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Sequential optimization of integrated climate change models

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Abstract

A sequential optimization approach is applied to optimize the behavior of a complex dynamical system. It sequentially solves a large set of mathematical equations and next optimizes the behavior of a reduced-system, fixing certain variables of the larger original problem. These two steps are repeated till convergence occurs. The approach is applied to the problem of identifying response strategies for climate change caused by antropogenic emissions of different trace gases. The convergence properties are analyzed for this example. © 2001 IMACS. Published by Elsevier Science B.V. All rights reserved.

Keywords: Sequential optimization; Sequential reduced-system programming; Dynamical system

1. Introduction

Human activities may change the environment in unpredictable ways. A human-induced climate change is one of the major global environmental problems, which may have severe implications for the well being and welfare of coming generations. Various complex simulation models have been developed which try to project the possible consequences of human activities. Those models represent the state-of-the-art knowledge of the integrated social, economic and ecological system [1]. Although those models produce highly uncertain projections, they are the best tools we have to produce quantitative visions of the future. The aim of this paper is to discuss how such complex dynamic models can be used in an optimization framework to generate efficient mitigation strategies.

Dynamic systems are applied in different fields of science, such as physics, engineering, economics, biometrics, cybernetics and environmental sciences. Often the state space approach presents an adequate descriptive mechanism. The properties of the state variables of the system can be analyzed by considering the consequences of changes in the forcing terms.

These models may be used to derive optimal strategies for the control of the system according to a well-defined objective function. When the set of mathematical equations representing the

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dynamical system is too large or too sophisticated in order to solve them along mathematical analytical lines, numerical optimization is the next tool. Therefore, the input values of the system, at every iteration, will be changed in such a direction that the objective function reaches the best feasible level.

In practice, the efforts needed for the derivation of an "optimal solution" is often a matter of balancing the costs of deriving the solution against the benefits from the optimal control. In this paper, an approach will be used which can be applied to optimal control problems for large-scale systems and which is called sequential reduced-system programming (SRSP) [2]. The algorithm tries to solve an optimization problem by iteratively solving a large dynamical system of differential equations and next optimizing the behavior of a reduced version of the dynamical system. These two steps are repeated until convergence occurs. The reduced version of the dynamical system is chosen such that it represents the core of the unrestricted dynamical system. Parameters within the reduced version of the dynamical system are updated after each simulation of the original dynamical system.

The approach is related to solving a simultaneous problem of parameter identification and optimization. Such an approach is necessary when estimates of parameter values may, besides depending on the values of the control variables, on the other hand, influence the feasible range of the control variables. A general approach to solve this problem is the two-step approach that sequentially estimates the parameter values of the dynamical system and optimizes the criterion of the system [3–5]. Instead of parameter estimation and optimization, the two steps of the sequential reduced-system optimization method consist of solving a dynamical system of equations and optimization of a subsystem.

First, the methodology will be described in a more formal way, and some properties for convergence of the algorithms and the solution will be discussed. Next, a practical application of SRSP, to the optimization of response strategies for climate change, will be presented.

2. Methodology

2.1. Problem formulation

Dynamical systems will be considered which can be written as the following set of differential equations:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = h(x(z), z) = Ax(t) + N(x(t)) + F(z(t)), \tag{1}$$

where $x(t) \in \Re^s$ is the state vector of the system that describes all the characteristic quantities, 'A' a square matrix and F a vector-valued function of $z(t) \in \Re^n$, the input vector of the system. N represents the non-linear terms. If A, N, x_0 and $z(t)$ for every t are known, then $x(t)$ is assumed to be uniquely determined. Clearly, A, N and x_0 are defined once and for all and $x(t)$ is a function of $z(t)$:$x(z(t))$.

Suppose that the aim is to optimize the behavior of the system. Therefore, the elements of the input vector $z(t)$ have to be chosen so that an objective function $f : \Re^{s+n} \to \Re$ is maximized taking into account the k constraints represents by the vector valued function $g : \Re^{s+n} \to \Re^k$. Vector $z(t)$ can, thus, be considered as the decision vector.

The optimal control problem can mathematically be stated as

$$\max_{z(t)} \int_{t_0}^{t_1} f(x(z(t)), z(t))\,\mathrm{d}t, \qquad z(t) \in \Im = \begin{cases} z(t) \in \Re^n : g(x(z(t)), z(t)) \le 0 \\ \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + N(x(t)) + F(z(t)) \end{cases}, \tag{2}$$

where $\Im \in \Re^n$ is the decision space. All functions are supposed to be continuously differentiable.

2.2. Algorithm

The algorithm proposed here to solve (2) is based on the idea to reduce the system (1) into a smaller system, fixing certain variables, and which allows the application of optimization methods in order to derive an optimal solution for this smaller system. Next, this solution is used as input for the next iteration of the original system, etc.

The state space will now be split up into $x^1(t) \in \Re^m$ and $x^2(t) \in \Re^{s-m}$, with $m < s$. The algorithm will treat the variables $x^1(t)$ as endogenous and $x^2(t)$ as given exogenous values which are updated every iteration. Observe that, for the small system, $x^1(t)$ is steered by the input variable $z(t)$, and the fixed input $x^2(t)$. The other part of the system, $x^2(t)$, is only influenced indirectly by changes in the input variable $z(t)$ through $x^1(t)$.

The resulting algorithm is, therefore, choose some start vector $z_0(t)$. Set the iteration number $q = 1$.

Step 1 (Simulation): Using $z_{q-1}(t)$ solve

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + N(x(t)) + F(z_{q-1}(t)) \tag{3}$$

to derive $x_q(t) = \{x_q^1(t), x_q^2(t)\}$.

Step 2 (Optimization): Fixing the estimates of $x_q^2(t)$ from step 1, obtain $z_q(t)$ by solving

$$\max_{z(t)} \int_{t_0}^{t_1} f(x^1(z(t)), z(t))\,\mathrm{d}t,$$

$$z(t) \in \Im_q = \begin{cases} z(t) \in \Re^n : g(x^1(z(t)), z(t)) \le 0 \\ \dfrac{\mathrm{d}x^1(t)}{\mathrm{d}t} = A_1 \begin{pmatrix} x^1(t) \\ x_q^2(t) \end{pmatrix} + N_1(x^1(t), x_q^2(t)) + F_1(z(t)) \end{cases}. \tag{4}$$

Set $q = q + 1$ and go to step 1.

In step 2 of the algorithm, A_1 is the appropriate submatrix of A, while N_1 and F_1 refer to the x^1-components of N and F, respectively. The steps of simulation and optimization are performed sequentially until no further improvement can be observed, or until the convergence criterion is met.

2.3. Convergence of the algorithm

The algorithm generates a sequence $x_q^2(t)$ and $z_q(t)$, $q = 0, 1, 2, \ldots$. Some conditions have to be fulfilled in order to guarantee the convergence of this sequence. Since $x_q(t)$ is dependent on $z_{q-1}(t)$ and $z_q(t)$ on $x_q(t)$, the solution $x_q^2(t)$ of the system (3) can be substituted into the problem formulation (4). Therefore, a map $M : \Re^n \to \Re^n$ can be defined which represents the relation between z_q and z_{q-1}, or

$z_q = M(z_{q-1})$, where M denotes the optimal solution of problem (4) using the solution $x_q^2(t)$ of system (3). Obviously, any convergence point of algorithm coincides with a fixed point of M. On the other hand, it can easily be seen that any optimal solution of the system (2) also coincides with a fixed point of M. Let, $z^*(t)$ be a fixed point of M. If $z_0(t)$ is started in the attraction space $\Im^*$ of $z^*(t)$, then M is assumed to be a continuous map.

The map M is a contraction map on a compact subspace $S \subset \Re^n$, whenever $M(z) \in S$ for any $z \in S$ and $||M(z_1) - M(z_2)|| \leq \lambda ||z_1 - z_2||$ for any $z_1, z_2 \in S$ and $\lambda \in (0, 1)$ in some well-defined norm $\|\cdot\|$. In such a case, M has a unique fixed point on S. A sufficient condition for the existence of a fixed point is as follows (e.g. [6]):

$$\left|\frac{\partial m_i(z)}{\partial z_j}\right| \leq \frac{L}{n}, \quad \text{whenever } z(t) \in \Im^*, \tag{5}$$

where $0 < L < 1$ and for each $j = 1, 2, \ldots, n$ and each component function m_i of M. Then, the sequence $\{z_q(t)\}$, defined by a starting vector $z_0(t)$ in the attraction set $\Im^*$ of $z^*(t)$ and generated by $z_q(t) = M(z_{q-1}(t))$ converges to the unique solution $z^*(t)$.

Condition (5) can be rewritten as

$$\left|\sum_{w=1}^{s-m} \frac{\partial m_i(z^1(t))}{\partial x^2(w, t)} \frac{\partial x^2(w, t)}{\partial z_j^1(t)}\right| \leq \frac{L}{n}, \tag{6}$$

where $x^2(w, t)$ denotes the wth component of the vector $x^2(t)$. Inequality, (6) says that if $x^2(t)$ is not “too sensitive” to changes in the decision variable $z(t)$ and if an optimal z of the problem (4) for a new iteration is not “too sensitive” to changes in $x^2(t)$, then the algorithm will converge. Translated back to the separation of the state vector x into x^1 and x^2 this means that x^1 should be chosen so that it represents the core dynamics of the model.

Although the above conditions can hardly be checked for large-scale problems, it points to the fact that if the solution of an iteration does not largely change in comparison to the solution of the preceding iteration, the algorithm converges to $z^*(t)$. However, if $z_q(t)$ converges, the solution will not necessarily be a global or even a local optimum of (2). Conditions for a local optimum will be formulated next.

2.4. Convergence of the optimal solution

If z^* is a fixed point of the algorithm M which generates the system state x^*, what are the conditions for z to be a (local) optimum of (2)? The following equation of the Hamiltonian of problem (2) can be formulated.

Define

$$H(x, z, \lambda) = f(x(z), z) + \sum_{i=1}^{s} \lambda_i h_i(x(z), z) + \sum_{i=1}^{k} \mu_i g_i(x(z), z), \tag{7}$$

where $\lambda(t) = (\lambda_1(t), \ldots, \lambda_n(t))$ and $\mu(t) = (\mu_1(t), \ldots, \mu_k(t))$ are assumed to be continuously differentiable functions and where h_i is the ith equation of the dynamical system.

If the vector control function z^* and the corresponding state vector x^* solve problem (2), then, z^*, x^*, λ, μ together simultaneously should satisfy

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \frac{\mathrm{d}H}{\mathrm{d}\lambda} = h(x(z), z), \tag{8}$$

$$g(x(z), z) \leq 0, \tag{9}$$

$$0 = \frac{\partial H}{\partial z_j} = \frac{\partial f}{\partial z_j} + \sum_{i=1}^{s} \lambda_i \frac{\partial h_i}{\partial z_j} + \sum_{i=1}^{k} \mu_i \frac{\partial g_i}{\partial z_j}, \quad j = 1, \ldots, n, \tag{10}$$

$$\frac{\mathrm{d}\lambda_j}{\mathrm{d}t} = -\frac{\partial H}{\partial x_j} = -\left(\frac{\partial f}{\partial x_j} + \sum_{i=1}^{s} \lambda_i \frac{\partial h_i}{\partial x_j} + \sum_{i=1}^{k} \mu_i \frac{\partial g_i}{\partial x_j} \right), \quad j = 1, \ldots, s, \tag{11}$$

$$\mu \geq 0, \quad \mu g = 0. \tag{12}$$

It is clear that if $m = s$, in case that the system is not reduced, the solution found by the algorithm is equal to a local optimum satisfying the necessary conditions as above. It might occur that, in case $m < s$, a convergence point of the algorithm does not coincide with a local optimum. A set of sufficient conditions of the reduced-system will be traced for which a solution within an area of tolerance around a local optimum is found by SRSP. In a fixed point z^* of M, a local optimum is found for problem (4), so, conditions (8), (9) and (12) are satisfied.

If g and f do not directly depend on z and if $F(z) = Bz$, where B is a $(s \times n)$-matrix of coefficients, then

$$\frac{\partial f}{\partial z_j^*} = 0, \quad \text{for } j = 1, \ldots, n, \tag{13}$$

$$\frac{\partial g_i}{\partial z_j^*} = 0, \quad \text{for } j = 1, \ldots, n \quad \text{and} \quad i = 1, \ldots, k, \tag{14}$$

$$\frac{\partial h_i}{\partial z_j^*} = b_{ij}, \quad \text{for } i = 1, \ldots, s \quad \text{and} \quad j = 1, \ldots, n, \tag{15}$$

While in the local optimum of (4), there are λ_i^* and μ_i^*, $i = 1, \ldots, m$, such that

$$\frac{\partial f}{\partial z_j^*} + \sum_{i=1}^{m} \lambda_i^* \frac{\partial h_i}{\partial z_j^*} + \sum_{i=1}^{k} \mu_i^* \frac{\partial g_i}{\partial z_j^*} = 0, \quad \text{for } j = 1, \ldots, n. \tag{16}$$

It follows that

$$0 = \sum_{i=1}^{m} \lambda_i^* b_{ij}, \quad j = 1, \ldots, n. \tag{17}$$

An obvious solution to (17) is the time independent choice $\lambda_i^* = 0$, $i = 1, \ldots, m$. Observing the fact that $\mathrm{d}\lambda_i/\mathrm{d}t = 0$, for $i = 1, \ldots, m$, it follows that from the optimality conditions of problem (2), the following condition is obtained:

$$0 = -\left(\frac{\partial f}{\partial x_j^*} + \sum_{i=1}^{m} \lambda_i^* \frac{\partial h_i}{\partial x_j^*} + \sum_{i=1}^{k} \mu_i^* \frac{\partial g_i}{\partial x_j^*}\right), \quad j = 1, \ldots, n \tag{18}$$

in the fixed point. Consequently, Eq. (11) can now be reduced to

$$0 = \sum_{i=m+1}^{s} \lambda_i \frac{\partial h_i}{\partial x_j^*}, \quad j = 1, \ldots, n. \tag{19}$$

Finally,

$$\frac{\mathrm{d}\lambda_j}{\mathrm{d}t} = -\left(\frac{\partial f}{\partial x_j^*} + \sum_{i=m+1}^{s} \lambda_i \frac{\partial h_i}{\partial x_j^*} + \sum_{i=1}^{k} \mu_i \frac{\partial g_i}{\partial x_j^*}\right), \quad j = n+1, \ldots, s \tag{20}$$

which can be written as

$$\frac{\mathrm{d}\lambda_j}{\mathrm{d}t} = \sum_{i=m+1}^{s} a_{ij}(t)\lambda_j + b_j(t), \quad j = n+1, \ldots, s \quad \text{with} \quad a_{ij}(t) = -\frac{\partial h_i}{\partial x_j^*},$$
$$b_j(t) = -\frac{\partial f}{\partial x_j^*} - \sum_{i=1}^{k} \mu_i \frac{\partial g_i}{\partial x_j^*}. \tag{21}$$

According to Kamien and Schwartz [7 page 313], there is a unique solution for the $(s-n)$ differential equations if the derivatives and μ_i's are continuous on the considered time horizon. Therefore, it may be concluded that if g and f do not directly depend on z, if the control is linear $F(z) = Bz$, and if conditions (19) and (20) are fulfilled, then a solution can be found which satisfies (8)–(12) for this class of problems using SRSP. Notwithstanding, it can be proved that SRSP can successfully be applied to a large class of problems, it might occur that in practice problems do not satisfy the conditions in order to be certain of nice convergence properties. However, in some cases, SRSP still appears to be a suitable algorithm, as will be shown in the next case study for optimization model for economic and greenhouse assessment (OMEGA).

3. Application: climate change response strategies

The possibility of a human-induced climate change is one of the major global environmental problems, and such a change might have profound implications for economic and social development achieved by subsequent generations. Therefore, international effort is needed to develop strategies in order to reduce the risks of an anticipated climate change. Recently, optimization has been used to support the search for sustainable strategies in a more structured way. Although most of the optimization studies use biophysical system representations which describe only the basic relations by regression equations (e.g. [8–10]), recently, several studies have appeared on the use of integrated assessment models which describe the biophysical system in terms of physical processes [1,11]. In this case study, the optimization model OMEGA as designed by Janssen [11] and Filar et al. [12] is used, which is based on the economic

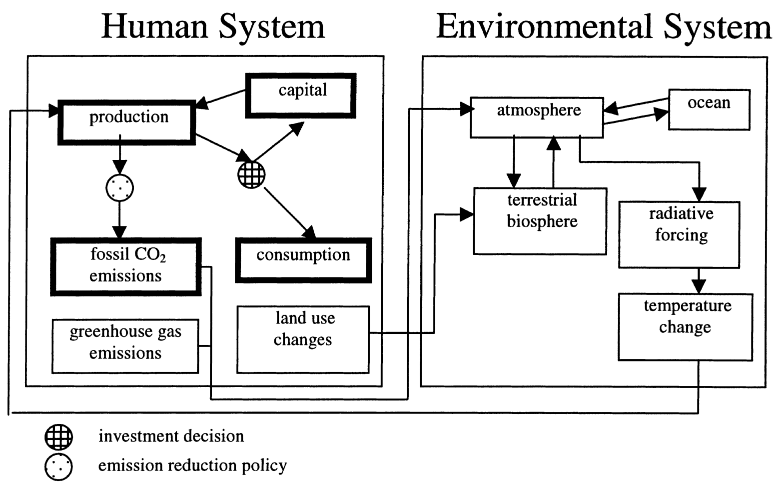

Fig. 1. Scheme of the basic relations within the OMEGA model. The blocks with the bold lines come from DICE, the other blocks from IMAGE 1.0.

component of the optimization model dynamic integrated model for climate and the economy (DICE) [8,9] and the biophysical system formulated as a non-linear dynamical system of the integrated assessment model for climate change of integrated model to assess the greenhouse effect (IMAGE 1.0) [13–15]. The combined model contains a conventional economic component in which the world economy is assumed to produce a composite commodity. Output is produced by a Cobb–Douglas production function of capital, labor and technology. The biophysical model contains besides a carbon cycle, in which the ocean, the terrestrial biosphere and the atmosphere are distinguished, also an atmospheric chemistry model and a land use model. A scheme of the basic relations within OMEGA is shown in Fig. 1.

3.1. OMEGA

Since a climate change policy must consider numerous objectives any formulation of an optimization problem will necessarily neglect specific aspects. Therefore, the test problems, which are used in this study can only be seen as possible examples of a climate policy and are only used to illustrate some characteristics of SRSP.

The general case study is one in which economic activities are restricted by an environmental constraint. Investments and emission reduction rates are selected as decision variable by means of which the goals might be realized. Nordhaus assumes that the purpose of long-term policies is to improve living standards or consumption of humans in the long run and, therefore, he maximizes a discounted sum of the utilities of consumption,

$$\max_{z} \int_{1990}^{2100} \mathrm{pop}_t \ln(Cpc_t(z))(1+\rho)^{(1990-t)}\,\mathrm{d}t. \tag{22}$$

Here, $\ln(Cpc_t(\cdot))$ is the flow of utility or social well-being and $Cpc_t(\cdot)$ the consumption per capita, pop_t the population size at time t and ρ the pure rate of social time preference. To clarify the characteristics of the algorithm, we consider only one decision variable z, which determines the force of policy measures.

The emissions of fossil fuel E are defined as a function of economic output $Y(t)$ times, a time dependent emission quotient per unit of output, $\sigma(t)$, and a forcing term to start emission reduction policy.

$$E(t) = \frac{\alpha + 1}{\alpha + \mathrm{e}^{z(t-1990)}} \sigma(t) Y(t). \tag{23}$$

OMEGA is implemented in *M*, a modeling environment and simulation language which converts the code ultimately in C++ [16]. The dynamical system is solved by second order Runga–Kutta with a fixed time step of 0.05 year [15]. The algorithm, which is used to solve the optimization problem for the reduced-system, is a quasi-Newton algorithm [17]. The constraints are handled by using the absolute-value penalty function [18]. Within the stop-criterion we use a relative tolerance level of 0.001.

The test problems, which are distinguished here, all strive for the same objective: maximize the discounted sum of utility of consumption. They differ in the set of (environmental) constraints [11]:

Case 1 (no constraints): This is a straightforward cost-benefit problem.

Case 2 (maximum temperature increase): The absolute temperature limit of 2°C above pre-industrial global mean temperature for the next century [19], which is based on vulnerability of ecosystems to a climate change.

3.2. Results

The original model consists of 163 equations, but has been reduced to a system of nine essential equations [11]. Because, the optimization problem only consists of one decision variable, the exact solutions can be traced. In Figs. 2 and 3, the relations are plotted between the decision variable and the value of the objective function and the maximum temperature increase. The optimal solutions for the test cases are $z = 0.00950$ (case 1) and $z = 0.02955$ (case 2).

If the original SRSP code is used to solve two test problems, the algorithm converges only for test case 1 (see Fig. 4). Checking condition (5) of the algorithm leads to the conclusion that SRSP applied on test case 2 do not result in a value of condition (5) below one instead of SRSP applied on test case 1 (Fig. 5) where the derivative is smaller than one for the whole range of *z*. However, in order to find a fixed point of the algorithm, I introduce a heuristic solution to avoid divergence from the fixed point.

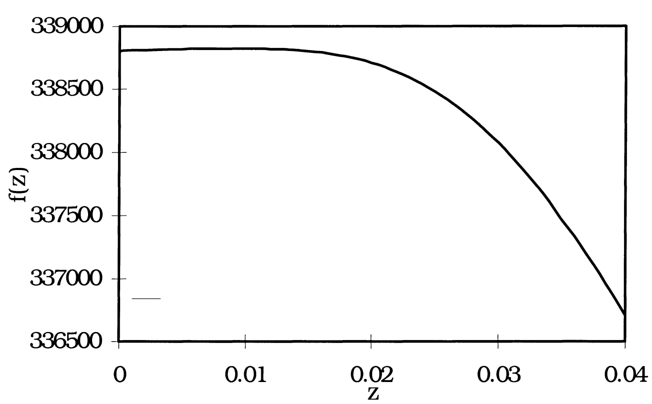

Fig. 2. Value of the objective function *f*(*z*).

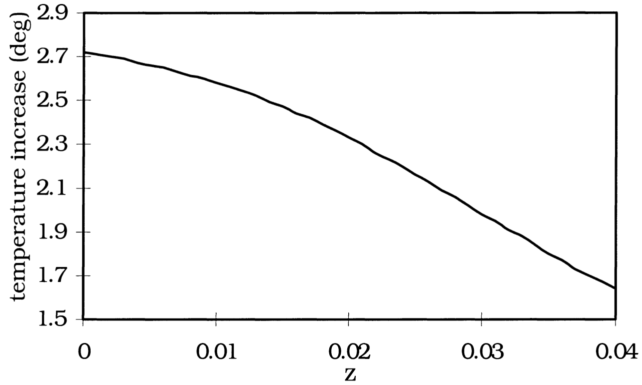

Fig. 3. Maximum temperature increase for the next century as a function of *z*.

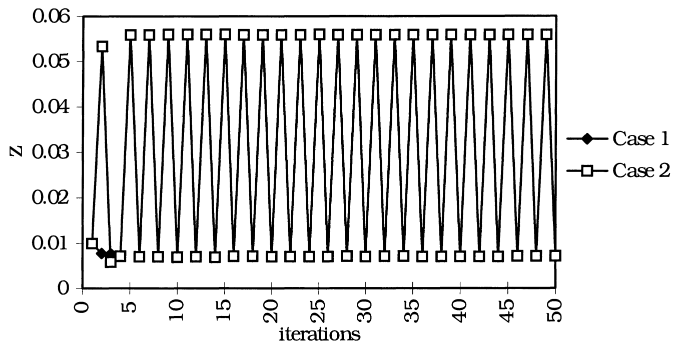

Fig. 4. Objective value during running the algorithm.

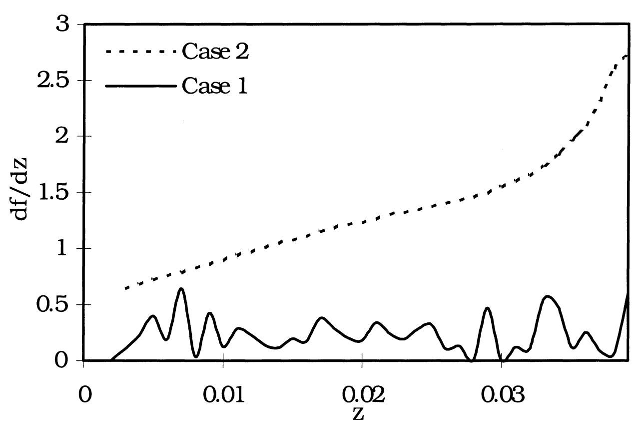

Fig. 5. Condition (5).

It is known that for every q

$$z_{q-1} = \alpha z_{q-2} + \beta,$$

$$z_q = \alpha' z_{q-1} + \beta'.$$

While in the fixed point $z^* = \alpha z^* + \beta$, the following equation can be derived:

$$\alpha = \frac{z_q - z_{q-1}}{z_{q-1} - z_{q-2}},$$

$$\beta = z_{q-1} - \alpha z_{q-2}.$$

Expected value of $z^* = \beta/(1 - \alpha)$.

So, the heuristic adaptation of the algorithm will be the following check after optimizing problem (4):
if $q = 1$, $z_1 = z$;
if $q \geq 2$, $z_2 = z_1$, $z_1 = z$;
if $q \geq 3$, $\{z_3 = z_2, z_2 = z_1, z_1 = z\}$;
if $||z_2 - z_1|| < ||z_3 - z_2||$, then $\{$set z_3 to the expected value of z^* and $q = 1\}$.

This adaptation forces the algorithm to a fixed point. Fig. 6 shows the results of SRSP improved when the solution is forced to a fixed point. Table 1 shows the results of the algorithm of the two cases compared to the optimal solution and to a usual quasi-Newton algorithm for solving problem (2).

Because $F(z)$ is a non-linear function of z, convergence to a unique solution cannot be proved. However, the algorithm finds a good, but not necessary, optimal solution of the original system. Fig. 7 shows the relation between the objective value and the decision variable z of the original problem and the reduced problem in the fixed point. The reduced problem is a little perturbed and reaches a maximum in a somewhat smaller value (0.007644) than the original problem (0.00950).

The approximation of the temperature increase (in 2100) in the fixed point of test case 2 is plotted in Fig. 8 and is related to the real values of the original system. The approximation in the fixed point is perfect, but a perturbation of z leads already to a significant difference with the actual values, which may be the cause of divergence of the original version of SRSP.

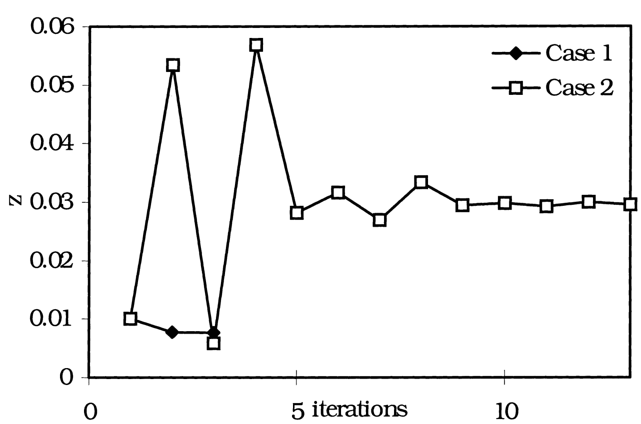

Fig. 6. Objective function value by forced convergence.

Table 1
Comparison of the optimal solutions of SRSP, quasi-Newton and the real solution of the two test cases[a]

	Test case 1			Test case 2		
	SRSP	Optimal	Quasi-Newton	SRSP	Optimal	Quasi-Newton
z^*	0.007644	0.0093	0.009189	0.029545	0.02955	0.029549
$f(z^*)$	338824.19	338826.03	338826.03	338113.51	338118.97	338118.2
Time	15.5		1273.7	183.9		3953.4
#fe[b]	3 (79)		401	13 (1608)		1228
$\sum g(z^*)$				0.000573	0.0	0.0
$\mathrm{d}f(z^*)/\mathrm{d}t$	807.3	0.0	78.1			

[a] The run time is in seconds on a Silicon Graphics (Indy) Workstation.

[b] The number of function evaluations (#fe) of the original systems are given next to the number of evaluating the reduced-system which is given between brackets.

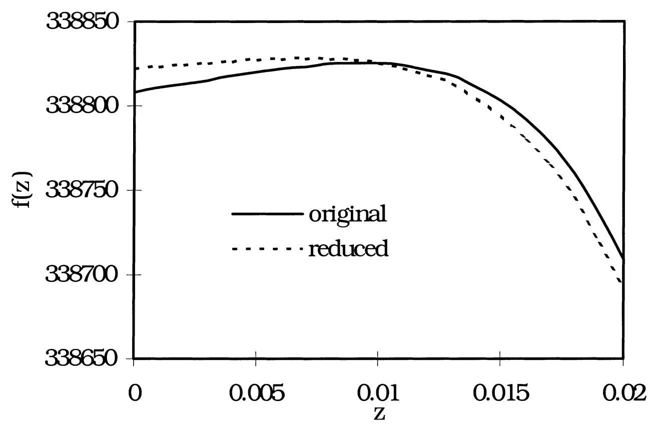

Fig. 7. Original vs. reduced model results.

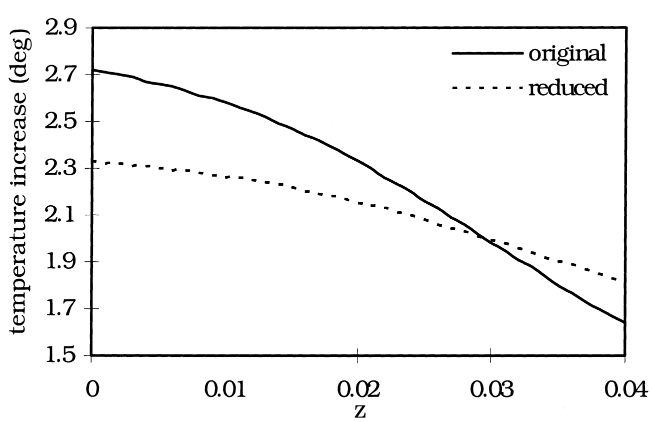

Fig. 8. Original vs. reduced model temperature change in 2100.

4. Conclusion

Sequential reduced-system programming is designed to solve optimization problems for large non-linear dynamical system. By sequentially solving a dynamical system and an optimization problem for a reduced-system, a fixed point can be found which for a large class of problems is an optimal solution for the original problem. Also, for problems outside this class of problems, SRSP might be a suitable algorithm as is illustrated in the case study for climate change response strategies.

An example of applying SRSP on the search for response strategies for climate changes is analyzed. The derived optimization problem fell out of the class of problems of which convergence of a solution was proved. However, the solutions derived with SRSP are close to the real solutions because of the close approximation of the system by the reduced-system.

Although convergence of the algorithm has not occurred in all cases, a heuristic adaptation is used to force the algorithm to converge to a fixed point of the algorithm.

The main benefit of SRSP is the efficiency in run time. Therefore, SRSP is expected to be suitable for a large class of applications in practice where complex simulation models are used to describe the system to be optimized.

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