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4 **THE EVOLUTION OF RULES IN SHEDDING-TYPE**  
5 **OF CARD GAMES**

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14 Shedding-type of card games are used as a fruit fly to study the evolution of institutional  
15 arrangements. Eleven types of rules are identified which leads to a spectrum of 2048  
16 possible shedding games. Each game can be evaluated by the length and difficulty of the  
17 game and as such a fitness landscape of possible shedding games can be constructed.  
18 Building on cultural group selection simulations are performed with 100 groups which  
19 start with randomly throwing cards and evolving to games similar to UNO. Finally,  
20 experiments have been performed where characteristics of agents co-evolve with the  
21 rules of the game.

22 *Keywords:* Institutions; card games; evolution of rules.

23 **1. Introduction**

24 Institutional rules are the prescriptions that humans use to organize all forms of  
25 repetitive and structured interactions or situations that humans get involved in at  
26 different levels of scale [8]. Rules are defined as shared understandings that refer  
27 to enforced — by a third party — prescriptions about what actions are *required*,  
28 *prohibited*, or *permitted* [4]. Those rules (e.g. law, regulations, contracts) can be  
29 defined explicitly on paper or not. In contrast, norms are shared understandings  
30 but are not *enforced* prescriptions, meaning that it is not explicitly defined to a  
31 third party what to do when a prescription is not met.

32 Institutional rules facilitate people making decisions in complex situations.  
33 Given that people often pursue multiple objectives, which may be conflicting, limits  
34 of people’s mental capacity compared with the complexity of the decision environ-  
35 ment prevents him/her from exploring all the alternatives. Bounded rationality is  
36 assumed to explain that people adopt a “satisficing” rather than an optimizing  
37 strategy, searching for solutions that are satisfactory, given some aspiration levels  
38 [12]. Institutional rules and social norms may facilitate the coordination among  
39 actors and reduce transaction costs [8].

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1 Institutions, rules and norms which govern the interactions between people,  
2 evolve over time [8, 9]. Institutions affect the functioning of markets, traffic flows,  
3 appropriation of common resources, use of the internet, social order in a classroom,  
4 etc. The interest in studying institutions relates especially in understanding when  
5 and why institutions do not function: overharvesting of common resources, ineffec-  
6 tive educational system, illegal downloading, etc. However, we cannot easily fix an  
7 institutional arrangement. The history of the institutional arrangements plays an  
8 important role in how they can change. This so-called path-dependency affects the  
9 options and effectiveness of institutional change. One way to derive a better under-  
10 standing of the evolution of institutional arrangements is to develop computational  
11 models which can be used to do controlled simulation experiments.

12 It is important to understand the difference between norms and rules [4]. Norms  
13 are statements which define what activities by whom are allowed or not. Rules are  
14 statements like norms, but explicitly include the consequences if the rule is broken.  
15 For example, a norm can be “You must not park your car at this location”. But this  
16 does specify what happens if you park on that spot. A rule in this situation will be  
17 “You must not park your car at this location or else you have to pay a fine of \$60  
18 to the parking office”. It much be noted that the distinction between norms and  
19 rules include some gray zone. Even if one breaks a norm, this does not mean that  
20 there are no consequences. Norm breakers can be excluded or experience violent  
21 confrontations. With a rule, a third party, say police, can step in and enforce the  
22 rule.

23 Institutional rules are imbedded in a nested system of rules [8]. At the lowest  
24 level, operational rules exists which define the day-to-day activities of human inter-  
25 action. At the collective choice level, a subset of the actors of the operational level  
26 come together and adjust the operational rules within the context of the constitu-  
27 tional rules. The constitutional rules are the highest level of rules that define the  
28 basic principles in which social order needs to be governed. In fact, the constitutional  
29 choice level defines the rules and how to change the rules at the collective choice  
30 level. Changes in constitutional rules are a slow process with additional checks and  
31 balances. The model discussed in this paper only includes the rules of the opera-  
32 tional and collective choice level, assuming a fixed set of constitutional rules.

33 There has been considerable history on using models to understand the condi-  
34 tions in which certain norms evolve [1]. Typically these models study the evolution  
35 of cooperation and whether norms evolve to cooperate or not with which type of  
36 agents. There have also been game theoretical studies on the evolution of rules [11]  
37 which assume perfect rational agents selecting equilibria with constraints.

38 We are interested to develop a formal model of the evolution of institutional  
39 rules using rationally bounded agents in order to improve our understanding of  
40 societal dynamics of the past, present and future. Such a formal analysis of rules is  
41 lacking (but see Janssen [6] and Smajgl *et al.* [13]).

42 We have limited information available how institutions evolve over time. Only  
43 in unique circumstances, highly accurate data over time is available [7]. Besides

1 the empirical difficulties, institutional arrangements of real societies become too  
2 complex for an initial study of evolving rule systems. To start developing a model  
3 of rule evolution we would like to use a problem that is defined by a limited  
4 set of rules, but is relatively simple in order to explore the whole spectrum  
5 of possible rule configurations. Card games are chosen here as such a problem.  
6 The rules of the game define how players play the game and there are official  
7 descriptions of the rules. Each change of the rules can lead to distinctly differ-  
8 ent outcomes of how the game is played and how players evaluate the game.  
9 It must be mentioned that the use of games has a distinguished history in the  
10 study of collective action since the pioneering work of Jean Piaget with marble  
11 games [10].

12 Children are tinkering with rules of marble games, as well as card games, which  
13 make them interesting candidates for the study of rule evolution. The goal of this  
14 exercise is to provide a proof of concept on the evolution of actual rule sets, instead  
15 of abstractions of rules, using rationally bounded agents. Can we evolve in a sim-  
16 ulated environment a population of agents who play card games which mimic the  
17 principles of actual card games? We focus here on a simple class of card game, a  
18 shedding-type of games like UNO and crazy eights.

19 In the next section, we analyze the rules of the shedding-type of card games.  
20 Then we present a model to simulate a group of agents playing such card games  
21 and show how crazy eights and UNO are within a fitness landscape of variations of  
22 the card game. This brings us to the core of the paper, the evolution of rules for  
23 which we describe the model and present model analysis. The paper concludes with  
24 a discussion how this work can be extended to the study of institutional change in  
25 social-ecological systems.

## 26 **2. Rules of the Shedding-Type of Card Games**

27 The shedding-type of card games is a family of games played by two or more players  
28 who receive a specific initial number of cards with the objective of being the first to  
29 “shed” all of their cards.<sup>a</sup> An example is crazy eights, and there are many variations  
30 including the popular UNO game.<sup>b</sup> The large diversity of rules of shedding games  
31 show that this kind of games allow for tinkering of the rules and make interesting  
32 variations of the same base game.

33 The basic idea is to have a pack of  $N$  cards and randomly distribute an initial  
34 amount to each player. The undealt stock is placed face down on the table, and  
35 the top card of the stock is turned face up and placed beside the stock to start  
36 the discard pile. In order, the player must either play a legal card face up on  
37 top of the discard pile, or draw a card from the undealt stock. A card is legal  
38 when it matches the rank or suit. There are various special cards. Some cards

<sup>a</sup>[http://en.wikipedia.org/wiki/Category:Shedding-type\\_card\\_games](http://en.wikipedia.org/wiki/Category:Shedding-type_card_games) accessed June 21, 2008.

<sup>b</sup><http://www.pagat.com/eights/> accessed June 21, 2008.

Table 1. A description of the cards and their roles in crazy eights and UNO.

| Card                    | Crazy eights   | UNO   |
|-------------------------|--|---|
| Number of cards         | 52   | 108   |
| Number of suits/colors  | 4  | 4   |
| Initial amount of cards | 5  | 7   |
| Normal cards            | 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace                   | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9  |
| Skip card               | —  | Skip the next player's turn   |
| Reverse card            | —  | The direction of play switches  |
| Wild card               | —  | Might be played on any card and player calls color to be played by next player  |
| Draw two                | —  | The next player draws two cards and skip his/her turn   |
| Wild draw four          | —  | Might be played on any card, and the next player draws four cards and skips his/her turn. Wild Draw Fours may only be played if you do not have a card of the current color. The player might bluff and play a Wild Draw Four illegally, though the player drawing the cards may "challenge" the card. If the card was played illegally, that player is penalized with drawing two cards, if not, the challenger is penalized |
| Eight card              | Can always be put and player can define suit (like Wild card of UNO) | —   |
| End game                | —  | Must say "uno" when one card is left  |
| Drawing cards           | May draw card if one can play. Must draw card until one can play     | May draw card if one can play. Must draw one card if one cannot play  |

1 lead other players to have to pick up more cards from the stock, or skip a turn  
 2 (see Table 1).

3 There are many variations of the shedding-type of card games.<sup>c</sup> We define now  
 4 a more general shedding game which covers some of this diversity in a systematic  
 5 way. By defining a systematic way to create shedding games, we can develop com-  
 6 putational models of rule evolution.

7 The basic rule is that a player must play a legal card when it is his/her turn.  
 8 We assume that players use one or two decks of cards with four suits of thirteen  
 9 cards per deck. These thirteen cards included numbers 1–10, Jack, Queen and King.  
 10 Some cards may have special meanings:

- 11 — 10 can become a Wild card (we use 10 instead of 8 to make it more fluid)
- 12 — Jack can become a Reverse card

<sup>c</sup>[http://www.pagat.com/invented/uno\\_vars.html](http://www.pagat.com/invented/uno_vars.html) accessed June 21, 2008 and [http://www.pagat.com/invented/eights\\_vars.html](http://www.pagat.com/invented/eights_vars.html) accessed June 21, 2008.

- 1 — Queen can become a Skip card  
 2 — King can become a draw two card
- 3 If there is a second deck, 10s of the second deck become a Wild 4 card.  
 4 Besides the special cards, the following rules may vary:
- 5 — One or two decks of cards  
 6 — Random order or fixed order or turns. With random order we mean that a  $N$ -  
 7 sided dice is thrown each round to determine which of the  $N$  agents is allowed  
 8 to play  
 9 — The color of the card must match color or not of the card on top of the discard  
 10 pile  
 11 — Wild Draw Four can be drawn legally any time, or only when there is no card  
 12 of the same color  
 13 — One must draw until one can play or only draw one

14 We now can define a spectrum of the shedding-type of card games as variations  
 15 of a set of rules. Eleven rules are varied which leads to  $2^{11}(= 2048)$  variations of  
 16 shedding games in which Crazy Eights and UNO are special cases.

17 Since we are interested in rule evolution, it is important to recognize that there  
 18 is only one point in the game where we explicitly include the breaking of a rule.  
 19 Namely the use of the Wild Draw Four which can only be used when one has  
 20 no card of the same color as that on top of the discard pile. We may consider  
 21 that people do not draw cards if they have to, or that they peek at the cards of  
 22 other players. However, this kind of behavior is difficult to properly include into  
 23 a formal model, and we assume that the agents play the game in an orderly way.  
 24 A drawback of using the card games as an analogy for institutional change is the  
 25 constant monitoring of the players, which is not happening in most other action  
 26 arenas.

Table 2. The rule space is build up from 11 types of rules. The well-known shedding games are 2 of the 2048 possible games within the rule space.

| Rules           | Options    | “Crazy eights” | UNO   |
|-----------------|------------|----------------|-------|
| Random turn     | True/False | False          | False |
| Initial # cards | 5/7        | 5              | 7     |
| 4 colors        | True/False | True           | True  |
| Color change    | True/False | True           | True  |
| Decks of Cards  | One/Two    | One            | Two   |
| Reverse         | True/False | False          | True  |
| Draw 2          | True/False | False          | True  |
| Draw 4          | True/False | False          | True  |
| Draw 4 Anytime  | True/False | False          | False |
| Skip            | True/False | False          | True  |
| Draw till play  | True/False | True           | False |

### 1 **3. Model of Card Game<sup>d</sup>**

2 We present now a computational model where  $N$  artificial agents play shedding card  
3 games to a set of specified rules. This model enables us to explore the consequences  
4 of changes in the rules on various performance indicators as discussed below.

5 How do we evaluate the different rule configurations of the game? For insti-  
6 tutional rules about natural resource management equality of appropriation and  
7 overharvesting and logical indicators of performance. For card games, we have to  
8 come up with something quite different that cannot relate directly back to many  
9 institutional arrangements.

10 The basic assumption is that for card games, the evaluation is based on how  
11 players evaluate the game. Fortunately, some research has been done on what peo-  
12 ple enjoy in play and games. Csikszentmihalyi [5] developed the concept of “flow”,  
13 which represents the feeling of complete and energized focus in an activity with  
14 a high level of enjoyment and fulfillment [3]. The flow is derived by balancing the  
15 challenges and the ability by the player to overcome it. If an activity is too challeng-  
16 ing given the ability it generates anxiety. When an activity is not very challenging  
17 given the ability, it generates boredom. To define indicators to quantify the trade-  
18 off between challenge and ability, we first recognize that playing the shedding-type  
19 card games is not extremely challenging. It is no surprise that those card games are  
20 popular among children. We define the trade-off between boredom (length of game)  
21 and challenging (number of types of cards a player can play). The last indicator  
22 requires some explanation. If all legal cards are normal cards of the same color, it  
23 does not matter much which card to put. If one has some special cards like Draw 2  
24 or Reverse, the number of types of cards that can be played increases. The more  
25 types of cards are available, the more important the choice of the card becomes.

26 We assume a simple strategy for agents to decide which card to put on the deck.  
27 First, one evaluates which cards are legal to play. We assume that with probability  
28  $p_c$  an agent may consider to put the Wild draw 4 card when he still has cards of  
29 the same color. This is the probability of cheating.

30 If the amount of legal cards is equal to 1, play the card. If there are more legal  
31 cards, there is a probability  $p_r$  that a card is drawn randomly. With probability  
32  $1 - p_r$ , the player will evaluate whether there is a card that one can play that can  
33 prevent the next player from playing. Note that this is the extreme case where there  
34 is perfect knowledge and the cards of the other player are known. By varying the  
35 level of  $p_r$ , we can see the impact of perfect information. It will first look at the  
36 possibilities using a normal card. If no good card is available, one will consider a  
37 card that can change the color of the cards. If that does not work, one will consider  
38 the reverse card, skip card, draw 2 card or draw 4 card.

<sup>d</sup>Netlogo version of the model versions used in this paper can be found at <http://www.openabm.org/model-archive/sheddinggames> and <http://www.openabm.org/model-archive/evosheddinggames>.

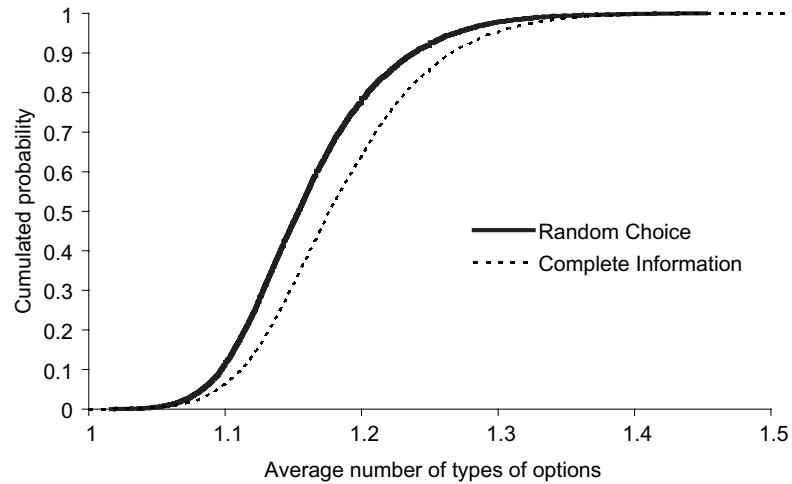


Fig. 1. The cumulated probability of the number of types of options in 10,000 games of UNO. The distributions differ if a different type of agent is used: random choice versus complete information.

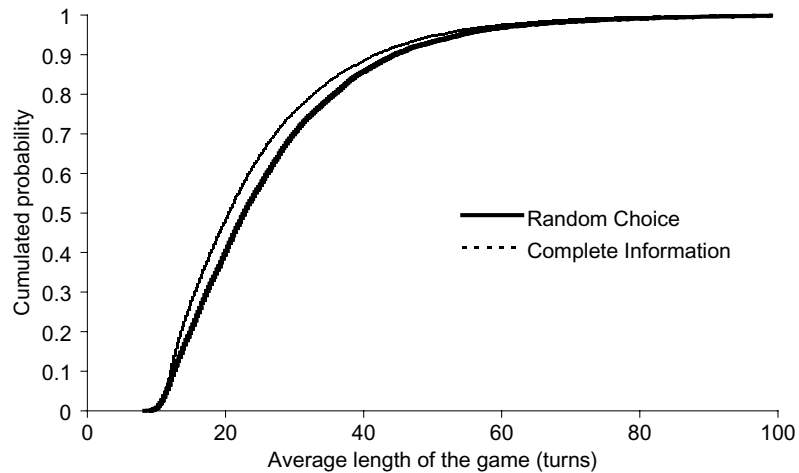


Fig. 2. The cumulated probability of the number of turns it takes to finish a game in 10,000 games of UNO. The distributions differ if a different type of agent is used: random choice versus complete information.

1 We use as a default strategy where  $p_c$  is 0 and  $p_r$  is 0. We also have explored  
 2 different versions of the model and found that there is no major difference in the  
 3 results in this section. In Figs. 1 and 2, we show the distributions of the average  
 4 number of types of options and the average number of turns per person for playing  
 5 the UNO game. In each figure, we show that the distribution for  $p_r$  is 1 (random  
 6 choice) and  $p_r$  is 0 (complete information). The average length of the game is  
 7 slightly shorter for the situation that agents play the game with full information.

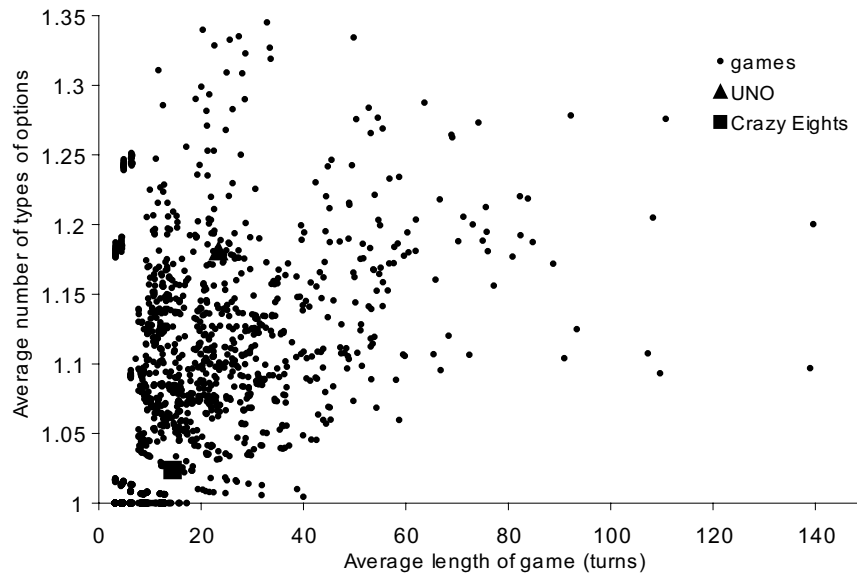
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Fig. 3. The performance of all 2048 possible card games expressed among the dimensions of the number of types of options players experienced and the average length of the game per person. For each possible card game, 10,000 runs are performed for a group size of 4 players.

1 Furthermore, the players have more types of options during games if they have  
 2 complete information instead of random choice.

3 However, these differences are not significant. This is not surprising since most  
 4 of the time, agents have only one option to play a card legally.

5 There are 2048 ( $= 2^{11}$ ) variations of the game. We run the model 10,000 times  
 6 for each of the 2048 for a group size 4 (Fig. 3).<sup>e</sup> In these games, we assume agents  
 7 have complete information ( $p_r = 0$ ). We see a large diversity of outcomes. A few  
 8 games have only one option per turn and an average length of a few turns. These  
 9 are games where there are no rules that restrict players to put a card on the table.  
 10 In other games, the average duration of the game is 140 turns (all including the  
 11 rule “draw till play”). In some games, the duration of the game is relatively short,  
 12 but the average number of types of options is around 1.35. There is no direct  
 13 relation between one individual rule and the performance since it depends on the  
 14 interactions with other rules.

15 The Crazy Eights game is relatively short and has a low average number of  
 16 types of options. The UNO game has more turns on average and is more exciting  
 17 (more types of options on average per turn). This is in line with the history of  
 18 UNO, which is developed as a follow up of Crazy Eights.

<sup>e</sup>When we vary the group size between 2 and 10 players, we see an increase in the average length of the game as well as the number of types of options, but the shape of the distributions remain the same.

#### 1 4. Evolution of Rules

2 Now we have seen the spectrum of possible games, can we simulate evolution of  
 3 games? What kind of games will evolve? The simulation of rule evolution in this  
 4 paper is based on cultural group selection [2]. Groups who do better — have more  
 5 exciting games — are more likely to be copied. Hence successful innovations of rules  
 6 will spread from group to group.

7 We now like to start with agents randomly throwing cards and have them evolve  
 8 meaningful shedding-type of card games. We are given is a landscape of 100 cells  
 9 using a regular grid of  $10 \times 10$  cells. The borders are wrapped around to derive a  
 10 torus and avoid border effects. Each cell contains a group of size  $N(= 4)$  who plays  
 11  $n(= 10)$  shedding games before evaluating their performance.

12 When all groups have played  $n$  games, the groups will look around to see whether  
 13 there is a nearby neighboring group — eight surrounding cells — who has performed  
 14 better. A better performance is considered to be a group with a higher number of  
 15 types of options as long as the average length of the game is not more than  $x$  times  
 16 the current game. We use  $x$  equal to 10 in the simulations discussed below. Hence  
 17 this means that groups derive a small sample of possible outcomes of a set of rules.

18 The best neighboring group, if any, is imitated. The bigger the difference in  
 19 the performance between the own group and the best neighboring group, the more  
 20 likely rules of the better group are copied. For each of the 11 rules, the rule is  
 21 adopted with probability  $1 - O_i/O_{\max}$ , where  $O_i$  is the average number of options  
 22 of neighboring group  $i$ .

23 Independent of the imitation process, groups tinker their rules. For each of the  
 24 11 types of rules with an independent probability  $\mu(= 0.01)$ , a rule is changed for  
 25 the next generation.

26 In the evolutionary process described, we will expect that groups will evolve  
 27 towards card games with high number of types of options per turn. However, the  
 28 variability in the outcomes of the games is high (Figs. 1 and 2). As a result, groups  
 29 may experience that their neighbors had more interesting games than theirs, and  
 30 imitate the rules, but will not experience an improvement in the next generation.  
 31 Groups evolve their rules in a noisy fitness landscape and it is not known in advance  
 32 if this leads to a convergence of a certain set of rules.

33 Figure 4 depicts the average performance of games evolving in the fitness land-  
 34 scape of possible games. The starting point is at one option per turn and about  
 35 6 or 7 turns since they just throw any card on the stack. Over time, the length  
 36 increase as well as the number of types of options. The average number of types of  
 37 options that evolve is similar to the UNO game, although the length of the game is  
 38 shorter (Table 3). Since only 10 games are played in each generation, groups receive  
 39 a noisy signal. This limits the ability of the evolutionary process to find those rule  
 40 combinations that leads to the highest number of types of options in the long run.

41 In Fig. 5, we see for 3 examples' rule how they are adopted over time. The  
 42 “four colors” as well as the ability to “change color” emerge quickly as rules in

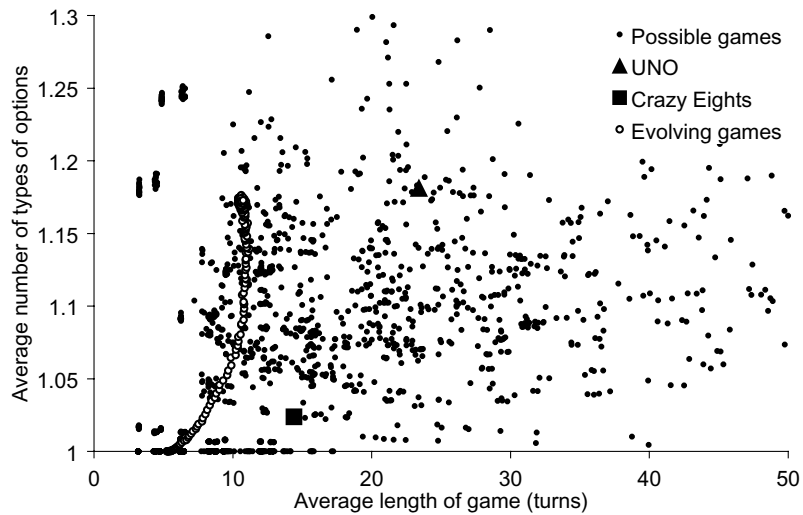
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Fig. 4. The trajectory of the evolving games (open dots) starting at the left bottom of the screen and evolving to about 10 turns and 1.18 number of types of options (averaged over 10 runs). We included the fitness landscape of Fig. 3 as a reference.

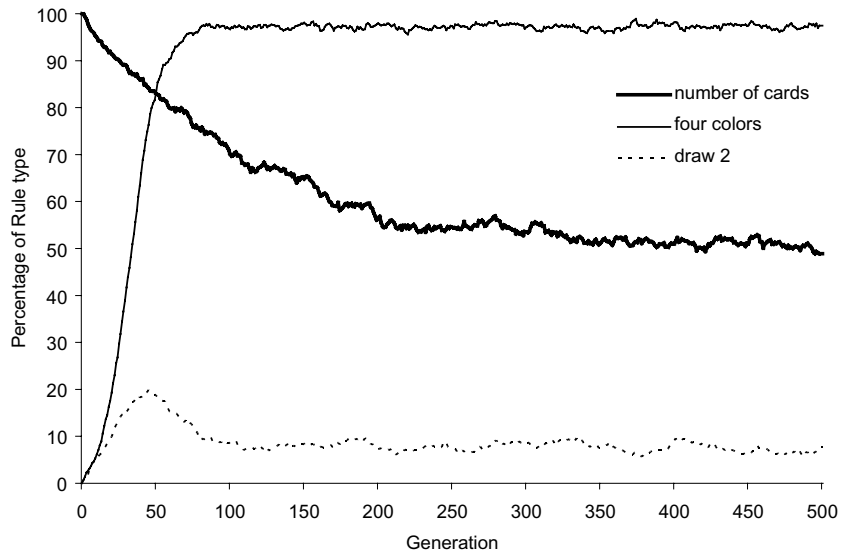


Fig. 5. Examples of three rules used by the 100 groups over 500 generations (average over 10 runs). The rule of distinguishing four colors, and the rule to use the draw 2 card.

- 1 the games. Other rules that evolve quickly are the “reverse card”, “to draws cards
- 2 until one can play”, and the “skip card” as well as the “initial number of 7 cards”.
- 3 Other rules such as a “draw 4 cards” or having one or two decks of cards seem not
- 4 relevant for the agents. Half of the groups have such a rule, while the other half

Table 3. The evolved rule sets for two types of experiments. The standard experiment assumes that agents view which card the other puts on the stack, while in the blind case agents need to challenge the agent who put a card on the stack if one likes to check whether a valid card is used. The percentage is the percentage in the last 100 generations of the 100 groups in which groups use a particular rule.

| Rules           | “Crazy eights” | UNO   | Evolved (standard) | Evolved (blind) |
|-----------------|----------------|-------|--------------------|-----------------|
| Random turn     | False          | False | 58%                | 15%             |
| Initial # cards | 5              | 7     | 91% (7)            | 96% (7)         |
| 4 colors        | True           | True  | 98%                | 99%             |
| Color change    | True           | True  | 97%                | 98%             |
| Decks of Cards  | One            | Two   | 49% (two)          | 63% (two)       |
| Reverse         | False          | True  | 95%                | 97%             |
| Draw 2          | False          | True  | 6%                 | 3%              |
| Draw 4          | False          | True  | 52%                | 33%             |
| Draw 4 Anytime  | False          | False | 45%                | 33%             |
| Skip            | False          | True  | 95%                | 96%             |
| Draw till play  | True           | False | 93%                | 41%             |
| Options         | 1.03           | 1.18  | 1.17               | 1.51            |
| Length          | 14.4           | 23.4  | 10.7               | 26.60           |
| $p_c$           |                |       |                    | 0.38            |
| $p_{ch}$        |                |       |                    | 0.48            |

1 have turned off such rules. After about 200 generations, the evolved rules start to  
 2 converge. Compared to UNO there are a few interesting differences. The evolved  
 3 game has the rule to draw cards until the agent can play, but does not include the  
 4 “draw 2” cards rule (Table 3).

5 Since the shedding-type of card games have limited opportunities to test the  
 6 monitoring and enforcement mechanisms, we also ran experiments where agents  
 7 cannot directly see the card which has been put on the deck by the other agents  
 8 until it is one’s turn. One can challenge an agent who just put a card on the deck.  
 9 If one rightly challenges the agent, the agent who cheated has to draw two cards,  
 10 otherwise the challenging agent needs to draw 2 cards. An agent will only cheat  
 11 when one has no valid card.

12 The probability of agents to cheat ( $p_c$ ) and to challenge ( $p_{ch}$ ) are individual  
 13 attributes. These attributes evolve over time. Agents who win more games are  
 14 more likely to get offspring compared to agents who win fewer games. Thus besides  
 15 the rules that may change each generation, attributes of agents may change each  
 16 generation. Hence we will simulate the co-evolution of agent attributes as well as  
 17 the rules of the game. Figure 6 shows that initially the games become more and  
 18 more lengthier and has also more types of options. After about 500 generations, the  
 19 peak of the length of the game is reached. The reduction of the length of the game  
 20 from 35 to 30 turns happens in the 500 next generations and then stabilizes. The  
 21 number of types of cards played keeps going up slowly to 1.5 coinciding with an  
 22 increase of cheating and challenging. More cheating and challenging leads to more  
 23 cards and more options of cards available.

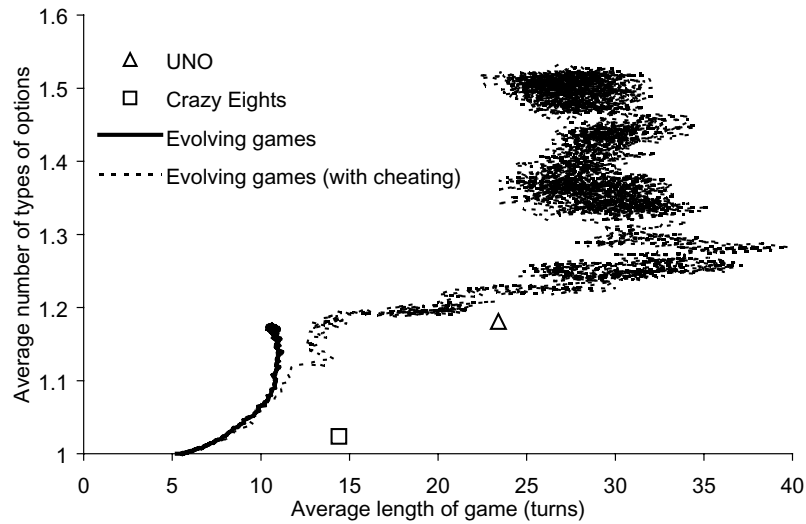
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Fig. 6. The evolution of the rules when agents cannot see whether the other agents put valid cards on the stack. The small black dots represent average performance of individual generations.

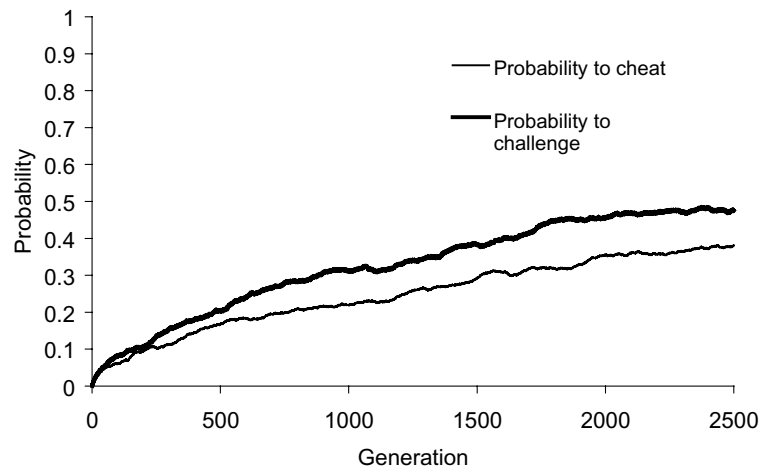


Fig. 7. The evolved probability to cheat and to challenge agents who put cards on the stack.

1        Figure 7 shows the average attributes of agents that evolve over time. Agents  
 2        have a modest tendency to cheat, around 40% of the time they cannot play a legal  
 3        card, and they have 50% probability to challenge agents. Using the indicators we  
 4        defined for the performance of the games, cheating when agents run out of legal  
 5        options is an evolved option to increase the number of types of cards agents can  
 6        play. Groups where agents do not cheat frequently will end sooner and have a low  
 7        number of types of options.

1 Rules evolve where agents have more options, but also agents evolve who  
2 are challenging other agents more frequently. When agents have evolved who are  
3 challenging other agents often, the rule that leads agents to draw cards until they  
4 can play becomes less important for groups. In the end of the simulations, the same  
5 type of game evolves as with the game where everybody sees which card are put  
6 on the table, except for a lower use of the rule that lead agents to draw cards until  
7 they can play and the game is more likely to follow a predefined order of play.

## 8 **5. Conclusion**

9 In this paper, we use shedding games to create a full fitness landscape of rule con-  
10 figurations. This enables us to explore the development of a model on the evolution  
11 of rule structures. Using cultural group selection, shedding games evolve somewhat  
12 similar to UNO, but are shorter and experience similar choices for the players. When  
13 we artificially allow agents to cheat and monitor each other, we derive a model in  
14 which agents and rules co-evolve. This leads to outcomes that were not foreseen in  
15 the original fitness landscape, since agents evolve with a large probability of cheat-  
16 ing and challenging other agents. Such an outcome can happen since diversity of  
17 options to play was rewarded, not whether agents are honest and follow the rules.

18 Although this exercise is somewhat artificial, and the assumptions of perfor-  
19 mance of groups of agents drive the results, there are a number of insights we  
20 derived. Cultural group selection is a useful way to study the evolution of institu-  
21 tional rules. This assumes that attempts to imitate practices of other groups are  
22 important sources for innovation. Certain components of the genome of institu-  
23 tional rules may be found to be “junk” rules, hence are not driving the differences  
24 of outcomes between groups. The evolved rule sets led to game performance that  
25 was to be expected given that we could explore the whole fitness landscape. There-  
26 fore, we may start applying this method to more comprehensive problems for which  
27 the fitness landscape cannot fully be explored.

28 Most studies in the evolution of cooperation assume a static institutional land-  
29 scape in which agent characteristics evolve. This paper is an initial step towards  
30 the study of the evolution of institutional rules, and its co-evolution of agent-  
31 characteristics. Agents may play social dilemma games that are defined by a simple  
32 grammar of rules [4, 6]. Changes in the rules will affect the pay-off structure, the  
33 order of play, the information agents derive, the monitoring and enforcement, etc.  
34 If we use social dilemma games related to public good provision, the level of the  
35 generated public good and the level of inequality might be used as indicators to  
36 evaluate group performance. Groups that have higher levels of public goods will  
37 do better in group competition. Groups who have higher public good levels and  
38 are more equal in the distribution of resources might be copied more frequently by  
39 neighboring groups.

40 The development of such comprehensive models of rule evolution enables us to  
41 start addressing questions on the emergence of complex societies in which both

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1 agent types and its institutional context evolve. What is the role of information,  
2 constitutional rules, diversity in environmental context between groups, and the  
3 level of path dependency? Furthermore, it enables us to start exploring questions  
4 on the transitions in complex societies in which not only institutions change but  
5 also the attributes of the agents. In sum, this paper provides some interesting initial  
6 steps towards a research agenda of the co-evolution of agents and institutional rules.

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