# A Theory of Local Menu Preferences*

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Abstract: Standard choice models cannot explain data that arises from settings in which the choice problem itself, i.e. the 'context', affects the decision-maker's preferences over the objects of choice. This paper develops a theory of context-dependent preferences. The primitive is a preference order on menus, or sets of consumption opportunities. We introduce a model where the menu of consumption opportunities itself affects the consumer's preferences over consumption choices within the menu. There are two main results in the paper - a representation theorem for this model in the setting where the DM has preference for flexibility, and a companion representation in the setting where he has preference for commitment. Our interest in these two behavioral phenomena, *per se*, is that context-dependence can provide a source for flexibility and (resp.) commitment that cannot be captured by models which take a state-space approach. Additionally, the parameters of the model are behaviorally identified, so that it is rich enough to permit comparative statics analysis.

*Keywords*: Menu Choice, Preference for Flexibility, Preference for Commitment, Context Effects.

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# 1 Introduction

The standard approach to revealed preference uncovers a decision-maker's (DM) preferences by subjecting him to a sequence of choice experiments. Hence, one proceeds as if a preference relation is akin to an object sitting behind a curtain, with choice experiments designed to unveil the object piece by piece. A crucial assumption that underpins this procedure is that the decision-maker comes to the choice experiment with a preconceived preference relation. In other words, it is assumed that the process of subjecting the DM to choice experiments cannot by itself alter the preferences that are revealed through these experiments. This hypothesis is problematic as it cannot be reconciled with evidence that suggests that there is co-dependence between the choice problem and preferences over choice objects.[1] When preferences exhibit this dependence, we say that the DM has *context-dependent preferences.*

There is a large literature on context-dependent preferences, within which the same term is used to address disparate behavioral phenomena. When we use the term 'context-dependence', we have in mind a decision-maker who develops his ranking on choice objects as they are offered to him. This is in contrast to prevailing models, which we refer to as 'context-independent' models, where it is as if the ranking is developed prior to participation in choice experiments. The aim of this paper is to reexamine two important behavioral phenomena – preference for flexibility and preference for commitment – from the vantage point of context-dependent preferences. Both of these phenomena have received considerable attention within the decision theory literature. However, most of the models that have been developed to understand flexibility and commitment fall into the context-independent category. Taking context-dependence into account is important since – we will argue – it provides a *source* for preference for flexibility and, respectively, commitment that is distinct from the primary source studied in existing models, e.g. uncertainty about future tastes.

To illustrate this idea, consider two examples of behavior. The first is a type of expression of preference for flexibility, which we refer to as *preference for diversity.* The second example is an expression of preference for commitment, which we refer to as *preference for compromise* since it bears some similarity to the classical compromise effect (e.g. see Simonson and Tversky (1993)). Let us start with the notion of preference for diversity. Imagine a DM is placed in two scenarios. Either (i) he can invest in a safe asset or (ii) he has the option of investing in either the safe asset or a risky asset. He may make the same consumption choice, say the safe asset, in

[1]See Kahneman and Tversky (1986), Simonson (1989), Kahneman et al. (1991), Kahneman and Tversky (1991), Shafir et al. (1993), Simonson and Tversky (1993), Bateman et al. (1997).

both choice problems. However, the second problem implicitly involves two choices. First, a choice to invest in the safe asset and, second, a choice to forego the more risky investment. Note that the latter "choice" is not available in the problem where only the safe asset is offered. Yet, the final utility of the agent may well depend on the utility of both what he chooses and from the presence of the additional, and unchosen, investment opportunity. Consequently, he exhibits what is referred to in the literature as a "preference for flexibility" even though the source of this preference has nothing to do with uncertainty about future tastes - the traditional explanation for preference for flexibility, after Kreps (1979). Rather, the source of the evident preference for flexibility is merely that the DM likes having more choices because it makes him feel less constrained to ultimately make a choice. This type of behavior has received a title in the economics literature - *preference for freedom of choice* - after Sen (1991), Sen (1993), where it is first argued that freedom of choice should be incorporated into welfare analysis. For brevity, in this paper we denote the same concept with the term 'preference for diversity'.

A related phenomenon occurs in the setting where a decision-maker expresses preference for commitment, also referred to as 'temptation preference'. To illustrate, let us take an (variant of) example from Gul and Pesendorfer (2005). A decision-maker is offered the following sequence of choices. First, a choice to invest in a highly risky asset or a safe asset. Second, a choice between the safe asset and a somewhat risky asset. Third, a choice from one of the three. In the first choice problem, the DM picks the highly risky asset. From the second, he picks the safe asset, and from the third choice problem he picks the medium-risk asset. Examples of this kind have received the title of 'the compromise effect'.[2] The idea is that the decision-maker wants to invest in the safe asset, but is tempted by high risk assets whenever offered. Moreover, he has a preference for commitment in the sense that he would rather be committed upfront to only make safe (i.e. risk less) investments. The choice of the mid-level risk asset is then a 'compromise' between his conflicting desires to make a safe investment and obtain a high return. Notice that these choices cannot be generated by a single ranking on assets.

As in the example of preference for diversity, the issue is that the decision-maker doesn't come to the choice experiment with a preconceived ranking, or even a collection of (possible) rankings, as is the case with the state-space models of Kreps (1979) and Dekel et al. (2009). Rather, he only ranks objects that are available in the choice problem he is currently facing. This idea can account for the switch from the safe option to the medium risk option when the very risky option is also added to the choice set. Indeed, the status of the medium risk option as a 'compromise' is

[2] For some experiments where this effect has been documented, see Simonson (1989), Simonson and Tversky (1993).

itself a relative notion. The DM is choosing the medium risk option to mitigate the tension between what he should choose (the safe option) and what he is tempted to choose (the high risk option). Whether he ranks the medium-risk option higher than the safe option is dependent on the presence of the high risk option. Hence, when we allow rankings over consumption choices to vary with the choice problem we obtain a natural explanation for the behavior described by these examples.

This paper rationalizes both preference for diversity and preference for compromise via a model of context-dependent reasoning. We model context-dependence with a system of (subjective) rankings, $\{\succeq_A\}_{A\in\mathcal{M}}$ - where the index set $\mathcal{M}$ is the set of choice problems (i.e. menus). The relation $\succeq_A$ denotes a ranking over consumption choices available in the choice problem $A$. The collection $\{\succeq_A\}$ is what we refer to as *local menu preferences* since the ranking $\succeq_A$ is specific - or *local* - to the menu. Moreover, these rankings need not be compatible with the ranking, $\succeq_X$ - where the choice problem includes all feasible options.[3] Our heuristic for context-dependence is that the decision-maker develops his ranking over choice objects as they are offered to him. How should we formally model this idea? There are surely several plausible approaches[4], but to motivate the one taken in this paper let us first ask: How should we model context-*independence*, i.e. where the decision-maker comes to the choice problem with a preconceived ranking over the universe of choice objects? The following definition is a formalization of the intuition behind context-independence - in fact, this is how Sen (1997) defines context-independent choice.

$$\textbf{Deductive Consistency}: (\succeq_X)|_A \ = \succeq_A, \forall A \in \mathcal{M}$$

This property captures the idea that the DM enters a choice experiment with a fixed ranking (this is $\succeq_X$ in the notation above) that is determined prior to any particular choice problem he faces.[5] In contrast, we would like our formal 'definition' of context-dependent reasoning to capture the idea that the decision-maker develops his rankings over consumption choices as they are offered to him, rather than prior to participation in any choice experiments. The following restriction adds some formal discipline to this idea[6]

$$\textbf{Inductive Consistency}: A \subseteq B \text{ and } x \succeq_A y \Rightarrow x \succeq_B y$$

[3] That is, $A \subseteq X, \forall A \in \mathcal{M}$.

[4] We exposit some of the alternative approaches to this problem in the related literature section.

[5] The statement $x \succeq_B y \Rightarrow x \succeq_A y, \forall x, y$ means: the exact ranking on the pair $(x, y)$ in the menu $B$ is carried down to the menu $A$. So, strict preferences remain strict and weak preferences remain weak. A formal definition is given in section 2.

[6] In other words, we require that $(\succeq_B)|_A$ coarsens $\succeq_A$. That is, the order $\succeq_B$ (restricted to $A$) is obtained from $\succeq_A$ by just grouping together (without reversing any strict preferences) $\succeq_A$-indifference classes – so that the induced partition of $A$ is a coarsening of the partition of $A$ induced by $\succeq_A$. A formal definition is given in section 2.

Given two experiments $A$ and $B$ where we sequentially make more choices available to the decision-maker, the criterion says that the ranking on the larger choice set coarsens the ranking on the smaller set. Let us motivate these concepts with a general example of deductive vs. inductive reasoning. Imagine we ask two different decision-makers a question, "Should I invest in technology stocks or pharmaceuticals?" The first decision-maker might base his response on his own investment experience, e.g. if he personally has earned a better return on technology stocks, then he would recommend investing in technology stocks. This is an illustration of inductive reasoning. For an example of deductive reasoning, consider a second decision-maker who develops a recommendation as follows. First, he computes the average return over all types of stocks, e.g. technology stocks, pharmaceuticals, and others - including those that he might not have any personal experience investing in. This yields a 'global' ranking on the universe of all stocks. He then makes a recommendation on which among any two stocks to purchase based on this global ranking.

Why should rankings over alternatives be coarsened as we add choices to the option set? The idea is that each menu provides a context, e.g. investment experience, for the (local) ranking of one option over the other. Now take two choice problems $A$ and $B$. In the first, $x$ is (locally) ranked strictly higher than $y$. In the second, both are in the same indifference class. Coarsening requires that in the grand menu $A \cup B$, $x$ is indifferent to $y$. The decision-maker in weighing $x$ against $y$ reasons that there are situations, *viz.* menu $A$, in which $x$ is better and situations, *viz.* menu $B$, in which neither is clearly better. Hence, the only way to be consistent with both rankings is to put $x \sim_{A \cup B} y$. Notice that the term "consistency" is being applied in a weak sense. The ranking on the superset menu does not have to agree with the ranking on the subset, it just can't reverse a weak preference, say $x \succeq_A y$, into a strict preference $y \succ_B x$ (where $A \subseteq B$).

Finally, note that the Inductive Consistency condition allows strict preferences to be coarsened to weak preference, e.g. it is possible that $x \succ_A y$, but $x \sim_B y$ when $A \subseteq B$. For example, it may be the case that for all subsets $A$ of the menu $B$ containing $x, y$ we have $x \succ_A y$, yet $x \sim_B y$. We suggest two reasons why this may occur. The first is based on "complexity" - the decision-maker uses a coarser ranking procedure when there are more objects to rank. This seems consistent with the idea of inductive reasoning - where decision-makers solve a given problem by reasoning by analogy from solutions to simpler (sub)problems. This complexity-based interpretation of inductive consistency is also the one given in Tyson (2008). However, Tyson's approach is formally quite different than ours - in his model, rankings become incomplete, as opposed to coarsened, as we enlarge the domain of the choice problem.

A second motivation for coarsening is via the examples of preference for compromise and preference for diversity. An inductively consistent decision-maker might rationalize these examples as follows. First take the preference for compromise example. When the menu consists of just the high risk and the safe asset, he prefers the safe asset. Similarly, when the menu consists of the safe choice and the medium risk choice he prefers the safe choice. But when all three are present, the medium risk asset looks more attractive by virtue of contrast with the high risk asset, e.g. it also provides a higher return than the safe asset, but without as much volatility. Consequently, his ranking is coarsened and he ranks both the safe and medium risk choice equally in the choice problem consisting of all three asset choices. A companion intuition for coarsening is motivated by the idea of preference for diversity. In this setting, the decision-maker might rank the safe asset higher than the the middle asset when the option set consists of only these assets. However, when we add the high risk asset to the choice set this "triggers" the DM's desire for flexibility as he now feels he has genuine freedom of choice in available investment opportunities. Consequently, the strict ranking between the safe and middle risk asset is coarsened, with both ranked equally when we add the high risk option to the choice set.

The inductive consistency condition is our criterion for context-dependent choice. Thus, the key ingredient of our model is a system of menu-indexed orders, $\{\succeq_A\}$, that satisfies this condition. This collection of rankings, $\{\succeq_A\}$, is subjective, i.e. it is inferred from the observable of the environment, which is a preference relation on menus. The utility model is described as follows. For each choice problem (menu), the decision-maker groups choice objects into tiers, where objects in the same tier are of comparable "type". It will turn out that the inductive consistency implies that the local order $\succeq_A$ is a coarsening of the order on singleton menus. Hence, objects in a fixed tier can have different $u(\cdot)$-rank, but no objects in lower tiers have higher $u\cdot)$-rank. In this way, the process by which choice objects are grouped into tiers is consistent with the singleton ranking. Once objects are sorted into tiers, the utility of a given menu is taken to be the numerical value of a top tier object times the size of the top tier. This yields the following expression:

$$U(A) = |\Phi_u(A)| \cdot u(x, A)$$

where (i) $u(\cdot,\cdot)$ is a cardinal representation of the system of local menu preferences, $\{\succeq_A\}$, (ii) the set $\Phi_u(A)$ denotes the top $\succeq_A$-indifference class, and (iii) $U(\cdot)$ is the utility on menus. The utility representation is hence completely determined by the function $u(\cdot,\cdot)$.

Notice that only the top types "count", i.e. only the elements of the set $\Phi_u(A)$, contribute to the utility of a given menu $A$. However, types are not static across choice problems. Hence, an object that is not a top type in a smaller menu may obtain top type status in a larger choice problem. This is the mechanism through which we can rationalize the behavior expressed in preference for diversity and the preference for compromise examples. Thus, a *local menu preferences* representation, which is the paper's namesake, is a system of menu-indexed (*local*) preferences, $\{\succeq_A\}$, with a given cardinal representation $u(\cdot,\cdot)$.

The main result of the paper is a behavioral characterization of the representation $u(\cdot,\cdot)$. We consider two families of representations - one where the function $u(\cdot,\cdot)$ is monotone increasing in the menu argument and the other where it is monotone decreasing. The first class corresponds to the preference for flexibility setting and the second class corresponds to the preference for commitment case.[7] These two classes are linked together by the inductive consistency condition on the representation $u(\cdot,\cdot)$.[8] In this way, we provide a unified model of context-dependent menu preferences.[9]

The idea of inductive consistency has previously appeared in the literature in choice theory. Two important predecessors in which a similar idea is present are Gilboa and Schmeidler (2003) and Tyson (2008).[10] We share with these papers the notion that preferences across choice problems are assembled on a case-by-case basis, where preferences in larger problems are built-up from preferences in smaller problems. The differences lie in the choice of behavioral primitive and in the formal models themselves. For instance, there is a close parallel between our inductive consistency condition and a key behavioral postulate in the Gilboa and Schmeidler (2003) model, the combination axiom. Recall that the primitive in that paper is a collection of 'case-based' orders, $\{\succeq_M\}$, where there is a (suppressed) index set of possible cases. The analogue of a case in our setting would be a choice problem (menu). The combination axiom then says, (there is a disjointness requirement on cases which we omit in the statement below)

$$\textbf{Combination}: \ x \succeq_M y, x \succeq_N y \Rightarrow x \succeq_{M\cup N} y$$

Note that this is exactly (see footnote) the statement that the ranking on the case $M \cup N$ is a common coarsening of the rankings on the sub-cases $M$ and $N$. Hence, via the inductive consistency condition, our results draw a connection between two rich literatures in decision theory - the work on menu choice and the literature on case-based decision-making.

[7] In both cases, we also impose a growth condition on the cardinal representation $u(\cdot,\cdot)$. Details may be found in the formal definitions in section 2.

[8] More precisely, this is a condition on the underlying (ordinal) system of orders, $\{\succeq_A\}$.

[9] The families are dichotomous - so we don't study menu preferences which exhibit flexibility at some menus and strict commitment at others. It is either one or the other, corresponding (resp.) to whether $u(\cdot,\cdot)$ is monotone increasing or monotone decreasing in the menu argument.

[10] I am grateful to an anonymous referee for bringing Tyson's work to my attention.

A version of the inductive consistency condition is also present in Tyson (2008). Tyson models DM's who face cognitive constraints in evaluating choice problems. These constraints are revealed through a family of menu-indexed (semi-transitive) relations $\{\mathcal{R}_A\}_A$ that satisfy a *nestedness* condition:

**Nestedness**: If $A \subseteq B$ and $x\mathcal{R}_B y$, then $x\mathcal{R}_A y$.

Borrowing Tyson's intuition, the idea behind the condition is that the DM is only able to rank an $x, y$ in the larger (and hence more complex) choice problem $B$ if he can rank them in the smaller problem $A$. Since the larger menu presents a more complex choice problem, there are no new rankings developed over choice objects that were unranked in the smaller problem. Note that this allows the relations $\mathcal{R}_A$ to be incomplete. This is important for Tyson's model since he is interested in modeling satisficing behavior (that arises in response to complexity considerations).[11] Consequently, if the relations $\mathcal{R}_A$ are complete (and transitive) his model reduces to a standard, Arrowian DM. In our case, each of the relations $\mathcal{R}_A$ (equiv. $\succeq_A$) are complete - but our DM's are, nevertheless, not Arrowian.

The source of the difference is in the way we define rationalization of a choice correspondence. In Tyson (2008), the primitive is a choice correspondence and the nested system $\{\mathcal{R}_A\}$ is said to rationalize the correspondence if

$$C_{\text{Tyson}}(A) = \{x \in A : \neg(y\mathcal{R}_A x), \forall y \in A\}$$

In contrast, a rationalization for us is defined via

$$C(A) = \{x \in A : x\mathcal{R}_A y, \forall y \in A\}$$

If $\mathcal{R}_A$ is a strict relation (i.e. $x\mathcal{R}_A y \Rightarrow \neg(y\mathcal{R}_A x)$), then there is no difference in the definitions. However, if $\mathcal{R}_A$ exhibits ties (at the top indifference class), then there is a difference. In particular, $C_{\text{Tyson}}(A)$ is empty in these cases, whereas $C(A)$ picks out the top $\mathcal{R}_A$-indifference class. Beyond formalities, ties in the local relation are what drive the representation. Inductive consistency doesn't allow full preference reversals. What it does allow is a weak reversal, where $x \succ_A y$ and $x \sim_B y$ (where $A \subseteq B$). This is the gap between Inductive and Deductive Consistency and, more importantly, it is what provides a source of preference for flexibility (resp. commitment). See the comparative statics section where this is made more explicit. Hence, if we do not allow ties (and we require the local relations to be orders) then our DM's become Arrowian as well.

[11] Satisficing is where the DM selects choices using the following formula: $C(A) = \{x \in A : f(x) \geq \alpha\}$, where $\alpha$ is some threshold and $f(\cdot)$ is some numerical ranking on alternatives. If a nested system rationalizes a choice rule that admits a satisficing representation, then the local relations will typically not be complete.

The forthcoming sections of the paper are organized as follows. In the next section, we lay out the model and derive the main representation theorems, Theorems 1 and 2. While these are listed as two results, they should be taken together as a single, unified representation for context-dependent preferences. The first theorem provides a representation in the preference for flexibility setting and the second presents a companion representation when the DM has preference for commitment. Together they provide a coherent explanation for the two behavioral effects we discussed, (i) preference for diversity and (ii) preference for compromise. The next section presents identification results for our model. This then takes us to the last section where we carry out a comparative analysis between our model of context-dependence and the models in Dekel et al. (2009) and Kreps (1979), culminating with some comparative statics results. Proofs may be found in the appendix.

## 1.1 Related Literature

This paper fits into the program in choice theory that seeks to understand framing issues in choice problems. What this usually refers to is the behavioral phenomenon where the manner in which choices are presented affects what actually gets chosen. This topic has a rich history, and has seen a revival of interest due, in part, to an infusion of new models that have arisen from the study of novel behavioral phenomena such as flexibility, temptation, regret, shame (e.g. Kreps (1979), Gul and Pesendorfer (2001), Sarver (2008), Dekel et al. (2009), Dillenberger and Sadowski (2012)) among others. Let us briefly summarize the different approaches that have been taken to analyze this problem. To keep this tractable, the summary is biased towards papers which take an axiomatic approach.

There have been essentially two approaches to the phenomenon of framing, which can be roughly classified according to the choice of behavioral primitive. With some recent exceptions (see Masatlioglu et al. (2010)), papers in both groups take the choice domain to be a collection of framed choice problems, i.e. a set of consumption choices coupled with a 'frame' which may affect how the DM evaluates the choices. Papers in one group, e.g. Plott (1973), Fishburn and Lavalle (1988), Bandyopadhyay (1989), Masatlioglu and Ok (2005), Masatlioglu and Ok (2006), Kőszegi and Rabin (2006), Rubinstein and Salant (2008), Kochov (2009), take the modeler's observable to be a choice correspondence mapping from framed choice problems to actual choices.

Papers in the second group take the behavioral primitive to be a welfare ranking on framed choice problems, e.g. framed Anscombe-Aumann acts in Ahn and Ergin (2010) and case-based choices in Gilboa and Schmeidler (2003) and Chambers and Hayashi (2010). A key objective of papers in this group is to show how framing effects can distort the parameters of the utility representation. We take a similar approach to the papers in this group. Indeed, the way in which we model context-dependent choice - the inductive consistency condition - is inspired by the approach taken in Gilboa and Schmeidler (2001) and Gilboa and Schmeidler (2003). The main distinction between these two papers and ours is that in our setting the frames are menus and the observable is a preference relation on menus, whereas in Gilboa and Schmeidler (2003) the frames are cases and the observable is a preference relation on case-based choices.

There has also been axiomatic work that addresses the choice behavior exhibited in the two examples we have discussed, viz. preference for diversity and the compromise effect. In addition to Sen (1991), Sen (1993) where the term 'preference for freedom' was coined, see also Bossert et al. (1994), Nehring (1996), Puppe (1993), Pattanaik and Xu (1998), and Nehring (1999). The primary distinction between our paper and those in this group is in the way we model the preference for diversity phenomenon. Our goal is to develop a model of context-dependent reasoning and to use this model to provide a unified explanation for phenomena such as preference for diversity. There is also recent work in menu choice, e.g. Nehring (2006), Chatterjee and Krishna (2008), Barbos (2010), and Noor and Takeoka (2010), which models context-dependent temptation. The models in these papers capture, in different ways, the distortions to commitment costs that can arise from menu-effects. While our interest in preference for commitment is motivated by similar examples (namely, a version of the compromise effect), we explain this behavior with a general model of context-dependent choice - hence, we simply tell a different story than the models in this group.

# 2 Model

The behavioral primitive assumed for the results in this paper is described by the following choice environment.

- Let $X = \{x_1, .., x_n\}$ be a set with $n$ elements (the prize space).
- Let $\mathcal{M}$ denote the set of all non-empty subsets of $X$ (i.e. menus).
- Let $\mathbf{P}(\mathrm{X}) :=$ the set of complete and transitive preference relations on $\mathcal{M}$.

Henceforth, when we mention a binary relation $\succeq$ on menus, we will implicitly have in mind an order on menus, i.e. $\succeq$ is always assumed to be element of the collection $\mathbf{P}(X)$. We also use the following notation at various points in the paper. Given $\succeq \in \mathbf{P}(X)$ let $(\succeq)|_X$ denote the order on singleton menus and let $\sup(A)$ denote the $(\succeq)|_X$-maximal elements in the menu $A$ (resp. $\inf(A)$ denotes the $(\succeq)|_X$-minimal elements in the menu $A$).

Following the approach in Gul and Pesendorfer (2005), for us a menu is a choice problem[12] so that by taking the primitive to be a relation on menus we are assuming that one can observe the DM's welfare ranking over choice problems. There are two main benefits to this approach. First, it allows us to fully recover and (weakly) identify the local menu preferences that are generating the menu choice. Moreover, the identification is meaningful in that we can carry out comparative statics. This formalizes our idea that context-dependence provides a source for flexibility and (resp.) commitment. Second, by taking a common primitive with other menu choice models we can make behavioral (axiomatic) comparisons between existing models and ours. These benefits notwithstanding, in taking the observable to be a welfare ranking over choice problems we are explicitly taking the frame of the choice problem (i.e. the menu) to be observable. It is reasonable to ask whether the frame itself should be subjective and derived from a coarser observable. We defer a discussion of this issue to the final section of the paper. For now, let us turn to a description of our model.

**Definition 1.** Given two orders $\succeq$, $\succeq'$ on a set $X$, we say that $\succeq'$ *coarsens* $\succeq$ if, for all $x, y \in X$, $x \succeq' y$ whenever $x \succeq y$.[13]

**Definition 2.** A system of orders $\{\succeq_A\}_{A \in \mathcal{M}}$ is *inductively consistent* if whenever $A \subseteq B$, the order $(\succeq_B)|_A$ coarsens $\succeq_A$.

Note the distinction with Deductive Consistency, which says: $\succeq_B |_A = \succeq_A$ (when $A \subseteq B$). In contrast, Inductive Consistency allows disagreement between $\succeq_B |_A$ and $\succeq_A$ in the following form: If $x \sim_B y$, then it may be the case that either $x \succ_A y$ or $y \succ_A x$ (it will turn out that only one of these possibilities can occur). Hence, it does allow non-trivial dependence of the local order $\succeq_A$ on the choice problem. For an illustration of how this dependence can arise in the preference for diversity setting please see Example 1. We now use these definitions to describe the utility model.

The main ingredient in our utility representation is a collection of inductively-consistent menu-indexed orders on elements of the menu, denoted as $\{\succeq_A\}_{A \in \mathcal{M}}$. This object is *subjective* in that it is derived from the order on menus. Depending on whether the menu preference exhibits preference for flexibility or preference for commitment, we will require the collection satisfy two of the following three properties: ($u(\cdot, \cdot)$ is a cardinal representation of the collection $\{\succeq_A\}_{A \in \mathcal{M}}$)

- **Inductive Consistency**: $(\succeq_B)|_A$ coarsens $\succeq_A$ whenever $A \subseteq B$
- **Downwards Monotonicity**: $A \subseteq B \Rightarrow u(x, A) \geq u(x, B)$

[12] as opposed to a (subjective) Anscomne-Aumann act as in Dekel et al. (2001).

[13] This implies that if $x \sim y$, then $x \sim' y$. Coarsening occurs when $x \succ y$ and $x \sim' y$ - so that some strict preferences are switched to weak preferences going from $\succeq$ to $\succeq'$.

- **Upwards Monotonicity**: $A \subseteq B \Rightarrow u(x, A) \leq u(x, B)$

Notice that the first condition is ordinal, i.e. it is a condition that must be satisfied by all cardinal representations of the system $\{\succeq_A\}_{A \in \mathcal{M}}$. The latter two conditions are cardinal, i.e. each one is a condition that must be satisfied by some cardinal representation of the system $\{\succeq_A\}_{A \in \mathcal{M}}$. A system of orders $\{\succeq_A\}_{A \in \mathcal{M}}$ will be called a *local menu preference.* If it satisfies Inductive Consistency and Upwards Monotonicity (along with an additional technical condition on the cardinal representation $u(\cdot, \cdot)$, which we omit here) it will be called a *local preference for flexibility.* Similarly, if it satisfies Inductive Consistency and Downwards Monotonicity, then we refer to it as *local temptation preference.*[14]

Let $u(\cdot, \cdot)$ be a cardinal representation of the system $\{\succeq_A\}$. Denote the set of local arg-maxima via

$$\Phi_u(A) := \{x \in A : x \succeq_A y, \forall y \in A\}$$

This set depends only on the underlying orders $\{\succeq_A\}$, but we maintain the "$u$" in the subscript for economy of notation. From a pair $(u, \Phi_u)$, we define a utility of a menu $A$ to be the aggregate of the $u$-values of the elements in the set of local arg-maxima. The formal definition of the utility representation imposes one additional condition on the function $u(\cdot, \cdot)$.

**Definition 3.** A function $u : X \times \mathcal{M} \to \mathbf{R}_+$ is a *local preference for flexibility* (LPF) representation of $\succeq \in \mathbf{P}(X)$ if $U(A) = |\Phi_u(A)| \cdot u(x, A)$ (for $x \in \Phi_u(A)$) represents $\succeq$ and, additionally, satisfies the following properties:

- (Inductive Consistency) $A \subseteq B$ implies $(\succeq_B)|_A$ coarsens $\succeq_A$, where $\{\succeq_A\}$ is the ordinal system with cardinal representation $u(\cdot, \cdot)$.
- (Upwards Monotonicity) If $A \subseteq B$, then $u(x, A) \leq u(x, B)$.

[14] This terminology suggests that the two sets of preferences comprising (respectively) local temptation preferences and local preference for flexibility are distinct. However, this isn't immediately obvious from definition since all that is required for monotonicity is that there exists a cardinal representation of the local orders $\{\succeq_A\}_{A \in \mathcal{M}}$ that satisfies upwards monotonicity. Since there are also cardinal representations that satisfy downwards monotonicity, the designation of a system $\{\succeq_A\}$ as a "local preference for flexibility" (resp. "local temptation preference") might not seem, ex ante, to be well-defined. What if there are two cardinal representations of the same system $\{\succeq_A\}$, where one is upwards monotonic and the other is downwards monotonic, and both represent the same menu preference $\succeq$? Luckily, it turns out that this cannot happen. This is demonstrated in the section on identification.In fact, it cannot even be the case that there are two different systems $\{\succeq_A\}, \{\succeq'_A\}$ where (i) $\{\succeq_A\}$ admits an upwards monotonic representation, (ii) $\{\succeq'_A\}$ admits a downwards monotonic representation, and (iii) both represent the same menu preference. Hence, the terminology is well-defined.

- (Technical Condition I) The function $u(\cdot,\cdot)$ satisfies the following two conditions:

  i. $u(x,A) > |X| \cdot u(y, A\backslash x)$ if $\Phi_u(A) = \{x\}$.

  ii. $u(x,A) = u(x, A\backslash y), \forall x \in \Phi_u(A), \forall y \in A\backslash\Phi_u(A)$.

Inductive consistency and Upwards Monotonicity, taken together, are conditions on the underlying local menu preference $\{\succeq_A\}$. The definition above adds to this one additional property – which we label simply as a technical condition. Both properties are growth conditions on the function $u(\cdot,\cdot)$, and both admit a straightforward interpretation. The second property says that if we remove an element from the menu which was not providing any flexibility, then this doesn't change the utility of any of the terms that were providing flexibility. The first condition applies only to menus in which the top ranked indifference class is singleton. In this case, it requires that the cardinal gaps, $u(x,A) - u(y, A\backslash x)$, are sufficiently large. The idea is that if $(u, \Phi_u)$ is a putative representation, then this difference should be positive since $x$ was providing flexibility and has now been deleted. The growth condition requires this difference in utility be large enough so that we can (ordinally) detect that it is, in fact, providing flexibility to the DM. Moreover, any constant greater than or equal to $|X|$ suffices, there is nothing special about the one we have given.

We now describe the companion model when the decision-maker has a preference for commitment. Say that the local order $\succeq_A$ is *non-constant* if there is more than one $\succeq_A$-indifference class. Similarly, say an associated cardinal representation $u(\cdot,\cdot)$ is *non-constant* if $u(\cdot, A)$ is non-constant (for all non-singleton menus $A$).

**Definition 4.** A non-constant function $u : X \times \mathcal{M} \to \mathbf{R}_+$ is a *local temptation preference* (LTP) representation of $\succeq \in \mathbf{P}(X)$ if $U(A) = |\Phi_u(A)| \cdot u(x,A)$ (for $x \in \Phi_u(A)$) represents $\succeq$ and, additionally, satisfies the following properties:

- (Inductive Consistency) $A \subseteq B$ implies that $(\succeq_B)|_A$ coarsens $\succeq_A$.

- (Downwards Monotonicity) If $A \subseteq B$, then $u(x,A) \geq u(x,B)$.

- (Technical Condition II) Let $K_A(z) := \frac{|\Phi_u(A\backslash z)|}{|\Phi_u(A)|}$.[15] The function $u(\cdot,\cdot)$ satisfies the following two cardinal restrictions:

  i. $K_A(x) \cdot u(y, A\backslash x) < u(x,A), \forall x \in \Phi_u(A), \forall y \in A\backslash x$

  ii. $u(x,A) \leq K_A(y) \cdot u(x, A\backslash y), \forall x \in \Phi_u(A), \forall y \in A\backslash\Phi_u(A)$.

[15] Note that the constants $K_A(z)$ depend only on the underlying ordinal orders $\succeq_A$, so that the growth conditions on the cardinal representation depend on the constants $K_A(z)$, but not vice-versa. In other words, there is no circularity in the definition.

There are three differences between an LTP utility and an LPF utility. The first, and most important, is the monotonicity property. In the LPF case, the function $u(\cdot,\cdot)$ is increasing in the menu argument, whereas in the LTP case it is monotone decreasing. Second, we require that the function $u(\cdot,\cdot)$ defining the LTP representation be non-constant. That is, for every menu $A$, the order $\succeq_A$ is not constant. Finally, the technical condition on the $u(\cdot,\cdot)$ map for the LTP utility is different than in the case of the LPF map and, admittedly, not as intuitive.

The condition is more complicated due to an additional subtlety that arises when modeling preference for commitment. For such preferences, there is no preference for flexibility on the level of singletons. However, there can be preference for flexibility on the level of subsets (see the discussion preceding the representation result for this model for elaboration). For example, a decision-maker may strictly prefer one particular subset of a menu to the menu itself, yet strictly prefer the full menu to all of its other subsets. This phenomenon cannot occur in the LPF case. Hence, due to the added richness of the preference, the model is more difficult to pin down behaviorally in the case of preference for commitment. We now turn to the behavioral characterization of the LPF and LTP models.

Recall that the order axiom is a standing assumption for all preferences studied in this paper. In addition to the order axiom there will be three more axioms that characterize the LPF and (resp. LTP) utility. Two of these axioms are shared between both models. To introduce these, consider a map $C : \mathcal{M} \to \mathcal{M}$ defined by,

$$C(A) = \begin{cases} \{x \in A : A \backslash x \prec A\}, & \text{if } |A| > 1 \\ \{x\}, & \text{if } A = \{x\}. \end{cases}$$

That is, $C(A)$ is the set of elements in $A$ such that the DM is strictly worse off when any of these elements is deleted from $A$. This map was introduced in Puppe (1993) and also enters in some of the results in Ergin (2003). We will follow Puppe's terminology and refer to the elements of $C(A)$ as the 'critical' elements of the menu $A$. The following two axioms, which both involve the map $C(\cdot)$, are the key behavioral conditions defining the LTP and LPF models.

**Local $C$-Monotonicity (A2)** : If $A \subseteq B$ and $C(B) \cap A \neq \emptyset$, then $C(A) \subseteq C(B)$.

**Non-Emptiness (A3)** : $C(A) \neq \emptyset, \forall A \in \mathcal{M}$.

The second axiom, Non-Emptiness, appears in Puppe (1993) (under the moniker 'freedom of choice'). The interpretation of this axiom is tied in with our interpretation of what it means to be a member of the set $C(A)$. Hence, let us postpone an interpretation of Non-Emptiness until after we discuss Local $C$-Monotonicity.

This latter axiom is the key behavioral restriction in the entire paper. To provide an interpretation, note the similarity between this axiom and Sen's $\beta$ - which is a classical example of an 'expansion consistency' condition. We will borrow Sen's interpretation of this axiom, as described in Kreps (1988). The axiom says the following (where $C(\cdot)$ is just an arbitrary choice correspondence)

**Sen's** $\beta$: If $A \subseteq B$ and $C(A) \cap C(B) \neq \emptyset$, then $C(A) \subseteq C(B)$.

Sen's interpretation of this axiom is: *If the world champion in some game is a Pakistani, then all champions (in this game) are also world champions.* To elaborate on Sen's intuition, the menus $A$ and $B$ are the games, with the set $B$ being the set of all players. Thus, an element of $C(B)$ - the presumptively top ranked element of the set $B$ - is a world champion. Hence, if there is common top-ranked element in both the smaller match $A$ and the world match $B$, then the axiom says that all top-ranked elements in the smaller match $A$ must also be top-ranked in the world match $B$. Note that if we replace cricket players with stocks, then we arrive at exactly the analogy of the introduction. Namely, Sen's $\beta$ (and similar expansion consistency conditions) formalizes a form of inductive reasoning. In particular, the Local $C$-Monotonicity axiom admits the same interpretation.

Now let us interpret the axiom through the representation. In particular, let us match the necessity proof of the axiom with Sen's intuition for his axiom. Given, say, an LPF representation we let $\{\succeq_A\}$ be the system of local orders represented by the function $u(\cdot,\cdot)$. Let $\succeq$ be the menu preference generated by the LPF utility and define the map $C(\cdot)$ of critical elements induced by this menu preference. One checks that we have the equality

$$C(A) = \Phi_u(A)$$

That is, the set of arg-maxima w.r.t. the function $u(\cdot,\cdot)$ is exactly $C(A)$ - the set of critical elements in the menu $A$. Once we have this, then we apply inductive consistency of the system of orders $\{\succeq_A\}$. Say there is a $\succeq_B$-maximal element of $B$ in a sub-menu $A$. In Sen's example, a world cricket champion (i.e. the top-ranked element in the menu $B$) participates in the regional cricket match denoted by the menu $A$. Consider any other champion of the regional match $A$. By definition, the $\succeq_A$-rank of the champion is weakly higher (in the regional match) than the $\succeq_A$-rank of the world champion. Inductive consistency requires that this relative ranking is carried over to the larger menu $B$, so that the regional champion must also be a world champion - implying that $C(A) \subseteq C(B)$. This is Sen's intuition for his expansion consistency condition and is the essence of the proof for why Local $C$-Monotonicity is implied by the representation. Hence, if one believes that inductive consistency is a reasonable way to model context-dependence, then we are led very naturally to the correspondence $C(\cdot)$ and to the Local $C$-Monotonicity restriction on this correspondence.

With these remarks, we can interpret the Non-Emptiness condition as well. Since, via the representation, the set $C(A)$ is the set of arg-maxima of the local order $\succeq_A$, the axiom amounts to saying that this arg max set is non-empty. The obviousness of this latter claim might give an impression that the axiom is totally innocuous. When we turn to the section on comparative analysis, we will see that there are some surprising implications of the Non-Emptiness condition when it is applied to some benchmark temptation models - namely, Gul and Pesendorfer (2001).

## 2.1 Preference for Flexibility

The representation of the LPF model relies on one more axiom, initially introduced by Kreps (1979) in his seminal study of preference for flexibility. We take this occasion to briefly review the Kreps model and the axioms introduced in the characterization. This will also make it easier to see the contrast (on both the functional form and behavioral side) between the Kreps utility and the LPF model. Kreps axiomatized the following utility function on menus,

$$U(A) = \sum_{s \in \mathcal{S}} \max\nolimits_{x \in A} u_s(x)$$

There are two ingredients to this utility representation, (i) a subjective state space $\mathcal{S}$ of tastes, and (ii) a collection of state-dependent (cardinal) consumption preferences, $\{u_s(\cdot)\}_{s \in \mathcal{S}}$. In addition to the order axiom, Kreps shows that the following axioms characterize this utility model:

**Monotonicity** $(\mathbf{A1}^*)$ : If $A, B \in \mathcal{M}$ and $A \subseteq B$, then $B \succeq A$.

**Modularity** : If $A, B \in \mathcal{M}$, $A \subseteq B$ and $A \sim B$, then for any $C \in \mathcal{M}$, $A \cup C \sim B \cup C$.

Kreps motivates the topic of preference for flexibility with an example of a DM whose period 1 problem is to choose a dinner reservation at a restaurant. Consumption only takes place once dinnertime arrives, however, due to the uncertainty in his taste preferences the DM values restaurant menus that offer flexibility. To fix ideas, imagine that we have the ranking, $\{\text{chicken}, \text{fish}, \} \sim \{\text{chicken}\}$. Modularity requires that we then exhibit the preference

$$\{\text{chicken}, \text{fish}, \text{steak}\} \sim \{\text{chicken}, \text{steak}\}$$

In other words, if the option of fish does not offer flexibility in the smaller menu, then it does not offer flexibility in any larger menu.

To see where this axiom comes from, consider the source of preference for flexibility in the Kreps utility. According to the model, the option value of flexibility comes from uncertainty about consumption preferences. Moreover, in formulating a state-space, the DM accounts for all scenarios where fish could possibly provide flexibility. The menu ranking $\{\text{chicken}, \text{fish}\} \sim \{\text{chicken}\}$ then implies that in all these scenarios, chicken is preferred to fish. Hence, adding steak as an option could only have the effect of switching the DM's choice in those states where steak is ranked higher than chicken. It cannot alter his predetermined ranking of chicken over fish in every ex post state.

Our model of preference for flexibility keeps the monotonicity axiom, but jettisons modularity. The main value of this alternative model is that it provides a *source* of preference for flexibility that is distinct from uncertainty about future tastes. To this end, consider the following simple example.

**Example 1** (*Preference for Diversity)*. Let $a, o$ respectively denote apple and orange. Let $u(\cdot)$ denote the ranking on singleton menus and assume $u(a) > u(o)$. The following menu preference is a canonical example of what we call preference for diversity,

$$\{a, o\} \succ \{a\} \succ \{o\}$$

In this example, the DM prefers the choice problem where he has the chance to consume either the apple or the orange to either of the problems where he only has the opportunity to consume one of the two. There need not be any uncertainty about consumption preferences in this example. Indeed, while the given preference does not constitute a violation of Kreps' modularity condition, it is easy to construct similar examples that tell the same story of preference for diversity, and that do violate modularity. Take an anstract triple $\{x, y, z\}$ with $\{x\} \succ \{y\} \succ \{z\}$ and assume we have

$$\{x, y\} \sim \{x\}, \{x, z\} \sim \{x\}, \text{ yet } \{x, y, z\} \succ \{x\}$$

The idea is that the DM has preference for diversity and that this desire for diversity is increasing in the size of the option set. Hence, with two options there isn't enough diversity for him to feel unconstrained but with more options he feels like he has genuine freedom of choice, to borrow the term coined by Sen (1991). Notice that this example does violate modularity, so that it cannot be explained by Kreps' model. As in example 1 above, the source of preference for flexibility is evidently that the DM feels less constrained to make a choice when more choices are available. Consequently, his welfare is higher when option sets are richer even though he may ultimately end up making the same choice that he would when his options are constrained. It is straightforward to construct an inductively consistent local order with an associated LPF representation $u(\cdot, \cdot)$ that can capture the example. However, this may not help explain *why* we are able to obtain a representation where the

Kreps model cannot. In other words, what does context-dependence (viz. inductive consistency) have to do with it?

Notice that there is no uncertainty, so that choices are governed by the singleton order. However, welfare can increase depending on the quality of the set of choices against which the selected option is being compared. For a concrete decision process, imagine the DM separates the consumption space into tiers. Options in lower tiers are unambiguously dominated by options in higher tiers. Within tiers, options are still ranked but they are relatively closer in quality. Hence, whenever the menu contains (implicitly unchosen) elements in the same tier the DM's welfare is higher since these elements raise the overall quality of the option set. Also imagine that membership criteria for tiers are not refined as the option set increases. With example 1 in mind, tiers may be determined by levels of sweetness of the fruit, where some apples and some oranges are of comparable sweetness but the sweet apple, nevertheless, is more sweet than the sweet orange - hence, is chosen. This behavior is very naturally captured using an inductively consistent system of local orders, $\{\succeq_A\}$ ($\succeq_A$-indifference classes are determined using the 'comparable sweetness' criterion in the example). Hence, one way to interpret the phenomenon of preference for flexibility (and, specifically, preference for diversity) is via a model where the DM ranks options on a menu-by-menu basis, using an inductively-consistent local order $\succeq_A$. We now turn to the representation theorem for this model.

**Theorem 1.** *A preference $\succeq \in \mathbf{P}(\mathrm{X})$ satisfies A1*, A2, and A3 if and only if it admits an LPF representation.*

## 2.2 Preference for Commitment

We now turn to a utility representation for the LTP model. Note that the key conceptual distinction between this model and the LPF utility is that the map $u(\cdot,\cdot)$ is required to satisfy Downwards Monotonicity in the LTP case and Upwards Monotonicity in the LPF case. The intuition for Upwards Monotonicity is simply that the DM has a preference for diversity, so that when the option set is larger he is happier with what he chooses. With a more diverse option set, his choices are presumptively more closely calibrated to his consumption preferences than they would otherwise be when the option set is smaller and perhaps only includes second-best choices.

To explain the intuition for Downwards Monotonicity condition, we first recall the notion of preference for commitment, or 'temptation preference'. To fix ideas, let $x =$ work, $y =$ shirk. The normative preference is to work, but the DM is tempted to put off work when opportunities to shirk are available. Resisting these opportunities is costly, e.g. the DM loses utility by resisting shirking and following through on work. This might lead to the following preference on menus,

$$\{x\} \succ \{x, y\} \succ \{y\}$$

This behavior is captured by the following axiom on menu preferences, introduced in Gul and Pesendorfer (2001):

**Set-Betweenness**: $A \succeq B \Rightarrow A \succeq A \cup B \succeq B$.

Menu preferences that exhibit this property have been given the moniker "preference for commitment". This paper models preference for commitment in a manner that is distinct from - but inspired by - the Gul and Pesendorfer (2001) model. In the preference for flexibility case, objects of choice within a menu are just the singleton elements comprising the menu. However, in the preference for commitment case we think of the objects of choice as a collection of 'consumption plans'. For instance, in the preceding example, the menu $\{x, y\}$ itself implicitly represents two plans: (i) the DM can plan to commit to work $x$ and incur the cost to avoid shirking, or (ii) the DM can simply plan to shirk. Hence, in the case of preference for commitment we imagine that the DM subjectively views a menu $A$ as a collection of consumption plans, $(x, A)$. We formally denote this association via,

$$A \leftrightarrow \{(x, A)\}_{x \in A}$$

The pair $(x, A)$ denotes a 'plan' to (i) consume $x$ and (ii) resist options in the menu $A$ make it costly to follow through on $x$. In the Gul and Pesendorfer (2001) model this association occurs via the formula,

$$U(A) = \max_{x \in A} \left[u(x) - \max_{y \in A} \left(v(y) - v(x)\right)\right]$$

If the DM evaluates menus using this formula, he converts a menu $A$ to a set of consumption plans, each one denoted $(x, A)$, where

$$(x, A) := \{\arg\max_{y \in A} v(y)\} \cup \{x\}$$

Each pair $(x, A)$ implicitly denotes a commitment choice. Under a Gul-Pesendorfer-type criterion, the utility of the menu $A$ is the utility of its best commitment plan, $(x, A)$. The Gul-Pesendorfer model provides a nice way to motivate the downwards monotonicity condition: As we add options to the menu we possibly add more temptations (e.g. shirking opportunities), thereby making it harder for the DM to follow through on any given commitment choice $x$. Hence, the net value of the commitment plan $(x, A)$ decreases as we enlarge the option set.

We now extend our model of context-dependent choice to the setting where the DM has a preference for commitment. The behavior we wish to model here is the same as in the 'pure' preference for flexibility case, i.e. we imagine the DM expresses a desire for flexibility for commitment plans $(x, A)$ – so that, all else equal, the commitment options there are, the better off he is. The distinction with the preference for flexibility case is that the DM has a non-trivial preference for commitment, so that there is always some element of the menu that does not provide flexibility. This is exactly what the following axiom captures:

**Commitment (A1)** : If $A$ is not singleton, then $A \backslash x \succeq A$ for some $x \in A$.[16]

This axiom tries to capture two behavioral phenomena. First, the DM values commitment at every menu - in the sense that there is a subset of the menu that is weakly preferred to the full menu. Second, the designation of a choice object as a commitment is relative to the menu at hand. An element that is tempting in a smaller menu might be a lesser of evils when the menu is enlarged by adding more harmful temptations. On a formal level, it is implied by the Set-Betweenness axiom of Gul and Pesendorfer (2001) and, in turn, implies the Desire for Commitment axiom of Dekel et al. (2009).[17] There is also a symmetry between the Commitment axiom and the Monotonicity axiom. To see this, let us rephrase Monotonicity from the previous section:

**Monotonicity**: If $A$ is not singleton, then $A \succeq A \backslash x$ for all $x \in A$.

Say that a decision maker has *a preference for flexibility at $A$* if Monotonicity holds at $A$. Then, saying that the DM values commitment at $A$ is nearly the logical negation of having preference for flexibility at $A$.[18] The symmetry between the axioms is reflected in the utility functions through the monotonicity property of the local utility index, $u(\cdot, \cdot)$. The following representation provides a companion characterization of local menu preferences in the case of preference for commitment.

**Theorem 2.** *A preference $\succeq \in \mathbf{P}(\mathrm{X})$ satisfies A1, A2, and A3 if and only if it admits an LTP representation.*

# 3 Identification

The key subjective data of the model is the collection of local preferences, $\{\succeq_A\}$. This is the object to which we would like to apply comparative statics analysis. For example, a natural comparative statics question is whether we can detect which

[16] Equivalently, $C(A) \neq A$ for non-singleton $A$.

[17] This is the content of Lemma 3.

[18] The actual negation of having preference for flexibility at $A$ would be: $A \backslash x \succ A$ for some $x \in A$.

among two agents, say agents $a$ and $b$, has greater desire for flexibility by comparing the respective collections of local orders, $\{\succeq^a_A\}, \{\succeq^b_A\}$. This would show that context-dependence (viz. inductive consistency) does indeed provide an (alternate) source of preference for flexibility.

To carry out this comparative static we need to establish that there is a well-defined, unique local order $\{\succeq_A\}$ that we can attach to each menu preference $\succeq$. In both the LPF and LTP case, it turns out that there is some multiplicity in the collection of local menu preferences that represent a given menu preference. However, there is (in both cases) a unique system $\{\succeq^*_A\}$ with the property that every local menu preference that represents $\succeq$ is a coarsening of $\{\succeq^*_A\}$.[19] We also characterize exactly which coarsenings occur as representations of $\succeq$. However, the characterization is notationally involved and, hence, is in the appendix.

An LPF representation is comprised of a (ordinal) local system $\{\succeq_A\}$ along with a cardinal representation of this system $u(\cdot,\cdot)$ that satisfies Upwards Monotonicity, resp. Downwards Monotonicity for an LTP representation. Hence, the identification result comes in two steps. First, we verify that – unless a menu preference is trivial, in a sense defined below – it cannot admit a representation with an upwards monotonic utility index and a representation with a downwards monotonic utility index. Hence, fixing a menu preference $\succeq$, to prove that there is a "most refined" local menu preferences representation, we need only search within the class of local preference for flexibility representations or (resp.) within the class of local temptation representations. The second step of identification shows that within the class of (ordinal) LTP (resp. LPF) representations $\{\succeq_A\}$ there is a unique element $\{\succeq^*_A\}$ such that *any* other LTP (LPF) representation is a coarsening of $\{\succeq^*_A\}$.

**Definition 5.** Let $\{\succeq_A\}, \{\succeq'_A\}$ be two systems of menu-indexed orders. Say that the system $\{\succeq'_A\}$ *coarsens* the system $\{\succeq_A\}$ if, for every menu $A$, we have that the local order $\succeq'_A$ coarsens the local order $\succeq_A$.

Call a preference $\succeq \in \mathbf{P}(X)$ a *trivial* preference if $U(A) = \max_{x\in A} u(x)$, where $U(\cdot)$ is any cardinal representation of $\succeq$ and $u(\cdot)$ is the restriction of $U(\cdot)$ to singleton menus. Let $\Sigma_*$ denote the set of trivial preferences in $\mathbf{P}(X)$ that satisfy the Non-Emptiness axiom. Also let $\Sigma_{LTP}$ (resp. $\Sigma_{LPF}$) denote the set of preferences that admit an LTP (resp. LPF) representation. Now consider preferences in this class that additionally admit a representation $u(\cdot,\cdot)$ where (i) $u(\cdot,\cdot)$ is upwards monotonic and (ii) $\Phi_u(A) = C(A)$. Call this subclass $\Sigma_{LTP}(f)$. Similarly define the subclass $\Sigma_{LPF}(t)$ to be those preferences in $\Sigma_{LPF}$ that additionally admit a representation

[19] I say a system $\{\succeq_A\}$ is a *coarsening* of another system $\{\succeq'_A\}$ if, for each menu $A$, the order $\succeq_A$ coarsens the order $\succeq'_A$.

$u(\cdot,\cdot)$ where (i) $u(\cdot,\cdot)$ is non-constant and downwards monotonic and (ii) $\Phi_u(A) = C(A)$.

**Theorem 3.** *$\Sigma_{LTP}(f) = \Sigma_*$ (resp, $\Sigma_{LPF}(t) = \Sigma_*$) and each element of $\Sigma_{LTP}$ (resp. $\Sigma_{LPF}$) admits a unique representation $\{\succeq^*_A\}$ with the property that any other LTP (resp. LPF) representation, $\{\succeq'_A\}$, is a coarsening of $\{\succeq^*_A\}$.*

Let us unravel the notation. First, the monotonicity property is an identifiable property of preference for flexibility (resp. preference for commitment) - so that if a menu preference admits a local temptation preference representation and, at the same time admits another representation with an upwards monotonic map, then the menu preference is trivial, i.e. it collapses to the value function of the ranking on singletons. A similar statement holds for menu preferences which admit a local preference for flexibility representation. Second, in both the LPF and LTP case, for any fixed menu preference $\succeq$, if there is a local menu preferences representation, then there is a unique system of local menu preferences with the property that any other representation of $\succeq$ is a coarsening of this system. The identification result is tight. In the proof of Theorem 2 we actually (i) show how to construct the full family of LTP representations and (ii) identify the unique, dominant (i.e. the most refined system $\{\succeq_A\}$) element in this family. Hence, the form of identification given in the proposition is the strongest form available.

# 4 Comparative Analysis and Comparative Statics

In this paper, we have suggested an approach to modeling preference for flexibility and preference for commitment that departs from the prevailing state-space approach, pioneered by Kreps (1979) Gul and Pesendorfer (2001), and Dekel et al. (2001). The state space approach consists of the following ingredients: (1) a state space $\mathcal{S}$, (2) a collection of state-dependent utilities, $u_s(\cdot)$, and (3) a monotone aggregator $\mathcal{A} : \mathbf{R}^{|\mathcal{S}|} \to \mathbf{R}$ that aggregates state-dependent utilities into a composite utility. The aggregator $\mathcal{A}$ is often taken to be an integral against some (possibly signed) measure $\pi$ on the state space, in which case the utility on menus is given by the following formula:

$$U(A) = \int_{s\in\mathcal{S}} \mathrm{max}_{x\in A}\, u_s(x)\, d\pi_s$$

This is an archetype of the class of models that represent context-*independent* preferences.

Our heuristic definition of context-independent reasoning is that the decision-maker develops a ranking over all possible consumption choices prior to being offered any actual choices. Let us match this heuristic with the decision procedure described by the state-space model. Break up the utility formula into the following sequence of steps:

$$\mathcal{S} \overset{\text{preference-formation}}{\longmapsto} u_s(\cdot) \overset{\text{aggregation}}{\longmapsto} \{\pi_s\}_{s\in\mathcal{S}} \overset{\text{participation}}{\longmapsto} U(A) = \int_{s\in\mathcal{S}} \max_{x\in A} u_s(x)\, d\pi_s$$

According to this decomposition, the DM first makes a list of all (subjective) contingencies $\mathcal{S}$. Next, he compiles a list of state-contingent preferences. This is the main step that we refer to as context-independent reasoning. In formulating the list, $\{u_s(\cdot)\}$, the DM imagines that state $s$ has just been realized and that the universe of feasible choices is the full consumption space $X$. He then formulates a state-dependent ranking over the objects in this space. Finally, he attaches probabilities to these states and, for a given choice problem $A$, attaches a cardinal value to $A$ given by the expected utility formula given above. We have called this last step 'participation' since we view the choice problem as something that the DM is presented with - as in a choice experiment. Since preferences are developed prior to participation, we view this as a model of context-independent choice.

This is merely one interpretation of the Dekel et al. (2001) model. The decomposition given above is a construct we are using to understand the decision-maker's choices. It is entirely possible that he computes values of choice problems as follows. First, he participates. Next, he formulates states, state-dependent preferences, probability weights, and assembles into an expected value. If the (menu-dependent) states, state-dependent preferences, and probability weights can be reconciled to yield a well defined and menu-independent triple $(\mathcal{S}, u_s(\cdot), \pi_s)$, then the two procedures are observably indistinguishable. In order to vindicate our chosen interpretation, we need to present an observable (i.e. revealed-preference) distinction between the Dekel et al. (2001) model and our model of local menu preferences.

In the setting of preference for flexibility, we obtain a relatively complete picture of the observable differences between our model of preference for flexibility and the Kreps (1979) model.[20] Moreover, we concretely characterize those menu preferences that lie at the intersection of these two models. This is then used to carry out a comparative statics exercise. In fact, we present two comparative statics results for the preference for flexibility case. The first is for the class of preferences that lie at the intersection of the LPF model and the Kreps model. The second result is a more general comparative static for the full class of LPF preferences. Why the two separate results? While the second result is more general, the parameter on which the comparative static is formulated is the system of local orders, $\{\succeq_A\}$. This is a less tractable object than the corresponding parameter for the first comparative static,

[20] It is a theorem in Dekel et al. (2001) that, in the presence of monotonicity and modularity, the monotone DLR utility and the Kreps model have (roughly) the same axiomatization.

which is the set of maximal fixed points of the critical points map $C(\cdot)$. These fixed points admit a concrete characterization and, relative to the comparative static for the full LPF model, are governed by the single order $\succeq_X$ (as opposed to a family of orders $\{\succeq_A\}$).

We next compare the LTP model with two of the benchmark models in the temptation literature, Gul and Pesendorfer (2001) and Dekel et al. (2009). Just as Kreps (1979) is the 'canonical' state-space model of preference for flexibility, we view Dekel et al. (2009) as the 'canonical' stat-space model of preference for commitment. However, the connections between these models and ours are less complete than in the flexibility setting, primarily due to the fact that there is no axiomatization of Dekel et al. (2009) in the discrete menus setting. Near the end of the section, we make some heuristic arguments that suggest some tighter axiomatic contrasts between Dekel et al. (2009) and the LTP model, but in the absence of a discrete menus axiomatization these remain merely heuristics.

## 4.1 Comparison with Kreps (1979)

Our primary goal is to try to understand the the revealed preference differences between the LPF and Kreps utility models. This leads us to two results. First, we develop some implications of our core axioms *A2*-*A3* in the setting of flexibility. This will provide a 'parameter' of the preference relation upon which we can execute comparative statics. Second, this parameter is used (along with a result of Ergin (2003)) to provide a concrete characterization of the intersection of our class of preferences with those of Kreps.

The key step is provided by the following Proposition. This result provides a characterization of the set of maximal fixed points of the critical points correspondence, $C(\cdot)$. Recall that a set $B \subseteq X$ is an *order interval* in $X$ if, writing $X := \{x_1 \succeq x_2 \cdots \succeq x_n\}$, we have $B = \{x_k, x_{k+1}, \ldots, x_l\}$ for $1 \leq k \leq l \leq n$.

**Proposition 1.** *(Partition Structure) Assume* $\succeq \in \mathbf{P}(\mathrm{X})$ *satisfies A1*, A2, and A3. Put* $\Phi := \{A \subseteq X | C(A) = A\}$ *and let* $\{B_1, .., B_k\}$ *be a list of the maximal elements of* $\Phi$*. Then, the collection* $\{B_1, \ldots, B_k\}$ *partitions* $X$ *into disjoint order intervals.*

Put $B_i = \{x_{l_{i-1}+1}, \ldots, x_{l_i}\}$, where the set $\{l_1, l_2, \ldots, l_{k-1}\}$ denotes the cut-points of the partition $\{B_i\}_{i=1}^k$. The proof of the proposition actually shows a little more than the statement above. Put $\Phi(i) := \{A \in \mathcal{M} : C(A) \subseteq B_i\}$ and take any $A \in \Phi(i), B \in \Phi(j)$ where $i < j$. From the proof of the proposition (in the appendix) we deduce that $A \succ B$. That is, the sets $\Phi(i)$ are totally ordered in the sense that menus whose choice sets lie in $\Phi(i)$ are strictly preferred to menus whose choice sets lie in $\Phi(j)$ for any pair $(i, j)$ with $i < j$.

Using a result from Ergin (2003) we can also use the proposition to obtain a crisp comparison between the axioms that characterize the Kreps model and the ones that we have given. Let $\Sigma_{KPF}$ denote the class of menu preferences that satisfy the Kreps (1979) axioms ($A1^*$ and $A2^*$) and let $\Sigma_{LPF}$ denote the class of local menu preferences that satisfy $A1^*$. Recall the following well-known axiom:

**Sen's** $\alpha$: If $A \subseteq B$, then $C(B) \cap A \subseteq C(A)$.

This axiom is usually applied to an abstract choice correspondence, whereas above it is being applied to the correspondence $C(\cdot)$ of critical elements - which is itself derived from the menu preference. Notice that, in contrast to Sen's $\beta$, this is a 'contraction consistency' condition. Borrowing Sen's intuition (and example): if the champion of the world (i.e. the world cricket championship is symbolized by the players comprising the set $B$) is from Pakistan (an element $x \in A$), then he is also a regional champion, i.e. he beats all players in the collection $A$. This is a canonical example of deductive reasoning. The following proposition, taken from Ergin (2003), demonstrates that the Kreps model can equivalently be characterized if we replace Modularity with Sen's $\alpha$.

**Proposition 2.** *(Ergin (2003)) Let $\Sigma_{KPF^*}$ be the subset of* $\mathbf{P}(\mathrm{X})$ *satisfying Monotonicity and Sen's $\alpha$. Then, $\Sigma_{KPF^*} = \Sigma_{KPF}$.*

The appendix reproduces Ergin's proof of this result. Note that in conjunction with Proposition 2, Ergin's result gives a concrete characterization of the intersection of the class $\Sigma_{LPF}$ with the class $\Sigma_{KPF}$. Define a new class of preferences, $\Sigma_{LSPF}$ (the subscript stands for "locally strict preference for flexibility"), as follows. Let $\{B_i\}$ be a partition of $X$ into disjoint order intervals, put $\Sigma_i := \{A | A \subseteq B_i\}$ and define an element $\succeq \in \mathbf{P}(X)$ by the following properties:

- $\succeq|_{\Sigma_i}$ is strictly monotonic, that is, $B \succ A$, whenever $A \subsetneq B$ and $A, B \in \Sigma_i$.
- $A \sim A \cap B_{i^*}$, where $i^* = \min_i \{i : A \cap B_i \neq \emptyset\}$.

In words, the class of preferences designated $\Sigma_{LSPF}$ is characterized by two properties. First, the order is strictly monotonic on any set of menus contained in a partition cell $B_i$. Second, the indifference class of a menu is determined by the highest rank partition cell which intersects the menu non-trivially. Note that neither of these restrictions pins down the ranking on pairs $(A, B)$ where neither $A \subseteq B$ nor $A \supseteq B$. Thus, the preference relation is not trivialized by these two restrictions.

The following corollary shows that this class of preferences is exactly the intersection of those preferences that satisfy the axioms of both local preference for flexibility and Kreps' preference for flexibility. The proof is immediate from Proposition 2 and 3, and hence omitted. We use the corollary to carry out a comparative statics exercise on the class $\Sigma_{LSPF}$.

**Corollary 1.** $\Sigma_{LPF} \cap \Sigma_{KPF} = \Sigma_{LSPF}$.

Notice that preferences in the class $\Sigma_{LSPF}$ satisfy both Sen's $\alpha$ and *A*2, which is itself a strengthening of Sen's $\beta$. Hence, critical points maps,$C(\cdot)$, induced by menu preferences in $\Sigma_{LSPF}$ satisfy a strengthened form of expansion consistency and contraction consistency. It follows that the base relation, $\mathcal{R}_C$, induced by $C(\cdot)$ rationalizes $C(\cdot)$.[21] The base relation, in this case, turns out to be exactly the ranking on singleton menus, $u(\cdot)$, so that $C(A))$ just picks out the $u(\cdot)$-maximal indifference class in the menu $A$. If we interpret $C(A)$ as the set of (implicitly) second stage choices, then this says second choices are standard, in that they are induced by maximizing the first-stage ranking on consumption options. Nevertheless, the menu preference is not induced up from the singleton ranking, so that preferences over menus can be non-trivial.

**Definition 6.** Let $\succeq_1, \succeq_2 \in \Sigma_{LSPF}$ agree on $X$. Say that $\succeq_1$ has *more preference for flexibility* (hereafter, MPF) than $\succeq_2$, written ($\succeq_1$ MPF $\succeq_2$), if whenever $A \subsetneq B$ and $B \succ_2 A$, then $B \succ_1 A$.

The restriction that $\succeq_1$ and $\succeq_2$ agree on the singleton ranking is required to have a well-defined comparison. Otherwise, the condition that one partition be a refinement of another cannot be used as a test of MPF. To illustrate, consider $\succeq_1, \succeq_2 \in \Sigma_{LSPF}$ with $x \succ_1 y \succ_1 z \succ_1 w$ and take

$$\{B_i^1\} \equiv \{\{x\}, \{y\}, \{z\}, \{w\}\}$$

Now put $x \succ_2 w \succ_2 z \succ_2 y$ and let agent 2's partition be given by

$$B_1^2 = \{x, w\}, B_2^2 = \{z, y\}$$

Notice that $\{B_i^2\}$ is a coarsening of $\{B_i^1\}$; however, it is neither the case that $\succeq_1$ MPF $\succeq_2$ nor that $\succeq_2$ MPF $\succeq_1$. To see this put

$$A = \{y\}, B = \{z, y\}$$

and note that $B \succ_2 A$, but $A \sim_1 B$, so that agent 1 does not have greater desire for flexibility than agent 2. Similarly, put

$$\hat{A} = \{w\}, \hat{B} = \{z, w\}$$

and note that $\hat{B} \succ_1 \hat{A}$ but $\hat{A} \sim_2 \hat{B}$ so that $\neg(\succeq_2$ MPF $\succeq_1)$.

[21] Recall that $xP_Cy$ iff $x \in C(\{x, y\})$. When expansion and contraction consistency hold, this is a well-defined and transitive relation.

**Proposition 3** (Comparative Statics, I). *Let* $\succeq_1, \succeq_2 \in \Sigma_{LSPF}$*, where* $(\succeq_1)|_X = (\succeq_2)|_X$ *and with associated partitions* $\{B_i^1\}, \{B_i^2\}$*. Then,* $(\succeq_1 \text{ MPF } \succeq_2)$ *if and only if* $\{B_i^1\}$ *is a coarsening of* $\{B_i^2\}$*.*

Let $\succeq_1, \succeq_2 \in \Sigma_{LPF}$ with associated partitions $\{B_i^1\}, \{B_i^2\}$. The proof of the proposition shows that if $\succeq_1$ MPF $\succeq_2$, then $\{B_i^1\}$ is a coarsening of $\{B_i^2\}$. That is, the coarsening condition is necessary to determine which among two agents with LPF preferences has more preference for flexibility. The addition of Sen's $\alpha$ is needed to close the gap, so that if the preferences are in $\Sigma_{LSPF}$, then the coarsening condition is both necessary and sufficient to make this determination. Note that the partitions $\{B_i\}$ are (at least in principle) computable from the choice map $C(\cdot)$. Also note that since $\Sigma_{LSPF} = \Sigma_{LPF} \cap \Sigma_{KPF}$ these preferences also have a representation by Kreps' utility functions. However, since the state space in this utility representation is not identified (see Dekel et al. (2001) for examples and discussion on this point), it is not possible to carry out any kind of comparative statics exercise using a Kreps utility, even for preferences as simple as those in the class $\Sigma_{LSPF}$.

The following result, our main comparative static, holds across the full class of LPF preferences, albeit with a less tractable parameter as it requires knowledge of the full system of local menu preferences, $\{\succeq_A\}$. We implicitly maintain the requirement that the two preferences being compared must agree on the singleton ranking. As noted in the example above, without this assumption comparative statics cannot be carried out. Moreover, when we associate a collection of local menu preferences, say $\{\succeq_A^1\}$, to a given menu preference $\succeq^1$ we are implicitly taking the (unique) most refined system of local orders, $\{\succeq_A^*\}$, that represents the given menu preference $\succeq$.

**Theorem 4** (Comparative Statics, II). *Let* $\succeq_1, \succeq_2 \in \Sigma_{LPF}$ *with associated local menu preferences* $\{\succeq_A^1\}, \{\succeq_A^2\}$*. Then,* $(\succeq_1 \text{ MPF } \succeq_2)$ *if and only if* $\{\succeq_A^1\}$ *coarsens* $\{\succeq_A^2\}$*.*

We asserted in the introduction that context-dependence could provide a source for flexibility that was alternative to the source in the Kreps (1979) model. To this end, the comparative statics exercise makes an explicit connection between context-dependence (i.e. coarseness of the system $\{\succeq_A\}$) and desire for flexibility. In particular, one decision-maker exhibits greater desire for flexibility than another if and only if his local rankings on consumption choices exhibit more context-based reasoning than the other decision-maker.

## 4.2 Comparison with Gul and Pesendorfer (2001)

We now study the implications of $A1 - A3$ on preferences that admit a GP representation. It will be convenient for proofs to express the GP utility with the pair

$(u, v)$ consisting of the "choice utility" $u$ and the temptation utility $v$, as opposed to $(u', v)$ where $u = u' + v$ - as is more common.

**Lemma 1.** *Let* $\succeq \in \mathbf{P}(\mathrm{X})$ *admit a GP utility representation with pair* $(u, v)$*. Also assume that* $\succeq$ *satisfies A3. Then,* $\succeq$ *satisfies A2 as well.*

It is hard to gauge the value of this lemma without having a clear understanding of what restriction *A3* imposes on the collection of GP pairs $(u, v)$. The following lemma characterizes the restrictions on pairs of mappings $(u, v)$ that are necessary and sufficient for the GP preference generated by the pair to satisfy *A3*. Consider the following two conditions

$$P_1 : u(x) \neq u(y), \forall x, y \in X$$

and

$$P_2 : u(x) > u(y) \Rightarrow u(x) - u(y) > v(x) - v(y)$$

Note that these are independent conditions on $(u, v)$ and, moreover, neither one by itself guarantees that *A3* holds.

**Lemma 2.** *Assume* $\succeq \in \boldsymbol{P}(X)$ *admits a GP representation with pair* $(u, v)$*. Then,* $\succeq$ *satisfies A3 if and only if the pair* $(u, v)$ *satisfies conditions* $P_1$ *and* $P_2$*.*

The GP model is usually written as follows

$$U^{GP}(A) := \max_{x \in A} (u'(x) + v(x)) - \max_{x \in A} v(x)$$

We are putting $u(\cdot) = u'(\cdot) + v(\cdot)$ in referring to a pair $(u(\cdot), v(\cdot))$. Translating into the original GP notation we obtain:

$$P_1 : u'(x) + v(x) \neq u'(y) + v(y), \forall x, y \in X$$

and

$$P_2 : u'(x) + v(x) > u'(y) + v(y) \Rightarrow u'(x) > u'(y)$$

Assume $P_1$ and $P_2$ hold. Note that if $u'(x) > u'(y)$, then these conditions together imply $u'(x) + v(x) > u'(y) + v(y)$. In other words, $P_1 + P_2$ in conjunction require that the normative ranking $u'(\cdot)$ and the ex post choice function $u' + v$ agree. The two lemmas together characterize the intersection of the class of LTP preferences with the class of GP preferences. Since GP preferences require the critical points map $C(\cdot)$ to be at most single-valued, the non-emptiness condition turns out to be quite restrictive insofar as GP preferences are concerned. However, it is less restrictive when applied to menu preferences where the critical points map is non-singleton, e.g. the temptation model developed in Dekel et al. (2009).

## 4.3 Comparison with Dekel et al. (2009)

We first establish a (heuristic) connection between the functional form of the DLR model and the LTP utility. By some simple manipulation of terms, we will suggest that the DLR model also satisfies a Sen's $\alpha$-type criterion. Hence, in analogy with the discussion of flexibility, we may think of this model as a 'context-independent' model. However, this is merely by way of suggestion as we are not making the connection with Sen's $\alpha$ via behavioral postulates. Next we present two variations on the classical 'compromise effect'. The first of these examples admits a Dekel et al. (2009) representation, but the second example does not. This is despite the fact that the examples are very close to each other, both formally and conceptually. We point to the general state-approach (described more generally above) as the culprit, arguing that notions such as compromise are manifestly settings where the DM does not use deductive reasoning in evaluating his choices.

To establish the (heuristic) connection between Dekel et al. (2009) and Sen's $\alpha$, we actually consider only a specialized version of Dekel et al. (2009). This model has recently been axiomatized (in the menus of lotteries framework) in Stovall (2010).

**Definition 7.** A utility $U : \mathcal{M} \to \mathbf{R}$ is said to be a *No-Aggregation* (NAG) DLR utility if there is a $k+1$-tuple $(u, v_1, .., v_k)$ of functions and a probability distribution $q = (q_1, .., q_k)$ such that: $U(A) = \sum_{i=1}^{k} q_i \cdot (\max_{x \in A} (u(x) + v_i(x)) - \max_{x \in A} v_i(x))$.[22]

Under the additional assumption that the state-dependent terms $u(x) + v_i(x)$ are strict, we can rewrite the DLR utility in a form that more closely resembles a LTP utility. Put $u_i := u(\cdot) + v_i(\cdot)$ and set $I := \{1, 2, .., k\}$. For each $x \in A$, put

$$I_A(x) = \{i \in I : x \in \text{argmax}_{z \in A} u_i(z)\}$$

By the strictness assumption, the sets $I_A(x)$ partition $I$. Let

$$S(A) = \{x \in A : I_A(x) \neq \emptyset\}$$

and put

$$u_{DLR}(x, A) := \sum_{i \in I_A(x)} q_i (u_i(x) - \max_{y \in A} v_i(y))$$

Observe that

i. $I_A(x) \supseteq I_B(x)$ if $A \subseteq B$, and

ii. The term $u_i(x) - \max_{y \in A} v_i(y)$ is downwards monotonic.

[22] I call this the No-Aggregation model since the more general model axiomatized in Dekel et al. (2009) takes the form $U(A) = \sum_i q_i \cdot [u(x) - \sum_{j=1}^{k_i} (\max_{z \in A} (v_j(z) - v_j(z))]$, where the state-dependent terms have aggregation of self-control costs.

It follows that the function $u_{DLR}(x, A)$ is downwards monotonic. Moreover, we may re-formulate the DLR utility as

$$U(A) = \sum_{x \in S(A)} u_{\text{DLR}}(x, A)$$

Thus, in the case that the terms $u(x) + v_i(x)$ are strict, the difference between a (NAG) DLR utility and an LTP utility comes down to the difference between the sets $S(A)$ and $C(A)$. We can push the comparison between the two utility functionals a little further by noting that $S(B) \cap A \subseteq S(A)$, so that the $S(\cdot)$ operator satisfies a Sen's $\alpha$-like property - a fact that mirrors the distinction between the Kreps utility and the LPF utility. This analogy, while highly suggestive, is imperfect for two reasons. First, we have assumed the state-dependent "commitment utility" terms $u(\cdot) + v_i(\cdot)$ are strict. Second, and more substantively, the map $S(\cdot)$ is derived from the utility and *not* from behavioral primitives. Turning to axioms, one of the key behavioral axioms in the Dekel et al. (2009) model is the following:

**Desire for Commitment**: $\forall A, \exists x \in A$ s.t. $\{x\} \succeq A$.

Hereafter refer to this axiom simply as DFC. To see that DFC is weaker than Set-Betweenness note that it is implied by Positive Set-Betweenness.[23] The following lemma shows that the Commitment axiom ($A1$) strikes a compromise between DFC and Set-Betweenness.

**Lemma 3.** *A1 implies DFC and Set-Betweenness implies A1. Moreover, Set-Betweenness implies $A \backslash x \succeq A$ for any $x \in \inf(A)$.*

The main motivation for $A1$ is that it explicitly captures the idea that commitment (and temptation) is itself a relative notion. That which is tempting in a smaller menu may provide commitment value when the menu of options is enlarged in a particular way. We now present two example that express this idea - both of which can be thought of as versions of the classical 'compromise effect'. The first part is taken verbatim from Dekel et al. (2009), and admits both a DLR and LTP representation. The second example, which also admits an LTP representation, mildly modifies the first and verifies that the modification admits no DLR representation.

**Example 2** (*Preference for Compromise*)**.** Let $X = \{\text{broccoli}, \text{frozen yogurt}, \text{cake}\}$ (hereafter, $\{b, y, c\}$) and consider the following preference,

$$b \succ \{b, y\}, \;\; b \succ \{b, c\}, \;\; \{b, y, c\} \succ \{b, c\}, \;\; \{b, y, c\} \succ \{y, c\}$$

[23] After Dekel et al. (2009), break up Set-Betweenness into two pieces: (i) $A \succeq B \Rightarrow A \succeq A \cup B$ and (ii) $A \succeq B \Rightarrow A \cup B \succeq B$. These individual implications are respectively titled *Positive Set-Betweenness* and *Negative Set-Betweenness*.

That is, yogurt is a (relative) temptation in the absence of cake, otherwise it becomes a (relative) commitment since it evidently mitigates the dessert craving when facing the menu $\{b, y, c\}$.

**Example 3** (*Preference for Compromise, Redux*)**.** We change yogurt from a (relative) strong temptation to a (relative) weak temptation in the menu $\{b, y\}$, everything else is the same as in the preceding example:

$$\{b\} \sim \{b, y\} \succ \{y\}, \{b, y, c\} \succ \{b, c\} \succeq \{y, c\}$$

I claim that the modified preference cannot be represented by a (NAG) DLR utility.[24] Otherwise, let $(\{q_i\}, u, v)$ be a representing triple and observe that $\{b\} \sim \{b, y\} \succ \{y\}$ implies

$$\max_{x \in \{b,y\}}(u(x) + v_i(x)) = u(b) + v_i(b), \forall i$$

That is, broccoli is weakly preferred in every state. To see this, first note that $\{b\} \succ \{y\}$ implies that $u(b) > u(y)$. Thus, if there is an $i$ such that $\max\{u(b) + v_i(b), u(y) + v_i(y)\} = u(y) + v_i(y)$, then, for this $i$, we must have $v_i(y) > v_i(b)$ so that

$$\max_{\{b,y\}}(u(x) + v_i(x)) - \max_{\{b,y\}} v_i(x) = u(y) < u(b)$$

Moreover, in the case where $\max_{\{b,y\}}(u(x) + v_i(x)) = u(b) + v_i(b)$ we also have $u(b) + v_i(b) - \max_{\{b,y\}} v_i(x) \leq u(b)$. Thus, $\{y\} \prec \{b, y\} \sim \{b\}$ implies $\max_{\{b,y\}}(u(x) + v_i(x)) = u(b) + v_i(b), \forall i$. On the other hand, we also know that

$$C(A) \subseteq \bigcup_i \arg\max_A(u(x) + v_i(x))$$

Since $y \in C(\{b, y, c\})$, there must be some $i$ such that $y = \arg\max_{\{b,y,c\}}(u(x) + v_i(x))$. But then, for this same $i$, $y = \arg\max_{\{b,y\}}(u(x) + v_i(x))$ - contradiction.

Both examples arguably convey the same story - when temptations are added to a menu, elements that are relatively less tempting than the newly added items obtain newfound commitment value. Nevertheless, only the first example admits a DLR representation. To explain, we follow Dekel et al. (2009) and explain the first example by thinking of the decision-maker as facing two ex-post states. In the first state, there is no temptation and he chooses to consume broccoli. In the second state, the self-control cost required to commit to broccoli and stay away from chocolate far exceeds the self-control cost required to commit to the compromise of

[24] The preference cannot be represented by the full Dekel et al. (2009) model either on account of violating their AIT axiom (the necessity argument presented in Dekel et al. (2009) for this axiom, without the 'almost' part, applies to the discrete DLR model as well). We present the explicit argument for the NAG-DLR model since it makes it clearer *why* the non-representability is occurring.

yogurt, and so, yogurt is consumed in this state.

This example provides a nice illustration of the subtle implications of the state space approach. In order to explain why the DM consumes $y$ even when $c$ isn't on the menu (as the DLR model requires), we imagine that the DM forms preferences by reasoning about he would choose in the counterfactual (i.e. hypothetical) scenario where his menu includes $c$. Since he would consume $y$ in this scenario, he concludes that he should consume $y$ even when $c$ isn't on the menu. However, this corrupts the explanation that $y$ is chosen as a compromise in the menu $\{b, y, c\}$ since there shouldn't be any need to compromise when the menu is $\{b, y\}$.

It is straightforward to construct a local menu preference that represents the menu preference in either example 3 or 4. Let us try to understand intuitively *why* our model of inductive reasoning can capture the behavior in this example. Define a local relation $\succeq_A$ as follows: $x$ locally (i.e. w.r.t. the menu $A$) dominates $y$ if either (i) $x$ normatively dominates $y$ or (ii) $x$ does not normatively dominate $y$, but $y$ is 'unavailable' since it is compromised (and, implicitly, $x$ is not compromised).[25] This constitutes our definition of the local menu order $\succeq_A$. Note that this relation captures the reasoning embodied in the example. Also note that the relation $\succeq_A$ satisfies inductive consistency. The coarsening condition accounts for the (weak) local preference reversals that can occur if, as we add options to the menu, previously available 'healthy' choices now become compromised. Hence, the LTP model provides an intuitive representation of the (version of) compromise effect.

# 5 Conclusion

This paper has developed an axiomatic model of context-dependent reasoning. The behavioral primitive is a binary relation on menus. We model context-dependence via the idea that the DM's ranking over consumption choices are relative, or local, to the particular menu in which these choices are available. Hence, as we vary choice problems we trace out a system of menu-indexed rankings, $\{\succeq_A\}$. The main characteristic of this system is that it satisfies an "inductive consistency" condition, where rankings between any two given objects are coarsened as we enlarge the domain of the choice problem. The main results of the paper, Theorem 1 and Theorem 2, find axioms on the menu preference, when the DM exhibits (resp.) preference for flexibility or preference for commitment, that characterize when it is generated by a system of local orders $\{\succeq_A\}$ satisfying the inductive consistency property. Moreover,

[25] We have introduced some jargon from the temptation literature: With the Dekel et al. (2009) example in mind, when we say normatively dominant we mean 'healthy' and when we say $y$ is compromised, we mean that there is some unhealthy option on the menu such that in the presence of this option the DM is unable to commit to the healthy option - if it is the only one available.

in both settings the family of subjective local orders $\{\succeq_A\}$ is behaviorally identified.

A challenge in modeling framing effects is to find a tractable way to disentangle the frame of a choice problem from the preferences induced by the frame. There are two approaches to this problem. The first approach, which is the one taken in this paper, is to take the frame as observable. More precisely, the subjective data we want to infer is the decision-maker's frame-dependent preference relation over consumption choices. To make this inference, we have assumed that we can observe the decision-maker's welfare ranking across choice problems - so that the 'frame' is implicitly the choice problem (menu) itself. The strong suit of this approach is that it allows us to obtain an identified model with good predictive power, e.g. one that is amenable to comparative statics. However, a second approach is to take a more frugal primitive. For example, one could take the observable to be choices from menus as opposed to a binary relation on menus, and attempt to derive both the frame and the frame-dependent consumption ranking simultaneously. However, it seems difficult (to us) to obtain the same results using this approach. Nevertheless, the issue raises an interesting question: what are the limitations on subjective inference when one observes only a choice correspondence, as opposed to a welfare ranking, on menus? We leave this as a topic for future research.

# 6 Appendix

## 6.1 Omitted Proofs from Section 2

*Proof of Theorem 1.* Let $(u, \Phi_u)$ denote any LPF representation. Let us make a preliminary observation that will be useful in the sufficiency argument below. Note that if we have an LPF representation $(u, \Phi_u)$, then the map $u(\cdot, \cdot)$ satisfies the following additional property: For each menu $A$, the function $u(\cdot, A)$ is a coarsening of the singleton ranking (w.r.t. the underlying menu preference). This will not be needed for the proof of necessity, but our construction in the sufficiency argument checks Inductive Consistency by first showing that the map we construct has this coarsening property. To verify the property, we use the fact (proven below) that $C(A) = \Phi_u(A)$. That is, the set of critical elements in a menu are precisely the elements of the local arg maxima (w.r.t. the local order $\succeq_A$). Now take any doubleton menu $\{x, y\}$ where $x \succeq y$. I claim that $x \succeq_{\{x,y\}} y$. By Inductive Consistency, it then follows that $x \succeq_A y$ for any menu with $x, y \in A$. By A3, we have $C(\{x, y\}) \neq \emptyset$. If $\{y\} = C(\{x, y\})$, then we have $\{y\} \succeq \{x, y\} \succ \{x\}$ - contradiction. Hence, $x \in C(\{x, y\})$ implying that, since $C(A) = \Phi_u(A)$, $x \succeq_{\{x,y\}} y$.

**Necessity**
I claim that we have the equality

$$\Phi_u(A) = C(A)$$

To see this, take $x \in \Phi_u(A)$ and consider the utility of the menu $A \backslash x$. By definition, we have $U(A \backslash x) = |\Phi_u(A \backslash x)| \cdot u(y, A \backslash x)$, where $y \in \Phi_u(A \backslash x)$. Consider separately the case where $\Phi_u(A)$ is singleton and the case where it is non-singleton. In the case where it is singleton, by Technical Condition I(i) and the non-negativity of $u(\cdot, \cdot)$, we have

$$u(x, A) > |X| \cdot u(y, A \backslash x) > |\Phi_u(A \backslash x)| \cdot u(y, A \backslash x), \forall y \in A \backslash x$$

It follows that $U(A) > U(A \backslash x)$, showing that $\Phi_u(A) \subseteq C(A)$. In the case where it is non-singleton find $y (\neq x) \in \Phi_u(A) \cap (A \backslash x)$. Note that if $z \in \Phi_u(A \backslash x)$, then $z \succeq_{A \backslash x} y$ so that $z \succeq_A y$ - by Inductive Consistency. It follows that $\Phi_u(A \backslash x) \subsetneq \Phi_u(A)$. Hence,

$$U(A) = |\Phi_u(A)| \cdot u(y, A) > |\Phi_u(A \backslash x)| \cdot u(z, A \backslash x)$$

where $z \in \Phi_u(A \backslash x)$. To see the inequality note that $u(z, A) \geq u(z, A \backslash x)$ by Upwards Monotonicity and we just verified that, in the case where $\Phi_u(A)$ is non-singleton, we have $\Phi_u(A \backslash x) \subsetneq \Phi_u(A)$. Hence, in all cases, $\Phi_u(A) \subseteq C(A)$. For the reverse inclusion, take $x \in C(A)$ and towards contradiction say that $x \notin \Phi_u(A)$. By Technical Condition I(ii), we then have $u(y, A) = u(y, A \backslash x), \forall y \in \Phi_u(A)$. I claim that we have $\Phi_u(A) \subseteq \Phi_u(A \backslash x)$. To see this, say that $y \in \Phi_u(A)$ (where $y \neq x$). If there is some $z \in A \backslash x$ such that $u(z, A \backslash x) > u(y, A \backslash x)$, then by Upwards Monotonicity we have $u(z,A) > u(y,A\backslash x) = u(y,A)$ - contradicting the hypothesis that $y \in \Phi_u(A)$. Hence, $\Phi_u(A) \subseteq \Phi_u(A\backslash x)$. Now take any $y \in \Phi_u(A)$ and notice that we have

$$U(A) = |\Phi_u(A)| \cdot u(y,A) \leq |\Phi_u(A\backslash x)| \cdot u(y,A\backslash x) = U(A\backslash x)$$

contradicting the hypothesis that $x \in C(A)$.

Given the equality $C(A) = \Phi_u(A)$, necessity of A3 is trivial. For necessity of A1*, notice that by definition we have $A \succ A\backslash x$ for any $x \in C(A)$. If $x \notin C(A)$, then again applying the preceding argument we have $\Phi_u(A) \subseteq \Phi_u(A\backslash x)$. Inductive Consistency means that $\succeq_A$ coarsens $\succeq_{A\backslash x}$, hence given that $\Phi_u(A) \subseteq \Phi_u(A\backslash x)$ we must in fact have equality, $\Phi_u(A) = \Phi_u(A\backslash x)$. It follows that $U(A) = U(A\backslash x)$ - proving necessity of A1*. For necessity of A2, we will need to invoke the equality of $C(A)$ with $\Phi_u(A)$. Let $A \subseteq B$ and consider $y \in C(B) \cap A$. Then, $x \succeq_A y, \forall x \in C(A)$ (as $\Phi_u(A) = C(A)$). By Inductive Consistency, we obtain $u(x,B) \geq u(y,B)$. Since $u(y,B) \geq u(z,B), \forall z \in B$ this implies $x \in \Phi_u(B) = C(B)$, so that $C(A) \subseteq C(B)$.

**Sufficiency**

I break the the proof into two main steps, (i) construction of the map $u(\cdot,\cdot)$ and (ii) verification that the constructed map satisfies the inductive consistency condition (the verification of the other conditions that define an LPF representation will be evident from the construction).

**Construction of** $u(\cdot,\cdot)$

Let $\Phi := \{A \in \mathcal{M} : C(A) = A\}$ be the set of fixed points of the critical points map. Proceed in the three (sub)steps.

Step 1 - Reduction to $\Phi$.

We first claim that $A \sim C(A)$. Let $A\backslash C(A) = \{y_1, \ldots, y_k\}$ and put $A_i := A\backslash\{y_1, \ldots, y_i\}$. By A2, $C(A_i) \subseteq C(A), \forall i$. Iterative application of A1* then yields $A \sim A_1 \sim \cdots \sim A_k = C(A)$. Next, we check that $C(A) = C(C(A))$ via contradiction. Let $x \in C(A)\backslash C(C(A))$ and note that $C(C(A)) \sim C(A) \sim A \succ A\backslash x$. Since $C(C(A)) \subseteq A\backslash x$, by A1* we obtain $A\backslash x \succeq C(C(A))$. Thus, $C(C(A)) \sim C(A) \sim A \succ A\backslash x \succeq C(C(A))$ - a contradiction.

Step 2 - Constructing the Representation on $\Phi$.

Put $K := |X|^2$ and select a cardinal representation $U(\cdot)$ of $\succeq$ with the property that for any $A, B \in \Phi$ with $B \succ A$ we have $U(B) > K \cdot U(A)$. Now put $u_\Phi(x,A) := \frac{U(A)}{|A|}, \forall A \in \Phi$. Notice that if we have $A' \subsetneq A$ with $A', A \in \Phi$, then by definition of $U(\cdot)$ (and A1*) we have $u_\Phi(x,A) > u_\Phi(x,A')$ - so that $u_\Phi$ is upwards monotonic on $\Phi$. This defines an LPF representation of the preference on the set $\Phi$. Denote this representation as $u_\Phi(\cdot,\cdot)$.

Step 3 - Extension.
Now we extend the definition of $u_\Phi(\cdot,\cdot)$ to all menus. Put $u(x,A) := u_\Phi(x, C(A)), \forall x \in C(A)$ and set $U(A) := \sum_{x \in C(A)} u(x,A)$. Note that (i) $C(C(A)) = C(A)$ so that the utility is well-defined, and (ii) $U(A) = U(C(A))$ so that $U(\cdot)$ represents $\succeq$ by Step 1. We now check that the extended map is upwards monotonic. Put $A \subseteq B$. If $C(B) \cap A \neq \emptyset$, then A2 implies $C(A) \subseteq C(B)$. Since $U_\Phi(\cdot)$ is an LPF utility, $u(x,B) = u_\Phi(x, C(B)) \geq u_\Phi(x, C(A)) = u(x,A)$. Now extend $u(x,A)$ to $A \backslash C(A)$ as follows:

$$u(x,A) := \max\{u(x,A') : x \in C(A'), A' \subseteq A\}$$

and observe that $u(x,A)$ is upwards monotonic. This concludes the construction of the candidate LPF representation, $u(\cdot,\cdot)$.

**Verification of Inductive Consistency**
To conclude, we verify that (i) $C(\cdot)$ is the arg max of the kernel $u(\cdot,\cdot)$ (since we have extended the definition of the map $u(\cdot,\cdot)$ to non-critical elements, we need to formally verify that we haven't introduced non-critical elements into the arg max), and (ii) $u(\cdot, A)$ satisfies inductive consistency. The latter verification is carried out in two steps. First, we verify that $u(\cdot,\cdot)$ coarsens the singleton ranking. Second, we use this fact to verify that inductive consistency holds. A word on notation before we proceed. Note that, once we know $u(\cdot,\cdot)$ coarsens the singleton ranking, then the inductive consistency condition is equivalent to the statement that $u(\cdot,\cdot)$ is a monotone coarsening of the singleton ranking. Hence, I will at times refer to the inductive consistency property as 'monotone coarsening'.

First, note that we have $u(x,A) = u(y,A), \forall x,y \in C(A)$. Hence, we just need to check that $u(x,A) > u(y,A), \forall x \in C(A), y \in A \backslash C(A)$. Find $A' \subseteq A$ such that $A' = C(A')$ and $u(y,A) = u(y,A')$. Now consider two possible cases, (i) $A' \cap C(A) = \emptyset$ and (ii) $A' \cap C(A) \neq \emptyset$. In the latter case, by $A2$, we obtain $C(A') \subsetneq C(A) \subseteq A$ - contradicting the hypothesis that $y \in A \backslash C(A)$. Hence, consider case (i). By deleting elements of $C(A)$ and applying A1*, we obtain: $A \succ A \backslash C(A)$. By A1* we have $A \backslash C(A) \succeq A'$ (as $C(A) \cap A' = \emptyset$), whence:

$$C(A) \sim A \succ A \backslash C(A) \succeq A'$$

By selection of the cardinal representation $U(\cdot)$ it follows that $u(x,A) = u(x, C(A)) > u(y,A')$. This shows $C(A)$ is the arg max.

Now verify that $u(\cdot, A)$ is a monotone coarsening of the singleton ranking.[26] To this end, we first verify that it is a coarsening of the ranking on singleton

[26] For this part of the argument I will need to invoke the partition structure of the preference - see Proposition 2.

menus. To verify this, it suffices to show that $u(x) \geq u(y)$ implies $u(x, A) \geq u(y, A)$ (where $x, y \in A$). Note that if $x, y \in C(A)$, this is obvious as $u(x, A) = u(y, A)$. Similarly, if $x \in C(A)$ and $y \in A \backslash C(A)$, the preceding argument shows that $u(x, A) > u(y, A)$. By the partition structure, we know that $u(x) > u(y)$. Hence, consider $x, y \in A \backslash C(A)$. Towards contradiction, let us allege that $u(y, A) > u(x, A)$ and $u(x) \geq u(y)$. Find subsets $A_1, A_2 \subseteq A$ such that $A_1 = C(A_1), A_2 = C(A_2)$ and $x \in C(A_1), y \in C(A_2)$ with $u(x, A) = u(x, A_1), u(y, A) = u(y, A_2)$. Consider $A' = A_1 \cup A_2$. Note that $C(A') \subseteq A' = (A_1 \cup A_2) = (C(A_1) \cup C(A_2))$ so that $C(A') \cap A_1 \neq \emptyset$ or $C(A') \cap A_2 \neq \emptyset$. Thus, either $x \in C(A')$ or $y \in C(A')$. If the former holds, then $u(x, A') \geq u(y, A') \geq u(y, A_2) = u(y, A)$ - contradicting the hypothesis that $u(y, A) > u(x, A)$. Hence, consider the possibility that $y \in C(A'), x \notin C(A')$. Now consider the menu $\{x, y\} \subseteq A'$. By A2, we know that $C(\{x, y\}) \subseteq C(A')$ so that $x \notin C(\{x, y\})$. Hence, $\{y\} \succeq \{x, y\} \succ \{x\}$ - contradicting the assumption that $\{x\} \succeq \{y\}$. It remains to check the monotone coarsening property. That is, fix $A \subseteq B$ with $x, y \in A$. We want to show that $u(x, A) \geq u(y, A)$ implies $u(x, B) \geq u(y, B)$. Note that if $u(x, A) > u(y, A)$, then since $u(\cdot, A)$ is a coarsening of $u(\cdot)$ (the singleton ranking) we must have $u(x) > u(y)$, implying that $u(x, B) \geq u(y, B)$ as $u(\cdot, B)$ is a coarsening of $u(\cdot)$ as well. Thus, we need to check the case where $u(x, A) = u(y, A)$.

If $u(x) = u(y)$, then since $u(\cdot, B)$ coarsens $u(\cdot)$ we obtain $u(x, B) = u(y, B)$ as well. Hence, by symmetry, put $u(x) > u(y)$. Since $u(x, A) = u(y, A)$ find $A_1 \subseteq A$ with $y \in C(A_1)$ and $u(y, A_1) = u(y, A)$. Consider $A' = A_1 \cup \{x\}$ and note that, since $u(x) > u(y)$ we must have either (i) $\{x\} = C(A')$ or (ii) $C(A') \cap A_1 \neq \emptyset$. The former case cannot occur as $u(x, A) = u(y, A)$. Hence, $C(A') \cap A_1 \neq \emptyset$, which implies $y \in C(A')$. Since $u(x) > u(y)$ this then implies $x \in C(A')$. Now consider the menu $B$, where $A \subseteq B$. Let $B_1 \subseteq B$ be such that $x \in C(B_1)$ and $u(x, B_1) = u(x, B)$. Consider the menu $\hat{B} := B_1 \cup A'$. Note that either (i) $C(\hat{B}) \cap A' \neq \emptyset$ or (ii) $C(\hat{B}) \cap B_1 \neq \emptyset$. In the former case, we have $C(A') \subseteq C(\hat{B})$, so that $x, y \in C(\hat{B})$. Hence, $u(x, \hat{B}) = u(y, \hat{B})$. Since $u(x, B) = u(x, \hat{B})$ and $u(x, B) \geq u(y, B)$ we obtain $u(x, B) = u(y, B)$. In the latter case, we have $C(B_1) \subseteq C(\hat{B})$. Hence, $x \in C(\hat{B})$. As $x \in A'$ this implies that $C(\hat{B}) \cap A' \neq \emptyset$, so that $C(A') \subseteq C(\hat{B})$. Hence, $y \in C(\hat{B})$ implying that $u(x, \hat{B}) = u(y, \hat{B})$ - and, in turn, $u(x, B) = u(y, B)$. □

The following claim will be invoked in the proof of Theorem 2.

**Claim 1.** *Assume $\succeq \in \boldsymbol{P}(X)$ satisfies A1 - A3. If $A \subseteq B$ and $C(B)(\neq \emptyset) \subseteq A$, then $A \succeq B$.*

*Proof.* This only requires A2 and A3. Put $B \backslash A = \{y_1, \ldots, y_k\}$ and set $A_i = B \backslash \{y_1, \ldots, y_i\}$. Note that $A_1 \succeq B$ and, by A2, $C(A_1) \subseteq C(B) \subseteq A$. Moreover, $C(A_1) \neq \emptyset$ by A3. Iteratively apply the same argument to each $A_i$ and obtain $B \preceq A_1 \preceq A_2 \preceq \cdots \preceq A_k = A$. □

*Proof of Theorem 2.* We first present the proof of Necessity. This doesn't follow from the same argument as in the LPF model since the map $u(\cdot, \cdot)$ is downwards monotonic and the growth conditions are slightly different. As the reader will note, the role of the Technical Condition II is to allow us to obtain the equality of critical sets, $C(A)$, with the set of arg maxima of the LTP representation $u(\cdot, \cdot)$.

**Necessity**

Let $\succeq$ be the menu preference generated by the LTP representation $(u, \Phi_u)$. We first verify the equality $C(A) = \Phi_u(A)$. To see this, let $x \in \Phi_u(A)$ and note that by Technical Condition II(i) and non-negativity of $u(\cdot, \cdot)$ we have

$$|\Phi_u(A\backslash x)|\cdot u(y, A\backslash x) < |\Phi_u(A)|\cdot u(x, A), \forall y \in A\backslash x$$

Hence, $x \in C(A)$. Conversely, let $x \in C(A)$. Towards contradiction, say that $x \notin \Phi_u(A)$. Then, by Technical Condition II(ii), we have

$$|\Phi_u(A)|\cdot u(x, A) \leq |\Phi_u(A\backslash y)|\cdot u(x, A\backslash y)$$

Hence, $U(A\backslash x) \geq U(A)$ - contradiction. It follows that $C(A) = \Phi_u(A)$. Given this equality, we now verify necessity of the axioms. Note that $u(\cdot, A)$ is non-constant, so there is some $y \in A\backslash\Phi_u(A)$ (for $A$ non-singleton). Since $C(A) = \Phi_u(A)$, this implies $C(A) \neq A$ - so that A1 follows. Non-emptiness similarly follows as $\Phi_u(A)$ is always non-empty. For A2 let $A \subseteq B$ and put $x \in C(B) \cap A$. Take any $y \in C(A) = \Phi_u(A)$. By definition, $y \succeq_A x$. By inductive consistency, $y \succeq_B x$. As $C(B) = \Phi_u(B)$, this implies $y \in C(B)$. Hence, $C(A) \subseteq C(B)$.

**Sufficiency**

Now we turn to the sufficiency argument. The first step is the easy one - we define the map $u(\cdot, \cdot)$ on the critical elements of the menu. To this end, let $A_1, A_2, \ldots$ be any enumeration of $\mathcal{M}$ and identify a utility $U(\cdot)$ on $\mathcal{M}$ with a vector in the Euclidean space $\prod_{A_i} \mathbf{R}_{A_i}$ (here $\mathbf{R}_{A_i}$ denotes the coordinate which gives the utility of menu $A_i$). We will find an LTP representation of $\succeq$ in a neighborhood of $\mathbf{1} \in \prod_{A_i} \mathbf{R}_{A_i}$. Let

$$\kappa := \frac{1}{|X|} - \frac{1}{|X| + 1}$$

and choose a cardinal representation $U(\cdot)$ of $\succeq$ in $B_\epsilon(\vec{1})$ where $\epsilon < \kappa/2$. Put $U(A) = 1 + \epsilon_A$ and define,

$$u(x, A) = \frac{1 + \epsilon_A}{|C(A)|}, \forall x \in C(A)$$

This defines the map on elements of $C(A)$. To extend to other elements of the menu in a manner that satisfies the inductive consistency (monotone coarsening) condition constitutes the heart of the proof. The forthcoming argument leverages the rich structure on the menu preference induced by axioms $A1 - A3$. To uncover this structure, the key conceptual ingredient is a generalization of the partition structure of the preference in the flexibility case (see proof of Proposition 2). Also, as in the proof of Theorem 1 we break the forthcoming argument into two main steps. The first is the construction of the candidate LTP map, $u(\cdot, \cdot)$. The second step then verifies that the constructed map satisfies the defining properties of an LTP representation.

**Construction of $u(\cdot, \cdot)$**

We construct an equivalence relation on singletons $X$ which yields a partition of $X$ into order intervals. These order intervals have the property that, for any menu $A$, the set $C(A)$ is contained inside one of these order intervals. In this sense, the construction generalizes the one given in Proposition 2 to accommodate the presence of temptation. The mirror construction in the proof of Proposition 2, when there is preference for flexibility, is conceptually similar and makes for lighter reading - hence, it might be useful to go through that construction before reading what follows. Make the following definition: (since we are assuming $A1$ and this precludes ties in the singleton ranking, I will henceforth assume the singleton ranking is strict)

**Definition**: Say that $x\mathcal{R}y$ if there is a menu $A \subseteq X$ with $x, y \in C(A)$.[27]

If $x\mathcal{R}y$ I say that the pair $(x, y)$ are *not separated*. Clearly, $\mathcal{R}$ is a well-defined binary relation on the set $X$. I claim that $\mathcal{R}$ is transitive. First, notice that for any $(x, y)$ we must have either $x\mathcal{R}y$ or $\neg(x\mathcal{R}y)$. Take any $(x, y)$, say with $x \succ y$, and consider the menu $\{x, y\}$. By $A1$ and $A3$, we know that $C(\{x, y\}) = \{x\}$. If for every menu $A$ with $x, y \in A$ we have $C(A) \cap \{x, y\} \subseteq \{x\}$, then $x, y$ are separated, i.e. $\neg(x\mathcal{R}y)$. OTOH, if there is a menu $A$ with $\{x, y\} \subseteq C(A)$, then clearly $x\mathcal{R}y$. As this exhausts the possibilities we obtain either $x\mathcal{R}y$ or $\neg(x\mathcal{R}y)$. Next, we check transitivity. Take $(x, y, z)$ with $x\mathcal{R}y, y\mathcal{R}z$. We have the following three possibilities

1. $x \succ y \succ z$ (or $z \succ y \succ x$ - which is omitted by symmetry)
2. $x \succ y, z \succ y$
3. $y \succ x, y \succ z$

I verify that in each of the three cases we have $x\mathcal{R}z$. Say that $x \succ z$ wlog. In case (1), let $A_1, A_2$ be menus such that $\{x, y\} \subseteq C(A_1), \{y, z\} \subseteq C(A_2)$. Consider the menu $A' = A_1 \cup A_2$. We have either $C(A') \cap A_1$ or $C(A') \cap A_2 \neq \emptyset$. If the former holds, then $\{x, y\} \subseteq C(A_1) \subseteq C(A')$ by $A2$. Hence, $y \in C(A') \cap A_2 \neq \emptyset$ and we obtain $\{y, z\} \subseteq C(A_2) \subseteq C(A')$. Hence, $\{x, z\} \subseteq C(A')$, implying that $x\mathcal{R}z$. Now

[27]We will also make use of a relative form of this relation, $\mathcal{R}_B$, where for any fixed menu $B$ we say that $x\mathcal{R}_By$ if there is a subset $B' \subseteq B$ with $x, y \in C(B')$.

consider case (2). Choose menus $A_1, A_2$ with $\{x, y\} \subseteq C(A_1), \{z, y\} \subseteq C(A_2)$ and, as before, consider the menu $A' = A_1 \cup A_2$. If $C(A') \cap A_2 \neq \emptyset$, then $y \in C(A')$, implying $C(A') \cap A_1 \neq \emptyset$ - so that $x \in C(A')$. Hence, $x\mathcal{R}z$. Similarly argue if $C(A') \cap A_1 \neq \emptyset$ to obtain $x\mathcal{R}z$. The third case is dealt with similarly. Hence, the binary relation $\mathcal{R}$ is transitive. It follows that the $\mathcal{R}$ equivalence classes form a partition of $X$.

Let $\{B_1, \ldots, B_n\}$ be an enumeration of the $\mathcal{R}$-equivalence classes. I claim that the $B_i$ are order intervals. Let $x, y \in B_i$ with $x \succ z \succ y$. I check that $z \in B_i$. Let $A$ be such that $\{x, y\} \subseteq C(A)$ and consider the menu $A' = A \cup \{z\}$. Since $x \succ z$ and $x \in A$ it follows that $C(A') \cap A \neq \emptyset$, implying that $\{x, y\} \subseteq C(A')$. Since $C(A')$ is an order interval in $A'$ it follows that $z \in C(A')$, so that $x\mathcal{R}z$ and $z \in B_i$. Hence, the collection $\{B_1, \ldots, B_n\}$ partitions $X$ into order intervals. Let

$$\Sigma_i := \{A : C(A) \subseteq B_i\}$$

and note that $\Sigma_i$ is a partition of the space of all menus (exactly parallel to the preference for flexibility case - see Proposition 2). For each $B_i$ select

$$K_i < \frac{1}{2} \cdot \frac{1}{|X|}$$

Note that we have (i) $K_i > 0$ and (ii) $u(x, A) > K_i, \forall A \in \Sigma_i$ with $x \in C(A)$. Also note that, by making $K_i$'s smaller, if necesary, I can assume we have $K_1 > K_2 > \cdots > K_n$. Let $U(A) = 1 + \epsilon_A$ denote the cardinal utility selected earlier. Define (for, say, $A \in \Sigma_i$)

$$u(x, A) = \begin{cases} \frac{1+\epsilon_A}{|C(A)|}, & \text{if } x \in C(A) \\ u'(x, A), & \text{if } x \in (A \backslash C(A)) \cap B_i \\ K_j, & \text{if } x \in (A \backslash C(A)) \cap B_j, j \neq i \end{cases}$$

where $u'(x, A)$ is a function we define shortly. Note that we are defining $u(\cdot, \cdot)$ in three steps here. First, we identified its values on critical elements of the menu. Now we have identified its values on non-critical elements that lie in the 'lower' partition cells $B_j$. Next, we will define its values in the dominant cell $B_i$. Since A1 implies the singleton ranking is strict, let $u(x) > u(y)$ with $x, y \in A$. I claim that $u(x, A) \geq u(y, A)$. Note that if $y \in C(A)$, then $x \in C(A)$ (as $C(A)$ is an order interval in $A$ and $x \succ y$) - implying, by definition of $u(\cdot, \cdot)$, that $u(x, A) = u(y, A)$). Hence, consider the possibility that $x, y \notin C(A)$. As $A \in \Sigma_i$, consider $y \in B_j$ for some $j > i$. The fact that $u(x, A) \geq u(y, A) = K_j$ follows by construction of $u(\cdot, \cdot)$. Hence, we are down to $x, y \in B_i$, which takes us to the third step of the construction of the map $u(\cdot, \cdot)$.

I will now define values $u'(x, A)$ with the property that $u'(x, A) = u'(y, A)$ whenever $x, y \in B_i$ and $u'(x, A) > u'(y, A)$ if $x \in B_i, y \in B_j$ for $j > i$. This is a subtle aspect of the construction, so we first give some intuition for the formula we eventual write down for $u'(x, A)$.[28] The main idea is to apply the separatedness relation $\mathcal{R}$ to the menu $A$, call it $\mathcal{R}_A$. That is, say that $x\mathcal{R}_A y$ iff there is a subset, $A'$, of $A$ such that $x, y \in C(A')$. The same reasoning as given above, applied to the universe $A$, yields a partition of $A$ into order intervals, where the top cell of the partition is all of $C(A)$; hence, it yields a partition of $A \backslash C(A)$ (into $\mathcal{R}_A$-equivalence classes).[29] Let $\{B_1, \ldots, B_k\}$ denote this partition.

Consider the quantities, $\overline{u}(x, A) := \min\{U(A')/|C(A)| : A' \subseteq A, x \in C(A')\}$. Note that for each $y \in B_i$ the quantity $\overline{u}(y, A)$ yields an upper bound on the value $u'(y, A)$. For each $x \in \cup_{i>1} B_i$, let $\underline{u}(x, A) := \max\{U(A')/|C(A')| : A' \supseteq A, x \in C(A')\}$. Notice that this is a lower bound on the value of $u'(x, A)$. Hence, we need to be able to choose $u'(x, A)$ such that

$$\underline{u}(x, A) \leq u'(x, A) \leq \overline{u}(x, A)$$

To see that this can be done, let $A' \subseteq A$ be such that $C(A') = B_i$, where $x \in B_i$ and $B_i$ is the $\mathcal{R}_A$-equivalence class of $x$. Forgetting about the $1 + \epsilon_A$ terms in the numerator (these can always be adjusted ex post - for now equality should be read as approximate equality in the absence of $\epsilon$) we obtain that

$$\overline{u}(x, A) \leq \frac{1}{|B_i|}$$

Moreover, for any $x \in B_i$ with $A' \subseteq A$ and $x \in C(A')$ we have $C(A') \subseteq B_i$. Hence,

$$\overline{u}(x, A) = \frac{1}{|B_i|}$$

Note that this (approximate) equality holds for all $x \in B_i$. Next we find an upper bound for $\underline{u}(x, A)$. Consider $\hat{A} \supseteq A$ with $x \in C(\hat{A})$. Let $A'$ denote a subset of $A$ with $B_i = C(A')$. Note that if $x \in C(\hat{A})$, then $C(\hat{A}) \cap A' \neq \emptyset$. Hence, $C(A') \subseteq C(\hat{A})$. Also, since $C(A')$ is an order interval in $A'$ we obtain that (putting $x \in B_i$)

$$\cup_{j \leq i} B_j \subseteq C(\hat{A})$$

[28] The reader who wants to skip the intuition can pass directly to the definition of the map $u(\cdot, \cdot)$ without loss of continuity.

[29] To see this, (abusing notation) let $\{B_1, \ldots, B_k\}$ denote the partition of $A$ into its $\mathcal{R}_A$-equivalence classes. Since $C(A)$ is an order interval in $A$ with $\sup(A) \in C(A)$, it follows that $C(A) \subseteq B_1$. Now if $(A \backslash C(A)) \cap B_1 \neq \emptyset$, then let $z$ be in the intersection and find a menu $A' \subseteq A$ with $\{x, z\} \subseteq C(A')$, where $x \in C(A)$. By $A2$ this implies that $C(A') \subseteq C(A)$ - contradiction. Hence, $B_1 = C(A)$. A similar argument applies to other partition cells.

Hence, recalling that $B_1 = C(A)$, we obtain that

$$\underline{u}(x, A) \leq \frac{1}{|C(A)| + \sum_{j=2}^{i} |B_j|}$$

It follows that

$$[\frac{1}{|C(A)| + \sum_{j=2}^{i} |B_j|}, \frac{1}{|B_i|}] \subseteq [\underline{u}(x, A), \overline{u}(x, A))$$

for $x \in B_i$. Also note that if we choose a value of $u'(x, A)$ as the lower endpoint of the sub-interval, then we also satisfy $u'(x, A) < U(A)/|C(A)|$ (again, by choice of $\epsilon$). This now leads us to our definition of the extended map $u(\cdot, \cdot)$. In the definition below, I will need to make a distinction in notation between $\mathcal{R}$-equivalence classes and $\mathcal{R}_A$-equivalence classes. Denote the former with the partition $\{\hat{B}_i\}$ and the latter with the partition $\{B_i\}$. For $A \in \Sigma_m$ put

$$u(x, A) = \begin{cases} \frac{U(A)}{|C(A)|}, & \text{if } x \in C(A) \\ \frac{1}{|C(A)| + \sum_{j=2}^{i} |B_j|}, & \text{if } x \in \hat{B_m} \cap B_i \\ K_n, & \text{if } x \in \hat{B_n}, n > m. \end{cases}$$

This is our candidate LTP representation $u(\cdot, \cdot)$. Notice that (by selection of $\kappa$) (i) $U(A) = \sum_{x \in C(A)} u(x, A)$ (i.e. $C(A) = \Phi_u(A)$) and (ii) $\Phi_u(A) = C(A)$. Hence, we just need to check (i) the monotone coarsening condition (i.e. inductive consistency) and (ii) the downwards monotonicity property.

**Verification of Inductive Consistency and Downwards Monotonicity**

First, let us check that $u(\cdot, A)$ coarsens $u(\cdot)$. Take $u(x) > u(y)$ with $x, y \in A$. If $x, y \in C(A)$, the claim is obvious. Similarly, if $x \in C(A)$ and $y \in A \backslash C(A)$ (the reverse cannot hold as $C(A)$ is an order-interval in $A$). Hence, consider $x, y \in A \backslash C(A)$ and let $\{B_i\}$ denote the $\mathcal{R}_A$-equivalence classes. Note that if $x, y \in B_i$, then from definition of $u(x, A)$ we have $u(x, A) = u(y, A)$. If $x, y$ are in different partition cells but $x, y \in \hat{B}_n$ (i.e. they are in the same $\mathcal{R}$-equivalence class), then we must have $x \in B_i, y \in B_j, j > i$ (as $u(x) > u(y)$) and $u(x, A) > u(y, A)$ - again by definition of $u(\cdot, \cdot)$. If $x \in \hat{B}_n$ and $y \in \hat{B}_m$, then $m > n$ as $u(x) > u(y)$ and we obtain $u(x, A) > u(y, A)$, by choice of the constants $K_n$. This shows that $u(\cdot, A)$ coarsens $u(\cdot)$. To check that it is a monotone coarsening take $A \subseteq B$ and let $x, y \in A$. If $u(x, A) > u(y, A)$, then since $u(\cdot, A)$ coarsens $u(\cdot)$ we must have $u(x) > u(y)$. Hence, as $u(\cdot, B)$ coarsens $u(\cdot)$ we must have $u(x, B) \geq u(y, B)$. Thus, consider $u(x, A) = u(y, A)$. We check that $u(x, B) = u(y, B)$ as well. By symmetry, take $u(x) > u(y)$. Consider two cases, (i) $B \in \Sigma_n$ (where $A \in \Sigma_n$) and (ii) $B \in \Sigma_l, l < n$. Also notice that $u(x, A) = u(y, A)$ is only possible if either (a) $x, y$ are in the same

$\mathcal{R}_A$-equivalence class (and $x, y \in \hat{B}_n$), or (b) if $x, y \in \hat{B}_m$ for some $m > n$. Note that if $B \in \Sigma_l, l < n$, then we trivially have $u(x, B) = u(y, B)$. If $B \in \Sigma_n$, then note that if $x\mathcal{R}_A y$, then $x\mathcal{R}_B y$ (for $A \subseteq B$). Hence, in case (a), $x, y$ are also in the same $\mathcal{R}_B$-equivalence class - implying that $u(x, B) = u(y, B)$. In case (b), where $x, y \in \hat{B}_m$ for some $m > n$, then we have $u(x, B) = K_m = u(y, B)$. Hence, $u(\cdot, \cdot)$ is a monotone coarsening, i.e. it satisfies the inductive consistency condition.

The last thing we need to check is that $u(\cdot, \cdot)$ satisfies downwards monotonicity. Fix $A \subseteq B$ and let $x \in A$. Note that if $A \in \Sigma_n, B \in \Sigma_m, m < n$, then the claim is obvious. Hence, consider the case where $A, B \in \Sigma_n$. There are formally four cases to consider:

1. $x \in C(A), x \notin C(B)$.
2. $x \in C(B), x \notin C(A)$.
3. $x \in C(A), x \in C(B)$.
4. $x \notin C(A), x \notin C(B)$.

In the first case, let $\overline{B_i}$ denote the $\mathcal{R}_B$-equivalence class of $x$ and notice that, as $A \subseteq B$ we have $C(A) \subseteq \overline{B_i}$. Since $u(x, A) \approx \frac{1}{|C(A)|}$ and

$$u(x, B) = \frac{1}{|C(B)| + \sum_{j=2}^{i} |\overline{B_i}|} \leq \frac{1}{1 + |\overline{B_i}|}$$

it follows that $u(x, B) \leq u(x, A)$ in this case. In the second case, note that by A2 we have $C(A) \subseteq C(B)$. Moreover, if $x \in C(B)$, then $\cup_{j \leq i} B_j \subseteq C(B)$ where (again, abusing notation) $\{B_j\}_{j=2}^{i}$ denotes the $\mathcal{R}_A$-equivalence classes of elements of $A \backslash C(A)$ consisting of elements whose singleton rank is higher than $x$. It follows that

$$(*) \ |C(A)| + \sum_{j=2}^{i} |B_j| \leq |C(B)|$$

Hence, since $u(x, B) \approx \frac{1}{|C(B)|}$ we obtain $u(x, B) \leq u(x, A)$.[30] In the third case, we clearly have (by A2) $u(x, A) \geq u(x, B)$. This is obvious from choice of the neighborhood $B_\epsilon(\vec{1})$ if $C(A) \subsetneq C(B)$ and follows from an application of Claim 1 if

[30]There is an additional detail to check here if we have exact equality in the given inequality $(*)$. However, this only affects the $\epsilon$-factor that we place in the numerator of $u(x, A) = \frac{1+\epsilon}{|C(A)| + \sum |B_j|}$, which can be chosen, at the outset (see our choice of $\kappa$), to be a small amount (weakly) greater than $\epsilon_B$, where $u(x, B) = \frac{1+\epsilon_B}{|C(B)|}$. For example, for $x \in B_i \subseteq A \backslash C(A)$ put $\epsilon'_A = \kappa/2 \cdot 1_{\Sigma_A}$, where $\Sigma_A = \{B : A \subseteq B, C(B) = C(A) \cup \cup_{j=2}^{i} B_j\}$. Put $u(x, A) := (1 + \epsilon'_A)/(|C(A)| + \sum_{j=2}^{i} |B_j|)$. The indicator variable equals 0 if and only if $\Sigma_A$ is empty, in which case the equality case of $(*)$ doesn't occur.

we have $C(A) = C(B)$. Finally, in the last case consider two sub-cases, (i) $x \in \hat{B}_n$ and (ii) $x \in \hat{B}_m$ for some $m > n$. In the latter case, we have $u(x,B) = u(x,A)$. Hence, consider the former case. Notice that I have,

$$u(x,A) = \frac{1}{\sum_{j=1}^{i} |B_j|} = \frac{1}{|B_i| + |\{z \in A \backslash B_i : u(z,A) \geq u(x,A)\}|}$$

for any menu $A$ (again, ignoring the $\epsilon$ adjustment factors - see footnote on how we amend the construction to take care of these in equality cases), where $x \in B_i$ and the $\{B_j\}$ denote the $\mathcal{R}_A$-equivalence classes. In other words, we count how many elements in $A$ are in the same or higher ranked (w.r.t. $\mathcal{R}_A$) class and take the reciprocal of this quantity. Observe that for any $x, y \in A$ if we have $u(x,A) \geq u(y,A)$, then by monotone coarsening we have $u(x,B) \geq u(y,B)$ (with equality if we have equality $u(x,A) = u(y,A)$). Hence, if $y \in A$ is of the same or higher $\mathcal{R}_A$-rank than $x$, then it has weakly higher $\mathcal{R}_B$ rank than $x$. It follows that $u(x,B) \leq u(x,A)$ in this case. □

**Remark**: Note that there are two partitions at play in the proof: one induced by $\mathcal{R}_X$ and the other induced by $\mathcal{R}_A$ (where $\mathcal{R}_A, \mathcal{R}$ denote the 'separated-ness' relation). Using both relations is not strictly necessary for the argument, but serves two useful purposes. First, it shows how one can generate a family of LTP representations attached to a given menu preference (the parameters of variation in the family are the constants $K_1, \ldots, K_n$). Second, it shows that (more precisely, the proof shows that) essentially all LTP representations are obtained in this way (for some choice of constants $K_1, \ldots, K_n$).[31] Finally, with a view to the identification result note that we can immediately jump to the following definition:

$$u(x,A) = \begin{cases} \frac{U(A)}{|C(A)|}, & \text{if } x \in C(A) \\ \frac{1}{|C(A)| + \sum_{j=2}^{i} |B_j|}, & \text{if } x \in B_i \end{cases}$$

That is, simply ignore the 'outer' partition given by the relation $\mathcal{R}$. The same proof as given above shows that this map also yields an LTP representation. Moreover, *any* other LTP representation is clearly a coarsening of this map. Hence, we can concretely construct the unique, dominant LTP representation and since we have characterized the family of all LTP representations, this shows the identification result is tight.

## 6.2 Omitted Proofs from Section 3

*Proof of Theorem 3.* Break the proof into two parts as follows: (i) Identification of the Monotonicity Property, and (ii) Identification of the system of local orders.

[31] More precisely, the choice of constant can depend on the menu with the requirement being that the value $u(\cdot, A)$ is constant on $\mathcal{R}_A$-equivalence classes.

**Identification of Upwards/Downwards Monotonicity**

We first verify the equality $\Sigma_{LPF}(t) = \Sigma_*$. If $\succeq \ \Sigma_{LPF}(t)$, let $u(\cdot,\cdot)$ be a (non-constant) representation. For any menu $A$ we have:

$$|C(A)|u(x,A) = U(A) = U(C(A)) = |C(C(A))|{\cdot}u(x,C(A)), \forall x \in C(A)$$

Since $C(C(A)) = C(A)$ we must have $u(x,A) = u(x,C(A)), \forall x \in C(A)$. As $C(A) = \Phi_u(A)$, this implies that $u(\cdot,C(A))$ is constant on $C(A)$. Hence since $u(\cdot,\cdot)$ is non-constant, $C(A)$ must be a singleton, for all menus $A$ - implying that $\Sigma_{LPF}(t) = \Sigma_*$. Next, we check that $\Sigma_{LTP}(f) = \Sigma_*$. Let $\succeq \in \mathbf{P}(X)$ satisfy $A1 - A3$ and note that this implies the singleton ranking is strict. Let $A = \{x_1,\ldots,x_k\}$ be a top-down (w.r.t. $\succeq$) enumeration of the elements of the menu $A$. Proceed in 3 steps.

Step 1: $x_1 \in C(A)$ and $C(A)$ is an order interval in $A$.
This requires only $A2$ and $A3$. Let $x_i \succeq x_j$ and assume $x_j \in C(A)$. Consider the menu $A' := \{x_i, x_j\}$. We claim that $x_i \in C(A')$. Towards contradiction, if $x_i \notin C(A')$, then $\{x_j\} = A' \backslash x_i \succeq A'$. On the other hand, by $A3$, $C(A') \neq \emptyset$ so that $C(A') = \{x_j\}$. Thus, $A' \succ A' \backslash x_j = \{x_i\}$. Put together this yields: $\{x_j\} = A' \backslash x_i \succeq A' \succ A' \backslash x_j = \{x_i\}$ - which is a contradiction. Thus, $x_i \in C(A')$. Now since $x_j \in C(A) \cap A'$, $A2$ then implies $C(A') \subseteq C(A)$ so that $x_i \in C(A)$.

Step 2: $C(A) \succeq A$.
We prove the more general statement, $\{x_1\} \succeq \{x_1,x_2\} \succeq \cdots \succeq \{x_1,x_2,..,x_k\}$. By Step 1 and $A1$, the element of a menu with the lowest singleton ranking must be a (relative) temptation. This shows $x_1 \succeq \{x_1,x_2\} \succeq .. \succeq \{x_1,x_2,..,x_k\}$.

Step 3: Show that $A \sim C(A) = \{\sup(A)\}, \forall A \in \mathcal{M}$.
Assume that $\succeq \in \Sigma_{LTP}(f)$, and let $u(\cdot,\cdot)$ be an upwards monotonic function such that $U(A) = \sum_{x \in C(A)} u(x,A)$ represents $\succeq$. We will show the claim by induction on the cardinality of the menus. Consider any menu $A = \{x_1,x_2\}$, where $x_1 \succ x_2$. Note that $C(A) = \{x_1\}$ by Step 2 and $A1$. Since $U(A) = u(x_1,A)$, by upwards monotonicity of the function we have $A \succeq \{x_1\}$. On the other hand, $\{x_1\} = C(A) \succeq A$, so that $C(A) \sim A$. Inductively assume the claim holds for all menus of cardinality less than or equal to $k$ and let $A$ have cardinality $k+1$. Let $A = \{x_1,..,x_{k+1}\}$, where $x_1 \succ x_2 \succ \cdots \succ x_{k+1}$. If $x_j \in C(A)$ for some $j \neq 1$, then $A \backslash x_j \prec A$. By the induction hypothesis, $A \backslash x_j \sim C(A \backslash x_j) = \{x_1\}$. On the other hand, by Step 2 again, $C(A) \succeq A$. By $A1$, we know that $|C(A)| \leq k$ so that the induction hypothesis implies $C(A) \sim C(C(A)) = \{x_1\}$ . Putting these together we obtain, $\{x_1\} = C(C(A)) \sim C(A) \succeq A \succ A \backslash x_j \sim C(A \backslash x_j) = \{x_1\}$ - which is a contradiction. Thus, $C(A) = \{x_1\}$. Since $u(\cdot,\cdot)$ represents $\succeq$, we obtain $U(A) = u(x_1,A) \geq u(x_1,C(A)) = U(C(A))$. Since $C(A) \succeq A$ this then implies $A \sim C(A) = \{x_1\}$.

**Identification of $\{\succeq_A\}$**: LPF case.
We first verify identification, i.e. existence of a unique, dominant local system $\{\succeq_A^*\}$ for LPF preferences, then present a companion argument for LTP preferences. Let $\{\succeq_A^*\}$ be the system constructed in the proof of Theorem 1. That is, we take $u(x, A)$ to be defined on fixed point menus such that the technical condition is satisfied and extend to all menus via the formula:

$$u(x, A) := \max\{u(x, A') : x \in A' \subseteq A, C(A') = A'\}$$

Now let $\{\succeq_A\}$ be any other system that represents the same menu preference $\succeq$. Let $\mathcal{R}_D$ denote the dominance relation, i.e. $\{\succeq_A^*\} \, \mathcal{R}_D \, \{\succeq_A\}$ iff for all menus $A$, $\succeq_A^*$ is a refinement of $\succeq_A$. I claim that $\{\succeq_A^*\} \, \mathcal{R}_D \, \{\succeq_A\}, \forall \{\succeq_A\}$. To see this, fix a menu $A$ and recall the relations $\mathcal{R}_A$ constructed in the proof of Theorem 2. If $x, y \in A$ and $x\mathcal{R}_A y$, then we must have $x \sim_A y$ by the inductive consistency condition, for *any* local menu preferences $\{\succeq_A\}$ that represents $\succeq$. We now show that $x \sim_A^* y$ iff $x\mathcal{R}_A y$. The direction $x\mathcal{R}_A y \Rightarrow x \sim_A^* y$ follows from inductive consistency. For the converse, let $A_x, A_y \subseteq A$ be menus such that the maximum $u(x, \cdot)$ is attained on the menu $A_x$ (and similarly for $A_y$). Wlog say that $x \succeq y$. Consider the menu $A' := A_x \cup A_y$. Notice that $C(A') \neq \emptyset$ (by A3), so that $C(A') \cap A_x \neq \emptyset$ or $C(A') \cap A_y \neq \emptyset$. In the former case, we have (by A2) $x \in C(A')$, so that $x \succeq_{A'}^* y$. OTOH, taking $u(\cdot, \cdot)$ to be a cardinal representation of $\succeq_A^*$, we know that $u(x, A_x) = u(x, A) \geq u(x, A') \geq u(x, A_x)$ - the latter two inequalities following from upwards monotonicity. Hence, since $u(y, A) = u(x, A)$ we obtain $u(x, A') = u(x, A) = u(y, A_y) = u(y, A) \geq u(y, A') \geq u(y, A_y)$ - again, the latter two inequalities following from upwards monotonicity. It follows that $u(y, A') = u(x, A')$, implying that $y \in C(A')$, so that $x\mathcal{R}_A y$. If $C(A') \cap A_y \neq \emptyset$, then by A2 we have $y \in C(A')$. Since $x \succeq y$, this implies $x \in C(A')$ so that $x \, \mathcal{R}_A y$. It follows that $x \sim_A^* y \Leftrightarrow x\mathcal{R}_A y$. Hence, if $x \sim_A^* y$, then $x \sim_A y$ for any local system $\{\succeq_A\}$ that represents $\succeq$. Now consider the case where $x \succ_A^* y$. We check that $x \succeq_A y$. Note that $x \succ_A^* y$ implies that $x \succ y$ - as local menu preferences coarsen the singleton ranking. It follows that if $\{\succeq_A\}$ is a local menu preference that represents $\succeq$, then we must also have $x \succeq_A y$. This proves that $\succeq_A$ is a coarsening of $\succeq_A^*$.

**Identification of $\{\succeq_A\}$**: LTP case.
Let $\{\succeq_A^*\}$ denote the local menu preferences constructed in the proof of Theorem 2. As in the preceding proof, the key observation is

$$(*) \; x \sim_A^* y \Leftrightarrow x\mathcal{R}_A y$$

where $\mathcal{R}_A$ is the relation constructed in the proof of Theorem 2. This equivalence was non-obvious from the definition of the functions $u(\cdot, \cdot)$ in the LPF setting, but it is obvious from the definition in the LTP setting. Now let $\{\succeq_A\}$ be any other local menu preference that represents $\succeq$. We claim that $\succeq_A$ is a coarsening of $\succeq_A^*$. Note that the aggregation requirement in the LTP utility implies that if $x, y \in C(A)$, then we have $x \sim_A y$ for any LTP system, $\{\succeq_A\}$, that represents $\succeq$. It follows, from inductive consistency, that if $x\mathcal{R}_A y$ then $x \sim_A y$. Since $x \sim_A^* y \Leftrightarrow x\mathcal{R}_A y$ we obtain $x \sim_A^* y \Rightarrow x \sim_A y$. Now we verify that: $x \succ_A^* y \Rightarrow x \succ_A y$. Since $\succeq_A$ coarsens the singleton ranking this implies that $x \succ y$. Moreover, by $(*)$, we have $\neg(x\mathcal{R}_A y)$. Hence, $x \sim \{x, y\} \succ y$ - as in the LPF argument. This implies, by inductive consistency, that $x \succeq_A y$ for any $A$ with $x, y \in A$. Hence, $\succeq_A$ coarsens $\succeq_A^*$. □

### 6.3 Omitted Proofs from Section 4

*Proof of Proposition 2.* (1). First, we check that the maximal fixed points $\{B_i\}$ form a partition of $X$. Note that $\Phi \neq \emptyset$ so that maximal elements exist. We claim that the maximal fixed points, $B_i$, are disjoint. Via contradiction, let $B_i, B_j$ be maximal elements with non-empty intersection. Put $B^* = B_i \cup B_j$ and consider $C(B^*)$. Since $C(B^*) \subseteq B_i \cup B_j$ we must have either $C(B^*) \cap B_i \neq \emptyset$ or $C(B^*) \cap B_j \neq \emptyset$ (by A3). Say the former occurs. Then, $C(B_i) \subseteq C(B^*)$ by A2. On the other hand, since $B_i \cap B_j \neq \emptyset$ and $B_i = C(B_i)$ we obtain $C(B^*) \cap B_j \neq \emptyset$. A2 then yields $C(B_j) \subseteq C(B^*)$. Thus, $C(B^*) = C(B_i) \cup C(B_j) = B_i \cup B_j = B^*$. This contradicts the maximality of both $B_i$ and $B_j$. To show that $\cup_i B_i = X$ note that any singleton menu $\{x\}$ is in $\Phi$. Thus, $\{x\}$ is contained in some maximal $B_i$.

(2). We check the $B_i$'s are order intervals in $X$. Note that the above argument shows that $C(B_i \cup B_j) = B_i$ or $B_j$. Choose a labeling of the maximal elements such that $B_1 \succeq B_2 \succeq \cdots \succeq B_k$. By Claim 2, this implies $C(B_i \cup B_j) = B_i$ whenever $i < j$. Now consider *any* $A \subseteq B_i$ such that $C(A) = A$. Since $C(B_i \cup B_j) = B_i$, by A2 we obtain $C(A \cup B_j) \subseteq C(B_i \cup B_j) = B_i$. Since $(A \cup B_j) \cap B_i = A = C(A)$, it follows that $C(A \cup B_j) \subseteq C(A)$. On the other hand, since $C(A \cup B_j) \cap A \neq \emptyset$, A2 implies $C(A) \subseteq C(A \cup B_j)$. Thus, $A = C(A) = C(A \cup B_j)$. It follows that, for any $x \in A$, we have $A = C(A \cup B_j) \sim A \cup B_j \succ (A \backslash x) \cup B_j \succeq B_j$ (by A1*), so that $A \succ B_j$. We use this fact to check that $B_1$ is an order interval, a similar argument applies to all other $B_i$. Towards contradiction, if $B_1$ is not an order interval, then there is some pair $(x, y)$ such that $x \in B_1, y \in B_j$ (for some $j > 1$) and $y \succeq x$. However, applying the preceding argument with $A = \{x\}$ we obtain $x \succ B_j \succeq y$ (by A1*) - contradiction. □

**Claim 2.** *Assume* $\succeq \in \boldsymbol{P}(X)$ *satisfies* A1*, A2, *and* A3 *and let* $\Phi$ *be the set of fixed points of the map* $C(\cdot)$ *with maximal elements* $\{B_1, \ldots, B_k\}$. *If* $B_i \succeq B_j$, *then* $C(B_i \cup B_j) = B_i$.

*Proof.* Proceed via contradiction. If $C(B_i \cup B_j) = B_j$, then $B_i \cup B_j \setminus y \succeq B_i \cup B_j$, for any $y \in B_i$. By A2, $C(B_i \cup B_j \setminus y) \subseteq C(B_i \cup B_j) = B_j$. Thus, $B_i \cup B_j \setminus \{y, z\} \succeq$

$B_i \cup B_j \setminus y \succeq B_i \cup B_j$, for any $z \in B_i \setminus y$. Inductively, we obtain $B_j \succeq B_i \cup B_j$. On the other hand, if $C(B_j \cup B_i) = B_j$, then for any $x \in B_j$, $B_i \cup B_j \succ B_i \cup B_j \setminus x \succeq B_i$, the latter relation following from A1*. Thus, $B_j \succeq B_i \cup B_j \succ B_i$ which is a contradiction since the labeling was chosen such that $B_i \succeq B_j$. □

*Proof of Proposition 3.* (1) We first show that $\Sigma_{KPF^*} \subseteq \Sigma_{KPF}$. Let $A \subsetneq B$ with $A \sim B$. It suffices to show that for any $x \notin B$ we have $A \cup x \sim B \cup x$. Via contradiction, assume there is an $x$ for which this is false. That is, let $B \cup x \succ A \cup x$ for some $x \notin B$. Let $D^*$ be a minimal subset of $B \cup x$ containing $A$ such that $D^* \succ A \cup x$. Write $D^* = A \cup D' \cup \{x\}$. By A1* and minimality of $D^*$, $D' \subseteq C(D^*)$. On the other hand, since $D' \nsubseteq A$, let $z \in D' \setminus A$ and consider $A' = A \cup z$. Note that $A' \subseteq D^*$. By Sen's $\alpha$, $z \in C(A')$. Thus, $A \sim B \succeq A' \succ A$ - contradiction.

(2) We check the reverse containment $\Sigma_{KPF} \subseteq \Sigma_{KPF^*}$. Via contradiction again, let $B$ be a set at which Sen's $\alpha$ doesn't hold. That is, there is an $x \in C(B)$ and $A \subseteq B$ with $x \in A \setminus C(A)$. Let $A^*$ be a maximal (w.r.t set inclusion) such counterexample (in $B$). Let $y \in B \setminus A^*$. Note that (i) $y$ exists as $A^* \neq B$, and (ii) $y \neq x$ as $x \in A^*$. By maximality of $A^*$, $x \in C(A^* \cup y)$. Thus, $(A^* \cup y) \setminus x \prec A^* \cup y$. By the contrapositive of A2* (Modularity), this implies $A^* \setminus x \not\sim A^*$. Thus, by A1*, $A^* \setminus x \prec A^*$ so that $x \in C(A^*)$ - a contradiction. □

*Proof of Proposition 4.* (1). We prove ($\succeq_1$ MPF $\succeq_2$) $\Rightarrow$ ($\{B_i^1\}$coarsens$\{B_i^2\}$) via contraposition. Assume that $\{B_i^1\}$ is not a coarsening of $\{B_i^2\}$. Then, there is an $i^*$ and a collection $\{j_1, .., j_n\}$ $(n \geq 2)$ such that $B_{i^*}^2 \subseteq \cup_{k=1}^n B_{j_k}^1$ and $B_{i^*}^2 \nsubseteq \cup_{k \in S} B_{j_k}^1$, for any $S \subset \{1, .., n\}$. Assume the labelling of $\{B_i^1\}$ is such that $B_i \succ_1 B_j$ whenever $i < j$. Put $A := B_{i^*}^2 \cap B_{j_1}^1$, $B := B_{i^*}^2$. Then, $A \subsetneq B$ and $A \sim_1 B$, yet $B \succ_2 A$.

(2). Assume that $\{B_i^1\}$ is a coarsening of $\{B_i^2\}$. Let $A \subsetneq B$ and assume $B \succ_2 A$. Put $i_A^2 := \min\{i : A \cap B_i^2 \neq \emptyset\}$, $i_B^2 := \min\{i : B \cap B_i^2 \neq \emptyset\}$ and similarly define $i_A^1, i_B^1$. We consider two cases,

<u>Case 1</u>: $i_A^2 = i_B^2$.
Since $\{B_i^1\}$ is a coarsening, we must have $i_A^1 = i_B^1$. Thus, we obtain $B \sim_1 B \cap B_{i_B^1}^1 \succeq_1 A \cap B_{i_B^1}^1 \sim_1 A$. Since $B \cap B_{i_B^2}^2 \sim_2 B \succ_2 A \sim_2 A \cap B_{i_B^2}^2$ and $\succeq \in \Sigma_{LSPF}$ we must have $A \cap B_{i_B^2}^2 \subsetneq B \cap B_{i_B^2}^2$. This implies that $A \cap B_{i_B^1}^1 \subsetneq B \cap B_{i_B^1}^1$, so that $B \succ_1 A$, since $\succeq_1 \in \Sigma_{LSPF}$.

<u>Case 2</u>: $i_A^2 > i_B^2$.
Note that $i_A^1 \geq i_B^1$. If $i_A^1 > i_B^1$, then we clearly obtain $B \succ_1 A$. Thus, assume $i_A^1 = i_B^1$. Find the maximal $j$ such that $\cup_{i=i_B^2}^j B_i^2 \subseteq B_{i_B^1}^1$. If $j \geq i_A^2$, then since $i_A^2 > i_B^2$ we know that $A^* := A \cap (\cup_{i=i_B^2}^j B_i^2) \subsetneq B \cap (\cup_{i=i_B^2}^j B_i^2) := B^*$. Moreover,

$A^* \sim_1 A, B^* \sim_1 B$ so that $B \succ_1 A$. If $j < i_A^2$, then $A \cap B^1_{i^1_B} = \emptyset$ so that $A \subseteq \cup_{k > i^1_B} B^1_k$. Since $B^* \sim_1 (B^* \cup_{k > i^1_B} B^1_k) \succ_1 \cup_{k > i^1_B} B^1_k \succeq_1 A$ and $B^* \sim_1 B$, we obtain $B \succ_1 A$. $\square$

*Proof of Theorem 4.* ($\succeq_1$ MPF $\succeq_2$) $\Rightarrow$ ($\{\succeq^1_A\}$ coarsens $\{\succeq^2_A\}$):
To show this we check that, menu-by-menu, the subjective relation $\succeq^1_A$ is a coarsening of the relation $\succeq^2_A$. Note that, since both of these are the (unique) most refined local orders representing (resp.) $\succeq_1$ and $\succeq_2$ we know that they are (resp.) defined as follows (let $C^1, C^2$ denote the critical element correspondences for the two menu preferences)

$$u_1(x, A) = \max\{U^1(A')/|A'| : x \in A' \subseteq A, C^1(A') = A'\}$$

and similarly for $u_2(\cdot, \cdot)$. Since we are assuming ($\succeq_1$ MPF $\succeq_2$), we immediately obtain the containment

$$C^1(A) \supseteq C^2(A)$$

Now we use the observation (found in the proof of Proposition 1)

$$x \sim^i_A y \Leftrightarrow x \mathcal{R}^i_A y$$

where the superscript denotes the menu preference of agent $i$. It follows that if $x \sim^2_A y$, then $x, y \in C^2(A')$ for some $A' \subsetneq A$. Since $C^2(A) \subseteq C^1(A)$, this implies $x \mathcal{R}^1_A y$, so that $x \sim^1_A y$. Now consider $x, y$ with $x \succ^2_A y$. We verify that $x \succeq^1_A y$. Note that, since $\succeq^i_A$ coarsens the singleton ranking we must have $x \succ y$ (note that we use the hypothesis that $\succeq^1$ and $\succeq^2$ agree on the singleton ranking here). By inductive consistency, it now follows that $x \succeq^1_A y$.

($\{\succeq^1_A\}$ coarsens $\{\succeq^2_A\}$) $\Rightarrow$ ($\succeq_1$ MPF $\succeq_2$):
Notice that if $\{\succeq^1_A\}$ coarsens $\{\succeq^2_A\}$, then we must have $C^1(A) \supseteq C^2(A)$. First, note that we cannot have $C^1(A) \subsetneq C^2(A)$. Else, find $x \in C^2(A) \backslash C^1(A)$ and $y \in C^1(A)$ and note that, on the one hand this implies $y \succ^1_A x$. On the other hand, $x \sim^2_A y$ which implies, as $\{\succeq^1_A\}$ coarsens $\{\succeq^2_A\}$, that $x \sim^1_A y$ - contradiction. Now, if $C^1(A) \not\supseteq C^2(A)$, then find $x \in C^2(A) \backslash C^1(A), y \in C^1(A) \backslash C^2(A)$ - the latter element exists by the argument just given. Note that we have $x \succ^2_A y$. As $\{\succeq^1_A\}$ coarsens $\{\succeq^2_A\}$ this implies $x \succeq^1_A y$. Since $y \in C^1(A)$ (so that $y \in \arg\max_{z \in A} u^1(\cdot, A)$), this then implies $x \in C^1(A)$ - contradicting the choice of $x$. Hence, we obtain $C^1(A) \supseteq C^2(A)$. Now take $A \subsetneq B$ and assume that $B \succ_2 A$. We wish to show that $B \succ_1 A$. Note that there are two cases to consider. First, $C^2(B) \cap A \neq \emptyset$. In this case, we have $C^2(A) \subsetneq C^2(B)$ and, similarly, $C^1(A) \subseteq C^1(B)$. If $C^1(A) \neq C^1(B)$, then we immediately obtain $B \succ_1 A$. We claim that we cannot have $C^1(A) = C^1(B)$. To see this, note that there is some $x \in C^2(B) \backslash A$ as $B \succ_2 A$. Since $C^2(B) \subseteq C^1(B)$ this implies $x \in C^1(B) = C^1(A) \subseteq A$ - contradiction. This proves the result in the case where $C^2(B) \cap A \neq \emptyset$. The second case is where $C^2(B) \cap A = \emptyset$. In this

case, either $C^1(B) \cap A = \emptyset$ or $C^1(B) \cap A \neq \emptyset$. In the latter case, we must have $C^1(A) \subseteq C^1(B)$. Moreover, since $C^2(B) \subseteq C^1(B)$ and $C^2(B) \cap A = \emptyset$ we have $C^1(A) \subsetneq C^1(B)$. Hence, $B \succ_1 A$. In the former case, we have $C^1(B) \cap A = \emptyset$. Hence, find $x \in C^1(B)$ and note that $B \succ_1 B \backslash x \succeq_1 A$ - the latter by monotonicity (A1*), implying that $B \succ_1 A$. □

*Proof of Lemma 1.* Note that $C(A) \subseteq \arg\max_{x \in A} u(x)$, so that if $C(A) \neq \emptyset$, then $|C(A)| = 1$. Let $x_A = C(A)$ and note that since $x_A \in \arg\max_{x \in A} u(x)$, if $x_A \in A' \subseteq A$ and $C(A') \neq x_A$, then $U(A \backslash x_A) = u(x_{A'}) - \max_{x \in (A \backslash x_A)} v(x) \geq U(A)$. This contradicts $x_A = C(A)$. Thus, $C(A') = C(A)$ so that A2 holds. □

*Proof of Lemma 2.* Assume that $\succeq \in \mathbf{P}(X)$ admits a GP representation with pair $(u, v)$. First, we check necessity of conditions $P_1$ and $P_2$. Necessity of $P_1$ is straightforward. For $P_2$, note that if $x, y$ are such that $u(x) > u(y)$, then we must have $C(\{x, y\}) = \{x\}$ by A3. This then implies $U(\{x\}) = u(x) - v(x) \geq U(\{x, y\}) = u(x) - \max_{z \in \{x,y\}} v(z) > u(y) - v(y)$ which yields condition $P_2$. Now check sufficiency. For any menu $A$ let $x_A = \arg\max_{z \in A} u(z), y_A \in \arg\max_{z \in A} v(z)$. Note that $x_A$ exists by $P_1$. We claim that $C(A) = \{x_A\}$. To show this we need to check that:

$$U(A) = u(x_A) - v(y_A) > \max_{z \in A \backslash x_A} u(z) - \max_{z \in A \backslash x_A} v(z) = U(A \backslash x_A)$$

If $\{x_A\} \neq \arg\max_{z \in A} v(z)$, then we can select some $y_A \neq x_A$ so that $U(A) = u(x_A) - v(y_A) > \max_{z \in A \backslash x_A} u(z) - v(y_A) = U(A \backslash x_A)$ in this case. If $x_A = \arg\max_{z \in A} v(z)$, then let $x'_A = \arg\max_{z \in A \backslash x_A} u(z), y'_A \in \arg\max_{z \in A \backslash x_A} v(z)$ and note that

$$\begin{aligned} U(A) = u(x_A) - v(x_A) &> u(x'_A) - v(x'_A) \text{ (by } P2) \\ &\geq u(x'_A) - v(y'_A) = U(A \backslash x_A) \end{aligned}$$

It follows that $C(A) = \{x_A\}$, so that A3 holds.

□

*Proof of Lemma 3.* That A1 implies DFC is straightforward. We check that A1 is implied by Set-Betweenness. Rank the elements of $A$ top to bottom: $A = \{x_1, .., x_k\}$, where $x_1 \succeq x_2 \succeq ... \succeq x_k$. By Set-Betweenness, for any $l$, $x_{l-1} \succeq \{x_{l-1}, x_l\} \succeq x_l$. This yields the following chain: $x_1 \succeq \{x_1, x_2\} \succeq x_2 \succeq \{x_2, x_3\} \succeq x_3 \succeq \cdots \succeq x_{k-1} \succeq \{x_{k-1}, x_k\} \succeq x_k$. Thus, $\{x_1, x_2\} \succeq \{x_2, x_3\} \succeq \cdots \succeq \{x_{k-1}, x_k\}$. Successive applications of Set-Betweenness ($k - 1$ times) then yield: $\{x_1, x_2\} \succeq \{x_1, x_2, x_3\} \succeq \cdots \succeq \{x_1, x_2, .., x_{k-1}\} \succeq \{x_1, x_2, .., x_{k-1}, x_k\}$, so that $A \backslash x_k \succeq A$. □

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