CHAPTER 7
OPTIMAL LOCATION OF ISOLATION VALVES IN WATER DISTRIBUTION SYSTEMS: A RELIABILITY/OPTIMIZATION APPROACH

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7.1 INTRODUCTION

7.1.1 Before September 11, 2001

The cornerstone of any healthy population is access to safe drinking water. The goal of the United Nations International Drinking Water Supply and Sanitation Decade from 1981 to 1990 was safe drinking water for all. A substantial effort was made by the United Nations to provide drinking water and sanitation services to populations lacking those services. Unfortunately, however, the growth in developing country populations almost entirely wiped out the gains. In fact nearly as many people lack those services today as at the beginning of the 1980s (Gleick, 1993, 1998).

The lack of adequate water supply systems is due to both the deterioration of aging water supplies in older urbanized areas, and to the nonexistence of water supply systems in many areas that are undergoing rapid urbanization, such as in the southwestern United States. In other words, methods for evaluation of the nation’s water supply services needs to consider not only rehabilitation of existing urban water supply systems but also the future development of new water supply systems to serve expanding population centers. Both the adaptation of existing technologies and the development of new innovative technologies will be required to improve the efficiency and cost-effectiveness of future and existing water supply systems and facilities necessary for industrial growth.

An EPA survey (Clark et al., 1982) of previous water supply projects concluded that the distribution facilities in water supply systems account for the largest cost item in future maintenance budgets. The aging deteriorating systems in many areas raise tremendous maintenance decision-making problems that are further complicated by the expansion of existing systems. Deterioration of the water distribution systems in many areas has translated into a high proportion of unaccounted-for water due to leakage, which is not only a loss of a valuable resource, but also raises concerns of safe drinking water by possible contamination through cracked pipes.

The reliability of the existing aging systems is continually decreasing. Only recently have many municipalities been willing or able to finance rehabilitation of deteriorating pipelines, and are still deferring needed maintenance and replacement of systems components until a catastrophe or the magnitude of leakage justifies the expense of repair. Water main failures have been extensive in many cities.
As a result of governmental regulations and consumer-oriented expectations, a major concern now is the transport and fate of dissolved substances in water distribution systems. The passage of the Safe Drinking Water Act in 1974 and its amendments in 1986 (SDWAA) changed the manner in which water is treated and delivered in the United States. The U.S. Environmental Protection Agency (USEPA) is required to establish maximum contaminant level (MCL) goals for each contaminant that may have an adverse effect on people’s health. These goals are set to the values at which no known or expected adverse effects on health can occur. By allowing a margin of safety (Clark et al., 1987; Clark, 1987) previous regulatory concerns were focused on water as it leaves the treatment plant before entering the distribution system (Clark, 1987), disregarding the variations in water quality that occur in the water distribution systems.

### 7.1.2 After September 11, 2001

Water utilities are implemented to construct, operate, and maintain water supply systems. In the past the basic function of these water utilities was to obtain water from a source, treat the water to an acceptable quality, and deliver the desired quantity of water at the desired time. The events of September 11, 2001 and subsequent anthrax attacks have shown the vulnerability of basic civilian infrastructure, somewhat changing the function of water utilities to include protection from various threats. Drinking water supply systems around the world offer great opportunities for terrorism as these systems are extensive, relatively unprotected and accessible, and often isolated. The threats to these systems include cyber, physical, chemical, and biological threats.

The events of September 11, 2001 have significantly changed the approach to management of water utilities. Previously the consideration of terrorist threats to our nation’s drinking water supply was minimal. New trends in the technical management of water supply systems will be heavily focused upon security issues as related to the design, operation, and management of these systems. Of particular interest will be the evaluation of risk and reliability issues of the various components, the subsystems, and the systems as a whole from the viewpoint of their susceptibility to terrorism.

Within a very short time after September 11, 2001 we began to see a concerted effort at all levels of government to begin addressing issues related to the threat of terrorist activities to the U.S. water supply. We saw a number of acts passed such as the “Security and Bioterrorism Preparedness and Response Act” and the “Homeland Security Act” that addressed the nation’s water supply. These acts resulted in agencies such as the U.S. Environmental Protection Agency (EPA) developing new protocols to address their new responsibilities under these acts.

### 7.1.3 Water Distribution Systems: A Brief Description

Water distribution systems are composed of three major components: pumping stations, distribution storage, and distribution piping. These components may be further divided into subcomponents, which in turn can be divided into subsubcomponents. For example, the pumping station component consists of structural, electrical, piping, and pumping unit subcomponents. The pumping unit can be divided further into subsubcomponents: pump, driver, controls, and power transmission. The exact definition of components, subcomponents, and subsubcomponents depends on the level of detail of the required analysis and to a somewhat greater extent on the level of detail of available data. In fact, the concept of component-subcomponent- subsubcomponent merely defines a hierarchy of building blocks used to construct the water distribution system.

A water distribution system operates as a system of independent components. The hydraulics of each component is relatively straightforward; however, these components depend directly upon each other and as a result affect each other’s performance. The purpose of design and analysis is to determine how the systems perform hydraulically under various demands and operating conditions. These analyses are used for the following situations:

- Design of a new distribution system
- Modification and expansion of an existing system

Distribution systems, consisting of pipelines, pipes, pumps, storage tanks, and the appurtenances such as various types of valves, meters, and the like, offer the greatest opportunity for terrorism because they are extensive, relatively unprotected and accessible, and often isolated. The physical destruction of a water distribution system’s assets or the disruption of the water supply could be more likely than contamination. A likely avenue for such an act of terrorism is a bomb, carried by car or truck, similar to recent events. Truck or car bombs require less preparation, skill, or manpower than complex attacks such as those of September 11, 2001. However, we must consider all possible threats no matter how remote we may think that they could be.

### 7.2 DEMAND-DRIVEN ANALYSIS VERSUS PRESSURE-DRIVEN ANALYSIS

#### 7.2.1 Definitions

Hydraulics of a water distribution network can be approached from two different perspectives. The difference between them comes from the level of primacy given to nodal demands vs. nodal pressures. The first approach assumes that consumer demands are always satisfied regardless of the pressures throughout the system and formulates the constitutive equations accordingly to solve for the unknown nodal heads. This approach is called demand-driven analysis and is used by almost all the traditional network hydraulic solvers. Examples are EPANET by Rossman (1994) and KYPIPE by Wood (1980).

A water distribution network can be designed or operated using demand-driven analysis under normal operating conditions. This is usually done by adjusting such independent decision variables as simple or rule-based controls for pump operations, valve settings, reservoir/tank levels, and so on. If a proper design has been made for a given design demand loading, then nodal pressures throughout the system should be well above the minimum pressure required. On the other hand, if the same network were analyzed with a higher demand loading, it would not be surprising to see warning messages such as “Negative pressures at 6:00 hrs” from the EPANET model while nodal demands are fully satisfied at the same time period. Almost all demand-driven models possess a partial recognition of this weakness with similar warning messages when negative pressures are calculated from a network hydraulic analysis.

The major drawback of the demand-driven approach is that it fails to measure a partially failed network performance resulting from abnormalities in physical and nonphysical system components (e.g., pipe breaks, fire-fighting demands, etc.). In such cases, if demand-driven analysis is used, it may produce very unrealistic results; for example, negative pressures are calculated at some nodes in the system. It is not uncommon that a hydraulic analysis using a demand-driven approach yields large negative pressures at one or more junctions of a distribution network.

The second approach to a network hydraulic analysis is called head-driven (occasionally called pressure-driven or pressure-dependent) analysis. Here, the primacy is given to pressures. A node is supplied its full demand only if a minimum required supply pressure is satisfied at that node. If the minimum pressure requirement cannot be met, then only a fraction of the nodal demand can be satisfied. That fraction is determined by a predefined relationship between nodal head and nodal outflow.

As mentioned above, demand-driven models work well under normal operating conditions. Major applications of head-driven analysis, on the other hand, are for analysis of network performance under partially failed conditions. Because a network reliability assessment requires evalua-
tion of partially failed conditions of a network, pressure-dependent analysis is much more suitable for reliability assessment of water distribution networks.

One of the major problems with the pressure-driven approach is that it requires extensive field data collection to determine the relationship between nodal heads and nodal flows. Such a relationship may be unique for each junction of a network inasmuch as it depends on such physical characteristics as type of service connection and the development type that the junction serves (e.g., residential, high-rise, schools, hospitals, etc.). Calibration of models using field data is also part of the extensive effort needed to be able to use pressure-dependent analysis.

The second major drawback of head-driven analysis is that it lacks robust methods for the computational solution of the constitutive equations (Tanyimboh et al., 2001). Almost all the traditional network hydraulic models are built upon the demand-driven approach. Therefore, it is very desirable that reliability analysis techniques based on interpretations or transformations of existing demand-driven analysis from a pressure-driven approach be developed.

### 7.2.2 Hydraulic Equations for a Network Simulation Problem

Two sets of equations are needed to solve a network simulation problem. First, conservation of flow must be satisfied at each network junction. The second set of equations describes a nonlinear relation between flow and headloss in each pipe, such as the Hazen-Williams or Darcy-Weisbach equation. These equations form a coupled set of nonlinear equations whenever the network contains loops or more than one fixed-head source (Rossman, 2000). The fact that most distribution systems are looped and/or have multiple sources requires the aid of a computer to solve the nonlinear set of equations using an iterative procedure. The basic equations in a network model are described by Rossman (2000) for a network with \( N \) junction nodes and \( NF \) fixed grade nodes (tanks and reservoirs).

Let the flow-headloss relation in a pipe between nodes \( i \) and \( j \) be given as:

\[
H_i - H_j = h_{ij} = r Q_{ij}^n + m Q_{ij}^2
\]  

where:
- \( H \) = nodal head
- \( h \) = headloss
- \( r \) = resistance coefficient
- \( Q \) = flow rate
- \( n \) = flow exponent
- \( m \) = minor loss coefficient

The value of the resistance coefficient and the corresponding flow exponent depend on which friction headloss formula is being used. Most network models use one of the following flow-headloss relationships: (1) Hazen-Williams, (2) Darcy-Weisbach, or (3) Chezy-Manning.

In addition to the first set of equations described above, conservation of flow around all nodes must be satisfied, which forms the second set of equations

\[
\sum_{j} Q_{ij} - D_i = 0 \quad \text{for } i = 1, \ldots, N
\]

where:
- \( D_i \) = flow demand (or supply) at node \( i \)
- \( j \) = set of nodes directly connected to node \( i \)
- \( Q_{ij} \) = flow in pipe \( ij \)

By convention, flow is positive if it is into the subject node and negative, otherwise.

As previously mentioned the conventional network hydraulic solvers such as EPANET by Rossman (1994) and KYPipe by Wood (1980) are demand-driven. Demand-driven analysis seeks a solution for all heads \( H \) and flows \( Q \) that satisfy the two sets of equations, (7.1) and (7.2), for a set of known heads at the fixed grade nodes. The basic assumption in demand-driven analysis is that nodes are assigned demands that are assumed fully satisfied. For example, variables used in the solution algorithm of EPANET are divided into unknown and known variables (Rossman, 1994).
The unknown variables are
\[ y_s = \text{height of water stored at tank node } s \]
\[ Q_s = \text{flow into storage tank node } s \]
\[ Q_{ij} = \text{flow in pipe connecting nodes } i \text{ and } j \]
\[ H_i = \text{hydraulic grade line elevation at node } i \text{ (elevation plus pressure head)} \]

and the known variables (constants) are:
\[ A_s = \text{tank cross-section area} \]
\[ E_s = \text{elevation of node } s \]
\[ D_i = \text{demand (or supply) at node } i \]

The major assumption of demand-driven analysis, that demands are fully satisfied, works well under normal operating conditions. However, it is not uncommon that under abnormal conditions a solution arises where pressures are negative or unacceptably low, because demands are satisfied regardless of validity of calculated pressures at network nodes (Bouchart and Goulter, 2000). A few examples of such abnormal conditions may occur when demand loading on the network exceeds the design demand loading, fire-fighting demands for which the network was not designed, or closure of a portion of a network because of contamination or pipe main breaks. In reality, a shortfall in the volume of water actually delivered to consumers begins as the pressure in the network drops below some threshold value. Naturally and ideally, a lower bound for this threshold value is zero gauge pressure.

Water distribution networks are almost always skeletonized to some degree, depending on the modeling purpose forcing demands from secondary networks to be lumped into nearby junctions. Because of various headlosses associated with secondary networks and different elevations of outlets in these networks, it is very difficult, if not impossible, to define a relationship between the pressure at a junction and the flow available from that junction into the secondary network it serves. A tedious way of doing this is field data collection and calibration. A typical value for threshold pressure is below the minimum of 20 m in network studies (Bouchart and Goulter, 2000). Tanyimboh et al. (1999) state that pressures of 15 to 25 m are the minimum acceptable standards with the use of network models. All this implies that demand-driven algorithms cannot cope with situations where pressures are less than satisfactory at some demand nodes (Tanyimboh et al., 2001) and thus fail to depict deficient network performances.

### 7.2.3 Service Pressures

Criteria for assessing acceptable service pressures in a distribution network may vary from system to system. Therefore, there are no universally acceptable pressure ranges. Chin (2000) lists the following considerations for assessing the adequacy of service pressures:

1. Flow is adequate for residential areas when pressures are above 240 kPa (35 psi);
2. The pressure required at street level for excellent flow to a 3-story building is about 290 kPa (42 psi);
3. Adequate flow to a 20-story building would require about 830 kPa (120 psi) of service pressure at street level, which is not desirable because of the associated leakage and waste. Thus, very tall buildings are usually served with their own pumping system;
4. In general, pressures in the range of 410 to 520 kPa (60 to 75 psi) would be adequate for
   - Supplying normal water delivery to buildings up to 10 stories
   - Providing adequate sprinkler service in buildings of 4 to 5 stories
   - Supplying water for fire protection
   - Handling fluctuations in pressure caused by clogged pipes and excessive length of service pipes.
Overall, pressures in most cases should be maintained above 207 kPa (30 psi) and below 689 kPa (100 psi) during normal operations. Pressures above 689 kPa tend to increase water waste through undetected leaks and may cause damage to residential and commercial plumbing systems or pipe breaks. The minimum pressure requirement may be lowered to 138 kPa (20 psi) during emergency conditions, such as a fire. A 138 kPa pressure is sufficient to supply the suction side of pumps on a fire pumper truck. According to Walski (2000), the threshold pressure value below which a failure to satisfy the full demands occurs is usually assumed to be below the minimum of 20 m (≈196 kPa or 28.4 psi).

The Office of Water Services in England specifies a minimum acceptable static pressure of 7 m (≈68.5 kPa or 10 psi) below which customers may be entitled to compensation for less than satisfactory service. On the other hand, it is not uncommon that minimum acceptable pressure is specified as high as 25 m (≈245 kPa or 35 psi), to allow for possible increases in demand (Tanyimboh et al., 1999). In general, nodal pressures of 15 to 25 m will guarantee satisfactory service at all related stop taps in a distribution system. Pressures that are lower may cause a shortfall in supply and failure to supply the full demands. Tanyimboh et al. (1999) also suggest that in the absence of field test data, a good approximation to critical pressures can be obtained by the equivalent static pressure to the height above ground level of rooftop water tanks in typical two-story residential areas.

Water-using devices in residential houses may also require certain minimum pressures to operate. For example, most dishwasher manufacturers specify minimum working pressures anywhere from 20 to 40 psi. Another factor that should be considered while estimating the critical pressure is the degree of network skeletonization. Typically, smaller pipes in a network are excluded from the modeling. The result is that total demand from the consumers located along those smaller lines is lumped into the nearby junctions located on larger pipes. Thus, when choosing the critical pressure for a junction, such characteristics as outlet elevations and headlosses of the secondary network that the junction serves must also be taken into account.

### 7.2.4 Head-Driven Analysis

Demand-driven analysis explained in the previous section assumes that consumer demands in a distribution network are fully satisfied regardless of validity of calculated pressures. It can be argued that many modern water-using devices such as toilets, washing machines, and dishwashers are essentially pressure-independent provided a minimum threshold pressure is available, which supports the assumption of primacy of demands.

The philosophical basis of head-driven analysis—the counterapproach to demand-driven analysis—is explained by Rossman (2001) as follows.

...[A] distribution system can be thought of as a completely closed, pressurized system with a set of variable sized orifices that connect to various devices that are open to the atmosphere (e.g., sinks, toilets, water heaters, washing machines, pipe leaks, etc.). For a specified opening of these orifices, the flow out of them will be dependent on the pressure maintained in the system (which in turn is dependent on the setting of the demand orifices)....

Thus, unlike demand-driven analysis, the head-driven approach treats demands as random variables and recognizes the primacy of pressures by defining a relationship between the outflow and the pressure at each node. This relationship is given, in its general form, by (Tanyimboh et al., 1999)

\[
H_j = H_j^{\text{min}} + K_j Q_j^{n_j}
\]

(7.3)

where \(H_j\) = head at node \(j\) corresponding to demand \(Q_j\)

\(K_j\) = flow resistance coefficient

\(n_j\) = exponent

\(H_j^{\text{min}}\) = minimum required nodal head below which outflow at node \(j\) is unsatisfactory or zero

Because demands are unknown, Eq. (7.3) can be rearranged:
The head-driven analysis requires substitution of \( Q_j \) into Eq. (7.2) in the place of \( D_j \). The solution algorithms for solving the resulting problem have been described by Gupta and Bhave (1996) and Tabesh (1998).

As explained in the previous section, demand-driven analysis may produce unrealistic results (i.e., pressures being negative or less than satisfactory) especially under partially failed conditions of a system. In such cases, head-driven analysis is superior to the demand-driven approach in that the former is able to determine the nodes with insufficient supply and the respective magnitudes of the shortfalls. On the other hand, performing a head-driven analysis requires extensive field data collection and calibration to determine \( K_i \) and \( n_j \) values, possibly for each node (Gupta and Bhave, 1997).

In addition to these practical difficulties, lack of robust methods for computational solution of constitutive equations in head-driven analysis makes the demand-driven network models the primary choice for hydraulic analysis or reliability assessments.

### 7.2.5 Node Flow Analysis

In order to overcome the weaknesses of demand-driven analysis, Bhave (1991) developed a method called Node Flow Analysis (NFA) to predict performance when a distribution network is deficient. The method was based on the assumption that when a network is deficient and thus unable to satisfy the nodal demands, the network will try to meet the demands, as far as possible, under the given conditions. In other words, the flow in the network will adjust such that the total supply is maximized under deficient conditions. Thus, the network flow analysis problem is considered as an optimization problem in their node flow analysis subject to certain constraints that relate hydraulic gradeline at each junction to available flow at that junction.

The complete optimization problem is as follows:

\[
\text{Maximize} \quad \frac{\text{Total outflow}}{\sum_j q_j} \quad (7.5a)
\]

where \( q_j \) is the flow available at node \( j \).

Subject to:

\[
H_m = H_{om} \quad \text{for all } m \quad (7.5b)
\]

where \( m \) is number of source nodes and \( H_{om} \) is the specified hydraulic grade line (HGL) value at source node \( m \).

\[
q_j = q_j^{\text{req}} \quad \text{if } H_j \geq H_j^{\text{min}} \quad (7.5c)
\]

or

\[
0 < q_j < q_j^{\text{req}} \quad \text{if } H_j = H_j^{\text{min}} \quad (7.5d)
\]

or

\[
q_j = 0 \quad \text{if } H_j \leq H_j^{\text{min}} \quad (7.5e)
\]

\[
\sum_j Q_j - q_j = 0 \quad \text{for all } j \quad (7.5f)
\]

where \( i \) is the set of all nodes connected to node \( j \) and assuming flow is positive if it is into node \( j \).

\[
\sum_{\text{loop}} h_L = 0 \quad \text{for all loops} \quad (7.5g)
\]
The basic differences from the usual demand-driven formulation are the additional constraints defined by Eqs. (7.5c) to (7.5f). Unlike Eqs. (7.3) and (7.4), a discrete flow-head relationship is defined. The method still requires defining a minimum pressure threshold at each node.

The network-solvability rule known as the unknown-number rule requires that in order for a network problem to be feasible, the total number of unknowns must be equal to the total number of nodes in the network. Let the number of source nodes and junctions in a network be $S$ and $J$, respectively. For source nodes, the nodal flows are the only unknowns. However, neither the flows nor the HGL values at junctions are known in the NFA described above. Depending on the number of demand junctions, the number of these unknowns (junction heads / junction flows) will be between $J + 1$ (the network must have at least one demand node) and $2J$ (all junctions are assigned nonzero demands). Thus, the total number of unknowns for the entire network lies between $M + N + 1$ and $M + 2N$ which is greater than the total number of nodes in the network, $M + N$.

In order to bring down the total number of unknowns, NFA analysis treats either the demand or the HGL as unknown at each demand node. In other words, if the constraint (7.5c) is chosen, then nodal flow is treated as known, being equal to the nodal demand, and the nodal head as unknown. On the other hand, choosing the constraint (7.5d) implies a known nodal head but an unknown nodal flow, which is also constrained to being less than the nodal demand. Finally, choosing constraint (7.5e) indicates a strictly zero nodal flow and an unknown nodal head subject to being less than the pressure threshold.

The solution procedure of the NFA described above begins by imposing constraint (7.5c) on all demand junctions. Note that this is equivalent to the usual demand-driven analysis. Once the available HGL values are determined at all demand nodes, it is determined which one of the constraints [Eqs. (7.5c) to (7.5e)] actually applies to each demand node and a new solution is obtained. The solution should normally be stopped here if all the nodal constraints are indeed applicable, which can be verified from the results of the new solution. However, as Tanyimboh and Tabesh (1997) have also found, when a network with locally insufficient heads is simulated using the demand-driven approach, the deficiency appears to be far more serious and widespread than it is in reality. The result is that there are almost always some demand junctions in the network that are pressure deficient from an initial demand-driven analysis but then violate the constraint given by Eq. (7.5d), because of $q_j > q_j^{"m"}$, at the end of the next solution. Thus the solution proceeds in an iterative manner until a set of demand nodes is found in the deficient network that is truly pressure deficient. In other words, when no discrepancies are found between each nodal head and nodal flow and the constraint assigned to that node, the final solution is achieved.

7.3 **SEMI-PRESSURE-DRIVEN ANALYSIS (SPDA) FRAMEWORK**

Ozger (2003) developed a Semi-Pressure-Driven Analysis (SPDA) framework, using EPANET Toolkit functions and the C++ programming language, to facilitate the application of NFA (Bhave 1991) in the EPANET network hydraulics solver. The method can be used to predict deficient performance, in terms of Available Demand Fraction (ADF), under partially failed conditions of a network that may include one or more of the following conditions:

1. Fire events.
2. One or more sources in the system are shut down.
3. A pump station is out of service.
4. A control valve that has logistic importance in terms of flow delivery is broken.
5. A transmission line is damaged and taken out of service for repair.
6. A given portion of the network has to be isolated because of contamination.

The input requirements for any what-if scenario are an accurate description of the equipment/network component that will be out of service, the network isolation valve scheme, and, if an EPS analysis is needed, the duration of the repair/replacement process or emergency (e.g., fire-fighting).
Once a semi-pressure-driven analysis is performed for any what-if scenario, the following questions can be answered using the results:

1. Are there enough isolation valves so that the isolation can be reduced to as small an area as possible?
2. How much of the demand can be supplied at each junction?
3. How much of the networkwide demand can be satisfied?
4. What is the temporal variation of nodal and networkwide ADFs?
5. What portions of the network are the most vulnerable to the given what-if scenario?
6. What operational changes could lessen the deficiency in network performance?
7. How long does it take for the network to fully recover, in terms of hydraulics and/or water quality, once the repair/replacement work is finished?

The terms minimum required pressure, critical pressure, and threshold pressure are used interchangeably in the following discussion. By definition, critical pressure at a junction is the pressure value below which the system fails to supply the full demands lumped into that junction.

SPDA starts with a hydraulic analysis of the deficient network using EPANET2. In other words, the network is first simulated from a demand-driven point of view to identify junctions of the network with pressures below the predefined critical pressure values. Threshold values within a network may vary from junction to junction depending on the characteristics of the service connection and the type of development served by that node. Thus, ideally, the critical pressure value is a unique number for each junction in the network and must be determined empirically by field testing. However, in the absence of such field data, approximate values of critical pressures can be determined using one or more of the guidelines discussed in Sec. 7.2.1.

Once the pressure-deficient nonzero demand junctions are identified from an initial demand-driven analysis, the next step is to determine the available flows at those nodes, presumably knowing that the remaining demand nodes are fully satisfactory in terms of both pressure and demand, and the zero demand nodes have pressures above the cavitation limit. For this purpose, the following modifications are made to each pressure-deficient nonzero demand node:

1. [New node elevation] = [original node elevation] + {threshold pressure head}.
2. Set demand to zero.
3. Connect an artificial reservoir to the node by an infinitesimally short control valve (CV) pipe that allows flow only from the node to the reservoir.
4. [Artificial tank elevation] = [new node elevation].

With these modifications, demand at each pressure-deficient junction in the algorithm is treated as an unknown while a pressure threshold is imposed. Figure 7.1 shows a flowchart of the semi-pressure-driven algorithm. Note that the algorithm proceeds in an iterative manner. That is, if one or more artificial reservoirs receives more water than the nodes demand, those artificial reservoirs are removed from the network and the original elevations and demands at the corresponding nodes are restored. A performance index, later to be used for network reliability/availability assessment, can be defined at each node for a steady-state analysis as

$$ADF_j = \frac{Q_{avl}}{D_j}$$

(7.6)

where $ADF_j =$ available demand fraction at node $j$

$Q_{avl} =$ available flow to node $j$

$D_j =$ total consumer demand allocated at node $j$

A networkwide-available-demand fraction is then given by
Start with a given failure scenario
Remove the components that fail

Run hydraulics using EPANET

Beginning of demand-driven ADF method

Record network nodes at which pressures are below the minimum acceptable

Modify pressure-deficient node properties

Assign an artificial tank to each pressure deficient node.

Run hydraulics using EPANET

Record flow into artificial tank at each node

Do any of artificial tanks receive more water than needed? Yes

Are all pressures sufficient? Yes

Calculate Nodal and Network ADFs

STOP

No

Do any of artificial tanks receive more water than needed? Yes

Remove those artificial tanks from the system. Set corresponding node’s properties to original values

No

Display warning message: “Severe Failure Mode”

FIGURE 7.1 Flowchart of SPDA algorithm.
Similarly, for an extended period simulation (EPS) analysis, nodal ADFs are given by

$$ADF_j = \frac{\sum_{t=1}^{nt} Q_{j}^{opt}}{\sum_{t=1}^{nt} D_{j}} \quad (7.8)$$

where \( t \) is the time step and \( nt \) is the total number of time steps from the time of component failure to when the network starts its normal operation following restoration of the malfunctioning equipment. Random pipe breaks typically take from several hours to a day to repair.

Systemwide ADF for an EPS analysis is then given by

$$ADF_{net} = \frac{\sum_{t=1}^{nt} \sum_{all \ nodes} Q_{j}^{opt}}{\sum_{t=1}^{nt} \sum_{all \ nodes} D_{j}} \quad (7.9)$$

It can be shown, mathematically, that the systemwide ADF is equivalent to the demand weighted average of nodal ADFs. Thus, unlike conventional reliability models based on pressure deficiencies, there is a deterministic relationship between nodal reliabilities and the system reliability in this approach.

### 7.4 RELIABILITY MODELS

SPDA, developed by Ozger (2003) for predicting deficient performance of a network under partially failed conditions, can be used for network reliability assessment considering water pipe breaks. Water main failures can be modeled using the homogeneous Poisson process (HPP) if failure rates are stationary with respect to pipe ages. Alternatively, if deterioration causes an increase in pipe break rates with time, then the nonhomogeneous Poisson process (NHPP) can be used. Both models are applicable for extended period simulation as well as steady-state analysis for reliability assessment of a network.

#### 7.4.1 HPP-Based Reliability Model

Pipe break rates in a distribution system can be determined from historical break/repair data. If such data indicate a monotonic break rate, the network is said to be nondeteriorating and consequently, reliability assessment of the network can be done using HPP for modeling the pipe failures. Two types of reliability measures are available if a network is found nondeteriorating. First, the reliability measure is based on individual pipe failure probabilities. Here, the probability of failure of an individual pipe is given by

$$p_i = 1 - e^{-\beta_i} \quad (7.10a)$$

and
\[ \beta_i = \lambda_i L_i \]  

(7.10b)

where \( \beta_i \) = expected number of failures per year for pipe \( i \)
\( \lambda_i \) = expected number of failures per year per unit length of pipe \( i \)
\( L_i \) = length of pipe \( i \)

Then, the first type of reliability measure based on individual pipe failure probabilities is given, following the work of Shinstine et al. (2002), by

\[ R_{net} = 1 - \sum_{i=1}^{np} (1 - ADF_{net,i}) p_i \]  

(7.11)

where \( ADF_{net,i} \) = network available demand fraction resulting from pipe \( i \) failure
\( p_i \) = probability of failure of pipe \( i \)
\( np \) = the number of pipes in the network

The reliability measure in Eq. (7.11) does not consider that pipes are repairable components. A more appropriate measure can be evaluated if the mean time between failure (MTBF) and the mean time to repair (MTTR) for individual pipes are known. This measure is called mechanical availability (\( ma \)) and given for individual pipes by (Ross, 1985; Cullinane, 1986)

\[ ma = \frac{MTBF}{MTBF + MTTR} \]  

(7.12)

where MTTR is the mean time to repair and MTBF is the mean time between failures and is given by

\[ MTBF = \frac{1}{\lambda L} \]  

(7.13)

where \( \beta \) is the expected number of failures per year per unit length of pipe \( i \), and \( L \) is the pipe length. For simplicity, the constant value of 1 day can be assumed for MTTR.

Mechanical unavailability of a pipe is then given by

\[ mu_i = 1 - ma = 1 - \frac{MTBF}{MTBF + MTTR} \]  

(7.14)

The probability that all the pipes in a distribution network are operational is given by

\[ MA_{net} = \prod_{i=1}^{np} ma_i \]  

(7.15)

where \( np \) = total number of pipes. The probability of a failure of the \( i \)th pipe and all others remaining operational is given by:

\[ u_i = MA_{net} \cdot \frac{mu_i}{ma_i} \]  

(7.16)

The event of simultaneous failure of two pipes (the \( i \)th and \( k \)th pipes) has a probability of

\[ u_{ik} = MA_{net} \cdot \frac{mu_i}{ma_i} \cdot \frac{mu_k}{ma_k} \]  

(7.17)
Finally, using the zero-, first-, and second-order pipe failure probabilities above, network availability can be determined (Fujiwara and De Silva, 1990) from

\[ A_{\text{net}} = ADF_{\text{net}}^0 \cdot MA_{\text{net}} + \sum_{i=1}^{n_p} ADF_{\text{net}}^{-i} \cdot u_i + \sum_{i=1}^{n_p} \sum_{k=1}^{n_p} ADF_{\text{net}}^{-ik} \cdot u_{ik} \]

(7.18)

where

- \( ADF_{\text{net}}^0 \) = available demand fraction of the network with a fully functional network
- \( ADF_{\text{net}}^{-i} \) = available demand fraction of the network with the \( i \)th component removed
- \( ADF_{\text{net}}^{-ik} \) = available demand fraction of the network with the \( i \)th and \( k \)th components removed

Individual nodal availabilities can be obtained using the ADF for the node of concern.

It should be noted that there are two alternatives for calculating the ADFs above. First, they can be determined with static demands at each node. In this case, ADFs can be calculated at each node using a steady-state analysis and the resulting network ADF can be found from Eq. (7.7). If, on the other hand, a repetitive demand pattern exists (e.g., a diurnal pattern with 24-h cycles), nodal and systemwide ADFs can be calculated using the results from an extended period analysis. Nodal ADFs are then given by

\[ ADF_j = \frac{\sum_{t=1}^{n_t} Q_j^{\text{avl}}}{\sum_{t=1}^{n_t} D_j} \]

(7.19)

where \( t \) is the time step and \( n_t \) is the demand pattern length. Similarly, systemwide ADF is given by

\[ ADF_{\text{net}} = \frac{\sum_{t=1}^{n_t} \sum_{\text{all nodes}} Q_j^{\text{avl}}}{\sum_{t=1}^{n_t} \sum_{\text{all nodes}} D_j} \]

(7.20)

### 7.4.2 NHPP-Based Reliability Model

If historical break data indicate a deteriorating network, the reliability model should be based on pipe failure probabilities modeled using the nonhomogeneous Poisson process. The two most commonly applied, time-dependent models for the mean cumulative repair function are the power relation model and the exponential model (Tobias and Trindade, 1995).

The power relation model is given by

\[ M(t) = at^b \]

(7.21)

where \( M(t) \) is the expected number of failures between time zero and \( t \), and \( b \) and \( c \) are empirically determined parameters. The corresponding intensity function is

\[ \lambda(t) = \frac{dM(t)}{dt} = abt^{b-1} \]

(7.22)

Note that if the parameter \( b \) is between 0 and 1, the network is deteriorating. On the other hand, \( b > 1 \) indicates an improving network. The model parameters for the power relation model can be determined from either graphical or analytical analysis. Taking the natural logarithms of each side in Eq. (7.21),

\[ \ln M(T) = \ln(a) + b \ln(t) \]

(7.23)
from which it can be seen that, if the power relation model is appropriate, the plot \( \ln M(t) \) versus \( \ln t \) on linear by linear graph paper should approximate a straight line with intercept \( \ln a \) and slope \( b \).

For a nonrenewal process that can be modeled using a nonhomogeneous Poisson process, Crow (1974) has developed MLEs (Maximum Likelihood Estimates) for the parameters of the power relation model above (Tobias and Trindade, 1995).

Alternatively, the exponential model can be used to model the nonrenewal pipe failure processes. The intensity function then becomes

\[
\lambda(t) = e^{\gamma + \xi t}
\]

The mean repair function can be found from simple integration of the intensity function

\[
M(t) = \int_0^t \lambda(u) \, du = \frac{e^{\gamma + \xi t}}{\xi} \left( e^{\xi t} - 1 \right) = \frac{a}{b} \left( e^{bt} - 1 \right)
\]

where \( a = e^\alpha \).

Similar to the power relation model, parameters of the exponential model can be determined from graphical or analytical analysis. Taking natural logarithms of both sides in Eq. (7.25) yields

\[
\ln M(t) = \ln(a) + bt
\]

Thus, the plot of \( \ln M(t) \) versus \( t \) should approximate a straight line with intercept \( \ln a \) and slope \( b \), if the exponential model is appropriate. Alternatively, analytical approaches can be used to estimate model parameters from MLEs.

If the NHPP model parameters (for either the power relation model or exponential model) are estimated from the historical break data, then the expected number of failures between time \( t_1 \) and \( t_2 \) can be determined from the difference between the mean cumulative repair functions as

\[
E(t_1, t_2) = M(t_2) - M(t_1)
\]

where \( M(t) \) is given by Eq. (7.21) or (7.25). What is more useful than the expected number of failures for reliability assessment is the failure rate as a function of time given by the intensity functions, that is, Eqs. (7.22) and (7.24). Pipe failure probabilities and pipe availabilities at time \( t \) can be calculated from Eqs. (7.10) to (7.18) using the failure rates at time \( t \) given by the appropriate intensity functions. Then, Eqs. (7.11) and (7.18) would yield the instantaneous network reliability and availability at the given time \( t \).

The following derivation can be used if rather than an instantaneous measure, network availability is needed for a time horizon from a given base time. The expected volume of shortage at node \( j \) resulting from link \( l \) failures between time \( t_0 \) and \( t \) is given by

\[
\text{VS}_{jl}(t_0, t) = E_i(t_0, t) (1 - \text{ADF}_{j,l}) Q_{j,l}
\]

where \( \text{ADF}_{j,l} \) and \( Q_{j,l} \) are the available demand fraction and total demand at node \( j \), respectively, during each repair work on link \( l \). If a repetitive demand pattern exists for the duration of given time horizon at node \( j \), the availability for that node between \( t_0 \) and \( t \) is then given by

\[
A_j(t_0, t) = 1 - \frac{\sum_{\text{all links}} E_i(t_0, t) (1 - \text{ADF}_{j,l}) Q_{j,l}}{n_i Q_{j,c}}
\]

where \( n_i \) is the number of demand cycles from time \( t \) to \( t_0 \) and \( Q_{j,c} \) is the total demand at node \( j \) per demand cycle. Note that the above formulation assumes single-order pipe failures only. In other words, it is assumed that no simultaneous pipe failures occur for the given time horizon. Equation (7.29) can be further simplified with the following three assumptions:

- Each failure occurs at the beginning of a demand cycle.
- Repair times are identical for all links (i.e., \( Q_{jl} = Q_j \)).
- The duration of demand cycle and the repair time are equal (i.e., \( Q_{jl} = Q_j \)).
Then Eq. (7.29) reduces to

$$A_j(t_0, t) = 1 - \frac{\sum_{\text{all links}} E_j(t_0, t)(1 - ADF_j)}{n_c}$$

(7.30)

Systemwide availability can be determined by considering all the nodes in the system

$$A_j(t_0, t) = 1 - \frac{\sum_{\text{all links}} \sum_{\text{all nodes}} E_j(t_0, t)(1 - ADF_j)Q_{j,i}}{\sum_{\text{all nodes}} n_c \cdot Q_{j,c}}$$

(7.31)

### 7.5 OPTIMIZATION MODEL

#### 7.5.1 Effect of Valving on System Reliability

A valve is defined as a mechanical device by which the flow of fluid may be started, stopped, or regulated by a movable part that opens or obstructs passage. Valves are used in water distribution systems for a variety of purposes. Accordingly, different types of valves can be grouped into four (Ysusi, 2000): (1) isolation valves, (2) control valves, (3) blow-offs, and (4) air-release and vacuum-release valves.

Control valves are used to regulate flow or pressure at different points of the system by creating headloss or pressure differential between upstream and downstream sections. The mission of isolation valves is to isolate a portion of the system whenever system repair, inspection, or maintenance is required at that segment. The two most common types of isolation valves are gate valves and butterfly valves. Unlike the control valves, little attention has been paid to the layout and operation of isolation valves. Yet, from a reliability point of view, isolation valves are of great interest because they determine the extent of isolation should a portion of the system need repair, inspection, or maintenance. As an extreme example, a system with no valves would have to be shut down completely at the source for any maintenance. On the other hand, if there is sufficient valving in the system the outage or isolation can be limited to a small portion of the system.

Little is known about the reliability of isolation valves because such valves are fully opened under normal operating conditions and fully closed only when isolation of a portion of the system is needed. Valves that remain in either position for extended periods of time become difficult (or even impossible) to operate (Ysusi, 2000). Therefore, they should be exercised at least once a year or more often if the water is corrosive or dirty. According to Walski (2000), valve exercising is the single most important form of preventive maintenance for improving reliability.

Valves are typically located around junctions in a distribution system. Figure 7.2 shows a typical valve scheme for a typical city block. Their locations and numbers are usually determined using the rules of thumb that have evolved over the years. The most commonly used rule of thumb is to install a minimum of \( n \) valves around a junction to which \( n \) links (pipes) are connected. In other words, a minimum of three valves should be placed at every cross-intersection and a minimum of two valves is needed around every T-section.

Although regular exercising programs may increase their reliability, valves are not 100 percent reliable whenever they need to be closed for isolation of a distribution segment. Therefore, another consideration for valve placement is that no more than four valves should need to be operated when an isolation is needed. Consider, for example, pipe 1 in Fig 7.2. Should this pipe fail, six valves (valves a, b, c, d, e, and f) need to be closed in order to isolate the break from the remainder of the network. If the isolation is achieved through closing of all six valves, only those consumers whose service connection is linked to pipe segment 1 will be without water during the repair work. However, if there is a percent probability \( p \) that each valve can be operated (closed) successfully, the mission can only be successful with probability \( p^6 \). For example, for \( p = 0.90 \), the chance of operating all six is 53 percent \((0.90^6 = 0.53)\).

*The probability of that scenario occurring is \((1 - p)(p^5) = (1 - 0.90)(0.90^5) = 5.9 \text{ percent.}\)
be closed for complete isolation. That means customers receiving water from pipe segment 4 will also be without water during the repair work on pipe segment 1.

The reliability of valves themselves has not been implicitly or explicitly incorporated in reliability assessments to date. Perhaps the main reason for this is that the human factor is the most important factor in determining their reliability. The more frequent the valve exercising programs, the greater the chance that they will operate when needed.

Walski (2000) gives a few more rules of thumb for locating isolation valves in distribution systems. The AWWA’s Introduction to Water Distribution (1986) states that “Isolating valves in the distribution system should be located less than 500 ft apart (150 m) in business districts and less than 800 ft (240 m) apart in other parts of the system. It is a good practice to have valves located at the end of each block so that only one block will be without water during repair work.” Similarly, The Ten States Standards state “Where systems serve widely scattered customers and where future development is not expected, the valve spacing should not exceed one mile.” Finally, Walski (2000) points out a good practice, which requires placement of an isolation valve for every fire hydrant lateral.

The effect of valving in distribution system reliability has two aspects. First, some consumers will be out of service when a pipe break occurs in the system. The spatial extent of isolation is mostly determined by the locations of valves in the system. It is theoretically possible to determine the impact of isolation, induced by a link failure, by calculating the volume of water of which consumers located in the isolated portion of the system are deprived during the repair work. Let us call this the shut-off volume. Bouchart and Goulter (1991) studied this aspect of valving and expressed the reliability in terms of the expected volume of deficit along the isolated segment(s).

It should be noted, however, that network hydraulic models rarely possess the level of detail to determine the shut-off volume accurately. Typically, demands from city blocks are assigned to nearby junctions whereas in reality demands are withdrawn via the service connections located along the supply lines. The concept of lumping the continuous demands along the supply lines into single “point” loads at the network junctions using various demand allocation techniques is required in many cases (e.g., hydraulic simulation) for mathematical tractability (Bouchart et al., 1991).

The second implication of the valve scheme, besides determining the extent of direct shut-off (or volume of deficit), is the effect of isolation on system connectivity. The connectivity of a network is significantly affected by the valving scheme. For example, when pipe 1 fails in Figs. 7.3 and 7.4, connectivity among pipe segments 2, 3, and 4 would be preserved in Fig. 7.3 and it would be lost in the case of Fig. 7.4. The same goes for the links 5, 6, and 7.

Unlike the expected volume of deficit, the effect of the valve scheme on a network connectivity can only be assessed with hydraulic simulation of the deficient network.
7.5.2 Optimal Valve Scheme Problem

Perhaps the single most important factor affecting the extent of isolation resulting from a pipe failure in a water distribution network is the valve layout in the vicinity of the break. A network with no isolation valves would have to be shut down at the source for any maintenance work. The more valves are situated in a network, the less the impact of a failure because the isolation can be restricted to a smaller portion of the system. Thus, only by knowing the valve layout in the vicinity of a pipe break is it possible to determine which links are to be closed and the deficient performance of the remaining network.

A fully valved network from a modeling approach is a network where each pipe in the network has isolation valves at both ends. Because demands are assumed lumped into junctions for mathematical tractability in conventional network modeling approaches, it would not make any difference if there were intermediate valves or not from a modeling point of view. In real life, on the other hand, demands are almost continuously distributed along a pipeline (withdrawn via service connections). Thus, the more intermediate valves are located on a pipe, the smaller the number of out-of-service consumers will be during the repair work following a pipe break.

In formulating the optimal valve location problem, a decision variable can be defined for each junction and the pipe that connects that junction to the rest of the system as

\[ x_{ij} \]

is the decision on whether to place a valve at the end of pipe \( i \) connected to junction \( j \)

\[ x_{ij} = 1 \text{ if an isolation valve is placed and 0 otherwise} \]

Thus the total number of decision variables is twice the number of pipes in a given network. Note that a solution to the problem is a binary string resulting in a combinatorial optimization problem. Due to the complex nature of the problem it is impossible to write the objective function explicitly in terms of the decision variables. It is well established that heuristic optimization techniques are very efficient in finding a near-optimal solution to such complex problems.
Because the decision variables of our problem are binary, the solution space for a network with \( n \) pipes contains \( 2^n \) solutions. The multiplier 2 of \( n \) comes from the fact that each pipe carries two decision variables, whether to place a valve at each end of a pipe. For a water distribution network with 50 pipes, there would be a total of \( 2^{50} = 1.26765 \times 10^{15} \) solutions. As the number of pipes reaches thousands (typical for even midsize networks), problem solution space and the time required for complete enumeration becomes even immeasurable.

Let us now add the following rule of thumb as a constraint to our problem: install \( n - 1 \) valves around a junction to which \( n \) links (pipes) are connected. For example, three valves should be placed at every cross-intersection and two valves are needed around every T-section. Mathematically, the “one less valve” constraint can be expressed as

\[
\sum_{j=1}^{NJ} x_{ij} = L_j - 1 \quad \text{for } j = 1, \ldots, NJ
\]  

(7.32)

where \( x_{ij} \) = binary decision variable defined above

\( L_j \) = number of links around junction \( j \)

\( NJ \) = number of junctions in the network excluding dead-end nodes
It is assumed that this constraint does not apply to dead-end nodes where an isolation valve is always assigned unconditionally. The size of the solution space with the one less valve constraint is then given by

\[ NS = \prod_{j=1}^{NJ} L_j \]  

(7.33)

Thus, adding this constraint to the valve optimization problem would result in a significant reduction in the size of solution space. Note that the degree of this reduction depends largely on the level of looping in the network. Because total enumeration is impossible in many cases, local search heuristics such as simulated annealing or genetic algorithms can be used to find a near-optimal solution to the problem.

The objective function can be either the reliability or the availability of the given network

Maximize \( A_{\text{net}} \)

or

Maximize \( R_{\text{net}} \)

Note that the objective function does not include any cost term that depends largely on valve diameters in a given solution. The cost of a valve decreases as pipe diameter gets smaller. On the other hand, the probability of a pipe failure increases as the diameter gets smaller. This suggests that smaller diameter pipes are more likely to affect network reliability than larger diameter pipes and thus are more likely to have isolation valves in an optimal solution. However, this does not necessarily mean that isolation valves should be placed on all but the largest diameter pipe around each junction in an optimal valve scheme subject to the “one less valve” constraint, because there are other factors affecting a pipe’s break rate such as a pipe’s length. Connectivity also determines how important a pipe is from a reliability point of view. For example, a transmission main may have a low probability of failure but large supply deficiencies may result from its failure. Nevertheless, we expect that an optimal valve scheme subject to the “one less valve” constraint will also be one of the least-cost solutions for the reasons explained above and thus cost can be omitted from the objective function.

### 7.5.3 Simulated Annealing for Valve Optimization Problem

Simulated annealing (SA) has been termed a “biased random walk.” The simulated annealing algorithm requires defining a local search neighborhood of the current solution in order to make its random moves. Although there are no strict rules for defining a neighborhood, a neighboring solution should be only slightly different from the current solution in order for the algorithm to be efficient. In order to maintain solution feasibility with respect to the “one less valve” constraint in the valve optimization problem, a random move to a neighboring solution is achieved as follows.

A junction in the network is chosen randomly and an isolation valve is added to the only valveless pipe around that junction. Next, a random pipe around the same junction is selected and the valve on that pipe is removed. However, this random pipe cannot be the same pipe as the valveless pipe, which is just assigned an isolation valve. This is done in order not to repeat the current solution. With this strategy, the “one less valve” constraint is maintained and the next solution is guaranteed feasible. The number of neighboring solutions \( NS \) with this random move approach is given by

\[ NS = \sum_{j=1}^{NJ} (L_j - 1) \]  

(7.34)

where \( L_j \) is the number of pipes connected to junction \( j \) and \( NJ \) is the number of junctions in the network excluding dead-end nodes.

Figure 7.5 shows the flowchart of valve optimization problem where ADF-based hydraulic performance is measured using the SPDA explained in previous sections.
The primary advantage of SA is the ability to escape from local optima. Thus, the ability to find the global optimum is not related to the initial conditions (i.e., the starting solution). Analogous to the physical process, simulated annealing slowly reduces the temperature $T$ of the solution, thus gradually reducing the probability of accepting a poor move with time. The schedule by which $T$ is reduced is called the cooling schedule and is critical to the success of SA. Two important parameters governing the cooling schedule are the step size for perturbation and $T$.

The major difficulty (art) in implementation of the simulated annealing algorithm is that there is no obvious analogy for the temperature $T$ with respect to a free parameter in the combinatorial problem. If the initial temperature is too low or cooling is done insufficiently slowly the algorithm may trap in a local minimum. In order to avoid entrapment in local minima (quenching) the annealing schedule, the choice of initial temperature, number of iterations performed at each temperature, and how much the temperature is decremented at each step as cooling proceeds should be carefully planned.

The recommended parameter settings are step sizes and $T$ values that allow approximately 80 percent of the poor moves to be accepted. The initial temperature for the SA algorithm can be determined to allow a certain fraction of poor moves to be accepted. One way of doing this is to search the entire neighborhood of the initial solution before the optimization starts and determine the average decrease $\Delta_{obj}$ of the objective function (assuming the objective is to maximize) with poor moves. Then the initial acceptance rate using the probability of acceptance for the Metropolis algorithm $e^{\Delta_{obj}/T}$, is given by

$$P_{init} = e^{\Delta_{obj}/T_{init}} \quad (7.35)$$

where $\Delta_{obj} = \text{average decrease of objective value from all poor moves}$

$P_{init} = \text{initial acceptance rate}$

$T_{init} = \text{user-specified initial temperature}$

Similarly, the final acceptance rate can be estimated using the final temperature $T_{final}$ in the place of $T_{init}$ in Eq. (7.35). Rearranging Eq. (7.35), initial temperature required for $P_{init}$ is given by

$$T_{init} = \frac{\Delta_{obj}}{\ln(P_{init})} \quad (7.36)$$

For example, if approximately 60 percent of the poor moves are accepted at the beginning of SA, then Eq. (7.36) reduces to $T_{init} = -1.95762 \Delta_{obj}$. Because this approach of setting the initial temperature might be sensitive to the initial solution the algorithm starts with, this procedure can be repeated for several different initial solutions and taking the average of the objective function decrease with poor moves.

The way in which the temperature is decreased in SA is known as the cooling schedule. The two most commonly used cooling schedules, in practice, are a linear cooling schedule ($T_{next} = T_{previous} - \Delta T$) and a geometric cooling schedule ($T_{next} = C \times T_{previous}$) where $C < 1.0$. The choice of cooling schedule is usually problem specific and should be considered as another tune-up parameter for an efficient optimization run.

There are several ways of determining when to stop an SA optimization. One way is to stop when the incumbent solution has not changed for a predetermined number of iterations. Another approach is halting the process when the total number of iterations reaches a predetermined limit. Another criterion is to stop when neighbors are being accepted at a rate of $\lambda$ or less (e.g., $\lambda = 0.01$).

A less likely stopping criterion is when the incumbent value is sufficiently close to a good predetermined upper bound for the global optima. This is less likely because in most optimization problems, it is difficult, if not impossible, to find a good upper bound for the global optimal solution because the objective function is somewhat the combination of a problem-specific measure (e.g., reliability or availability) and the cost of the given solution. In the valve optimization problem, however, cost is eliminated from the objective function. Therefore, a good upper bound for the global optima can be obtained by determining the objective value (i.e., reliability or availability) of the fully valved network. Figure 7.6 shows the flowchart of valve optimization using simulated annealing.
Start with a random or user-specified feasible solution subject to “One less valve constraint”

Calculate initial network R/A using SPDA

Randomly choose a feasible move:
(1) Choose a random junction.
(2) Add a valve to the valveless pipe around that junction.
(3) Choose a random pipe around the same junction and remove the existing valve.

Calculate the new network R/A for the new solution using SPDA

If $\Delta \text{obj} \geq 0$, accept the new solution.
else, generate a random number $0 < r < 1$, accept the new solution if $e^{\Delta \text{obj}/T} \geq r$
reject otherwise and return to the previous solution.

$\Delta \text{obj} \geq 0$?

No

Yes

If the new solution has a superior R/A to that of the incumbent solution, set the new solution as the new incumbent solution.

At least one stopping criteria met?

Yes

STOP

No

Beginning a new loop?

Yes

Reduce temperature: $T_{\text{new}} = C \cdot T_{\text{previous}}$

No

No
The valve optimization using SA is illustrated using the example network shown in Fig. 7.7. The network consists of two fixed reservoirs, 21 pipes, and 13 junctions. Pipe characteristics and the junction properties are summarized in Tables 7.1 and Table 7.2, respectively. Demands are expressed in m$^3$/hour (CMH) and the calculated pressures are in meters. The total network demand for the steady-state simulation is 3146.4 cubic meters per hour.

A global value of 20 psi is assumed as threshold pressure for all nonzero demand junctions. Mean time to repair (MTTR) is assumed one day. The cooling schedule targets 1000 iterations with an initial temperature of 6, and 16 iterations per loop, and the temperature multiplier is 0.85. In other words, temperature is reduced by a factor of 0.85 every 16 iterations in the geometric cooling schedule. The initial solution is specified random and the optimization type is reliability-based.
### TABLE 7.1 Pipe Characteristics

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<th>Pipe ID</th>
<th>Length, m</th>
<th>D, mm</th>
<th>C, H-W*</th>
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*H-W = Hazen-Williams.

### TABLE 7.2 Node Characteristics

<table>
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<tr>
<th>Node ID</th>
<th>Elevation, m</th>
<th>Demand, CMH</th>
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<tr>
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<tr>
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<tr>
<td>13</td>
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</tr>
<tr>
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</tr>
<tr>
<td>RES2</td>
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A test run involving 50 randomly selected initial solutions reveals that the initial and final acceptance rates among poor moves will be approximately 99.97 and 0.2 percent. This indicates that almost all the prospective moves will be accepted at the beginning of the simulation and the rate of acceptance decreases significantly at the end. The test run also determines the size of the solution space as 1,492,992 with the “one less valve” constraint and the neighborhood size as 27. The upper bound, corresponding to a fully valved network, is 0.9537.

The results indicate that there were 497 improving moves that were unconditionally accepted because they had a superior objective value to that of the current solution. Out of the remaining 503 nonimproving prospective moves, 403 were accepted although the objective function decreased with each, and 100 were rejected by the Metropolis algorithm. Figure 7.8 shows the cumulative number of accepted, accepted-poor, and rejected moves at the end of each loop. Although almost all the nonimproving moves were accepted at the beginning, the latter part of Fig. 7.8 shows the impact of reducing temperature $T$ as the algorithm proceeds. The rejection rate starts going up at about the 30th loop. By the end of the simulation the acceptance rate among the poor-moves decreases to almost zero as also expected from the results of the test run. Overall, 80 percent of the poor-moves were accepted as targeted at the beginning.

The final incumbent value was 0.9492, a number within 0.5 percent of the upper bound found by assuming a fully valved network. Figure 7.9 shows the objective function value change with the solution progress. Note that about halfway through the simulation fluctuations in the objective value start diminishing, reflecting the effect of decreasing the temperature. In other words, fewer and fewer poor solutions are accepted as the algorithm proceeds. This consequence of the temperature effect can best be seen toward the end where the objective value variation has a much smaller range than the first two thirds of the simulation. This type of behavior is typical for SA. Gross features of the eventual solution appear at the higher temperatures with fine details developing at low temperatures (Kirkpatrick et al., 1983). The fact that the search escaped local optima many times and the final incumbent occurred toward the end of the simulation shows the effectiveness of simulated annealing.
Figure 7.10 shows the valve scheme for the final incumbent solution. Note that the “one less valve” constraint is preserved throughout the simulation and the optimal solution shown is different from the least-cost solution with respect to only two valve locations. In other words, the optimal solution is also one of the least-cost solutions as expected.

REFERENCES


FIGURE 7.10 Optimal valve layout for example network I from SA.


