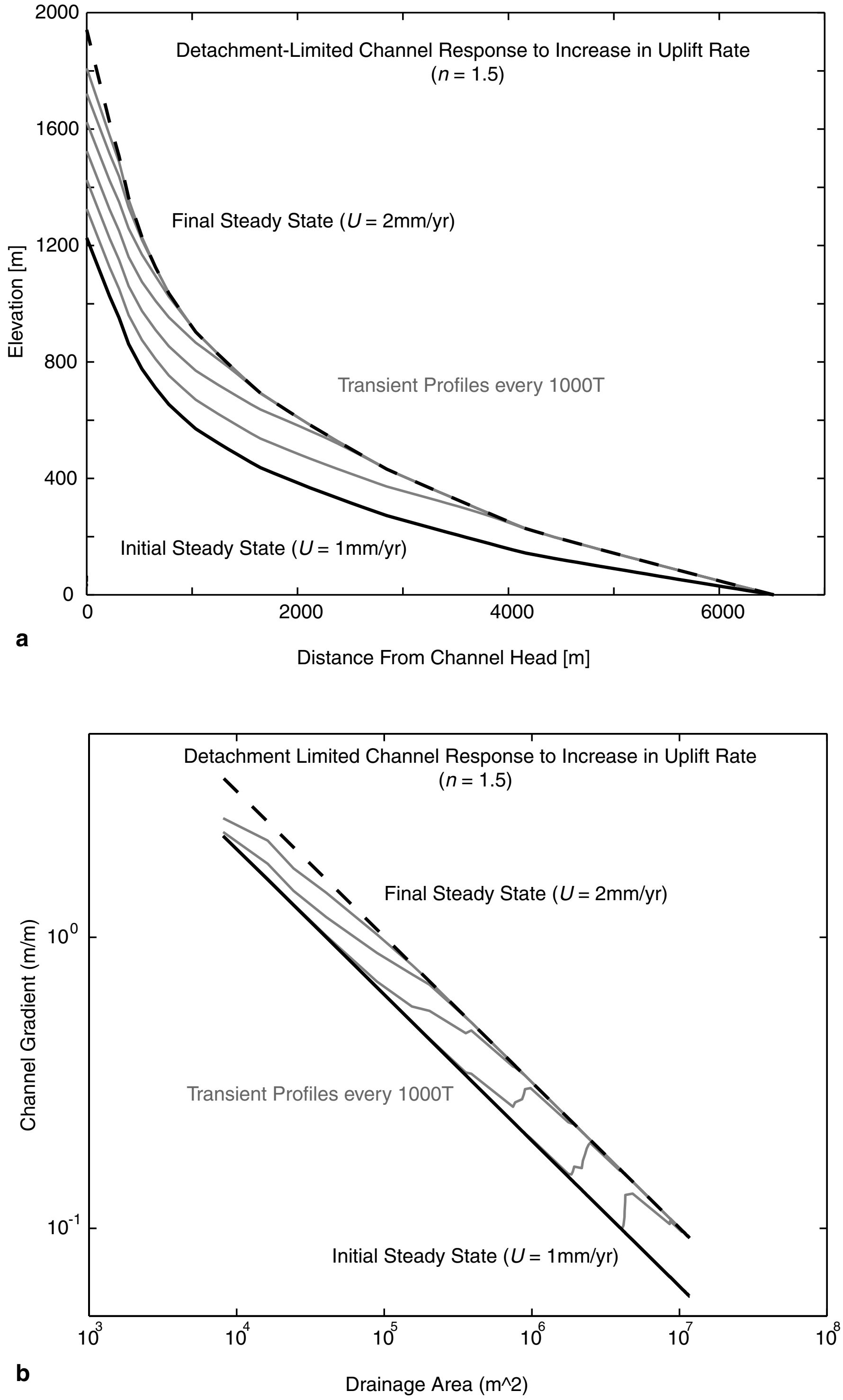


Figure 7. Transient response of a detachment-limited channel to an increase in rock uplift rate (U), shown for the case $n = 1.5$. (a) Longitudinal profiles. (b) Channel gradient versus drainage area (log-log scale). Note that the channel evolves by slope replacement following an upstream propagating kinematic wave, affected somewhat by numerical diffusion. Barring numerical diffusion, response time is $\sim 8000T$ (model time steps).

Figure 8) [*Tucker and Whipple*, 2002]. As described by *Howard et al.* [1994] and *Whipple and Tucker* [1999], the transient response of detachment-limited channels is characterized by upstream migration of an abrupt "knickpoint" (here used to denote a sharp break in channel gradient, rather than elevation as the term is also often used). The knickpoint marks the boundary between channel gradients that are in equilibrium with the final rock uplift rate (downstream) and channel gradients in equilibrium with the initial rock uplift rate (upstream). Thus the channel profile evolves through a mechanism of slope replacement rather than gradual adjustment (Figure 7). The converse is true of transport-limited channel response. In transport-limited systems, channel response

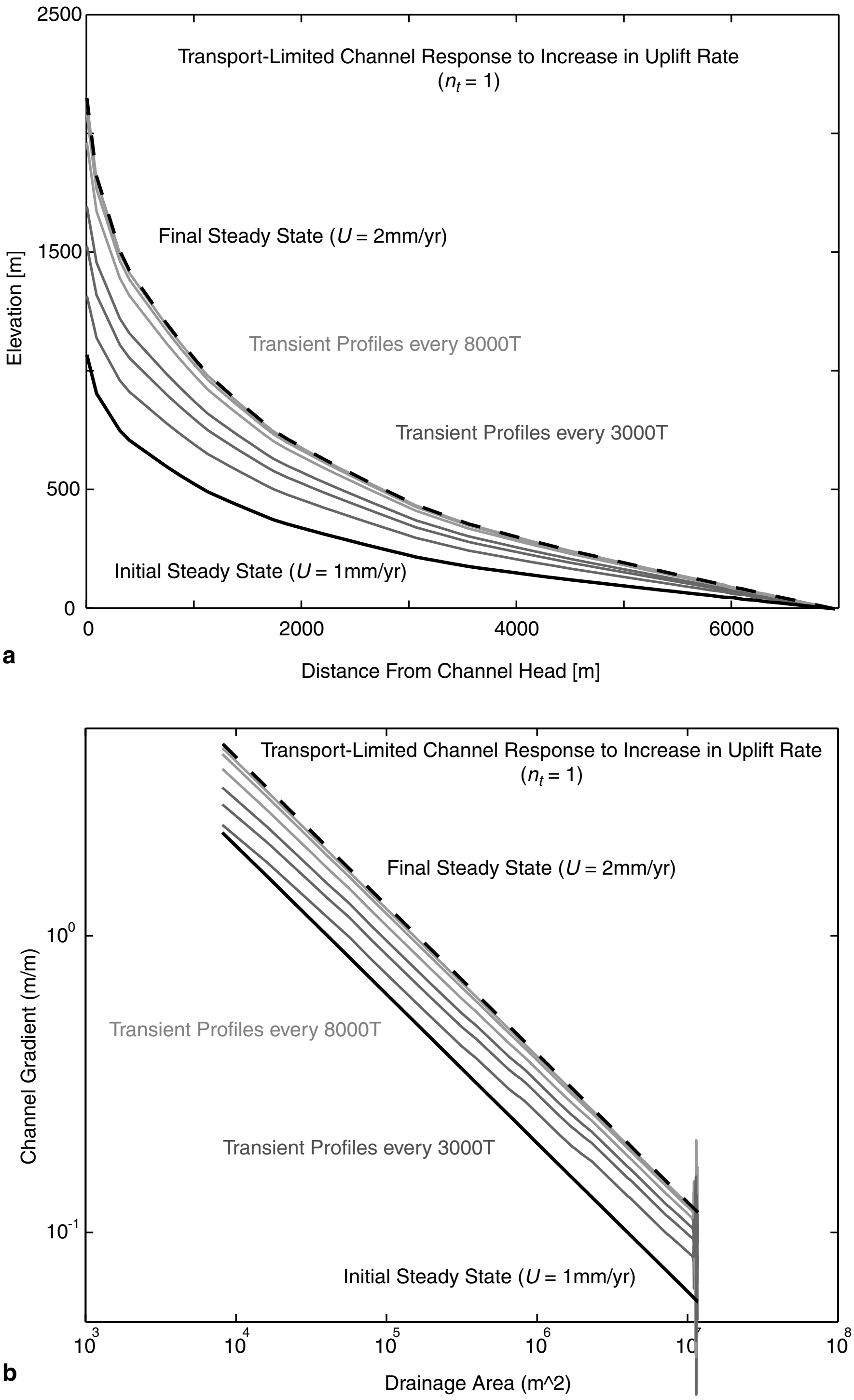


Figure 8. Transient response of a transport-limited channel to an increase in rock uplift rate (U), shown for the case n_t = 1. (a) Longitudinal profiles. (b) Channel gradient versus drainage area (log-log scale). Note that the channel evolves by a gradual steepening at an ever decreasing rate (difference from a detachment-limited channel is most evident in Figure 8b). Response time is $\sim$40,000T (model time steps).

also sweeps upstream from the outlet, but in a much more diffuse manner (Figure 8). Further, at all points along the channel,there is a progressive adjustment (e.g., steepening) toward the new steady state (Figure 8b). Consequently, the basin headwaters begin to respond well before the lower branches have fully adjusted to the new conditions. Thus, whereas for detachment-limited systems significant deviations from steady state are easily recognizable as distinct knickpoints or convexities in channel profiles (Figure 7), for transport-limited systems, transient and steady state channel profiles exhibit similar characteristic forms (Figure 8) [*Willgoose*, 1994]. Topographic analyses of transport-limited channels are therefore nondiagnostic in terms of whether the steady state assumption holds. Response times are greater for transport-limited systems and are in some sense undefined as the channel profile only asymptotically approaches the new steady state (see discussion by *Howard* [1982]).

[43] Simulations of systems initially at the threshold between detachment- and transport- limited conditions (N_{dt} = 1) are used to explore the behavior of hybrid channel systems. Much is revealed in numerical experiments with $n = n_t$ (both equal to unity in the cases explored here). As shown in (8) and (10), if $n = n_t$, changes in rock uplift rate will produce no lasting transition away from this threshold state. How then will the system respond? The answer depends on whether rock uplift rate is increased or decreased. In response to an increase in rock uplift rate, threshold channels behave entirely as detachment-limited systems despite the fact that the simulations begin and end at the threshold between detachment- and transport-limited conditions (N_{dt} = 1) (Figure 9a). This follows because at any given point in the system sediment flux lags behind channel response. Thus the instant the channel begins to steepen at the outlet it shifts into the detachment-limited regime ($N_{dt} < 1$). Consequently, channel response will be one of slope replacement and kinematic wave migration. Only at the final instant of basin response will sediment flux equal the steady state sediment flux. At this moment, the channel returns to the threshold between detachment- and transport-limited regimes. For exactly the same reason (the lag in sediment flux response), threshold channel response to a decrease in rock uplift rate will be governed by transport-limited behavior (Figures 9b, 10a, and 10b). This finding suggests that an interesting hysteresis in landscape evolution could result from differing channel response to increases and decreases in rock uplift rate (for a discussion of some implications, see *Baldwin and Whipple* [1999]).

[44] If $n \neq 1$, transient channel response of threshold channels differs in interesting, but predictable ways. If $n < 1$, one can anticipate from (8) and (10) that the channel will shift into the detachment-limited regime if uplift rate is increased and into the transport-limited regime if uplift rate is decreased. Since the lag in sediment flux would shift the incision regime in the same direction, transient response is simply detachment-limited if U increases and transport-limited if Udecreases. If $n > 1$, the opposite behavior is expected from (8) and (10), but an interesting complication arises. Analogous to the $n = 1$ case above, because of the lagged response of sediment flux, transient response to a decrease in U is entirely transport-limited (Figure 9b). Similarly, channel response to an increase in U is initially detachment-limited (Figures 9a, 10c, and 10d). Indeed, the propagating kinematic wave reaches most of the way to the divide before sediment flux at the outlet increases to the point that channel gradient locally becomes transport limited (Figures 10c and 10d). This sudden transition to transport-limited conditions then sweeps very rapidly upstream. From this moment on, channel response is entirely transport-limited (Figures 9a, 10c, and 10d). Thus, in this scenario, transient channel profile evolution can essentially be divided into two distinct stages: an initial, rapid kinematic wave response that increases channel slope by slope replacement and knickpoint propagation according to (2), followed by a slower diffusive wave response that gradually increases slope according to equation (5) (Figures 9a, 10c, and 10d). Nontrivial forms of the $f(q_s)$ term, of course, would lead to more gradual transitions in transient behavior, with some interesting potential complications due to the sensitivity of the parabolic $f(q_s)$ models to variations in N_{dt} caused by the lagged sediment flux response (equations (20) and (21)).

8. Discussion

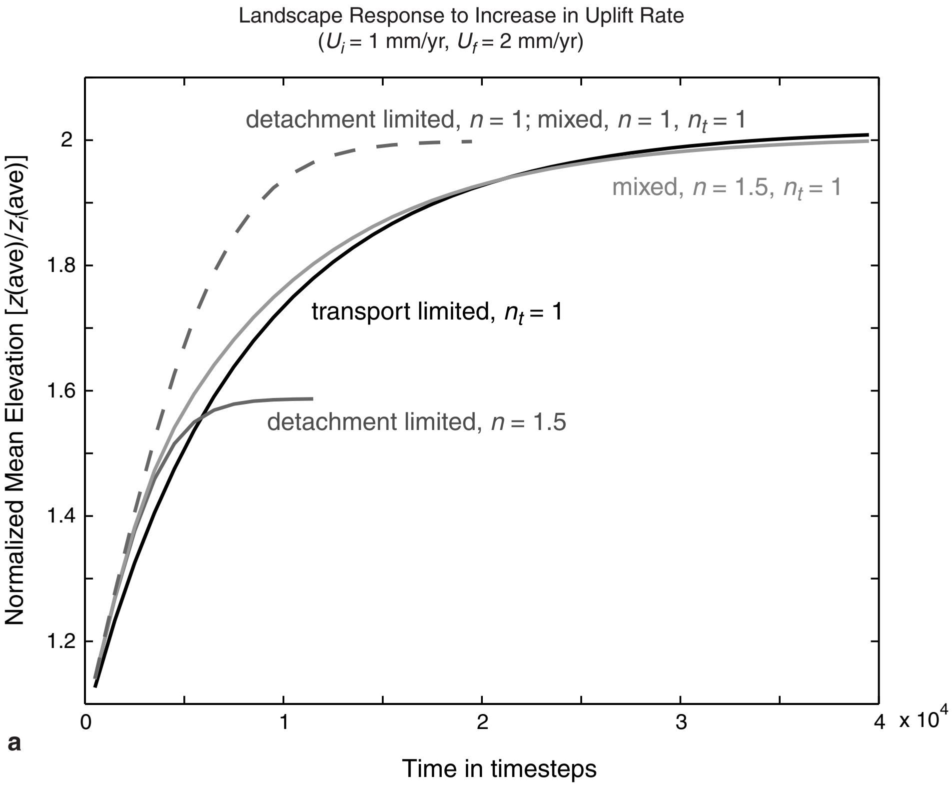
[45] Many of the direct implications of the analyses presented here have already been discussed in the pertinent sections above. Here we focus on some of the broader implications of this work. The discussion addresses three topics: (1) the widespread occurrence of mixed bedrock-alluvial channels, (2) implications of hybrid channels and transitions between detachment- and transport-limited regimes for the relationships among climate, lithology, tectonics, and topography, and (3) implications for orogen evolution.

8.1. Occurrence of Mixed Channels

[46] The immediate impetus to our analysis of the occurrence of mixed channels was the thoughtful discussion of the processes and occurrence of mixed bedrock-alluvial channels by *Howard* [1998]. The reader is advised to contrast Howard's discussion with ours as the conclusions are rather different. *Howard* 1998, [p. 308] presents a theoretical analysis that suggests that (1) "within a basin of uniform sediment yield, steep channel gradients should be associated with bedrock channels, whereas low-gradient channels should favor alluvial beds" and (2) "few channels should exhibit beds transitional between full alluvial and bare bedrock." He points out, however, that both these expectations commonly prove to be incorrect when examined in the field. Howard then proposes a number of plausible explanations for the failure of these model predictions. The explanations he offers fall into two groups: (1) temporal change (climate or landuse change, or a recent pulse of base level fall) and (2) episodic exposure of bedrock in partially alluviated, steady state channels. The former he discards as being unstatisfyingly ad hoc, particularly for such a widespread phenomenon. The latter can be further divided into three plausible explanations: (1) bedrock is only locally exposed in waterfalls or knickpoints because rock is locally more resistant [e.g., *Miller*, 1991; *Whipple et al.*, 2000a], (2) bedrock is only locally exposed because bedrock erosion only occurs by knickpoint migration (i.e., only in local oversteepenings is the critical shear stress for bedrock detachment exceeded [*Seidl and Dietrich*, 1992], or (3) bedrock is only episodically exposed (and eroded) due to the vagaries of episodic sediment supply, deep scour during floods, and bed form migration [e.g., *Benda and Dunne*, 1997; *Hovius et al.*, 2000].

[47] We concur that these factors likely contribute to the occurrence and dynamics of mixed bedrock-alluvial channels and that episodic flooding, stochastic sediment supply, bed form migration, land use change, or recent climate change potentially confound confident discrimination of transport- and detachment-limited channel types in the field. However, our analysis suggests that it may be no surprise that mixed bedrock-alluvial channels appear to be rather common. The divergence of interpretation derives from the one fundamental difference in our approach to the problem: Whereas we consider steady state landscapes in which sediment flux and river incision rate are directly and strongly coupled, *Howard* [1998] held sediment flux and incision rate as independent variables.

[48] Our analysis shows that transport-limited, bedrock-incising channels, which are most likely to exhibit the morphologic characteristics of mixed bedrock-alluvial channels, are expected wherever rocks are highly erodible (high K), sediment transport is inefficient (low K_t), gravel is abundant (high β), channels are in a declining (decreasing U), postorogenic state (e.g., as in the Appalachians), or, if $n > n_t$, wherever uplift rates are so rapid that channels are buried in sediment derived from rapidly eroding hillsides (see equation (8)). Such conditions may be quite common, especially if the intrinsic concavity indices of detachment- and transport-limited systems are roughly equal ($\theta_d = \theta_t$), as discussed above and suggested by the data presented for arguably transport-limited systems. Moreover, any model of the $f(q_s)$ term that includes a bed protection effect as sediment flux approaches transport capacity predicts a broader range of conditions that approach the transport-limited end-member than determined in (8)–(10). As a specific example, the parabolic model with $n = 1$ allows only a very narrow range of stable detachment-limited conditions, implying that most channels must either be close to transport-limited or must be eroded by another mechanism, such



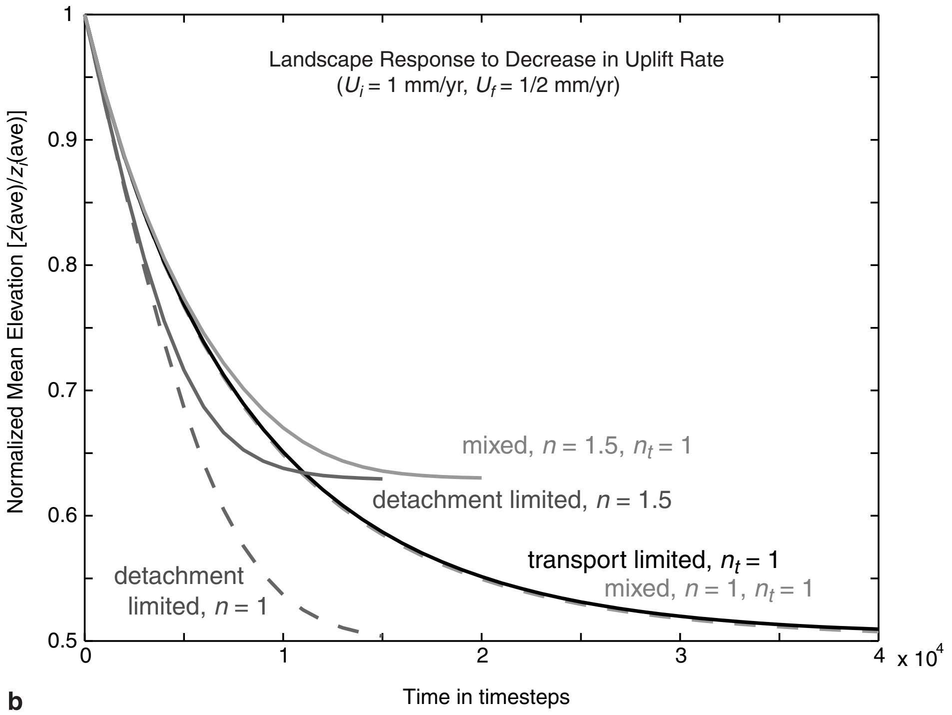


Figure 9. Time rate of change of normalized mean elevation for the cases shown in Figures 7, 8, and 10. (a) Transient response to an increase in rock uplift rate (U). Note that the hybrid case (n = 1.5; n_t = 1) initially follows the detachment-limited curve (for n = 1.5), then transitions over to the transport-limited curve as shown in Figures 10c and 10d. (b) Transient response to a decrease in rock uplift rate (U). Note that the hybrid case shown in Figures 10a and 10b (n = 1; n_t = 1) precisely follows the transport-limited case. Similarly, a hybrid case with $n > n_t$ (shown for n = 1.5) largely follows the transport-limited curve then abruptly converges on the final steady state dictated by the detachment-limited relation. This channel ends in a detachment-limited state, but as with n = 1, its transient response to a decrease in U is primarily transport-limited.

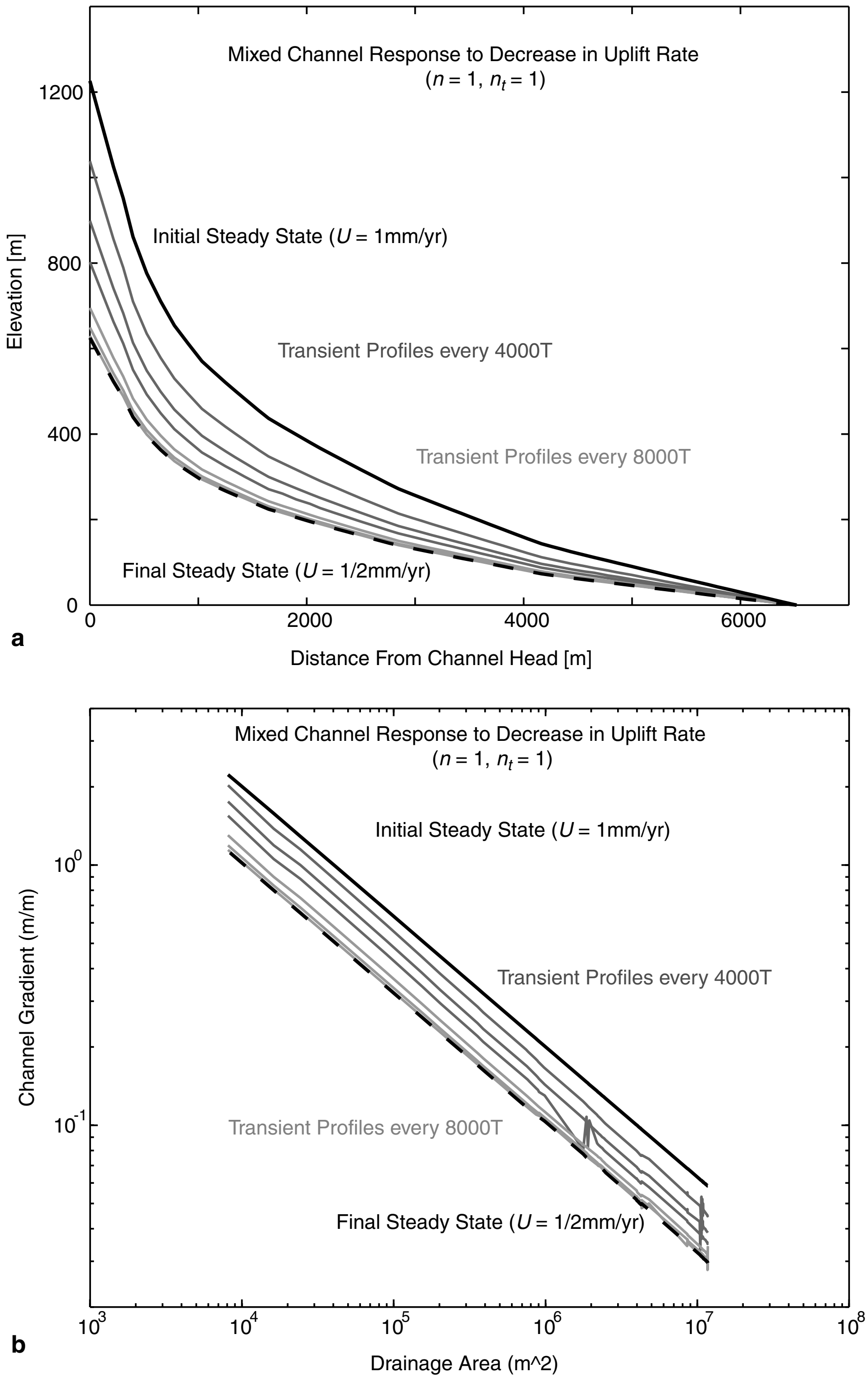


Figure 10. Transient response of a threshold channel ($\theta_d = \theta_t$; $S_d = S_t$ initially). (a) Longitudinal profiles and (b) channel gradient versus drainage area for a decrease in rock uplift rate (U), shown for n = 1. Note that transient response to a decrease in uplift rate is entirely transport-limited despite the fact that the channel begins and ends in a threshold condition. (c) Longitudinal profiles and (d) channel gradient versus drainage area for an increase in rock uplift rate (U) when n = 1.5. Note the distinct two-stage evolution: an initial, rapid detachment-limited kinematic wave response ($\sim$6000T) followed by a slower transport-limited diffusive response ($\sim$32,000T) (compare Figures 7 and 8).

as debris flows (for more discussion, see *Sklar and Dietrich* [1998]).

8.2. Implications for Relationships Among Climate, Lithology, Tectonics, and Topography

[49] *Whipple and Tucker* [1999] and *Tucker and Whipple* [2002] developed predictive relations among climate, lithology, tectonics, and fluvial relief for the detachment-limited case. They emphasized the need to refine our understanding of the controls on the slope exponent $n$because this parameter critically controls the sensitivity of fluvial relief and response time to climate, lithology and tectonics (e.g., equation (3a) and (3b)). However, by restricting the analysis to exclusively detachment-limited channels, their treatment neglects the potentially important role

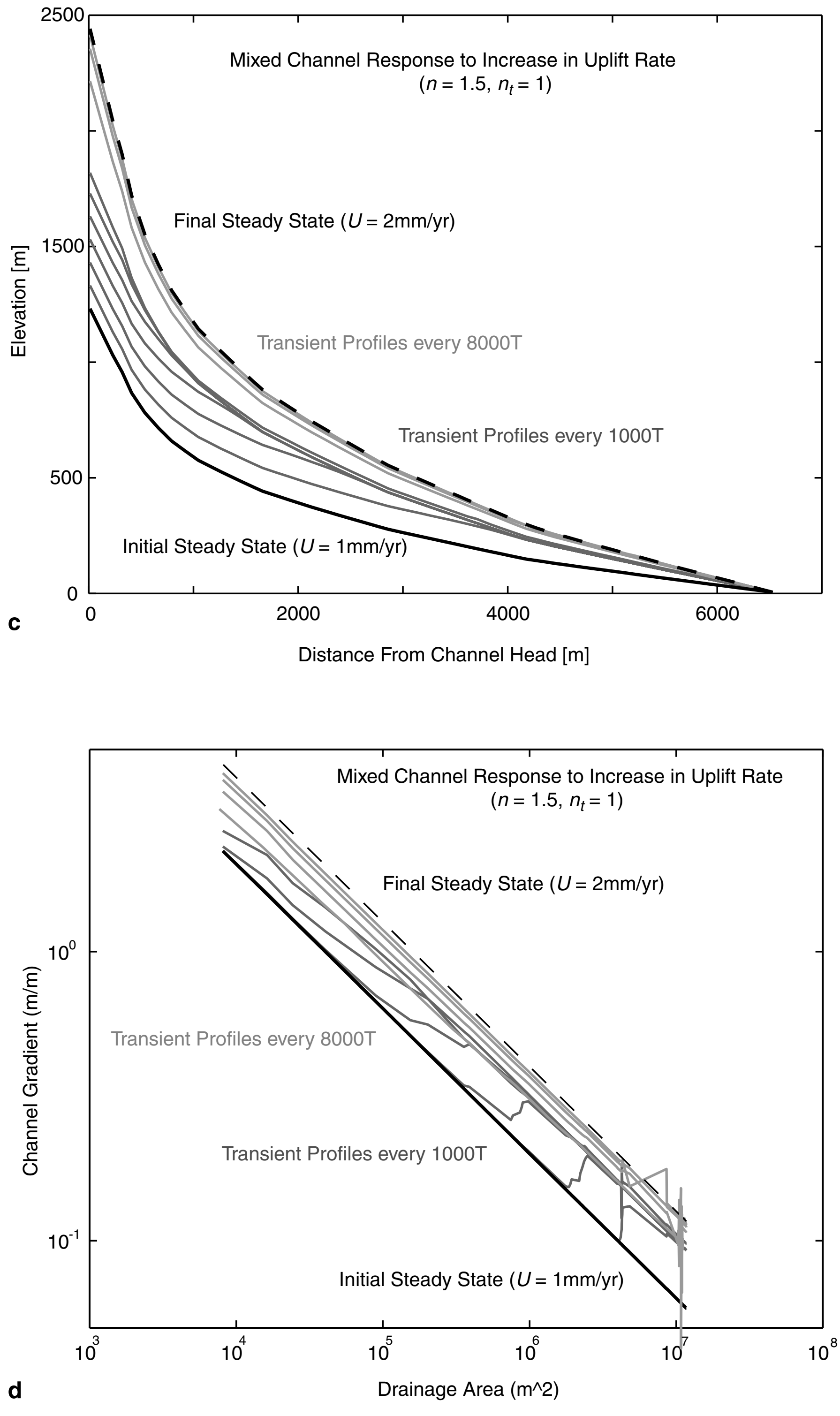


Figure 10. (continued)

of the dual requirement on steady state bedrock channels: erode the bed at a rate equal to rock uplift relative to base level and transport out the sediment flux delivered from upstream. As shown here, this second requirement can be of first-order importance, depending on the circumstances. This is particularly true if $n \neq n_t$. In essence, the requirement that all sediment be transported out restricts the steady state scaling of channel gradient with uplift rate to $1/n_t$ (rather than $1/n$) as uplift rate becomes small if $n < n_t$ and as uplift rate becomes large if $n > n_t$ (Figures 9 and 10). For example, in the latter case an initially detachment-limited channel eventually becomes transport limited as uplift rate increases (see Figure 4). In addition, all channels are expected to transition to a transport-limited condition during a declining state transient due to the lagged response of sediment flux. This finding is robust for all values of n and all models of the coupling between sediment flux and river incision.

[50] Allowing coupling of sediment flux and river incision rates further restricts the scaling between channel gradient and uplift rate. The linear decline model predicts steady state channel gradients that are essentially linear in rock uplift rate for both $n = 1$ and $n = 2$ (equations (14) and (15); see Figure 4). Similarly, the parabolic model predicts steady state channel gradients that are linear in U for $n = 1$ but become unstable as conditions move toward detachment-limited (equation (16)). The parabolic model with $n = 2$, on the other hand, does not exhibit this instability and predicts that steady state channel gradients are linear in Ubut that the dependence on U becomes weaker as conditions shift toward the detachment-limited end-member ($N_{dt} \gg 1$; equation (17) and Figure 4). These important differences between the $n = 1$ and $n = 2$ parabolic models suggest that details of the $f(q_s)$ term and the mechanics of the dominant erosion processes could be rather important to long-term channel evolution. For instance, the different mechanics of bedrock erosion by incremental abrasion and by block fracture and plucking [*Sklar and Dietrich*, 1998; *Whipple et al.*, 2000a] may lead to significant differences in landscape evolution dynamics. This is seen as strong motivation for further study of erosion processes and the coupling between sediment flux and channel incision.

8.3. Implications for Orogen Evolution

[51] An interesting implication of the observation that channels will tend toward transport-limited conditions during declining state transients is that a certain hysteresis will be imparted to landscape evolution through an orogenic cycle, with largely detachment-limited, kinematic wave behavior in the rising state and diffusive, transport-limited behavior during decline. This suggests that whereas deviations from steady state forms during the rising state may be readily recognizable in analyses of stream profiles, the characteristic forms identified by *Willgoose* [1994] for declining state equilibrium in transport-limited systems may confound any such analysis of postorogenic landforms [see also *Baldwin and Whipple*, 1999]. This may explain the smooth, concave up profiles of the mixed bedrock-alluvial channels characteristic of relict mountain ranges such as the Appalachian Mountains (Figure 5)[*Hack*, 1957]. Moreover, the transition to transport-limited conditions during decline may be expected to significantly lengthen the timescale of postorogenic topographic decay, perhaps providing one clue to the topographic persistence of ancient orogens [*Stephenson*, 1984; *Baldwin and Whipple*, 1999]. Finally, the suggested hysteresis will likely cause temporal variation in the quantitative relationships between relief and denudation rate during an orogenic cycle.

9. Conclusions

[52] In this contribution we have explored the controls on the occurrence, form, and dynamics of mixed bedrock-alluvial channels, for which the distinction between detachment- and transport-limited conditions is not easily made, in order to develop testable hypotheses to guide future investigations. Rather than evaluating the relative merits of the various river incision models that have been proposed, we instead have striven to establish, in a systematic manner, how the models differ and therefore how they may be tested. We highlight below the salient aspects of our analysis of this problem, including both new insights into the dynamics of landscape evolution and newly identified testable predictions of the various models considered.

1. Mixed bedrock-alluvial channels appear to arise from a transport-capacity limitation in incising bedrock channels; transport-limited conditions therefore do not necessarily equate to either a depositional state or erosion of unconsolidated materials, as is often assumed. Downstream transitions from detachment-limited to transport-limited, bedrock-incising channels are expected if the intrinsic concavity index of transport-limited systems is less than that of detachment-limited systems [*Howard*, 1980; *Howard and Kerby*, 1983; *Tucker and Slingerland*, 1996]. Our analysis shows that transport-limited conditions in bedrock channels are expected for erodible substrate (high K), abundant, coarse-grained sediment (high β, low K_t), during declining state transients (decreasing uplift rate) and in zones of high uplift rate if $n > n_t$.

2. In recent years, several investigators have independently argued that sediment flux strongly influences rates of river incision into bedrock [*Beaumont et al.*, 1992; *Sklar and Dietrich*, 1997; *Slingerland et al.*, 1997; *Howard*, 1998; *Sklar and Dietrich*, 1998; *Slingerland et al.*, 1998; *Whipple et al.*, 2000a]. We propose a generic formulation of the sediment flux term ($f(q_s)$) that allows us to place the various models in context and to explore the implications of each for steady state landscape morphology. Certain forms of the various models (i.e., the values of area and slope exponents n, m, n_t, m_t) may be ruled out as inconsistent with data as they fail to satisfy the commonly observed power law scaling between channel gradient and drainage area [e.g., *Flint*, 1974; *Howard*, 1980; *Tarboton et al.*, 1989].

3. Our analysis demonstrates that the intrinsic concavity index of transport-limited channels, closely related to downstream fining and transport of mixed-sized sediment [e.g., *Sinha and Parker*, 1996], largely determines whether or not steady state morphology is diagnostic of the various models that have been proposed. Although pronounced topographic differences are predicted in the case of differing intrinsic concavity indices, preliminary data analysis suggests that the intrinsic concavity indices of detachment- and transport-limited systems may often be similar. This implies that steady state landform morphology may not be diagnostic except where abrupt, downstream changes in either rock erodibility (K) or rock uplift rate (U) occur. Whereas detachment-limited systems exhibit abrupt gradient changes that coincide precisely with the change in lithology or rock uplift rate, transport-limited systems exhibit gradual channel gradient changes because their local gradient is controlled by the integrated sediment flux from upstream. Given the importance of the intrinsic concavity indices of transport-limited systems and the preliminary nature of the data presented, further work is needed. For instance, grain-size distributions (gravel fraction (β), sorting, and size) delivered to channels and the efficiency of downstream fining due to selective transport and abrasion may vary as a function of incision rate, suggesting additional internal feedbacks [*Howard*, 1980; *Howard et al.*, 1994; *Howard*, 1998].

4. Even if steady state morphology is nondiagnostic, the scaling behavior of topographic relief and the extent of detachment- versus transport-limited channels as a function of rock uplift rate may provide useful criteria for testing models. In particular, the parabolic models (and various forms of them) predict markedly different behavior than other models in these respects.

5. Interestingly, we find that for all n, channels in postorogenic decline eventually transition to transport-limited conditions. For channels initially at the threshold between detachment- and transport-limited conditions this transition is immediate, whereas for channels initially far from transport-limited conditions the transition will not occur until a significant proportion of the original relief has been eroded away. However, the eventual transition to transport-limited conditions implies that the characteristic form of declining transport-limited channels [*Willgoose*, 1994] should be quite common in landscapes in decline, even where channels are incising through competent bedrock. Thus similar slope-area scaling may be expected for channels of all types (transport-limited, detachment-limited, and hybrid channels), both at steady state and during postorogenic decline. Transient conditions may only be readily recognizable from topographic data in cases of detachment-

limited channels subjected to either a recent base level fall or an increase in rock uplift rate.

6. The richest source of information is in the transient response of channels governed by each of the different models to external forcings (change in rock uplift rate, climate, base level). Transport-limited systems are characterized by diffusive, gradual changes in channel gradient; detachment-limited systems by a wavelike response that sweeps rapidly upstream. Mixed bedrock-alluvial channels may exhibit transport-limited, detachment-limited, or hybrid behavior, depending on conditions. In exploring the transient response of channels at the threshold between transport- and detachment-limited conditions (N_{dt} = 1), we identify an interesting hysteresis in landscape evolution that stems from the inherent lag between channel response and sediment flux. The transient response of these threshold channels can be governed by either detachment- or transport-limited dynamics, depending on the directionality of the forcing. For $n \leq 1$, threshold channels respond in an entirely detachment-limited manner to an increase in uplift rate but entirely in a transport-limited manner to a decrease in U. If $n > 1$, the response is similar except that their response to an increase in uplift rate progresses through two stages: an initial, rapid kinematic wave detachment-limited response is followed by a slower, diffusive, transport-limited response.

Notation

A upstream drainage area [L^2].
A_{cr} critical drainage area for a transition from detachment- to transport-limited conditions [L^2].
β fraction of sediment delivered to channels as bed load, dimensionless.
E vertical erosion rate [$L\ T^{-1}$].
$f(q_s)$ erodibility scaling factor for sediment loading, dimensionless.
K coefficient of erosion [$L^{1\text{-}2m}\ T^{-1}$].
K_t bed load transport coefficient [$L^{3\text{-}2mt}\ T^{-1}$].
L_f bedrock erosion length scale [L].
λ_p bed sediment porosity, dimensionless.
n slope exponent, detachment-limited erosion rule, dimensionless.
n_t slope exponent, bed load transport rule, dimensionless.
N_{dt} detachment-transport transition number, dimensionless.
m area exponent, detachment-limited erosion rule, dimensionless.
m_t area exponent, bed load transport rule, dimensionless.
Q_c bed load sediment transport capacity [$L^3\ T^{-1}$].
Q_s bed load sediment flux [$L^3\ T^{-1}$].
S streamwise channel bed gradient, dimensionless.
S_d steady state gradient of detachment-limited streams, dimensionless.
S_{l1} steady state gradient for linear decline $f(q_s)$ and n = 1, dimensionless.
S_{l2} steady state gradient for linear decline $f(q_s)$ and n = 2, dimensionless.
S_{p1} steady state gradient for parabolic $f(q_s)$ and n = 1, dimensionless.
S_{p2} steady state gradient for parabolic $f(q_s)$ and n = 2, dimensionless.
S_t steady state gradient of transport-limited streams, dimensionless.
t time [T].
θ stream concavity index (scaling exponent in gradient-area relationship), dimensionless.
θ_d intrinsic concavity index of detachment-limited systems, dimensionless.
θ_t intrinsic concavity index of transport-limited systems, dimensionless.
U rock uplift rate [$L\ T^{-1}$].
U_{cr} critical rock uplift rate for a transition from detachment- to transport-limited conditions [$L\ T^{-1}$].
W channel width [L].
z elevation of streambed [L].

[53] **Acknowledgments.** Funding for this work was provided by NSF grants EAR-9614970, EAR-9725723, and EAR-0003571 (K.W.), and by the U.S. Army Research Office (G.T.). We thank P. Talling for sharing unpublished gradient and drainage area data for the Enza river and B. Gomez for sharing grain-size data for the Waipaoa River prior to publication. E. Kirby performed the analysis of Waipaoa channel profiles. Alan Howard and Rudy Slingerland provided helpful reviews.

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