

**CEE 341**

***Fluid Mechanics for Civil Engineers***

***Lab Manual***

Salt River Project Hydraulic Engineering Laboratory  
Department of Civil and Environmental Engineering  
College of Engineering and Applied Sciences  
Arizona State University

*by*

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# ***Dedication***

This work is dedicated to the memory of Paul F. Ruff. Without his hard work and continued dedication to the undergraduate experience at Arizona State University, the hydraulics laboratory, and this manual in particular, would not exist.

The authors would also like to acknowledge the contributions of Ms. Layla Hedayat, Mr. Paul Dahlen, Ms. Chandrika Manepally, and Ms. Laurie Marin, who provided valuable insight, experience, and comments.

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# ***Note to Students***

This lab manual is intended to guide you through five experiments to be performed in the Salt River Project Hydraulics Laboratory. Because of the nature of the course and the lab facilities, you will be required to perform some labs on material not yet covered in class. This will require an effort on your part to read the relevant sections of the textbook, as well as the lab manual *before* you come to lab. The experiments will make much more sense to you if you are prepared, and they will take much less time to complete.

The lab report will consist of several sections. Requirements of each section are explained in detail on page 6. In addition, the points assigned to each part are also shown in the right-hand margin. Your grade on each report will be based on this point scale.

In the “real world” engineers are required to follow standard specifications with regard to report writing, therefore part of the grade is dependent on your ability to follow directions. You may find it useful to have page 6 nearby when you are preparing your report.

A lab report is considered a technical report. As such, it must be professional and neat. The report must be written with a word processor of your choice. All drawings must be made with a straight-edge and clearly labeled or drawn on the computer. Plots are an integral part of technical reports and must be professional:

- Plots must be computer generated;
- Scales must be readable to the same accuracy as the data obtained during the test;
- Axes must be labelled;
- Units of variables must be shown;
- Various run results must be distinguished by different symbols and/or colors and each curve must be identified by a legend or a title;
- Curves derived from data must show experimental data points;
- Graphs must be drawn as smooth curves that represent an average of the experimentally determined data.
- Curves derived from an equation should contain no symbols, but show the equation of the curve;

# **Report Requirements**

*Each group member* will submit a laboratory report with the following components:

		Points (Out of 100)
a)	<u>Title page</u>	The following information <b>only</b> : Course number and title Descriptive name of experiment Date of submittal Your name  <b>(1)</b>
b)	<u>Table of Contents</u>	List of the report headings and contents of appendix <b>(1)</b> List of tables with titles <b>(1)</b> List of figures with titles <b>(1)</b> Corresponding page numbers and all pages numbered <b>(1)</b>
c)	<u>Objective</u>	Clear, brief statement of the investigation's purpose or goal. <b>(5)</b>
d)	<u>Theory</u>	Discussion of the principles behind the experiment, including assumptions made and their validity <b>(10)</b> Clarification of the equations to be used <b>(10)</b>
e)	<u>Anticipated results</u>	Numerical and qualitative expected results. Include expected values that you will compare your results to, expected shape of plots, etc. Hint: nearly everything in your results section should have an anticipated result. <b>(10)</b>
f)	<u>Apparatus</u>	List and describe the various tools and apparatus. Provide a drawing of the apparatus with all important dimensions and units of measurements <b>(5)</b>
g)	<u>Procedure</u>	Orderly concise list of the steps <b>your group</b> performed <b>(5)</b> <b>Do not simply copy the manual!</b>
h)	<u>Results</u>	Review the results obtained <b>(20)</b> NOTE: do not simply show tables or charts. Every item must have some explanation. Typically, this means pasting objects into your document!
i)	<u>Discussion</u>	Discuss the accuracy of <u>each</u> result, numerically if possible <b>(5)</b> Analyze probable sources of error (i.e. what effect did they have and how much?) <b>(10)</b>
j)	<u>Conclusion</u>	Was the initial objective met? Why or why not? <b>(5)</b>
k)	<u>Critique</u>	Briefly critique the experiment and the lab manual <b>(5)</b> Please give concrete suggestions if possible.

- l) References                      References from the body of the report                      **(2)**
- m) Appendix                      Original data and data control sheets                      **(3)**
- n) Sample hand calculations (Optional but beneficial)

# ***Dimensionless Fluid Parameters***

Several dimensionless parameters widely used in fluid mechanics are the Reynolds number, the Froude number, and the Mach number. These parameters are used to model the quality of flow characteristics (e.g. turbulent vs. laminar flow for the Reynolds number) and to reference certain flow conditions (e.g. subcritical vs. supercritical for the Froude number). These numbers will be referenced several times throughout the lab manual.

The Reynolds number is defined as the ratio of inertial forces to viscous forces and is given by:

$$R_e = \frac{Vd\rho}{\mu}$$

where  $V$  is the velocity,  $d$  is the diameter, and  $\rho$  is the density of the fluid, and  $\mu$  is the viscosity. The Reynolds number can be alternately described by:

$$R_e = \frac{VD}{\nu}$$

where  $\nu = \mu/\rho$  is the kinematic viscosity of the fluid.

The Froude number (rhymes with “food”) is defined as the square root of the ratio of inertial forces to gravity forces. It is given by the equation:

$$F_r = \frac{V}{\sqrt{gy}}$$

where  $V$  is the velocity,  $g$  is the gravitational constant, and  $y$  is the depth of flow.

The Mach number is the ratio between the velocity of flow and the speed of sound and is represented by the formula:

$$M = \frac{V}{c}$$

where  $V$  is the velocity and  $c$  is the speed of sound.



# Analysis of Log-Log Plots

While analyzing the results of experiments, you will often come across equations of the form:

$$y = bx^m \quad (1)$$

Consider two examples:

$$y_1 = 10x^2 \quad (2a)$$

$$y_2 = 5x^{50} \quad (2b)$$

These can easily be plotted using an arithmetic scale; as shown in Figure 1 and Figure 2. While Figure 1 is reasonably informative, Figure 2 is clearly not! For example, Figure 2 seems to indicate that  $5(25)^{50}$  is essentially zero, but obviously it is not. The trouble lies in the large range of the  $y$  values that results when the exponent  $m$  is large.

The usual approach to plotting these kinds of equations is to use log-scale for the  $x$  and  $y$  axes. The justification for this follows

- First take the logarithm of both sides of (1):

$$\log y = \log(bx^m) \quad (3)$$

- Then using logarithm rules (look these up if you don't remember them; they are used throughout CEE341):

$$\log y = m \log x + \log b \quad (4)$$

- Compare (4) to the usual equation for a line:

$$y' = m'x' + b' \quad (5)$$

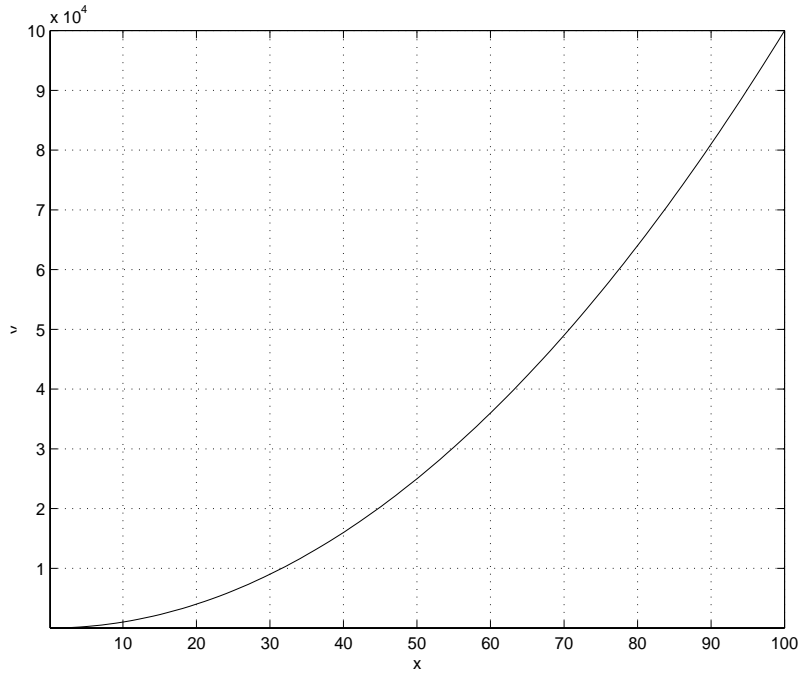
- The following relationships are clear:

$$y' = \log y$$

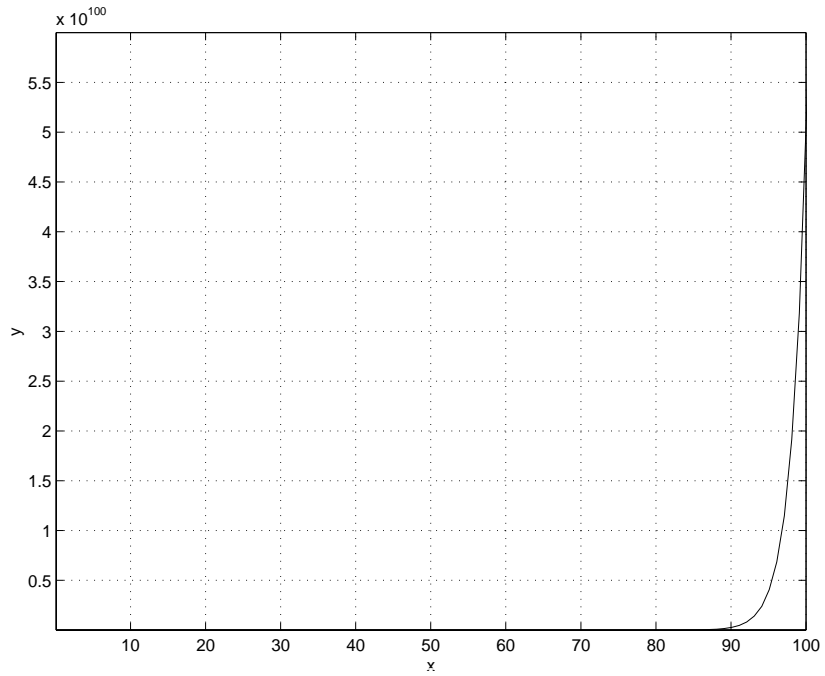
$$x' = \log x$$

$$b' = \log b$$

$$m' = m$$

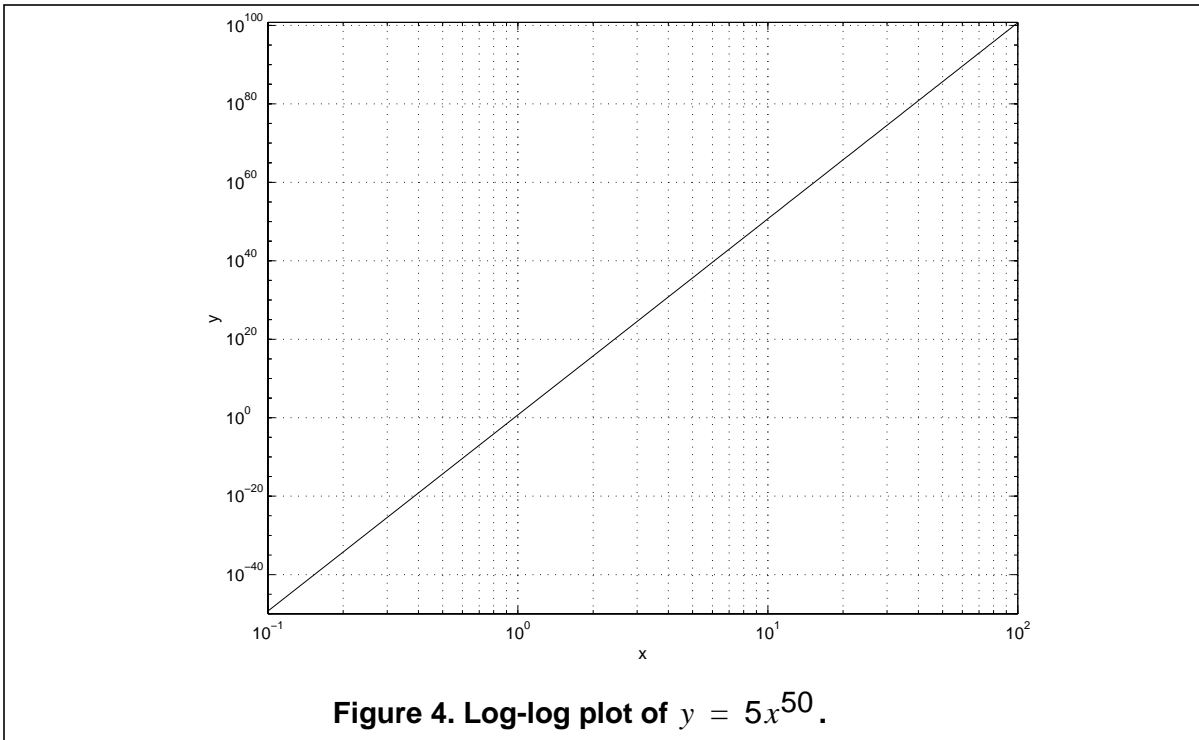
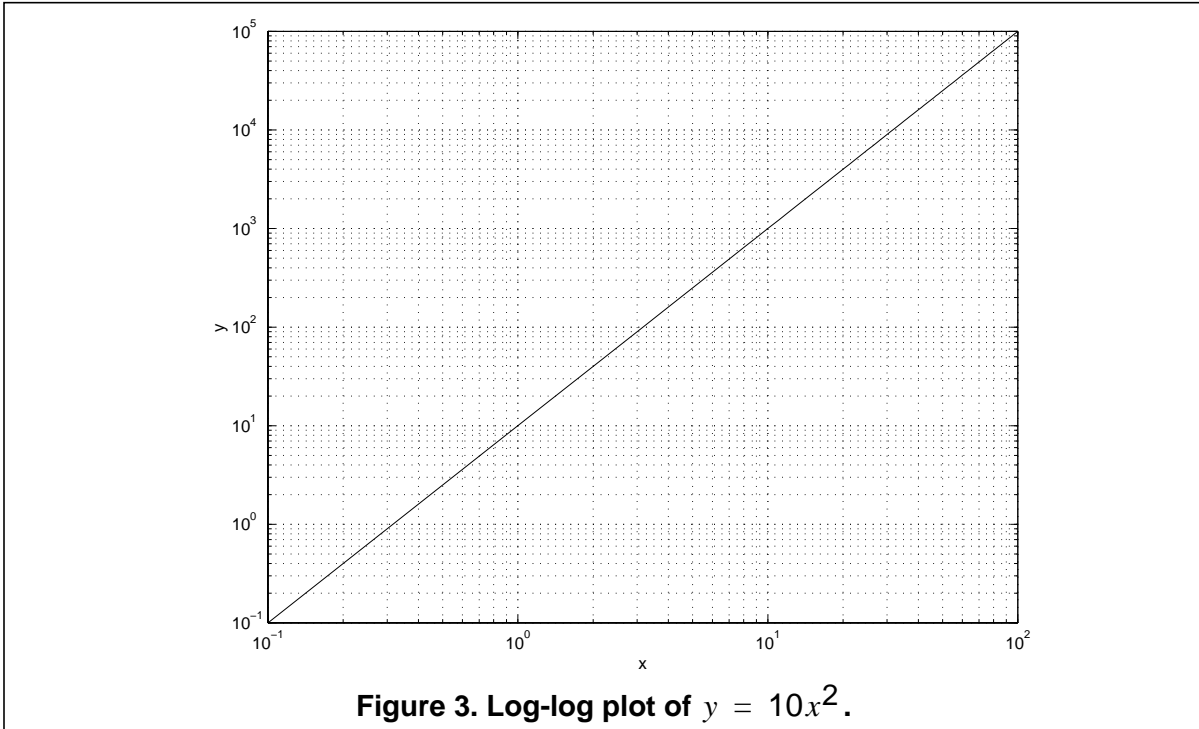


**Figure 1. Arithmetic plot of  $y = 10x^2$**

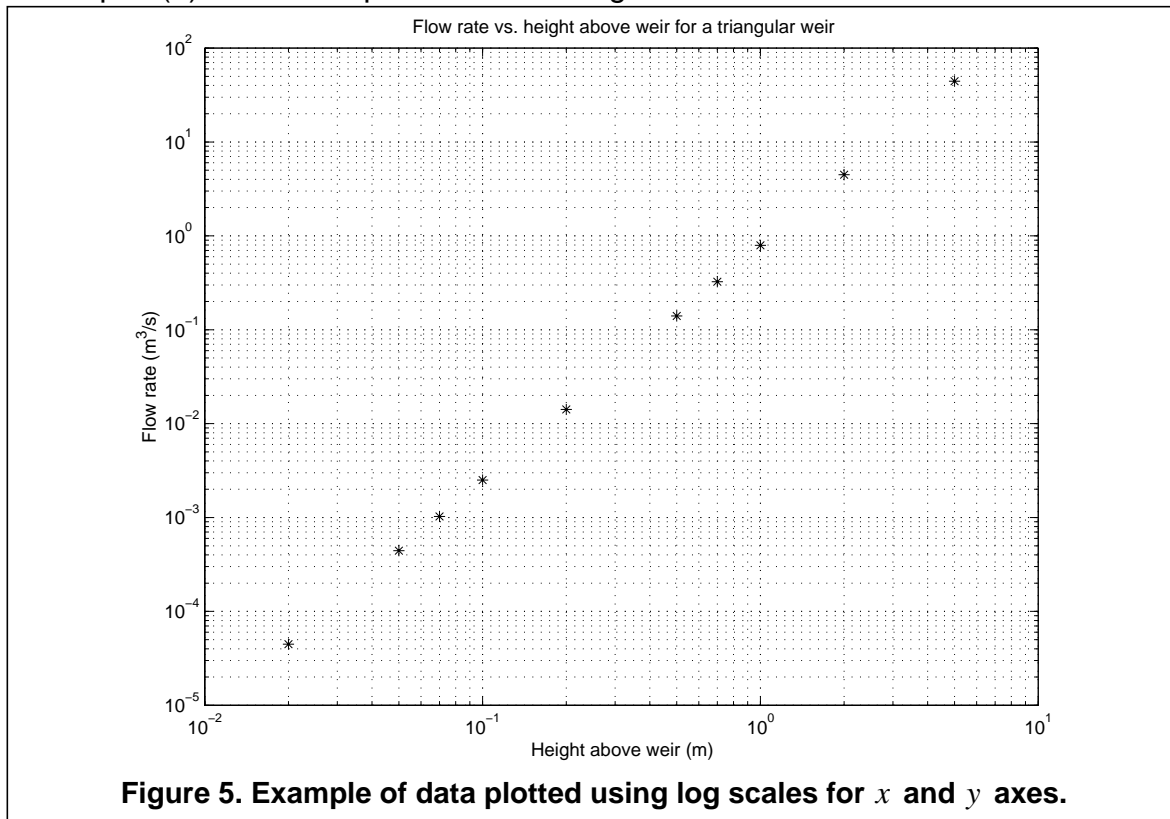


**Figure 2. Arithmetic plot of  $y = 5x^{50}$**

In other words, if one plots  $\log x$  versus  $\log y$  instead of  $x$  versus  $y$ , the result will be linear, regardless of the values of  $b$  and  $m$ . However, simply plotting  $y$  versus  $x$  using log-scales for the  $x$  and  $y$  axes accomplishes *exactly* the same thing (this way, there is no need to actually take the logarithms of  $x$  and  $y$ !) Figure 3 and Figure 4 demonstrate that plotting equations of the form (1) on log-log scales results in a straight line (although, of course, these two examples do not constitute a proof)



During your data analysis, you will often be asked to plot your data using log-log scales. If these plots resemble straight lines, then you might assume your data follows the power relationship in (1). One example is shown in Figure 5:



Of course, you can use a software package like Excel to determine the coefficients  $b$  and  $m$ , but it is absolutely essential that you know how to estimate  $b$  and  $m$  by eye! [This will be on the first test].

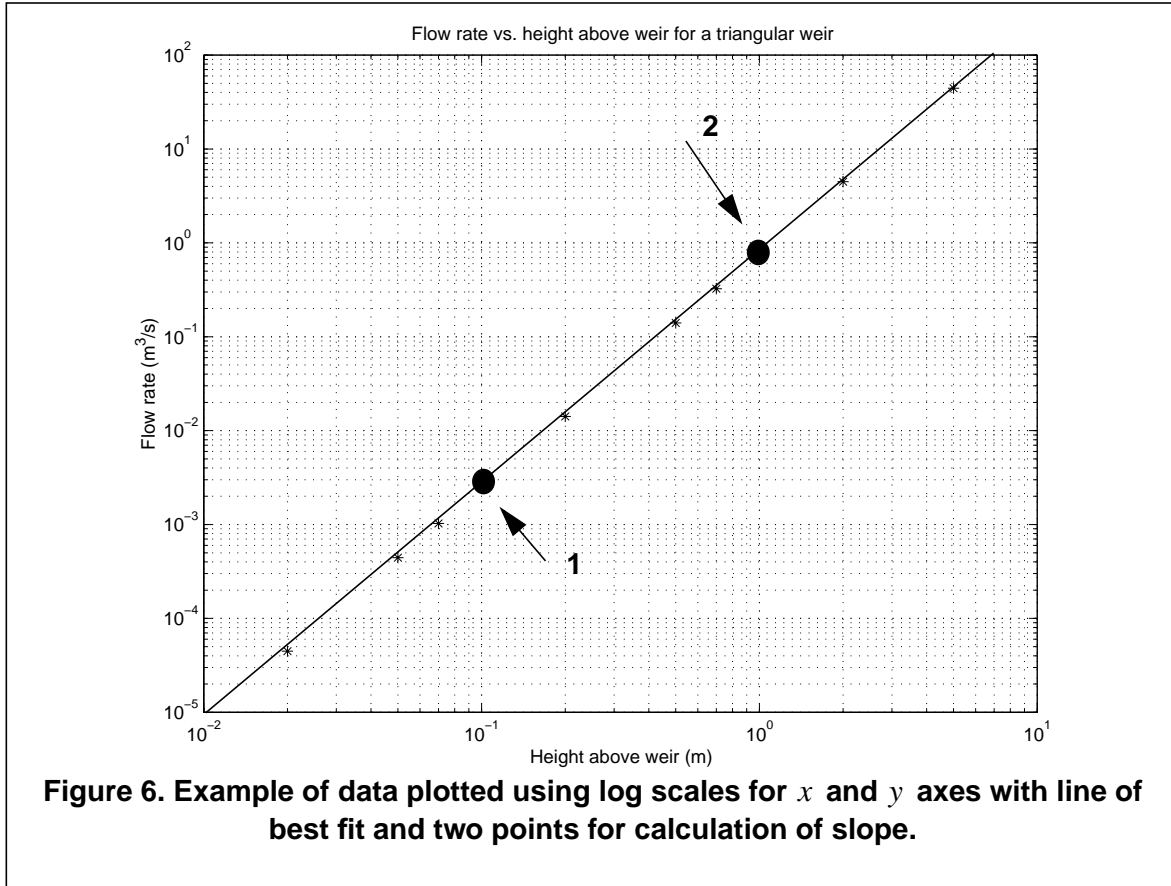
First, draw a line of best fit (by eye) through the points as shown in Figure 6. Given (5), it is clear that when  $x = 1$ ;

$$\log b = \log y$$

Therefore,

$$b = y$$

Looking at your plot for  $x = 1$ , read off the corresponding  $y$  value. This is  $b$ , the “intercept” of your log-log plot [Note: the intercept of the log-log plot is NOT the value of  $y$  where  $x = 0$ , but rather where  $x = 10^0$ . Why?] In the example in Figure 5 and Figure 6,  $b = 0.8$ .



Now, choose two points on your line of best fit as shown in Figure 6 [Note: it is most convenient to choose two points separated by an integer number of log cycles, but this is not necessary]. Write (5) for each point:

$$\log y_1 = m \log x_1 + \log b \quad (6a)$$

$$\log y_2 = m \log x_2 + \log b \quad (6b)$$

Subtract (6b) from (6a):

$$\log y_1 - \log y_2 = m(\log x_1 - \log x_2)$$

Solve for  $m$ :

$$m = \frac{\log y_1 - \log y_2}{\log x_1 - \log x_2}$$

In the example,  $m = 2.5$ .

# ***Points or Lines?***

When creating plots, it is absolutely essential that you know when to show the points and when to show the line connecting the points.

When plotting data, show the data points. You may also wish to show a trendline, but always show the points!

When plotting a functional relationship, show only the line connecting the points, not the points themselves. If the functional relationship is smooth, make sure you have enough points that your curve is smooth as well!

# Experiment 1: Fluid Properties

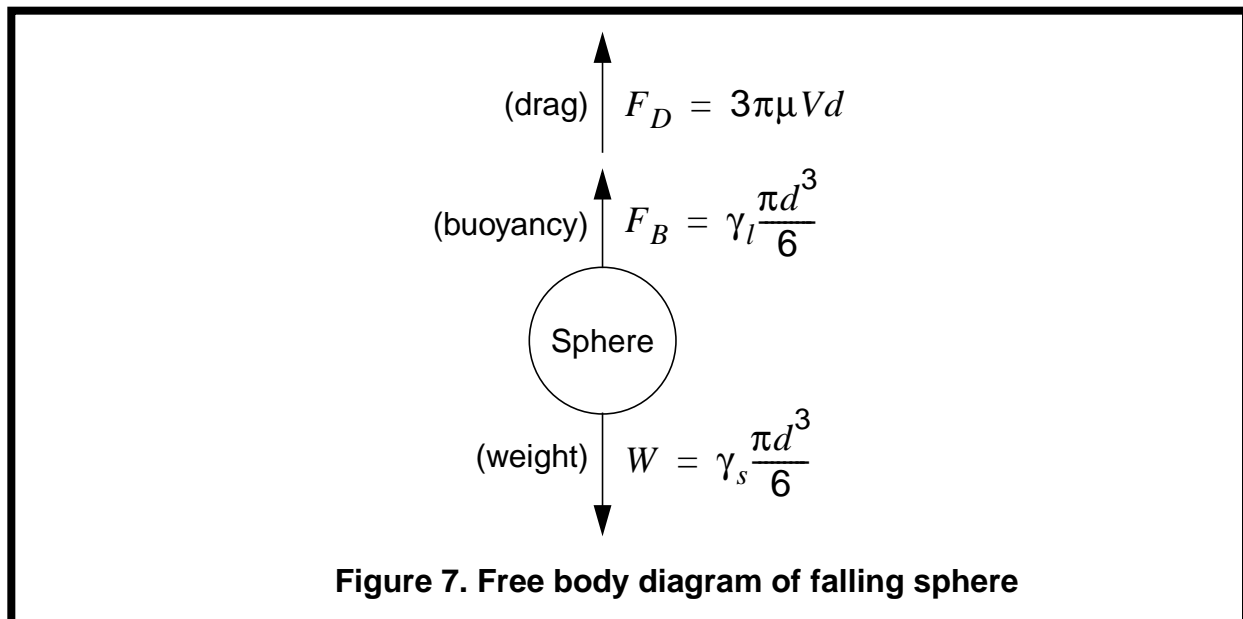
Key Concepts: Specific Weight, Dynamic Viscosity

Refer to: Roberson, Crowe & Elger, 7th ed., Chapter 2, pp. 13 -22

## I. Introduction

The transportation and accumulation of sediment in waterways and reservoirs, the movement of dust and other pollutants in the atmosphere, and the flow of liquids through porous media are examples of phenomenon in which specific weight and viscosity play important roles.

Consider a sphere with diameter  $d$  and specific weight  $\gamma_s$ , falling at a constant velocity  $V$  through a liquid with viscosity  $\mu$ , specific weight  $\gamma_l$ , and density  $\rho$ . The forces acting on the sphere are shown in Figure 1.



Note that the expression given in Figure 1 for the drag force is derived from Stoke's Law and is valid only for small Reynolds number (see "Dimensionless Fluid Parameters" on page 8).

According to Newton's Second Law (since the sphere is not accelerating):

$$\sum F = 0 \quad (7)$$

$$F_D + F_B - W = 0 \quad (8)$$

$$3\pi\mu Vd + \gamma_l \frac{\pi d^3}{6} - \gamma_s \frac{\pi d^3}{6} = 0 \quad (9)$$

Algebraic manipulation yields an expression for  $\mu$  in terms of  $\gamma_s$ ,  $\gamma_l$ ,  $d$  and  $V$ :

$$\mu = \frac{d^2(\gamma_s - \gamma_l)}{18V} \quad (10)$$

Equation (4) is valid for a sphere falling far from a wall. The 'wall effect' occurs when the falling sphere is close to a wall. The 'wall effect' affects the sphere when:

$$\frac{\text{sphere diameter } (d)}{\text{tube diameter } (D)} > \frac{1}{3} \quad (11)$$

The observed fall velocity,  $V_o$ , must then be corrected using:

$$\frac{V}{V_o} = 1 + \frac{9d}{4D} + \left(\frac{9d}{4D}\right)^2 \quad (12)$$

The drag force on a sphere may also be calculated by:

$$F_D = C_D A_P \rho \frac{V^2}{2} \quad (13)$$

where  $A_P$  is the projected area of the sphere and  $C_D$  is the coefficient of drag.

*In this experiment:* After measuring the terminal velocity of spheres falling through a fluid, the viscosity of a liquid will be determined according to (4). Equation (7) is then used to calculate the coefficient of drag.

## II. Objective

Determine the specific weight and the viscosity of liquids at room temperature. Also determine a relationship between the coefficient of drag and the Reynolds number.

## III. Anticipated Results

At a minimum, you should be able to anticipate:

- 1) What are the specific weight and viscosity of oil and glycerin?
- 2) What is the relationship between  $C_D$  and Reynolds number?

## IV. Apparatus

- 1) Two liquids contained in three transparent vertical tubes: two large tubes and



one small tube. The small tube and one large tube should contain the first liquid (oil), while the second large tube should contain the second liquid (glycerin). Inside each tube is a bail bucket to catch the falling spheres. There is also a hooked rod to retrieve the bucket by the handle.

- 2) Calibrated volumetric containers of the above liquids.
- 3) Tweezers.
- 4) Thermometer, micrometer, meter stick, stopwatch.
- 5) At least five spheres of varying density and/or diameter (use marbles, shot-filled balls, etc.).

## V. Procedure

- 1) Record the temperature of the liquids (use the ambient temperature if the liquids have been in the room for a long period of time).
- 2) Calculate the specific weight of each liquid by weighing a known volume. The tare weight of the calibrated containers (not including the stoppers) is scribed on the outside of each container.
- 3) Weigh each sphere and measure its diameter with a micrometer (to account for out-of-round conditions, take several measurements at various diameters and average the result). Calculate the specific weight of each sphere. NOTE: if the specific weight of the sphere is not greater than that of the fluid, it will float and not fall - choose another sphere.
- 4) Measure and record the inside diameter of the tubes.
- 5) Measure and record a vertical fall distance on each tube (the distance need not be the same for each tube). Use a scribed line or masking tape to locate the distance. There should be ample liquid above and below the lines so that the sphere will not be influenced by the bail bucket and to allow the person with the stopwatch an adequate distance to visually identify the sphere dropping. Check that the handle of the bucket will not interfere with the travel of the sphere.
- 6) Drop a sphere into the liquid using the tweezers and time the descent through the marked distance using the stopwatch. Record the travel time. The sphere should be dropped just at the fluid level so that the sphere will achieve terminal velocity prior to the marked distance.
- 7) Repeat item 6 for each sphere. When all spheres have been dropped, retrieve the bail bucket with the hooked rod. Remove the spheres from the bucket, cleaning them thoroughly with towels or rags. Push the bucket back down using the rod, then remove and clean the rod with towels or rags.
- 8) Repeat 6 and 7 for each tube.

## VI. Data Control

Data control consists of calculating the viscosity,  $\mu$ , for each drop. The consistency of the calculated data indicates the quality of the observed data.

## VII. Results

- 1) Plot, at log-log scale, the coefficient of drag,  $C_D$ , of the spheres vs. the Reynolds number from the laboratory data. Since neither variable depends on properties of the fluid or the spheres, use all of the data points on the same plot.

- 2) Determine the equation of the plot for item 1 and compare it to the expected value. Hint: use equation (13) and Figure 7.
- 3) Compare your calculated values of viscosity and specific weight with an authoritative source.

**VIII. Suggested Data Sheet Headings** ([ ] indicates units of measurement)

	<b>Liquid A</b>	<b>Liquid B</b>
<b>Volume [ ]</b>		
<b>Total mass [ ]</b>		
<b>Tare mass [ ]</b>		
<b>Mass of liquid [ ]</b>		
<b>Specific weight [ ]</b>		
<b>Temperature [ ]</b>		

<b>Sphere #</b>	<b>Sphere Diameter [ ]</b>	<b>Mass [ ]</b>	<b>Volume [ ]</b>	<b><math>\gamma_s</math> [ ]</b>

<b>Liquid A/B</b>	<b>Tube Diameter</b> [ ]	<b>Sphere #</b>	<b>Sphere Diameter</b> [ ]	<b>Fall Distance</b> [ ]	<b>Time of Fall</b> [ ]	<b>Terminal Velocity</b> [ ]	<b>Corrected Velocity</b> [ ]	<b>Viscosity</b> [ ]

# Experiment 2: Bernoulli's Equation

Key concept: Validity of Bernoulli Equation

Refer to: Roberson, Crowe & Elger, 7th ed., Chapter 5, pp. 158-180

## I. Introduction

This lab exercise tests the validity of the **Bernoulli equation**, one of the most widely used, and misused, equations in the analysis of fluid flow. The Bernoulli equation is derived from application of Newton's Second Law to a differential fluid element aligned with a streamline. It relates the pressure, elevation, and velocity between any two points on a streamline in an inviscid (ideal), constant density fluid flowing at steady state.

Consider a fluid element aligned *along a streamline*. A streamline is a line drawn in the flow field in such a manner that the velocity vector at each and every point on the streamline is tangent to the streamline at any instant.

In Figure 8,  $s$  is the coordinate along the streamline,  $\Delta h$  is the vertical height of the fluid particle,  $p(s, t)$  is the pressure,  $V(s, t)$  is the velocity of the fluid,  $W$  is the weight of the fluid particle,  $\Delta A$  is the cross-sectional area of the fluid particle (normal to the paper and the streamline coordinate), and  $\theta$  is the angle the fluid particle makes with the horizontal.

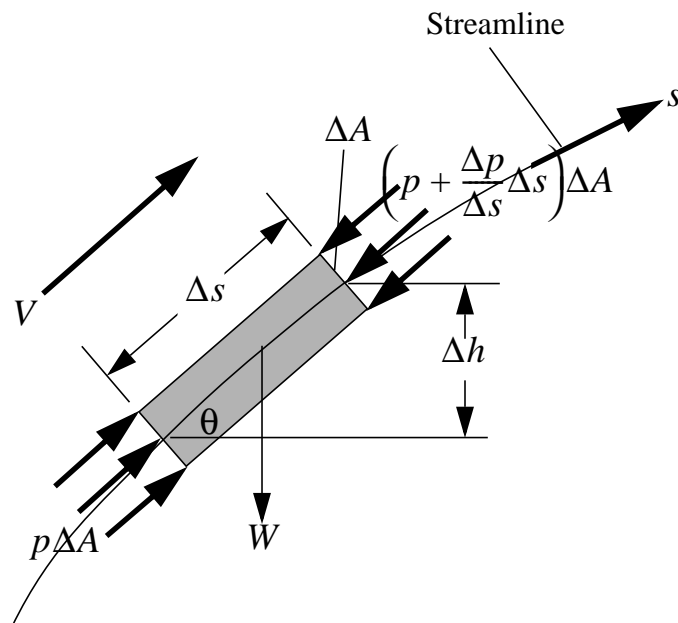


Figure 8. Differential fluid element oriented along a streamline

First, assume *the fluid is inviscid* (i.e., has no viscosity). How would this derivation differ if the fluid was viscous? Application of Newton's Second Law along a streamline yields:

$$\sum F_s = ma_s \quad (1)$$

where  $F_s$  are the external forces acting along the streamline,  $m$  is the mass of the fluid particle and  $a_s$  is the acceleration of the fluid particle along the streamline. Substitution of quantities from Figure 8 yields:

$$p\Delta A - \left( p + \frac{\Delta p}{\Delta s} \Delta s \right) \Delta A - \gamma \Delta s \Delta A \sin \theta = \rho \Delta s \Delta A a_s \quad (2)$$

where  $\gamma$  and  $\rho$  are the specific weight and density of the fluid, respectively. Furthermore, Figure 8 shows that  $\sin \theta = \Delta h / \Delta s$ . Algebraic manipulation yields:

$$-\frac{\Delta p}{\Delta s} - \gamma \frac{\Delta h}{\Delta s} = \rho a_s \quad (3)$$

In the limit, as  $\Delta s$  approaches zero:

$$-\frac{\partial p}{\partial s} - \gamma \frac{\partial h}{\partial s} = \rho a_s \quad (4)$$

Given that velocity is a function of time and position along the streamline ( $V(t, s)$ ), acceleration along the streamline is (by chain rule):

$$a_s = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial s} \quad (5)$$

For *steady flow*, (5) reduces to:

$$a_s = V \frac{\partial V}{\partial s} \quad (6)$$

Introduction of (6) into (4) yields:

$$-\frac{\partial p}{\partial s} - \gamma \frac{\partial h}{\partial s} = \rho V \frac{\partial V}{\partial s} \quad (7)$$

Use of the product rule,  $V \frac{\partial V}{\partial s} = \frac{1}{2} \frac{\partial V^2}{\partial s}$ , yields:

$$-\frac{\partial p}{\partial s} - \gamma \frac{\partial h}{\partial s} = \frac{\rho}{2} \frac{\partial V^2}{\partial s} \quad (8)$$

If *density is constant*, the differentials may be gathered to yield:

$$\frac{\partial}{\partial s} \left( p + \gamma h + \frac{\rho V^2}{2} \right) = 0 \quad (9)$$

Integration over  $s$  yields:

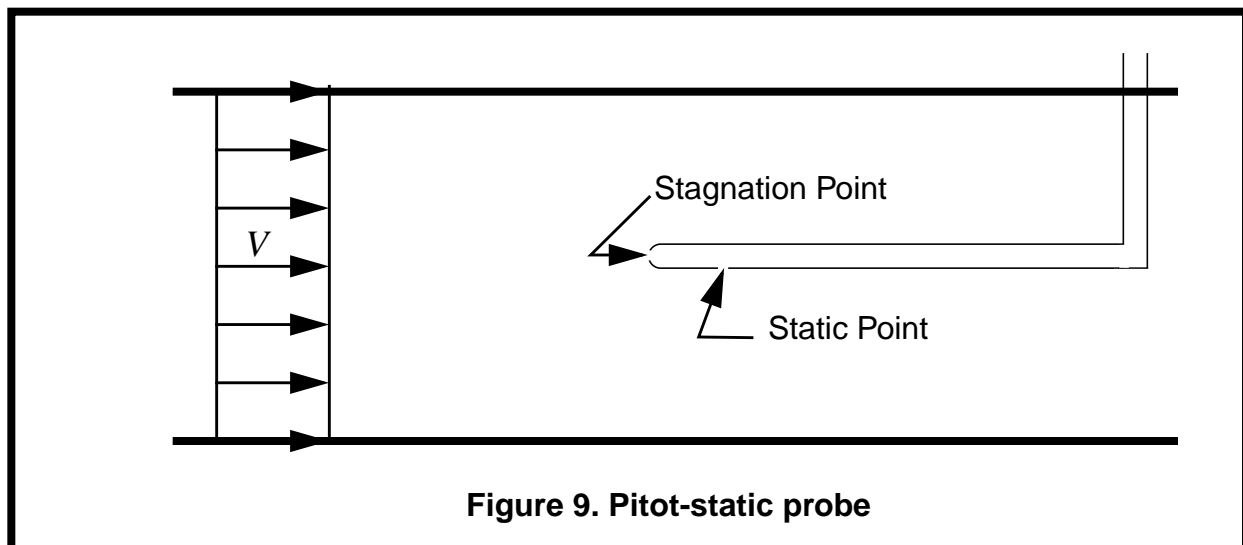
$$p + \gamma h + \frac{\rho V^2}{2} = \text{Constant along a streamline} \quad (10)$$

Finally, division by  $\gamma$  yields:

$$\frac{p}{\gamma} + h + \frac{V^2}{2g} = \text{Constant} = H \quad (\text{Bernoulli's equation}) \quad (11)$$

where  $H$  is total head. The first term on the left hand side of Bernoulli's equation is the pressure head, the second is the elevation head and the third is the velocity head; pressure head plus elevation head is called static head while velocity head is also referred to as dynamic head (static head plus dynamic head is total head). Therefore, Bernoulli's equation relates pressure, elevation and velocity between any two points **along a streamline** in a flow field that is **inviscid**, **steady**, and **constant density**. Each term in the equation has the dimension of length.

Consider the application of Bernoulli's equation to a pitot-static probe (Figure 9).



Bernoulli's equation may be written between any two points, 1 and 2, on a streamline:

$$\frac{p_1}{\gamma} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + \frac{V_2^2}{2g} \quad (12)$$

Let Point 1 be the stagnation point in Figure 9 and Point 2 be the static point. In a Pitot tube, the air velocity at the stagnation point is zero while the air velocity at the static point is constant. The elevation of the two points is the same. If the pressure at the stagnation and static points is measured, the velocity at the static point can be determined from equation (12) [Note: validate this yourself using the above definitions and equation (12)]:

$$V_2 = \sqrt{2 \frac{p_1 - p_2}{\rho}} \quad (13)$$

Indeed, Bernoulli's equation is a very useful relation. However, it must be used with great care! Remember that if the four assumptions required in the derivation of Bernoulli's equation are not met, then Bernoulli's equation is not valid. For example, if the fluid has significant viscosity, Bernoulli's equation can not be used to analyze fluid motion. In this case, one must turn to the general energy equation, which applies when any, or all, of the Bernoulli assumptions are not satisfied.

*In this experiment* the fluid, air, is assumed to have constant density. This assumption is valid for flow of gas at low velocity. Low velocity is defined when the Mach number (see "Dimensionless Fluid Parameters" on page 8) is less than 1/3. Furthermore, air has a small enough viscosity that it may be assumed to be inviscid for the purposes of this experiment. Be sure to consider the relative validity of these assumptions in analysis of your errors.

## II. Objective

Validate Bernoulli's assumptions and equation by determining if the summation of the terms in the Bernoulli equation at several locations along a streamline is a constant, H.

## III. Anticipated Results

At a minimum, you should be able to anticipate:

- 1) Will Bernoulli's equation apply?
- 2) What effect will the obvious violations of Bernoulli's assumptions have (and how much)?

## IV. Apparatus

- 1) Miniature wind tunnel device with variable flow control and uniform air supply.
- 2) Converging - diverging test section (Venturi).
- 3) Multiple tube inclined manometer.

- 4) Pitot-Static probe.
- 5) Measuring scale (mm), thermometer.

## V. Procedure

- 1) Measure and record the ambient air temperature.
- 2) Measure the dimensions of the test section.
- 3) Ensure that the manometer is leveled correctly. The fluid level should be equal in each tube. Level the manometer if required.
- 4) Adjust the flow rate to low and start the fan. Move the Pitot-Static tube vertically so that the manometer reads a high reading (low pressure). Then, carefully adjust the flow rate to high, but DO NOT ALLOW the manometer to overflow at any time. If the manometer is going to overflow, reduce the flow rate until it is safe to continue. Ensure that the high pressure (low readings) are within the scale of the manometer.
- 5) Position the Pitot-static probe vertically so that at least two readings can be taken in each test section (converging, constant, and diverging). Record the manometer readings [millibars] for the stagnation point, static point, and air box pressures. At each test location, measure the cross-sectional area of the test section and the distance from the top of the test section (use the static point).
- 6) Adjust to a low flow rate and repeat step 5.

## VI. Data Control

Data control consists of plotting  $\frac{p}{\gamma} + \frac{V^2}{2g}$  for the static point versus the distance from the entrance of the Venturi meter. A flat line indicates good observed data.

## VII. Results

- 1) Plot the stagnation pressure, dynamic pressure, static pressure, and total pressure versus the distance from the entrance of the Venturi. On a separate plot, plot static head, dynamic head, and total head versus the distance from the entrance of the Venturi. Hint: Pressure and head are related. Any pressure can be converted to an equivalent head (and vice-versa) by the relationship  $h\gamma = p$ , where  $h$  is the head and  $p$  is the pressure.
- 2) Repeat for the second air flow. Make sure there is a good legend to clarify the data.
- 3) Calculate the fluid velocity,  $V$  and the flowrate,  $Q$  for each reading. Compare to expected results. Hint: conservation of mass yields  $Q = VA$ , where  $A$  is the cross-sectional area at which the velocity is acting.
- 4) Is the flowrate constant? Is the Mach number in an acceptable range? Is the total head constant?



**VIII. Suggested Data Sheet Headings** ([ ] indicate the units of measurement)

<b>Flow</b>	<b>Distance</b>	<b>Pressure</b> [ ]			<b>Width</b>
[ ]	[ ]	<b>Stag Point</b>	<b>Static</b>	<b>Airbox</b>	[ ]

Length of Venturi section \_\_\_\_\_ [ ]

Air temperature \_\_\_\_\_ [ ]

# Experiment 3: Closed Conduit Flow

Key Concepts: Friction Factor, Reynolds Number,  
Minor Losses, Grade Lines

Refer to: Roberson & Crowe, 7th ed., Chapter 7, pp. 270-293

See also: Chapter 10, pp. 403-432

## I. Introduction

The losses of energy in conduits flowing full of a liquid usually result from the resistance of the conduit walls to the flow (pipe friction), or from pipe appurtenances (e.g. elbows, contractions, valves) which cause the flow velocity to be changed in magnitude and/or direction. These losses must be calculated so that, for example:

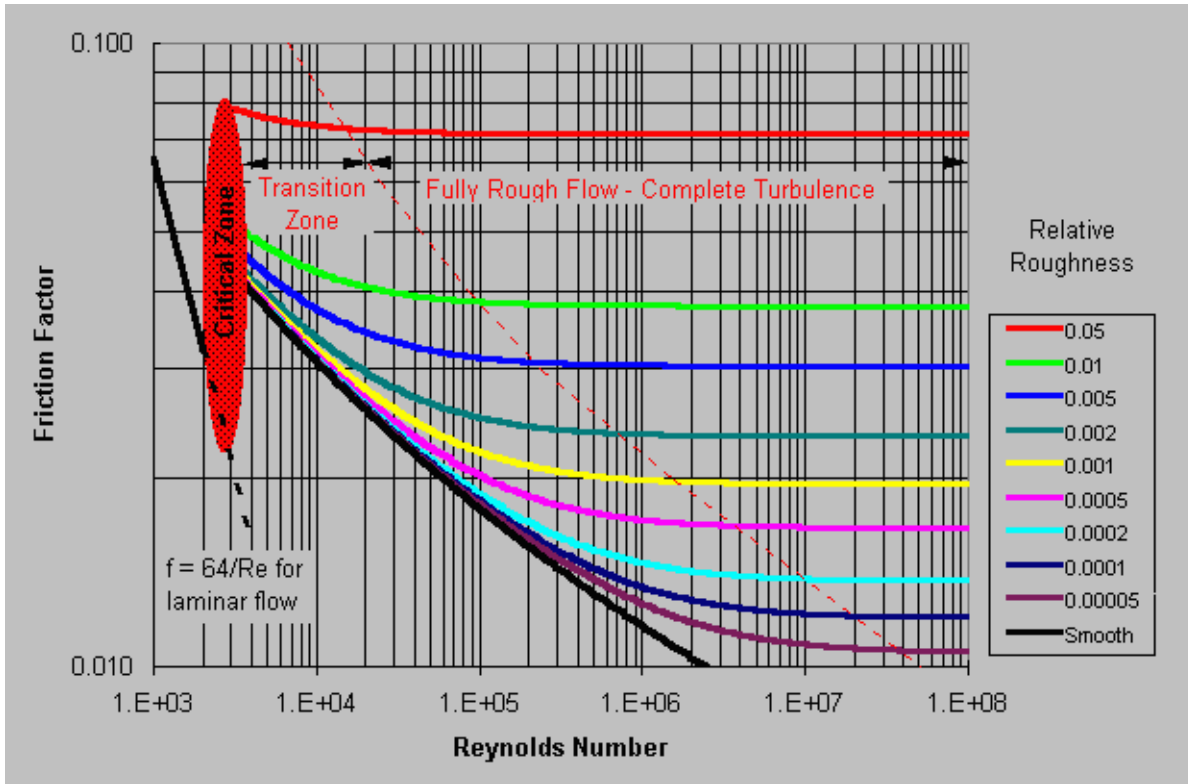
- the proper size and number of pumps can be specified in the design of a municipal water distribution system;
- the conduit size for a gravity-flow urban drainage project may be determined;
- the optimum size of valves and the radius of curvature of elbows can be stipulated in the specifications of a pipeline design.

When the ratio of the length of the pipeline,  $L$ , to the diameter,  $D$ , exceeds 2000:1, pipe system energy losses are predominantly the result of pipe friction. The energy losses resulting from pipe appurtenances are termed “*minor*” losses and are usually neglected in the calculation of pipe system energy losses. In short lengths of pipe, however, these minor losses can become *major* sources of energy loss.

The **Darcy-Weisbach** equation is used to express energy loss caused by pipe friction,  $h_f$ :

$$h_f = f \frac{LV^2}{D2g} \quad (1)$$

where  $f$  is the dimensionless Darcy-Weisbach friction factor,  $V$  is the average fluid velocity, and  $g$  is the acceleration of gravity.



**Figure 10. Moody Diagram [Note: this Moody Diagram is for illustration purposes only. Refer to the Moody Diagram in your text for all calculations.]**

For pipe flow, the **Moody diagram** (see Figure 10) was developed to show the relationship between the friction factor, the relative roughness of the pipe,  $\epsilon$ , and **Reynolds number** (see “Dimensionless Fluid Parameters” on page 8). On the Moody diagram there are three zones of flow: laminar, transitional, and turbulent. In lieu of the Moody diagram, the following equations (Table 1) may be used to determine  $f$  for *smooth* pipes ( $\epsilon \rightarrow 0$ ), where  $f$  is a function of the Reynolds number only:

	$R_e$	$f$
Laminar	$R_e < 2000$	$f = 64/R_e$
Transitional	$2000 < R_e < 10^5$	$f = 0.316/R_e^{0.25}$
Turbulent	$R_e > 10^5$	$1/\sqrt{f} = 2.0 \log[(R_e \sqrt{f}) - 0.8]$

**Table 1. Friction Factor equation for three flow regimes**

The energy loss may also be expressed as:

$$h_f = ZLV^n \quad (2)$$

Refer to "Analysis of Log-Log Plots" on page 9 for the procedure to determine  $n$  and  $Z$ . The values of  $n$  and  $Z$  are functions of the type of flow, i.e., laminar, transitional, or turbulent.

The losses due to appurtenances (minor losses) in pipe flow are usually expressed as:

$$h_L = K \frac{V^2}{2g} \quad (3)$$

where  $K$  is the dimensionless head loss coefficient related to the type of appurtenance. Typical  $K$  values can be found in most hydraulics textbooks.

Consider the energy equation for steady, incompressible, viscous, turbulent flow:

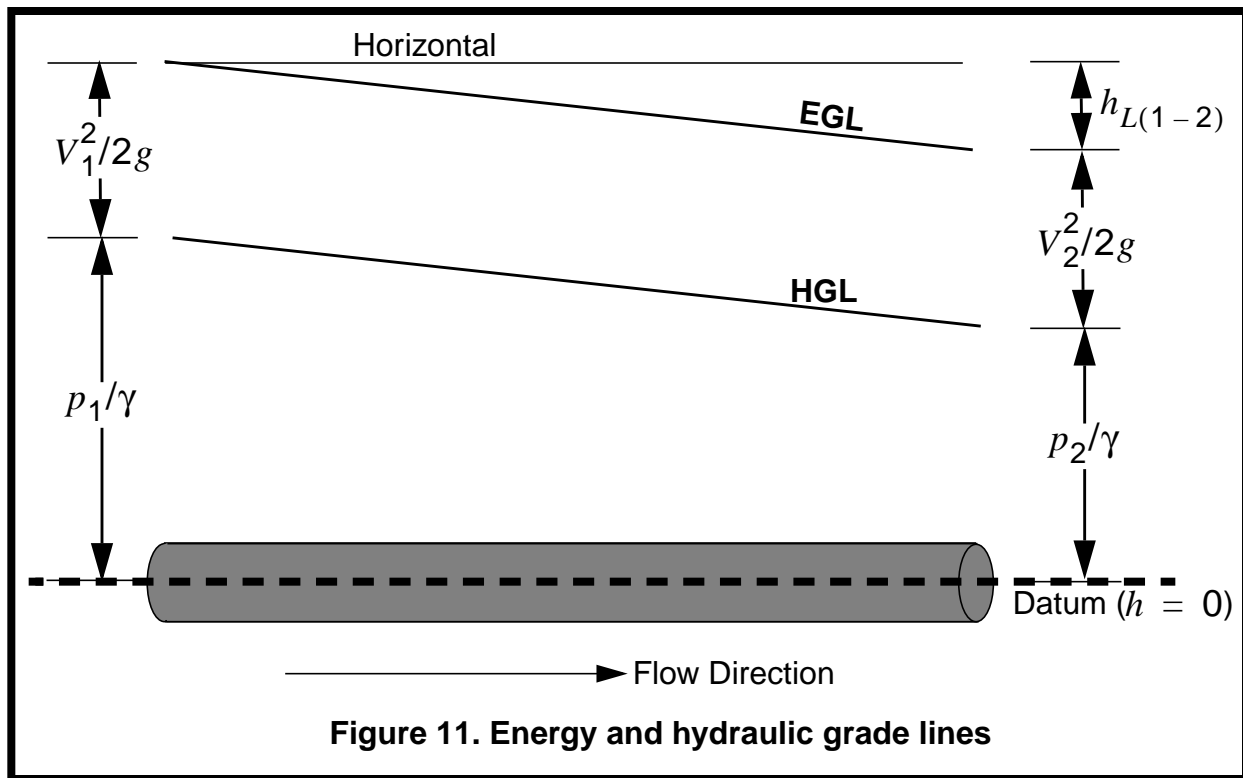
$$\frac{p_1}{\gamma} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + \frac{V_2^2}{2g} + h_{L(1-2)} \quad (4)$$

where  $p_i/\gamma$  is pressure head at point  $i$ ,  $h_i$  is elevation head at point  $i$ ,  $V_i^2/2g$  is velocity head at point  $i$ , and  $h_{L(1-2)}$  is the head loss between points 1 and 2. Each of these terms has units of length.

The sum of pressure head, elevation head and velocity head at a particular point is the total head at that point. A plot of total head vs. distance from the pipe entrance is an **energy grade line (EGL)**. In fact, an EGL is simply a spatial representation of the total energy. For a real fluid,  $h_{L(1-2)} > 0$ , so the EGL must slope downward in the direction of flow.

The sum of the pressure head and elevation head at a particular point is the piezometric or static head at that point. A plot of piezometric head vs. distance from the pipe entrance is a **hydraulic grade line (HGL)**. The vertical distance between the EGL and the HGL at a particular point is, by definition, the velocity head at that point.

Figure 11 shows the EGL and HGL for a horizontal constant diameter pipe with flow from left to right. Note the EGL slopes downward in the direction of flow and  $h_L = h_1 - h_2$  equals the head loss over the length of the pipe. Furthermore, the difference in total head between any two points represents the head loss between those two points.



Note also that the EGL and HGL are parallel, this implies that the velocity head ( $V^2/2g$ ) is constant over the length of the pipe. Since '2' and  $g$  are constants, clearly the velocity is also constant.

Finally, the distance between the HGL and the centerline of the pipe represents the pressure head. In this case, the loss in total energy is reflected in the loss of pressure head only, as elevation and velocity heads are constant over the length of the pipe.

## II. Objective

Determine the energy loss for pipelines in which a steady-state flow exists. Also determine the head loss coefficients for an orifice meter, a venturi meter, and a gate valve one-half open and full open. Compare measured  $K$  values to authoritative sources. Compare the head losses through different size pipes at the same flow rate.

## III. Anticipated Results

At a minimum, you should be able to anticipate:

- 1) Authoritative  $K$  values for the valves and the meters.
- 2) Mathematically determine the  $Z$  and  $n$  values for each pipe size.
- 3) The relative roughness of the pipe.

#### **IV. Apparatus**

- 1) Constant head tank and pump.
- 2) Copper drawn-tubing pipes with gate valves and pressure taps.
- 3) Piezometric tubes attached to a differential manometer. The differential manometer is read by taking the difference between the two readings. Note that at some points, one or both of the readings may be “negative” on the scale. Make sure that the total difference is being recorded.
- 4) Control valve.
- 5) Orifice and venturi meters.
- 6) Flow rate calibration chart.
- 7) Thermometer, yard stick.

#### **V. Procedure**

- 1) Measure the distance between the pipeline pressure taps. Measure and record the cross-sectional area of the meter test sections.
- 2) Check the water level in the reservoir and measure the water temperature (use ambient air temperature if it is not possible to measure the water temperature). There should be enough water in the tank to fill the pipes at full flow and maintain a sufficient amount of head above the pump inlet.
- 3) Study the conduit flow system starting at the pump and identify potential pathways for water flow.
- 4) Close all valves. Identify and open all valves which will route the water from the pump, through the 1” test section, through the meters, and back to the storage tank. The 1” line will be tested first. Ensure that the control valve (located immediately downstream from the meters) is open.
- 5) Start the pump. If water does not begin to travel through the meters within a short period of time, shutdown the pump and reevaluate the valve status.
- 6) Bleed the system of air (wait a few moments while the system runs). Visually inspect the meters to insure that air is out of the system.
- 7) Connect the manometer tubing to the calibration taps (the middle and upstream taps) of the Venturi meter. Determine the flow rate using the calibration chart.
- 8) For this experiment, four flowrates will be required. “Good” flow rates range from 250 to 420 gph. Choose four flowrates spread over this range (e.g. 250, 300, 350, 400). Adjust the control valve as needed to obtain the first flow rate.
- 9) Connect the manometer tubing to the calibration taps (the middle and upstream taps) of the orifice meter. Check the flow using the observed deflection. If the flow rates match, the flow rate has been calibrated successfully.
- 10) Connect the manometer tubing to the headloss taps (the outer taps) on one of the meters and record the manometer reading. Repeat for the second meter.
- 11) Connect the manometer tubing to the first and second pipeline taps of the 1” line. Record the deflection.
- 12) Connect the manometer tubing to the second and third taps of the same line and record the deflection.
- 13) Connect the manometer tubing to the pressure taps on both sides of the 1” gate valve. Record the deflections at fully open and half open. Use the sample valve hanging on the instrument to determine how many turns are needed for

half open.

- 14) Perform data control before changing the flow rate.
- 15) Open the valves for the 3/4" line and then close the valves for the 1" line. Attach the manometer tubing to the Venturi meter and readjust to the selected flow rate. Repeat steps 11 through 13 for the 3/4" pipe and gate valve.
- 16) Repeat steps 7 through 15 for four flowrates within the "good" range.

## VI. Data Control

Data control consists of a) obtaining the same discharge from each of the two meters; and b) obtaining the same rate of energy loss (head loss per unit length) along the length of each pipe.

## VII. Results

- 1) Calculate the Reynolds number and the friction factor from Table 1 for each run. Plot this data on a copy of the Moody diagram (a better copy of the moody diagram is in your textbook). What relative roughness does the pipe appear to possess? Compare to your expected value.
- 2) Determine the anticipated values of  $Z$  and  $n$  for the conditions of this experiment. Use (1), Table 1, and (2) to solve for  $Z$  and  $n$  algebraically (hint: start by setting (1) and (2) equal, remember  $ax^b = cx^d$ , only if  $a = c$  and  $b = d$ ), then compare to your lab data by using the method described in "Analysis of Log-Log Plots" on page 9. What does this say about the headloss in the 3/4" pipe as compared to the 1" pipe. How does this compare to your expected results?
- 3a) Calculate the head loss coefficient,  $K$ , for the venturi meter using (3) for each flow rate. Report a representative  $K$  value for the meter. Justify your representative  $K$  value based on your calculations. Compare to known values.
- 3b) Repeat step 3a for the orifice meter.
- 4a) Repeat step 3a for the half-open gate valve,
- 4b) Repeat step 3a for the fully-open gate valve.
- 5) Draw, to scale, the energy and hydraulic grade lines for the maximum and minimum flowrates in the 1 inch pipe. Show the velocity head, pressure head, elevation head and energy lost between taps 1 and 2 on the pipe. Hint: the manometer measures the relative static head; you may need to assume an elevation value.

**VIII. Suggested Data Sheet Headings** ([ ] indicate the units of measurement)

*Flow meters*

Run #	Flow Rate [ ]	Meter Type	Calibration Taps Manometer [ ]			Headloss Taps Manometer [ ]		
			RHS	LHS	$\Delta h$	RHS	LHS	$\Delta h$

*Pipes*

Run #	FlowRate [ ]	Pipe Diameter [ ]	Tap # ( ) to ( )	Manometer [ ]		
				LHS	RHS	$\Delta h$

*Gate Valves*

Run #	FlowRate [ ]	Pipe Diameter [ ]	Gate Opening 1/2 or full	Manometer [ ]		
				LHS	RHS	$\Delta h$



# Experiment 4: Open Channel Flow

Key Concepts: Flowrate Measurement, Weir Equations

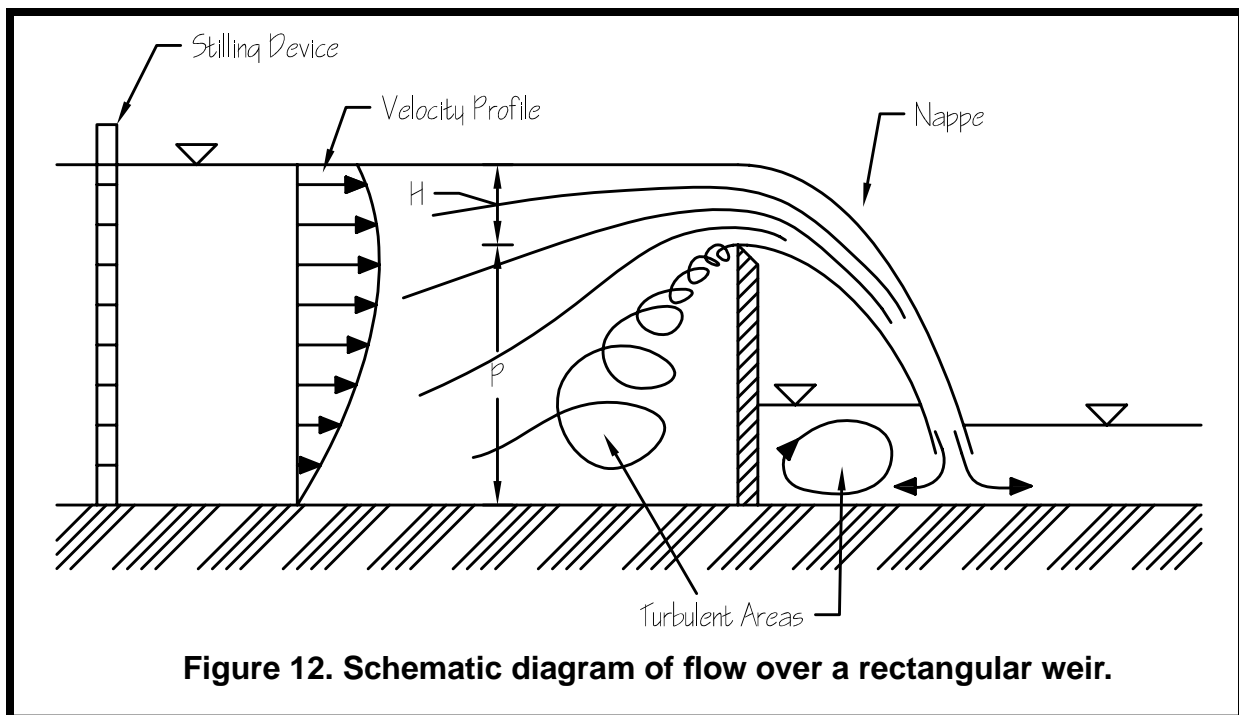
Refer to: Roberson, Crowe & Elger, 7th ed., Chapter 13, pp. 607-611

## I. Introduction

Determination of the flowrate of water in open channels is significant in many aspects of society. For example, urban and industrial water supplies must be measured so that demands are satisfied; the amount of water required for the dilution of pollutants being wasted into a river can be calculated mathematically, but metering devices are required to measure the supplied flow; and flood damage can be determined by correlating the depth of water passing over a dam spillway (a special type of weir) to the volume of water flowing downstream.

A **weir** is a vertical obstruction placed in an open channel, normal to the mean flow, thus forcing the flow over a crest designed to measure the flow rate. A well designed weir will exhibit subcritical flow upstream, accelerating to critical flow at the crest. For more information on subcritical and supercritical flow, refer to "Experiment 5: Hydraulic Jump" on page 39. This experiment will consider one class of weirs, known as *sharp-crested weirs*, which are smooth, vertical, flat plates with a sharpened upper edge. In particular, rectangular and triangular weirs will be studied.

Consider a schematic diagram of flow over a weir (Figure 12). Among the complicated features of the flow are:



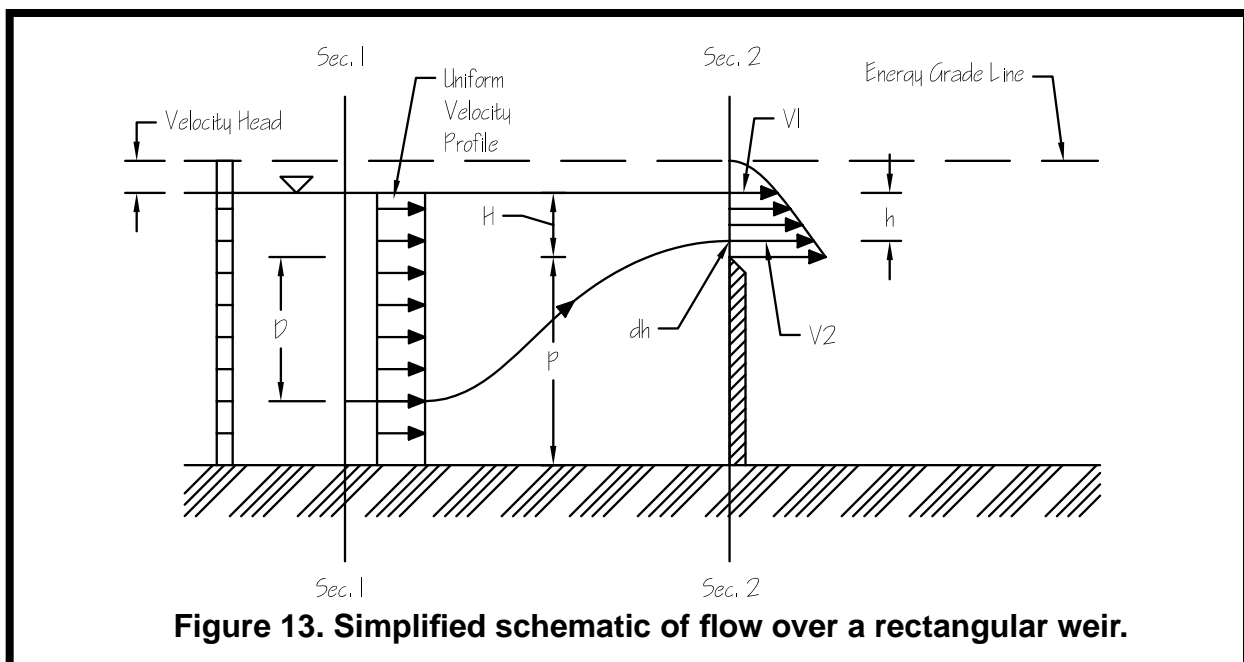
- (1) upstream velocity profile which varies over the vertical;
- (2) curved streamlines over the crest;
- (3) potentially inadequate ventilation under the nappe, which may result in subatmospheric pressure there;
- (4) secondary flows and other turbulent processes;
- (5) surface tension

For a first analysis, the problem is greatly simplified by neglecting these complicating features. A diagram of the simplified flow is shown in Figure 13.

Specifically, simplifications include:

- (1) uniform upstream velocity profile (generally valid for  $H/P < 0.4$ );
- (2) straight, horizontal streamlines over the crest;
- (3) good ventilation, and therefore atmospheric pressure, under the nappe;
- (4) neglect of secondary flows and other turbulent processes;
- (5) neglect of surface tension (generally valid for  $H > 3\text{ cm}$ ).

Simplifications (2) and (3) indicate that the flow over the weir may be treated as a jet. Note that the velocity profile over the crest is still not uniform.



**Figure 13. Simplified schematic of flow over a rectangular weir.**

An expression for  $Q$  can now be derived. Because velocity over the crest is not uniform:

$$Q \neq VA \quad (1)$$

Rather, the more general expression for  $Q$  must be used:

$$Q = \int_0^H V dA \quad (2)$$

The velocity can be determined using Bernoulli's equation. Consider the Bernoulli equation between the two points on the streamline indicated in Figure 13:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \quad (3)$$

Although the fluid is not ideal, viscosity may be neglected, for a first approximation, when the two points of interest are very close together, as they are here. For the simplified flow,  $z_1 = 0$ ,  $z_2 = H + D - h$ ,  $p_1 = (H + D)\gamma$ , and  $P_2$  is zero gage. Algebraic manipulation yields an expression for  $V_2$  in terms of  $V_1$ ,  $g$  and  $h$ :

$$V_2 = \sqrt{V_1^2 + 2gh} \quad (4)$$

If the upstream velocity is not only uniform, but also negligible, (4) becomes:

$$V_2 = \sqrt{2gh} \quad (5)$$

For the case of negligible upstream velocity, substitution of (5) into (2) yields:

$$Q = \int_0^H \sqrt{2gh} dA \quad (6)$$

Evaluation of the integral yields:

$$Q = \frac{2}{3} \sqrt{2g} LH^{3/2} \quad (7)$$

where  $L$  is the constant width of the weir. Letting  $D = 2/3 \sqrt{2g}$  (a different  $D$  than above), (7) becomes:

$$Q = DLH^{3/2} \quad (8)$$

Unfortunately, (8) doesn't work particularly well when applied to natural flow situations. Fudge factors,  $C$  and  $n$  are therefore introduced:

$$Q = CLH^n \quad (9)$$

where  $C$  is a coefficient of discharge, which is not necessarily equal to  $D$ , but rather is a function of the geometry of the weir, and  $n$  is also a function of weir geometry. For weirs constructed to standard specifications, the values of  $C$  and  $n$  are constants for each weir design and are given below. *These values are only valid for the assumptions of the equation's derivation.* Be sure to use consistent units such as fps and feet or mps and meters.

Similarly, the formula for the V - notch weir is:

$$Q = CH^n \quad (10)$$

where  $C$  is a function of the notch angle,  $\theta$ , and  $n$  is a constant. Refer to "Analysis of Log-Log Plots" on page 9 for the procedure to determine the values of  $n$  and  $C$ .

	Rectangular	90° V-notch
<b>C</b>	3.33 (fps) 1.84 (mps)	2.5 (fps) 1.38 (mps)
<b>n</b>	1.5	2.5

**Table 2. Weir coefficients for standard, sharp-crested weirs**

Note that if the upstream velocity is *not* negligible, the equation for a rectangular weir, analogous to (9), is:

$$Q = CL\left(H + \frac{V^2}{2g}\right)^{3/2} \quad (11)$$

The graphical method outlined above for the rectangular weir can be applied to a V-notch weir to determine  $C$  and  $n$  experimentally.

## II. Objective

Develop weir calibration curves and determine the  $C$  and  $n$  values for two types of weirs. Determine the influence of the velocity head in this experiment. Determine the most

appropriate flow conditions for rectangular and V-notch weirs.

### III. Anticipated Results

At a minimum, you should be able to anticipate:

- 1) Shape of  $Q$  vs.  $H$  curves (i.e. what is the relationship between  $Q$  and  $H$ ?).
- 2) Whether or not the velocity head is important or not.
- 3) Values of  $C$  and  $n$  for both types of weirs.
- 4) What flow characteristics are best for each type of weir (rectangular and V-notch)

### IV. Apparatus

- 1) Reservoir-Channel apparatus with no channel slope, a volumetric flow meter, and a measuring deck.
- 2) Hook gage, stopwatch, scale, and protractor.
- 3) One rectangular and one triangular weir plate for the above apparatus.

### V. Procedure

- 1) Check the reservoir water level. Open the drain in the tank basin.
- 2) Measure the upstream channel cross-section width.
- 3) Measure  $L$  or  $\theta$  for one of the weir plates. Install the weir in the channel.
- 4) Position the hook gage next to the weir plate and zero the scale at the weir crest elevation. For the rectangular weir, measure the crest height from the channel bottom.
- 5) Determine a maximum flow rate by opening the flow control valve, and turning on the pump. The maximum flow rate will fill the flume tank and will not overflow the channel. For more accurate measurements, the water surface upstream of the weir should not be too turbulent. Insert the stopper to close the drain and observe the volumetric flow meter. Note that the meter is marked from zero to 40 liters. When the basin is empty, the level of the meter is below zero.
- 6) For the first measurement, use the maximum flowrate found in step 4. Obtain a steady flow through the channel and use the hook gage to determine the water surface elevation above the weir crest.
- 7) Close the drain. As the water fills the tank the volumetric flow meter will start to rise.
- 8) Use the stopwatch to record the time it takes to fill the tank to a given volume. Do not start timing until the volumetric flow rate meter has reached zero.
- 9) Compute and record the flowrate. Use the data collected in step 7.
- 10) Decrease the flow rate and repeat steps 5 through 8 for five different flow rates.
- 11) Repeat steps 2 through 9 for the other weir plate.

## VI. Data Control

Data control consists of plotting  $Q$  vs.  $H$  (at log-log scale) for each weir. A linear relationship indicates good results.

## VII. Results

- 1) Plot on a graph (at log-log scale)  $Q$  vs.  $H$  (use similar units such as cfs and feet) for each weir from: (a) the laboratory data, and (b) weir formula calculations - use your laboratory data for  $H$  and solve for  $Q$  using (9) and (10).
- 2) Plot  $Q$  vs.  $H$  on one log-log plot for only the rectangular suppressed weir using (a) the laboratory data, and (b) the laboratory data, with  $H$  increased by the addition of the velocity head. Use your laboratory  $H$  values, the width of the upstream cross-section, and the laboratory  $Q$  value to determine the upstream  $V$  (using  $Q = VA$ ) and then calculate  $Q$  using (11).
- 3) Determine the  $C$  and  $n$  values of each weir using the laboratory data and the procedure outlined in the theory section. Compare to expected values.
- 4) What types of flows (e.g. high flow or low flow) are most appropriate for each type of weir shape?
- 5) Are the simplifying assumptions all valid? What effects might they have if not?

## VIII. Suggested Data Sheet Headings ([ ] indicate the units of measurement)

Crest Length ( $L$ ) \_\_\_\_\_ [ ]

Notch Angle ( $\theta$ ) \_\_\_\_\_ [ ]

Run #	Volume [ ]	Time [ ]	Discharge		Weir Type	Hook Gage [ ]		
			[l/s]	[cfs]		W.S.	"0"	H

# Experiment 5: Hydraulic Jump

Key Concepts: Validity of the Equation, Hydraulic Jump Phenomenon

Refer to: Roberson, Crowe & Elger, 7th ed., Chapter 15, pp. 675-697

## I. Introduction

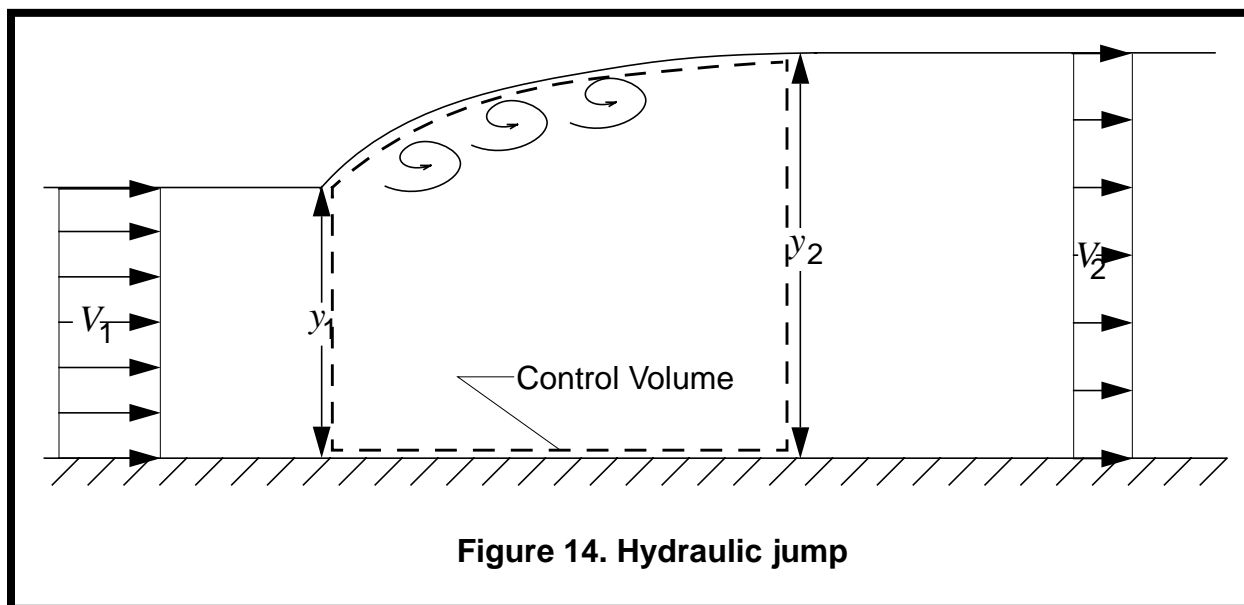
A hydraulic jump in an open channel of small slope is shown in Figure 14. In engineering practice the hydraulic jump frequently appears downstream from overflow structures (spillways) or underflow structures (sluice gates) where velocities are high. It may be used to effectively dissipate kinetic energy and thus prevent scour of the channel bottom, or to mix chemicals in a water or sewage treatment plant. In design calculations the engineer is concerned mainly with prediction of existence, size, and location of the jump.

A hydraulic jump is formed when liquid at high velocity discharges into a zone of lower velocity, creating a rather abrupt rise in the liquid surface (a standing wave) accompanied by violent turbulence, eddying, air entrainment, and surface undulations.

A flow is **supercritical** when:

$$F_r = \frac{V}{\sqrt{gy}} > 1 \quad (1)$$

where  $F_r$  is the Froude number (see "Dimensionless Fluid Parameters" on page 8),  $V$  is the fluid velocity,  $g$  is the gravitational constant, and  $y$  is fluid depth.



For a channel of rectangular cross-section and constant width,  $b$  :

$$F_r = \frac{q}{y} \frac{1}{\sqrt{gy}} \quad (2)$$

where  $q = Q/b$ , the flowrate per unit width of the channel. In supercritical flow, disturbances travel downstream, and upstream water levels are unaffected by downstream control. Supercritical flows are characterized by high velocity and small flow depth and are also known as *shooting flows*.

A flow is subcritical when:

$$F_r = \frac{V}{\sqrt{gy}} < 1 \quad (3)$$

In subcritical flow, disturbances travel upstream *and* downstream, and upstream water levels are affected by downstream control. Subcritical flows are characterized by low velocity and large flow depth and are also known as *tranquil flows*. In a hydraulic jump, supercritical flow changes to subcritical flow over a short horizontal distance.

**Specific energy ( $E$ )** in a channel section is the sum of the elevation head and velocity head, measured with respect to the channel bottom:

$$E = y + \frac{V^2}{2g} \quad (4)$$

For a rectangular channel of constant width  $b$  and constant discharge  $Q$ ,

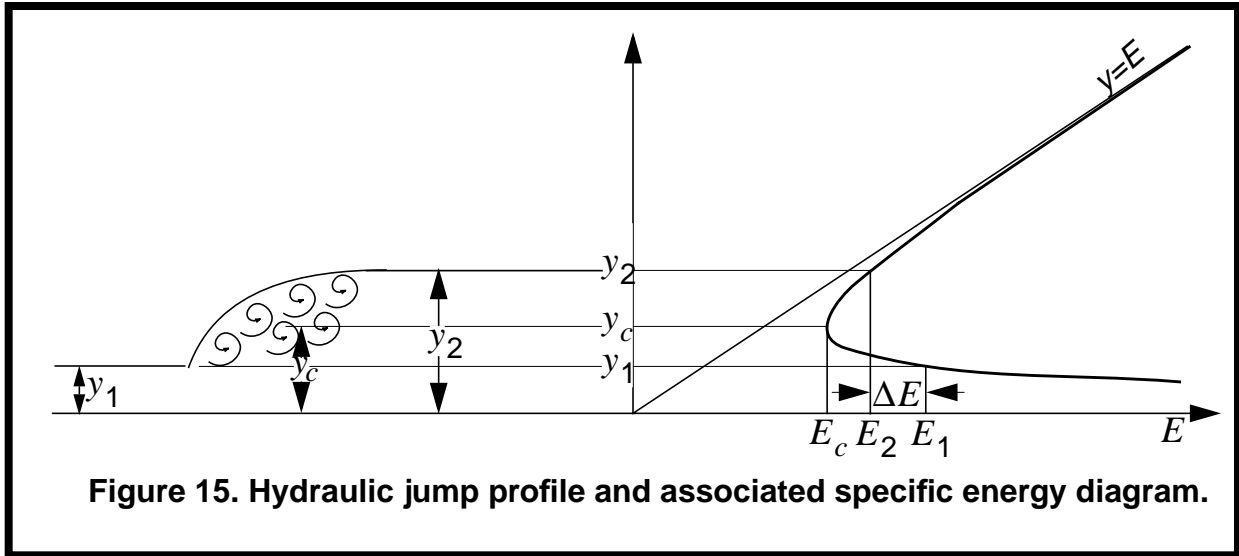
$$E = y + \frac{1}{2g} \left( \frac{q}{y} \right)^2 \quad (5)$$

Consider a plot of depth,  $y$ , vs. specific energy,  $E$ , for a given flow rate (Figure 16). This plot is known as a specific energy diagram. As the depth increases from a small value, the specific energy decreases to a minimum value,  $E_c$ . The depth associated with this minimum value of specific energy is called **critical depth**,  $y_c$ , and the associated Froude number is unity. As the depth continues to increase, the specific energy increases, eventually approaching the  $y = E$  line. For each value of specific energy greater than the minimum specific energy, there are two associated depths of flow. One,  $y_1$ , is less than the critical depth (supercritical), and one,  $y_2$ , is greater than the critical depth (subcritical).

Using equation (5), the energy loss through the jump may be determined:

$$\Delta E = E_1 - E_2 = \left[ y_1 + \frac{1}{2g} \left( \frac{q}{y_1} \right)^2 \right] - \left[ y_2 + \frac{1}{2g} \left( \frac{q}{y_2} \right)^2 \right] \quad (6)$$





Consider the integral momentum equation:

$$\sum \mathbf{F} = \int_{\text{c.s.}} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A} \quad (7)$$

where  $\mathbf{F}$  is an external force,  $\mathbf{V}$  is the velocity vector, positive to the left,  $\rho$  is the density of the fluid, and  $d\mathbf{A}$  is the area vector, positive outward from the stationary control volume. The control volume contains the jump, as shown in Figure 14. Assuming a uniform velocity distribution across the area  $\mathbf{A}$  yields:

$$\sum \mathbf{F} = \mathbf{V} \rho \mathbf{V} \cdot \mathbf{A} \quad (8)$$

Ignoring boundary friction and for small channel slopes,

$$F_1 - F_2 = \rho Q (V_2 - V_1) \quad (9)$$

where  $F_1 = \gamma y_1^2 / 2$  and  $F_2 = \gamma y_2^2 / 2$  are the hydrostatic forces upstream of the jump and downstream of the jump, respectively. After substitution for  $F_1$  and  $F_2$ , the momentum equation becomes:

$$\frac{q^2}{gy_1} + \frac{y_1^2}{2} = \frac{q^2}{gy_2} + \frac{y_2^2}{2} \quad (10)$$

Equation (10) is a quadratic in  $y_2/y_1$ , the solution of which may be written (the derivation is non-trivial!):

$$y_2 = \frac{y_1}{2} \left( \sqrt{1 + 8F_{r_1}^2} - 1 \right) \quad (11)$$

or

$$y_1 = \frac{y_2}{2} \left( \sqrt{1 + 8F_{r_2}^2} - 1 \right) \quad (12)$$

These equations show that  $y_2/y_1 > 1$  only when  $F_{r_1} > 1$  and  $F_{r_2} < 1$ , thus proving the necessity of supercritical flow for hydraulic jump formation. Another way of visualizing this is by defining a **momentum function**, also known as specific momentum or specific force,  $M$ :

$$M = \frac{q^2}{gy} + \frac{y^2}{2} \quad (13)$$

where the term  $q^2/gy$  is the momentum of the flow passing through the channel section per unit time per unit weight of water, and the term  $y^2/2$  is the force per unit weight of water.

Plotting  $M$  as a function of  $y$  for a constant flowrate (Figure 16), the solution of equation (10) occurs when  $M_1 = M_2$ . The depths  $y_1$  and  $y_2$  at which  $M_1 = M_2$  are called **sequent depths**.

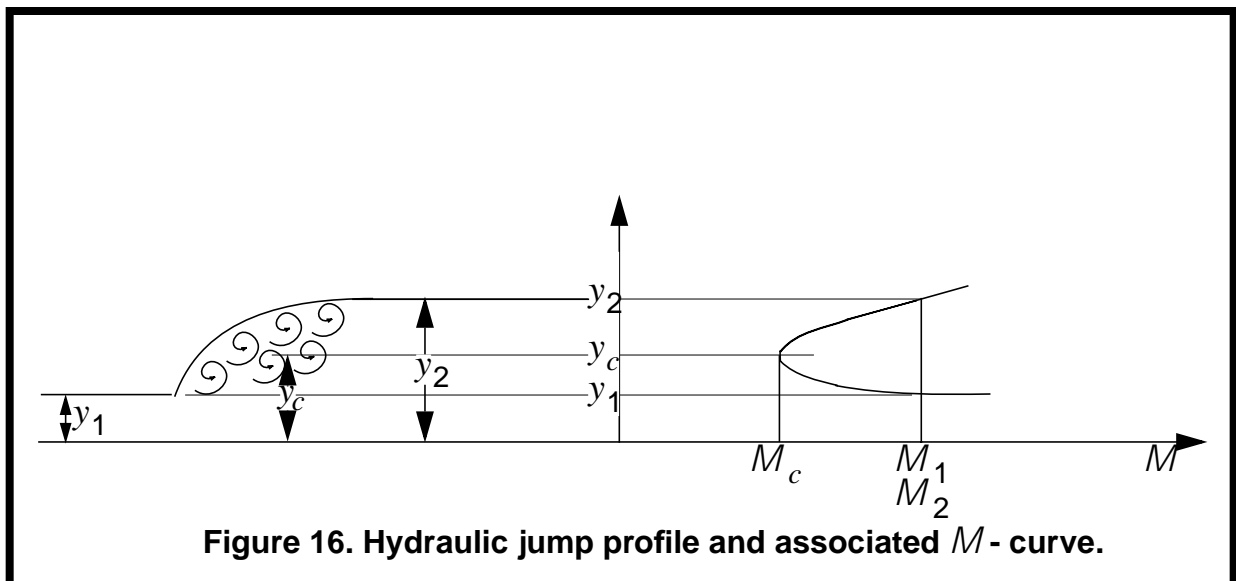


Figure 16. Hydraulic jump profile and associated  $M$  - curve.

A stable hydraulic jump will form only if the three independent variables ( $y_1, y_2, F_{r1}$ ) of the hydraulic jump equation conform to the relationship of equations (11) and (12).<sup>1</sup> The upstream depth,  $y_1$ , and the Froude number,  $F_{r1}$ , are controlled by an upstream headgate for a given discharge. In this experiment, the downstream depth,  $y_2$ , is controlled by a downstream tailgate and *not by the hydraulic jump*. Denoting the actual *measured* downstream depth as  $y_2$  and the sequent depth as  $y_2'$ , the following observations may be made:

- if  $y_2 = y_2'$ , a stable jump forms;
- if  $y_2 > y_2'$  the downstream  $M$  is greater than the upstream  $M$ , and the jump moves upstream;
- if  $y_2 < y_2'$  the downstream  $M$  is less than the upstream  $M$  and the jump moves downstream.

## II. Objective

Determine the validity of the integral momentum and specific energy equations for the hydraulic jump phenomenon. Determine the stability and characteristics of the jump obtained in the lab using the impulse-momentum and specific energy equations.

## III. Anticipated Results

At a minimum, you should be able to anticipate:

- 1) What happens to the energy curves as the flowrate increases?
- 2) What happens to the  $M$  curves as the flowrate increases?
- 3) Will the jump be stable?

## IV. Apparatus

- 1) Horizontal, glass-walled flume with headgate and tailgate controls.
- 2) Metered water supply.
- 3) Point gages.
- 4) Scale.

## V. Procedure

- 1) Check the reservoir water level.
- 2) Level the flume to horizontal if necessary. Measure the flume width.
- 3) Zero the gages to the bottom of the channel.
- 4) Lower the tailgate to approximately level. Start the pump and open the discharge control valve. Use flowrates between 85 gallons per minute and fully open (about 115 gpm)
- 5) Position the headgate so that the upstream water surface is near the top of the headgate. Do not overflow the headgate or channel.
- 6) Once steady-state conditions have been reached, record the discharge rate.
- 7) Position the tailgate (raise it) to create a hydraulic jump in the center of the flume. Very small adjustments are required once the jump is near the center of

the channel. Give the system a few moments to equilibrate before making more adjustments. Verify the jump is stable and not moving upstream or downstream. The jump will not remain perfectly steady but should not move during a period of several minutes.

- 8) Using the point gages, determine the water surface levels upstream and downstream of the jump.
- 9) Raise the headgate a small amount (only small amounts are required; give the system a few moments to equilibrate) and repeat steps 7 through 9 for at least five headgate positions.
- 10) Repeat steps 5 through 10 for three flowrates.

## VI. Data Control

Data control consists of plotting water surface depth (both upstream and downstream of the jump) versus  $E$  and versus  $M$  (on separate plots) for one set of flow data.

## VII. Results

- 1) Plot the specific energy curves for the three laboratory flowrates on the same graph. Denote on each curve the critical depth, and the loss of energy in one jump. Hint: critical depth occurs when  $E$  is a minimum and  $F_r = 1$ . Hint: plot the  $y=E$  line. What happens as the flowrate increases?
- 2) Plot the  $M$  - curves for the three flowrates on the same graph. Denote the sequent depths and the critical depth for one jump (use a  $y_1$  value from your lab data and equation (11)). What is the relationship between your calculated sequent depth and the measured depth? What is the relationship between the sequent depths and the critical depth? What happens as the flowrate increases?
- 3) Is the jump stable? Why or why not?

## VIII. Suggested Data Sheet Headings ([ ] indicate the units of measurement)

Run #	Discharge $Q$		$q$	Point Gage 1			Point Gage 2			
	[gpm]	[cfs]		[ ]	Bed	W.S.	$y_1$	Bed	W.S.	$y_2$

Flume width ( $b$ ) \_\_\_\_\_ [ ]

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