Effects of Beta Distribution and Persistent Factors on the Two-Pass Cross-Sectional Regression

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Abstract

This paper analyzes the finite-sample performance of the two-pass (TP) estimators of factor risk prices. Our results indicate that even if a model is correctly specified, the finite-sample properties of the TP estimators crucially depend on the structure of the beta matrix and the persistency level of the factors. We find that TP estimation can lead to biased statistical inferences when betas have small cross-sectional variations, and they are highly correlated. These biases can be as high as 60% of the true risk prices, for specific levels of correlation and variability of betas. We find that persistent factors can also distort the finite-sample properties of the TP estimators. Various pre-diagnostic methods are suggested in order to check the reliability of the inferences from the TP estimation. Our study is economically relevant since influential asset pricing studies contain these data characteristics.

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I. Introduction

The two-pass cross-sectional regression method, introduced by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), has been widely used to analyze linear factor pricing models, including the capital asset pricing model (CAPM), arbitrage pricing theory (APT), and their variants.[1] In the two-pass (TP) estimation, the matrix of asset betas is first estimated by a time-series OLS regression of asset returns on a set of common factors. Then, factor risk prices are estimated by cross-sectional OLS or GLS regressions of mean returns on the beta matrix. This methodology is simple and provides several convenient ways to test a given asset pricing model.

In this paper, we analyze the small sample proprieties of the TP estimation method under different characteristics of the beta matrix and under different persistency levels of the factors. Our main results show that the combination of highly correlated betas that have small cross sectional variation can produce very significant biases in the estimated risk prices. These biases can be as high as 60% of the true risk prices, for specific levels of correlation and variability of betas. We believe that our study is economically relevant since influential asset pricing studies use factor models where the estimated beta matrices have this kind of structures. Specifically, we show that Fama and French (1993) and Jagannathan and Wang (1996) factor models have estimated betas with low cross-sectional variation and high correlation.

The TP estimation method assumes that the matrix of asset betas is full column rank. This assumption implies that the columns in the beta matrix are linearly independent. Intuitively, since factor risk prices are estimated by cross-sectional regressions of mean returns on betas, linearity dependent columns in the beta matrix will lead to perfect multicollinearity. This full rank assumption also implies that the factor loadings cannot be equal for every asset. That is, betas cannot be constant over different assets since this will lead to a violation of the full rank condition. The compliance of the full rank assumption in the TP estimation has been ignored in the literature. This is because in empirical work, it is very unlikely to find perfect multicollinearity in the beta matrix and is even more difficult to find equal factor loadings for each asset. However, highly correlated betas with small cross-sectional variation are often found in empirical models.

We show that even with no perfectly multicollinear betas, near-multicollinearity in betas could

[1] See Campbell, Lo, and MacKinlay (1997) for a summary of the major models and research in this area since the original works in the 1970s.

hurt the finite-sample properties of the TP estimators. Similarly, even if the betas are not constant, the finite-sample properties of the TP estimators can be distorted when any beta has only small variations across assets (near-constant betas).

A second contribution of this paper is to measure the effect of persistent factors on the finite sample performance of the TP estimation. In general, TP estimation requires that factors have no unit root components. Even if the factors do not have a unit root component, highly persistent factors may distort the finite-sample properties of the TP estimators. Several asset pricing models use persistent factors, for example, the debt premium factor in Jagannathan and Wang (1996). We find that highly persistent factors produce large size distortions in the *t*-tests for risk prices. In particular, when factors are highly persistent, the *t*-tests for risk prices reject correct hypotheses too often and incorrect hypotheses too infrequently.

Given the importance of the two-pass methodology in testing asset pricing models and the potential problems analyzed in this paper, a relevant question is how we can detect these problems in the data. We suggest several pre-diagnostic methods in order to check the reliability of the inferences from the TP estimation.

Previous work on the TP estimation mostly focuses on the correction of the error in variables problem under different assumptions about the idiosyncratic errors (e.g., Shanken (1992), Kim (1995), Ferson and Harvey (1999), Jagannathan and Wang (1998a), Cochrane (2005) and Ahn, Gadarowski and Perez (2009)). Several other studies investigate the finite-sample properties of TP estimators. For example, Grauser and Janmaat (2005) investigate the finite-sample power property of a TP-based test of CAPM under the three factor model of Fama and French (1993); and Chen and Kan (2004) consider the finite-sample biases in TP point estimators and their asymptotic standard errors. To our knowledge, we are the first to investigate the finite-sample performance of the TP estimator under different specifications of the beta matrix and different persistency levels in the factors. In a related study, Kan and Zhang (1999) investigate the finite-sample properties of the TP estimators when the factors are uncorrelated with returns. They refer to these factors as "useless" factors. In their case, the betas of a factor are constant and equal to zero. A factor with a near-constant beta is not necessarily a useless factor, unless the beta is zero. In fact, the case with useless factors is a special case of near-constant betas.

This paper is organized as follows. In section 2, we discuss the basic asset pricing model, assumptions, departures from these assumptions, and the proposed pre-diagnostic measures. In section 3 discuss the simulation strategy and design. Section 4 is devoted to simulation results. Concluding remarks follow in section 5.

II. Analytical Analysis of the TP method

A. Basic Model and Assumptions

The basic model we consider is a multifactor model in which return data are generated by k common factors:

$$r_{\bullet t} = \alpha + \mathrm{B} f_t + \varepsilon_{\bullet t} \equiv \Lambda z_t + \varepsilon_{\bullet t}, \tag{1}$$

where $r_{\bullet t} = (r_{1t}, r_{2t}, ..., r_{Nt})'$, $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)'$, $\mathrm{B} = (\beta_1, ..., \beta_N)'$, $\Lambda = (\alpha, \mathrm{B})$, $z_t = (1, f_t')'$, $\varepsilon_{\bullet t}$ = $(\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$, r_{it} is the gross return of asset i (= 1,2,..., N) at time t (= 1,...,T), $f_t = (f_{1t}, ... , f_{kt})'$ is the vector of k factors at time t, α_i is the asset-specific intercept term, β_i is the vector of k betas of asset i corresponding to f_t, and ε_{it} is the idiosyncratic error for asset i at time t with zero mean.[2] The factors in f_t are strictly exogenous to the error terms; that is, $E(z_{\bullet t} \otimes \varepsilon_{\bullet s}) = 0_{(k+1)N\times 1}$ for all s and t. The number of time-series observations (T) is large, while the number of assets analyzed (N) is relatively small, and then asymptotics apply as T approaches infinity. The error vector $\varepsilon_{\bullet t}$ could be heteroskedastic or autocorrelated over time.

The usual restriction imposed on (1) by linear asset pricing models is given by:

$$H_o : E(r_{\bullet t}) = 1_N \gamma_o + \mathrm{B}\gamma_f \equiv X\gamma, \tag{2}$$

where $X = [1_N, \mathrm{B}]$, $\gamma = (\gamma_0, \gamma_f')'$, 1_N is the $N \times 1$ vectors of ones, γ_o is an unknown constant (*e.g.*, zero-beta return), and γ_f is the $k \times 1$ vector of factor risk prices. The traditional two-pass (TP) approach estimates the vector γ by regressing $\bar{r} = T^{-1}\Sigma_{t=1}^{T} r_{\bullet t}$ and $\hat{X} = [1_N, \hat{\mathrm{B}}]$ with an arbitrary positive-definite (and asymptotically nonstochastic) weighting matrix A, where $\hat{\mathrm{B}}$ is the OLS estimator of B:

$$\hat{\gamma}_{TP} = (\hat{\gamma}_{0,TP}, \hat{\gamma}_{1,TP}')' = (\hat{X}'A\hat{X})^{-1}\hat{X}'A\bar{r} . \tag{3}$$

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[2]If a risk-free asset yielding return r_{ft} is available, r_{it} may denote excess return ($r_{it} - r_{ft}$).

There are many possible choices for A. If we choose $A = I_N$, then the two-pass estimator $\hat{\gamma}_{\mathrm{TP}}$ becomes an OLS estimator. In contrast, with the choice of $A = [Var(\varepsilon_{\bullet t})]^{-1}$, the inversed unconditional variance matrix of the $\varepsilon_{\bullet t}$, the two-pass estimator $\hat{\gamma}_{\mathrm{TP}}$ becomes the GLS estimator considered by Shanken (1992) and Kandel and Stambaugh (1995).

A problem of the TP estimator (3) is that it uses the estimated beta matrix, $\hat{\mathrm{B}}$, because the true beta matrix, B, is not observed. This generates the well-known EIV problem. Shanken (1992) shows that despite this problem, the TP estimator is consistent and asymptotically normal. Further, under the assumption that $\varepsilon_{\bullet t}$ are independently and identically distributed (*i.i.d.*) over time, he provides the correct asymptotic variance matrix of the TP estimator explicitly incorporating the sampling errors generated by the use of the estimated beta. A more general variance matrix can be found in Jagannathan and Wang (1998a) and Ahn, Gadarowski and Perez (2009).

The most general set of assumptions used for the TP estimator are the following:

i) $r_{\bullet t}$ and f_t are covariance-stationary, ergodic, and have finite moments up to the fourth order. $E(\varepsilon_{\bullet t}\varepsilon_{\bullet s}' \mid f_1,\dots,f_T) = 0_{N\times N}$, for all $t \neq s$. That is, the errors $\varepsilon_{\bullet t}$ are serially uncorrelated given factors. $Var(\varepsilon_{\bullet t} \mid f_1,\dots,f_T) = \Sigma_\varepsilon$, for any t, where Σ_ε is the unconditional variance matrix of $\varepsilon_{\bullet t}$.

ii) Factors are strictly exogenous to the errors. $E(z_{\bullet t} \otimes \varepsilon_{\bullet s}) = 0_{(1+k)N\times 1}$ for all t and s.

iii) All columns in B are linearly independent and every element in β_i varies over different i. $X = [1_N, \mathrm{B}]$ is of full column

iv) $E(\varepsilon_{\bullet t}\varepsilon_{\bullet s}' \mid f_1,\dots,f_T) = 0_{N\times N}$, for all $t \neq s$. That is, the errors $\varepsilon_{\bullet t}$ are serially uncorrelated given factors.

v) $Var(\varepsilon_{\bullet t} \mid f_1,\dots,f_T) = \Sigma_\varepsilon$, for any t, where Σ_ε is the unconditional variance matrix of $\varepsilon_{\bullet t}$.

This set of assumptions is general enough to subsume most of the assumptions adopted in the literature. Under Assumptions (i) and (ii), $\mathrm{B} = E[(r_{\bullet t} - E(r_{\bullet t}))(f_t - E(f_t))'][Var(f_t)]^{-1}$. Thus, the matrix B has the usual beta interpretation. Assumption (ii) guarantees the consistency of OLS estimation of Λ. Assumption (i) rules out nonstationary factors and nonstationary idiosyncratic errors. But it allows the factors f_t and returns $r_{\bullet t}$ to be conditionally heteroskedastic and/or serially correlated over time. These assumptions also allow for cross-section dependence among the idiosyncratic errors in $\varepsilon_{\bullet t}$.

Assumption (iii) is necessary for the identification of factor prices by cross-section regression. The assumption rules out perfect multicollinearity in the beta matrix B. For the assumption to hold, $Var(f_t)$ should be nonsingular: that is, f_t contains no redundant factors, and all of the columns in $E[(r_{\bullet t} - E(r_{\bullet t}))(f_t - E(f_t))']$ are linearly independent.

Assumption iv) is stronger rules out autocorrelation in the errors, however, it is still a general and plausible assumption because it allows for heteroskedasticity and the autocorrelations in observed return data are generally weak. Assumption v) rules out heteroskedasticity as well as autocorrelation in the errors. Although this assumption is often empirically disputed, it is assumed in many studies. For example, under this assumption, Shanken (1985, 1992) derived the asymptotic distributions of the TP estimators.

B. Departures from the Basic Assumptions and Diagnostic Measures

In this paper we focus in three departures from the basic assumptions of the model: near multicollinearity of betas, near-constant betas and persistent factors. In this subsection we explain in detail each one of these departures and the possible effects on the TP estimation. We also propose pre-diagnostic measures in order to check the reliability of the inferences from the TP estimation.

Near Multicollinear Betas (NMB): Under assumption (iii) perfect multicollinearity of betas is ruled out. We argue that even with no perfect multicollinearity in B, near multicollinearity could hurt the finite-sample properties of the TP estimators. Depending on how portfolios are constructed, and the type of factors used, multi-factor betas can be highly correlated. As we document in Section IV, Fama and French (1993) and Jagannathan and Wang (1996) factor models have highly correlated estimated betas.

In order to measure the degree of multicollinearity in betas, we propose to use the *multicollinearity coefficient* (MC)[3]. MC is defined as the square root of the ratio of the maximum and minimum eigenvalues from the correlation matrix of betas. In regular OLS estimation coefficient of 15 is often used as the benchmark point of multicollinearity. Any value of MC greater than 30 is viewed as evidence for serious multicollinearity in data (Greene, 2003). To our knowledge there is no benchmark value of MC for the TP estimation. We will present a benchmark value based on our Monte Carlo simulations.

Near Constant Betas(NCB): Under Assumption (iii) it is required that all betas vary over different assets or portfolios. This is because, when the cross-section betas of a factor are constant, the matrix X is not a full column matrix. Even if the betas are not constant, the finite-sample properties of the TP estimators can be distorted when a beta has only small variations. We refer to such a beta as "near-constant" beta. Kan and Zhang (1999) have investigated the finite-sample properties of the TP estimators when the factors are uncorrelated with returns. They refer to these factors as "useless" factors. In this case the betas of a factor equal zero exactly. A factor with near-constant beta is not necessarily a useless factor, unless the beta is zero. In fact, the case with useless factors is a special case of near-constant betas.

In order to measure the degree of variation of each beta, we propose to use the *VB* coefficient:

$$VB_j = T\bar{\beta}_j^2 / \Sigma_{i=1}^N \beta_{ij}^2 = \bar{\beta}_j^2 / [\Sigma_{i=1}^N (\beta_{ij} - \bar{\beta}_j)/T + \bar{\beta}_j^2],$$

where β_{ij} indicates the beta of the j^{th} factor for portfolio *i* and $\bar{\beta}_j = N^{-1}\Sigma_{i=1}^N \beta_{ij}$ is the sample mean of the β_{ij}. This measure is always between zero and one. *VB* coefficient is equivalent to the uncentered R-square from a regression of 1_N on $(\beta_{1j}, \beta_{2j}, ..., \beta_{Nj})'$. When the β_{ij} are constant over *i*, $VB_j = 1$. As the cross-section variation in β_{ij} increases, VB_j decreases toward zero. The sample standard deviation of the β_{ij} could also be used as a variation measure. However, in simulation exercises, we found that *VB* is a more important determinant of the finite-sample performances of the TP estimators.

Persistent Factors(PF). Assumption (i) requires that factors have no unit root components.

[3] This coefficient is also known as *conditional number*.

Even if the factors have not a unit root component, highly persistent factors may distort the finite-sample properties of the TP estimators. The variance matrix of the risk price estimates depend on the variance matrix of factor means which should be estimated nonparametrically when factors are autocorrelated with unknown structure. It is a well known fact that the nonparametric methods tend to underestimate the variance matrix of variables severely when the variables are persistent. Thus, we can expect that the *t*-tests for the risk price with autocorrelated factors may suffer from finite-sample size distortions. In this case general autocorrelation measures and test can be used to check the degree of persistency in the factors. For this paper we just use the first order autocorrelation coefficient.

III. Simulation Design

A. Simulation Strategy

The objective of our Monte Carlo experiments is to analyze the effect of the departures from the basic assumptions described in the previous section (NMB, NCB and PF) on the TP estimation. In order accomplish this objective we use the following strategy:

1. First we apply our pre-diagnostic measures to actual data (preliminary results from actual data). We analyze the Fama and French (1993) and Jagannathan and Wang (1996) models (hereafter, FF and JW-96, respectively). This first step will give the reader a sense of the magnitude of our pre-diagnostic measures, and the level of departure from the basic assumptions of these two factors models.

2. We analyze the finite sample biases of the TP estimation when data is generated replicating the actual data from FF and JW-96 models. Results from this second step show the combined effect of NMB, NCB and PF on the TP estimation.

3. In order to analyze effects of NMB, NCB and PF separately, our third step consist in generating a clean base model. In this base model generated data does not suffer from any of the three problems analyzed in this paper. That is, betas are uncorrelated and have a high cross sectional variation and factors are *i.i.d.* over time. We analyze the finite sample biases of the TP estimation in this case and compare them with the results of step two.

4. Our last step consists in introducing one by one NMB, NCB and PF to our base model. This procedure allows us to identify the individual effect of each one of the three departures from the basic assumptions on the TP estimation.

B. Simulation Design

The foundations of our experiments are the two factor models by Fama and French (1993) and Jagannathan and Wang (1996) (FF and JW-96). The FF model is based on the (excess) market return (VW), SBM, and HML factors, while the JW-96 uses the debt premium (PRE) and labor return (LAB) factors, as well as the market return.

For our simulation exercises, we generate data mimicking the actual returns and factors as much as we can. Specifically, for a given three factor model (FF or JW-96), we estimate betas and risk prices by OLS using actual data. We compute the means, standard errors and correlations of the estimated betas, and use them to generate the betas used for simulations. Using different standard errors and correlations among betas, we can investigate the effects of correlated betas and near-constant betas. We use the estimated betas and risk prices from actual data to calculate the expected asset returns. Simulated return data are obtained by adding generated errors (ε_{it}) to these expected returns. Our simulation results are based on 1,000 trials.[4]

Since none of departures from the basic assumptions involves changes in the distribution of the idiosyncratic errors, our simulated data will just contain idiosyncratic errors that are *i.i.d.* over time. We take this approach in order to isolate the effects from biases caused by autocorrelated and/or heteroskedastic errors[5].

We use actual returns, not excess returns, in our simulation exercises. In unreported simulations, we also examined excess returns, but the results are not materially different from those reported below. The OLS estimates of factor prices from actual data are used as the true factor prices in simulations. The FF factors are excess returns. So, when excess, not raw, returns are used to estimate the FF model, the sample means of the FF factors are the efficient estimators of the risk prices (see Shanken, 1992). However, when factors are not returns as in the JW-96

[4]We also tried 5,000 trials for selected simulations and found qualitatively similar results.

[5] Ahn, Gadarowski and Perez (2009) included a complete analysis of the effects of autocorrelated and/or heteroskedastic errors on the asymptotic and finite sample properties of the TP estimation.

model, more reliable estimates of factor prices could be obtained by the GLS methods. That is, the efficient factor-price estimators are different for the two models. For fair comparison, we simply use the OLS estimates of factor prices from actual data.

We simulate both factors and return data for each trial. In order to facilitate our control of the degree of autocorrelation in each factor, we generate factor j (j = 1, 2, 3) using a simple AR(1) process:

$$F_{jt} = \xi_j + \rho_j F_{j,t-1} + w_{jt},$$

where the w_{jt} are drawn randomly from $N(0,(1-\rho_j^2)\,\mathrm{var}(F_j))$. We use the sample variance of actual factors for $\mathrm{var}(F_j)$, and for the parameters ξ_j and ρ_j, the estimates from simple AR(1) regressions of actual factors. More realistic factors could be simulated using the estimates from a vector autoregression (VAR) regression. However, using the VAR estimates, we were unable to successfully change the degree of autocorrelation of a factor while keeping the moments of simulated data close to those of actual data. In addition, some of our tentative simulation results indicate that the simulations based on VAR and simple AR(1) estimates produce qualitatively similar results.

Betas are generated by:

$$\mathrm{B} = 1_N m_\mathrm{B}' + V(s_\mathrm{B} C_\mathrm{B} s_\mathrm{B}')^{1/2},$$

where $m_\mathrm{B} = E[(\beta_{i1},\beta_{i2},\beta_{i3})']$, V is a $N \times 3$ orthogonal matrix such that $1_N' V = 0_{1\times 3}$, $V'V / N = I_3$, s_B is the 3×1 vector of standard errors of factor betas, and C_B is the 3×3 correlation matrix of betas. We use for m_B the means of estimated betas from actual data. We use this specification to investigate the effects of multicollinearity in betas and near-constant betas, respectively, by using different values of C_B and s_B.

We estimate the variance matrix of the residuals, Π, from the first-pass time-series regressions of actual data and compute a lower triangular matrix Γ such that $\Gamma\Gamma' = \Pi$. Then, we generate error vectors $\varepsilon_t = \Gamma v_t$, where the v_t are random vectors drawn from $N(0_{N\times 1}, I_N)$. This procedure generates cross-sectionally correlated errors. For the simulated data we replace all off-diagonal entries of Π with zeros.

The actual returns and factors we use are the data on raw returns for the FF portfolios, which JW-96 have created and used. JW-96 replicate the FF method of constructing 100 size/pre-beta decile portfolios for NYSE/AMEX firms from July 1963 to December 1990 (330 time-series observations).[6] To check that our data set matches that of JW-96, we replicate their OLS and Fama-MacBeth (FM) analysis with univariate betas for the models common to our analysis. We are able to replicate JW-96's univariate-beta FM estimation of point estimates and standard errors for their three factor model to within three significant digits for most variables. However, because their data set does not contain the FF factors, we use the data available from French's website. For the FF model, our estimates and *t*-statistics do not deviate more than 8% (in relative terms) from those reported by JW-96, but the OLS R^2 are identical to three significant digits. We suspect that these deviations are due to slightly different values for FF's factors in our respective data sets. Our results using FF's factors, however, appear close enough to theirs as to render any differences in inference immaterial. These results are available upon request.

For our simulations, we also use a set of 25 value-weighted portfolios constructed from JW-96's 100 portfolios. The portfolios are constructed as follows. First, we identify groups of 4 original portfolios to form 25 portfolios that are similar to the 5-by-5 size/pre-beta quintiles used by FF.[7] Second, while the 100 portfolios constructed by JW-96 are reported to be based on equally-weighted returns, it is common practice to evaluate 25 portfolios using value-weighted returns to avoid creating portfolios that are not representative of what an actual investor can realistically construct (see FF). To achieve value-weighting, we use the average firm size values reported for each 100 portfolios.

We consider two OLS estimators and one GLS estimator . First the Fama-MacBeth (FM) estimation, where the standard errors of OLS risk price estimates are computed ignoring autocorrelations in both factors and idiosyncratic errors. Since our data generation involves factors that are autocorrelated, our version of Fama-MacBeth estimator (OLS-FM) controls for the possible autocorrelations in factors, but not for autocorrelations in the idiosyncratic errors. Our

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[6]We obtained this data set through the FTP server at the University of Minnesota. We gratefully thank Jagannathan and Wang for access to their data.

[7]Strictly, we do so using neighboring size and pre-beta decile portfolios, while FF first sort firms by size. Because the average pre-betas in neighboring size deciles in JW-96's original 100 portfolios are similar, it is not likely that this difference in pre-beta sub-sorting results in materially different portfolios.

second OLS estimator (OLS-SH) computes the standard errors of OLS risk price estimates based on Shanken (1985, 1992). In addition to the OLS estimators we use the GLS type considered by Shanken (1992) and Kandel and Stambaugh (1995) as explained in the previous section, we refer to this estimator as GLS-SH[8].

IV. Simulation Results

A. Preliminary Results from Actual Data

Before introducing our simulation analysis, we present results of our pre-diagnostic measures applied to the FF and JW-96 models. We do so in order to have a sense of the magnitude of these measures in actual data. Table 1 reports the results from the TP estimation of the FF and JW-96 models using actual 25 (Panel A) and 100 (Panel B) portfolio returns. For each of the FF and JW-96 models, the table reports the cross-sectional averages and standard errors of estimated betas and the average R-square from the first round time-series regressions of returns on factors. We also report the values of our pre-diagnostic measures: for near constant betas (*VB*) and for near multicollinear betas (*MC*).

From Table 1, we can observe that in both the time-series regressions of the FF and JW-96 models, the *VB* (our measure of variation in a beta) values of the estimated VW betas are close to one. This result indicates that the variations in the VW betas are very small for both models. Currently, we do not have a benchmark value of *VB* that we can use to distinguish “small” from “large” variations. As discussed below, we find such a value through Monte Carlo experiments. The MC values from the analysis of 25 (100) assets are 5.95 (3.61) and 4.88 (2.98) for the FF and JW-96 factor betas, respectively. Thus, according to the conventional criteria used for MC, we do not find strong evidence for multicollinearity in the betas from the two models. Of course, however, there is no guarantee that the usual criteria (multicollinearity if MC $\geq$ 15) would apply to the TP regression.

We also are interested in the degree of persistence of the factors used in the FF and JW-96 models. Table 2 reports the VAR and unit-root test results for the FF and JW-96 factors. In Panel A, the estimated AR(1) coefficient of the PRE factor is 0.96. This result suggests that the PRE

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[8] Under our basic set of assumptions the idiosyncratic errors of the model are i.i.d. over time. This implies that GLS-SH is the optimal TP estimator in the sense of Ahn, Gadarowski and Perez (2009).

factor might be nonstationary. To explore this possibility, Panel B reports the Dickey-Fuller (1979) and Phillips-Perron (1988) test results for unit roots. The tests decisively reject unit roots in all of the three FF factors and the LAB factor, but do not with the PRE factor using either of the tests at the 5% significance level. Admittedly, unit root tests are known to have low power to reject unit roots. Additionally, it is intuitive that the PRE factor, the difference between the interest rates on BAA and AAA corporate bond rates, is likely to be stationary over a long time because the two rates are likely to be cointegrated. However, because our sample is only finite, the inference of the TP estimation could be materially perturbed by the presence of a near-unit root factor in a finite sample.

B. Finite-Sample Distortions in Two-Pass Estimators replicating actual data

In this subsection, we want to examine the finite-sample biases in the TP estimation when we replicate the data from FF and JW-96 models. We analyze the finite sample biases of the OLS and GLS two pass estimators of risk prices and the size and power properties of the t tests.

We simulate data using the beta and risk price estimates from the OLS estimation of the FF and JW-96 models. The errors are *i.i.d.* with variances equal to the estimates from the estimation of the two models with actual data. We do so because we wish to investigate the biases in the TP estimators caused by the problems other than heteroskedasticity or autocorrelation in errors.

Table 3 reports the results from 1,000 simulations. The reported biases are the relative biases computed by $(\hat{\gamma}_j - \gamma_j)/\gamma_j \times 100$, where the $\hat{\gamma}_j$ and γ_j are the estimated and true factor prices, respectively. Panel A reports the results from the cases of 25 portfolios. The OLS and GLS-SH estimates of factor prices are biased when 330 time-series observations of 25 portfolio returns are analyzed. For example, for the FF model, the relative biases in the OLS and GLS-SH estimators of the risk price of VW are 55.9 and 53.6%, respectively. For the JW-96 model, the biases are of similar magnitude.

We now consider the finite-sample size properties of the t-tests from the three TP estimators, OLS-FM, OLS-SH and GLS-SH. As expected, the t-tests from OLS-FM tend to reject correct hypotheses more often than those from OLS-SH. This is expected because the FM method underestimates the standard errors of the OLS estimators. For the data simulating the FF model, the t-tests from OLS-SH are reasonably sized despite the considerable relative biases in the TP estimates. The *t*-tests for SMB and HML somewhat over-reject correct hypotheses, but they are much better sized than the *t*-tests from the simulations of the JW-96 model. The sizes of the *t*-tests are severely distorted whatever estimation method is used for the data simulating the JW-96 model. Finally, the *t*-tests from GLS-SH also reveal size distortions. They are generally more biased than the *t*-tests using OLS-SH. Similarly to OLS-SH, GLS-SH generates more size distortions in the *t*-tests for the JW-96 model than for the FF model.

Panel A of Table 3 also examines the power properties of the *t*-tests. The incorrect hypothesis of zero price is tested with size-adjusted critical values at 5% significance level. These critical values are computed from the empirical distributions of the *t*-test statistics[9]. In general, the *t*-tests have low power for the FF model, except for the intercepts, whenever OLS-FM, OLS-SH or GLS-SH is used.

Panel B of Table 3 reports the simulations results for the cases with 100 portfolios. The results are qualitatively similar to those reported in Panel A although the biases in the risk price estimators are generally much smaller, except for PRE and LAB factors. The smaller biases in the TP estimators reported in Panel B do not necessarily imply that the TP estimators are more accurate for the analysis of a larger number of portfolios. This is so because the parameter values used for Panels A and B are different. For example, the values of *MC* coefficients for FF with 25 and 100 portfolios are 5.94 and 3.69 respectively.

The main results reported in Table 3 can be summarized as follows. First, the estimated risk prices from the simulations of the JW-96 and FF model are biased, whatever estimator is used. Second, the *t* tests have better finite-sample size properties for the FF model than for the JW-96 model. These results indicate that the biases in the TP estimators may be large depending on what factors are used. In the next subsection, we will attempt to isolate the determinants of biases.

C. Simulation of a Base Model

In this section, we examine the finite-sample properties of the TP estimators for a base model.

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[9] As reported in Table 1, the risk price estimates differ for each model and factors and these estimated risk prices are used to generate the data. Thus we can expect that the t-tests will have better power to reject the hypotheses of zero risk prices when the prices used to generate the data were larger than zero in absolute value.

By "a base model," we refer to a three-factor model in which (i) betas have zero multicollinearity, (ii) each beta has substantial variations (no near-constant beta) and (iii) factors are *i.i.d.* over time[10]. To simulate this base model, data are generated as follows. First, betas and factors are adjusted so that the average R-squares from the time-series regression of returns on factors equal 78% (which equals the average R-square from the estimation of the JW-96 model with actual 100 portfolio data). Second, the betas are orthogonalized so that the multicollinearity coefficient (MC) is one. The betas are generated only once and are used for 1,000 simulations. Third, the same mean and standard deviation values are used for all betas[11]. The risk prices used for data simulations are the estimates from the estimation of the FF model with actual data (see Table 1). For this reason, we continue to use the names VW, SMB and HML, to denote the generated three factors, although their generating processes are different from the actual FF factors. Fourth, for the cases with 25 portfolios, the mean and standard deviation values of each beta are fixed at 0.317 and 1, respectively. These numbers are chosen to make the average R-squares from time-series regression equal to 0.78 (which equals the average R-square from the estimation of the Jagannathan-Wang model with actual 100-portfolio data). Our generated data has an equal *VB* coefficient for all betas (0.094). For the cases with 100 portfolios, the mean and standard deviation values are set at 0.350 and 1.08, respectively. With this choice, the VB coefficient of each beta equals 0.097. Fifth, the idiosyncratic errors and factors are *i.i.d.* However, as discussed earlier, we still use the Newey-West estimator of the variance matrix of mean factors ($\bar{f}$) for OLS-FM, OLS-SH and GLS-SH. Because the idiosyncratic errors are *i.i.d.*, we only consider these three estimators.

Table 4 reports the results from the simulations of the base model. We consider both the cases of 25 and 100 portfolios. We note that the parameter values used to generate data are different for the two cases. Thus, the motivation of Table 4 is not to investigate the effects of the number of portfolios on the TP estimators, but to provide the benchmark finite-sample properties of the two pass estimators.

The major findings from Table 4 can be summarized as follows. First, for the base model, the OLS and GLS-SH estimators have much smaller biases compared to those obtained from the

[10] Also the idiosyncratic errors are i.i.d. over time.
[11] By this procedure we also ensure that our base model does not include a "useless factor" in the sense of Kan and Zhang (1999).

actual data structure of the FF and JW-96 models (Table 3). Second, GLS-SH generates slightly larger size distortions in *t*-tests than OLS-SH does, particularly for the intercept (Cst.). Third, not surprisingly, the (size-adjusted) *t*-tests using the three TP estimators have low power to reject the incorrect hypotheses of zero factor prices, when the true risk prices are small. For example observe that the *t*-test for HML has greater power for the cases of 25 portfolios and also the higher risk price (see Table 1).

In the following subsection, we consider how the empirical distributions of the two-pass estimators would change when each of the three restrictions (NMB, NCB and PF) imposed on the base model is relaxed.

D. Individual effects of NMB, NCB and PF

First we investigate the effect of different levels of near-multicollinearity of Betas on the TP estimation. The data generating process used for this analysis is the same as the base model, except that the betas now can have MC values different from one. To obtain the target MC values, we have adjusted the correlations between the betas of VW and HML. To save space, we only report the results for the estimated risk prices of VW and HML.

Reported in Table 5 are the results from the simulations of 25 portfolios. We can observe that the biases in OLS and GLS-SH estimators increase with MC. Also, for OLS-FM, OLS-SH and GLS-SH, the sizes of the *t*-tests are not sensitive to MC, but their power decreases with MC. Overall, MC does not have critical effects on the biases in the TP estimators and the sizes of the t_{tests} unless MC $\geq$ 10. Notice that the MC values of actual data reported in Table 1 are 5.945 and 4.885 for the FF and JW-96 respectively. This implies that MC alone cannot be the cause of the bias and size distortions reported in Table 3.

Second we focus on the effects of a near-constant beta on the distribution of the two TP estimators. For this case, data are generated with three different standard deviations of VW. The table reports the *VB* value of the VW beta corresponding to each of the three different standard deviations (*VB*= 0.13, 0.54 and 0.97). Other than these adjustments, the data generating processes used are the same as those used for the base model.

From Table 6, we observe the following. First, the biases in the TP estimators increase with *VB*. For the cases with *VB* smaller than 0.54, the biases in the OLS and GLS-SH estimators are not noticeable. However, at the level of 0.9790, the relative biases in the two estimators are 14.5% and 9%, respectively. Notice that the *VB* values of the VW betas from the estimation of the FF and JW-96 models with actual 25 portfolio data (reported in Table 1) are 0.9776 and 0.9531, respectively. The results reported in Table 6 suggest that the little variations in the VW beta may have caused biases in the TP estimators applied to actual data. Second, the sizes of the *t*-tests are not sensitive to *VB*. However, the power of the tests is inversely related to *VB*.

Third, we want to investigate, what is the combined effect on the distribution of the TP estimator of data generated with near-multicollinear and near-constant Betas. We generate data using betas with a MC=5, and with three different standard deviations of VW. The VB values of the VW betas corresponding to each of the three different standard deviations are 0.13, 0.54 and 0.97. Results are reported in Table 7. Consistently with our conclusions about near-constant betas reported in Table 6, the bias of VW increases with higher values of *VB*. However the magnitude of the bias is significantly larger than the case with uncorrelated betas (Table 7). In the case of *VB*=0.97, the bias for VW risk prices is as high as -74.1% for OLS and -67.8% for GLS estimators. These results suggest that the biases reported on Table 3 can be caused by betas that are near-multicollinear but also near-constant. Similarly than in table 6, the sizes of the *t*-tests are not sensitive to changes in *VB*, however, the power distortions are amplified by the correlation in betas.

Finally, Table 8 shows the effects of persistent factors on the TP estimators. The data generating processes are the same as the processes used for Table 4 (base model), except that the HML factor is generated by AR(1) processes using different autocorrelation coefficients (ρ_{HML}). We only report the results on the estimated risk prices of HML. The biases in the two pass estimators are not overly sensitive to the size of ρ_{HML}. However, the size distortions of the *t*-tests become more severe as ρ_{HML} increases. Overall, for the cases with $\rho \geq .5$, the *t*-tests reject the correct hypotheses more than 9% at the normal 5% significance level. For the cases with $\rho \geq 0.9$, all t-tests reject correct hypotheses more than 23.6% of 1,000 trials. Notice that actual PRE and LAB factors used for the JW-96 model have correlation coefficients of 0.961 and 0.565 respectively (Table 2). This suggests that the size distortions reported in Table 3 can be caused by highly persistent factors. This result is consistent with the well-known fact that the nonparametric estimates of variance matrices are downward biased when the analyzed variables are highly persistent. In addition, the power of the *t*-tests is negatively related to the degree of autocorrelations in HML.

Tables 9 – 12 report the results obtained by simulating the models with100 portfolios. We do not make detailed comments because the main results are qualitatively similar to those reported in Tables 5 –8. We can conclude that the effect of NMB, NCB and PF does not depend of the use of 25 or 100 of portfolios.

V. Conclusion

In this paper, we examined three major determinants of the reliability of the TP estimation: multicollinearity among betas, cross-section variations in each beta and persistency of individual factors. The main results from our simulations are as follows. First, severe multicollinearity ($\mathrm{MC} \geq 10$) could bias the two-pass estimators of risk prices and the resulting *t*-tests. Second, small variations in betas ($\mathrm{VB} \geq 0.95$) could also cause biased statistical inference. Third, the combinations of near-multicollinear and near constant betas produce very significant biases in the estimated risk prices. We documented that for correlated betas with $\mathrm{MC} \geq 5$, the effect of small variations in betas of $\mathrm{VB} \geq 0.97$ can bias the estimated risk prices in more that 60% of the true value. This is economically relevant since in our experiments, we find that the betas of VW have little variations from both the estimation of FF and JW-96 models. Our simulation results indicate that any model using VW as a factor may have to be analyzed with caution. Fourth, highly persistent factors can produce large size distortions in the *t*-tests for risk prices. The size distortions in the *t*-tests are not noticeable when the AR(1) coefficient of a factor is smaller than 0.5. However, the distortions may be severe when the AR(1) coefficient is greater than 0.90. Fifth, the combinations of near-multicollinear and near constant betas can also cause very low power in the *t*-tests, specially for $\mathrm{MC} \geq 5$.

We propose and recommend the use pre-diagnostic measures to check the reliability of the two-pass estimation results. The degree of multicollinearity and cross-section variations in individual betas estimated from these regressions need to be checked. Researchers are also advised to estimate persistency of each factor.

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Table 1
Diagnostic Results

The results by the two pass estimation of the Fama-French and Jagannathan and Wang models with 25 and 100 portfolios are presented below. We report the results from the first-pass time-series regressions of raw returns on model factors. R^2 is the average of the R-squares from the first-pass time-series regressions. MC is the multicollinearity coefficient of the estimated beta matrix. "Mean beta" refers to the average value of the betas of a factor. "Std. beta" is the standard deviation of the estimated betas of a factor. We also report the correlation matrix of estimated betas and the estimated risk prices from the second-pass OLS regressions.

Panel A: 25 Portfolios

		Fama-French				Jagannathan-Wang		
First Stage Regression								
R^2	0.902				0.784			
MC	5.945				4.885			
		VW	**SMB**	**HML**		**VW**	**PRE**	**LAB**
Mean beta		1.040	0.584	0.182		1.123	0.168	0.422
Std. beta		0.160	0.523	0.191		0.256	0.335	0.574
VB		0.978	0.565	0.484		0.953	0.208	0.360
Correlation Coefficients between betas								
		VW	**SMB**	**HML**		**VW**	**PRE**	**LAB**
	VW	1.000	0.456	-0.624	**VW**	1.000	-0.631	0.831
	SMB		1.000	0.335	**PRE**		1.000	-0.315
	HML			1.000	**LAB**			1.000
Second Stage Regression								
		VW	**SMB**	**HML**		**VW**	**PRE**	**LAB**
Risk price		-0.256	0.142	0.520		-0.045	0.491	0.179

Panel B: 100 Portfolios

		Fama-French				Jagannathan-Wang		
First Stage Regression								
R^2	0.780				0.780			
MC	3.609				2.987			
		VW	**SMB**	**HML**		**VW**	**PRE**	**LAB**
Mean beta		1.037	0.628	0.202		1.131	0.191	0.477
Std. beta		0.171	0.542	0.207		0.260	0.399	0.725
VB		0.974	0.575	0.490		0.950	0.191	0.3046
Correlations between betas								
		VW	**SMB**	**HML**		**VW**	**PRE**	**LAB**
	VW	1.000	0.343	-0.596	**VW**	1.000	-0.540	0.688
	SMB		1.000	0.360	**PRE**		1.000	-0.219
	HML			1.000	**LAB**			1.000
Second Stage Regression								
		VW	**SMB**	**HML**		**VW**	**PRE**	**LAB**
Risk price		-0.420	0.201	0.323		-0.231	0.303	0.199

Table 2
VAR and Unit Root Analysis of Factors

Panel A shows the vector autoregression (VAR) parameter estimates of coefficients and the error variance matrix estimated for each set of factors for the Fama-French (1993) and Jagannathan-Wang (1996) models for July 1963 to December 1990. The factors are the market index return per Jagannathan and Wang (1996) (VW), the small-minus-big firm (SMB) and high-minus-low book-to-market (HML) portfolio returns per Fama and French (1993), and the corporate bond premium (PRE) and labor market return (LAB) factors per Jagannathan and Wang (1996). The VAR parameters are estimated using one lag separately for each set of factors. The VAR coefficients for a given factor are shown in separate columns. Panel B provides results of unit root tests (persistence) using Dickey-Fuller (1979) and Phillip-Perrons (1988) for the each factor.

Panel A: Vector Autoregression (VAR) analysis

	VAR Coefficients					
	Fama-French (1993)			Jagannathan-Wang (1996)		
	VW	SMB	HML	VW	PRE	LAB
Cst	0.894	0.061	0.390	0.512	0.052	0.278
VW(-1)	0.014	0.170	0.004	0.021	0.003	0.003
SMB(-1)	0.123	0.095	-0.085			
HML(-1)	-0.059	0.027	0.172			
PRE(-1)				1.107	0.961	-0.029
LAB(-1)				-1.480	-0.017	0.565

	VAR Model Error Variance Matrix					
	Fama-French (1993)			Jagannathan-Wang (1996)		
	VW	SMB	HML	VW	PRE	LAB
VW	20.06	4.071	-4.010	19.61	0.061	-0.064
SMB	4.071	7.731	-0.409			
HML	-4.010	-0.409	6.290			
PRE				0.061	0.015	-0.001
LAB				-0.064	-0.001	0.080

Panel B: Unit root tests of factors (Null hypothesis: Factor has unit root)

	VW	SBM	HML	PRE	LAB
Dickey-Fuller (1979)*	-17.15 (a)	-15.08 (a)	-15.11 (a)	-2.42	-9.53 (a)
Phillips-Perron (1988)*	-17.13 (a)	-15.15 (a)	-15.07 (a)	-2.50	-9.30 (a)

* Significant at the following level per the note letter based on in the indicated critical values per MacKinnon (1991):

Note	Level	Critical Value
(a)	1.0%	-3.452
(b)	5.0%	-2.871
(c)	10.0%	-2.572

Table 3
Results from the Simulations of the Fama-French and Jagannathan-Wang

The results reported below are obtained from 1,000 simulations. The simulation data for 25 and 100 portfolio returns are generated using generated factors and the estimated risk prices and betas from the estimation of the Fama-French and Jagannathan-Wang models with actual data. The simulated errors are *i.i.d.* with variances equal to the estimated variances from the estimation of the Fama-French and Jagannathan-Wang models with actual data. Factors are generated with the autocorrelation levels equal the ones from the actual data. Reported are the relative biases (%) in the OLS and GLS-SH estimates of risk prices as percent of the true parameter values, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical size and power of a *t*-test are the rejection rate for the correct hypothesis of risk price being equal to its true value and the incorrect hypothesis of risk price being equal to zero, respectively. All hypotheses are tested at 5% significance level. Panel A and B show the results from the simulations of 25 and 100 portfolios, respectively.

Panel A: 25 Portfolios

Relative bias

	Fama-French				Jagannathan-Wang			
	Cst.	VW	SMB	HML	Cst.	VW	PRE	LAB
OLS	12.63	55.9	22.8	-26.6	8.9	46.8	-61.6	-47.9
GLS-SH	11.83	53.6	24.3	-26.4	5.8	60.5	-51.7	-41.9

Rejection rate (size) of the *t*-test for H_o: price = true value

	Fama-French				Jagannathan-Wang			
	Cst.	VW	SMB	HML	Const.	VW	PRE	LAB
OLS-FM	7.1	5.1	6.4	7.4	19.5	8.4	75.9	24.8
OLS-SH	5.1	4.4	6.1	6.3	12.0	6.4	70.9	20.3
GLS-SH	7.2	5.3	6.0	7.4	14.6	6.4	65.4	23.3

Rejection rate (power) of the *t*-test for H_o: price = 0

	Fama-French				Jagannathan-Wang			
	Cst.	VW	SMB	HML	Cst.	VW	PRE	LAB
OLS-FM	76.7	24.8	8.7	30.3	77.6	11.7	95.6	51.8
OLS-SH	77.6	26.9	10.2	32.3	81.3	10.7	96.4	57.7
GLS-SH	80.2	22.4	8.9	28.1	89.5	14.6	94.0	57.1

Table 3 continues…

Panel B: 100 Portfolios

Relative bias

	Fama-French				Jagannathan-Wang			
	Cst.	**VW**	**SMB**	**HML**	**Cst.**	**VW**	**PRE**	**LAB**
OLS	1.6	4.9	2.5	-9.7	-2.9	-31.7	-57.4	-38.1
GLS-SH	1.4	4.3	2.4	-9.1	-4.6	-35.0	-53.2	-40.4

Rejection rate (size) of the *t*-test for H_o: price = true value

	Fama-French				Jagannathan-Wang			
	Cst.	**VW**	**SMB**	**HML**	**Cst.**	**VW**	**PRE**	**LAB**
OLS-FM	6.4	5.4	6.2	4.3	27.0	7.3	54.8	44.3
OLS-SH	6.0	4.8	5.0	2.4	20.5	6.4	48.4	43.4
GLS-SH	16.5	7.0	5.2	6.0	35.3	7.8	52.1	54.2

Rejection rate (power) of the *t*-test for H_o: price = 0

	Fama-French				Jagannathan-Wang			
	Cst.	**VW**	**SMB**	**HML**	**Cst.**	**VW**	**PRE**	**LAB**
OLS-FM	100.0	38.3	20.1	43.2	100.0	13.5	82.8	99.9
OLS-SH	100.0	36.5	18.9	44.8	100.0	12.5	84.6	99.9
GLS-SH	100.0	29.2	17.1	38.4	100.0	6.8	75.3	99.9

Table 4
Simulations of Base Model

1,000 simulations are used. Data are generated using the risk price and error-variance estimates from the estimation of the Fama-French model with actual data. Idiosyncratic errors and factors are *i.i.d.* Betas are generated so that they are orthogonal to each other (MC = 1). They are generated only once, and the same betas are used for all simulations. The mean and standard deviation values of each beta used for simulation are reported below. The values are determined so that the average R-Square from the regressions of generated returns on generated factors equals 0.78. Reported are the means and standard deviations of betas, the relative biases (%) in the OLS and GLS-SH estimators of risk prices and the empirical sizes and power (%) of the two-tailed *t*-tests using three TP estimators (OLS-FM, OLS-SH ,and GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 3.

	25 Portfolios				**100 Portfolios**			
R^2	0.78				0.78			
MC	1.00				1.00			
	Cst.	**VW**	**SMB**	**HML**	**Cst**	**VW**	**SMB**	**HML**
Mean beta		0.317	0.317	0.317		0.350	0.350	0.350
Std. Beta		1.00	1.00	1.00		1.08	1.08	1.08
VB		0.094	0.094	0.094		0.097	0.097	0.097

Relative bias

	25 Portfolios				**100 Portfolios**			
	Cst.	**VW**	**SMB**	**HML**	**Cst**	**VW**	**SMB**	**HML**
OLS	0.2	1.3	-1.9	-0.2	0.0	0.4	0.2	0.2
GLS-SH	0.2	1.8	-1.9	-0.1	0.0	0.4	0.4	0.1

Rejection rate (size) of the *t*-test for H_o: price = true value

	25 Portfolios				**100 Portfolios**			
	Cst.	**VW**	**SMB**	**HML**	**Cst**	**VW**	**SMB**	**HML**
OLS-FM	6.0	4.7	5.7	6.7	4.8	6.2	5.3	6.7
OLS-SH	5.6	4.3	5.1	5.9	4.3	6.0	4.8	6.4
GLS-SH	7.8	4.5	5.1	6.4	15.7	6.2	5.2	6.4

Rejection rate (power) of the *t*-test for H_o: price = 0

	25 Portfolios				**100 Portfolios**			
	Cst.	**VW**	**SMB**	**HML**	**Cst**	**VW**	**SMB**	**HML**
OLS-FM	100.0	20.0	18.3	94.7	100.0	39.7	28.1	59.7
OLS-SH	100.0	19.3	18.2	93.3	100.0	39.7	26.0	55.5
GLS-SH	100.0	20.8	17.4	93.1	100.0	39.1	25.5	56.4

Table 5
Simulations with Near-Multicollinear Betas: 25 Portfolios

The data generating process used is the same as the process used for Table 4, except that the betas of VW and HML are generated with different levels of correlation reflected in four different values of the multicollinearity coefficient (MC). Only the results for VW and HML are reported since they are the only ones that change. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices and the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Relative bias

	MC=1		**MC=5**		**MC=10**		**MC=15**	
	VW	**HML**	**VW**	**HML**	**VW**	**HML**	**VW**	**HML**
OLS	1.3	-0.2	-3.6	-1.6	-18.0	-5.6	-39.3	-11.5
GLS-SH	1.8	-0.1	-2.3	-1.3	-15.1	-4.8	-34.5	-10.2

Rejection rate (size) of the *t*-test for H_o: price = true value

	MC=1		MC=5		MC=10		MC=15	
	VW	**HML**	**VW**	**HML**	**VW**	**HML**	**VW**	**HML**
OLS-FM	4.5	6.7	4.3	4.7	4.6	4.9	5.6	5.1
OLS-SH	4.3	5.9	4.3	4.3	4.6	4.5	5.1	5.1
GLS-SH	4.5	6.4	4.3	5.6	5.0	5.7	5.7	6.2

Rejection rate (power) of the *t*-test for H_o: risk price = 0

	MC=1		**MC=5**		**MC=10**		**MC=15**	
	VW	**HML**	**VW**	**HML**	**VW**	**HML**	**VW**	**HML**
OLS-FM	20.0	94.7	14.5	92.0	5.9	79.2	3.1	64.4
OLS-SH	19.3	93.3	13.8	91.5	5.9	81.1	2.7	65.5
GLS-SH	20.8	93.1	17.2	89.9	7.1	80.2	3.0	67.2

Table 6

Simulations with Near-Constant Betas: 25 Portfolios

The data generating process used is the same as the process used for Table 4, except that different standard deviation values are used for the VW betas in order to have a corresponding Near-Constant Beta measures *VB* = 0.13, 0.54 and 0.97. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Relative Bias

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS	0.2	1.3	-1.9	-0.2	0.2	1.1	-1.9	-0.2	-1.3	-14.5	-1.8	-0.2
GLS-SH	0.2	1.8	-1.9	-0.1	0.3	1.9	-1.9	-0.1	-0.9	-9.9	-1.9	-0.1

Rejection rate (size) of the *t*-test for H_o: price = true value

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	6.0	4.7	5.7	6.7	7.2	4.8	5.5	6.7	6.1	4.2	5.6	6.8
OLS-SH	5.6	4.3	5.1	5.9	6.3	4.2	5.1	5.9	4.6	3.9	5.1	6.0
GLS-SH	7.8	4.5	5.1	6.4	8.6	4.3	5.5	6.8	6.9	5.1	5.5	6.6

Rejection rate (power) of the *t*-test for H_o: price = 0

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	100.0	20.0	18.3	94.7	100.0	16.7	17.4	92.2	100.0	7.7	17.2	92.3
OLS-SH	100.0	19.3	18.2	93.3	100.0	18.8	18.3	93.3	100.0	7.3	18.2	93.7
GLS-SH	100.0	20.8	17.4	93.1	100.0	19.2	16.5	92.0	100.0	7.0	16.4	92.1

Table 7

Simulations with Near-Constant and Near Multicollinear Betas: 25 Portfolios

The data generating process used is the same as the process used for Table 6, except that the betas of VW and HML are generated with correlation reflected in the value of the multicollinearity coefficient MC=5. Different standard deviation values are used for the VW betas in order to have a corresponding Near-Constant Beta measures VB = 0.13, 0.54 and 0.97. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Multicollinearity coefficient, MC=5

Relative Bias

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS	-0.0	-3.6	-1.9	-1.6	-0.0	-7.7	-1.9	-1.7	-6.6	-74.1	-1.7	-2.2
GLS-SH	0.0	-2.3	-1.8	-1.3	0.0	-5.6	-1.8	-1.4	-6.1	-67.8	-1.6	-2.1

Rejection rate (size) of the *t*-test for H_o: price = true value

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	6.4	4.7	5.5	4.7	6.6	4.8	5.5	5.0	6.8	5.7	5.7	5.8
OLS-SH	5.7	4.3	5.2	4.3	5.5	4.6	5.2	4.4	5.0	4.1	4.9	5.7
GLS-SH	7.7	4.6	5.6	5.6	7.2	5.2	5.6	6.2	6.2	5.3	5.6	6.1

Rejection rate (power) of the *t*-test for H_o: price = 0

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	100	14.5	17.4	92.0	100	8.3	17.5	90.3	91.9	2.5	17.5	89.9
OLS-SH	100	13.8	16.8	91.5	100	8.2	16.6	90.6	92.5	2.2	17.1	89.8
GLS-SH	100	17.2	16.5	89.9	100	11.4	16.5	89.7	94.0	2.8	16.3	91.4

Table 8
Simulations with Persistent Factors: 25 Portfolios

The data generating process used is the same as the process used for Table 4, except that the AR(1) coefficient (ρ_{HML}) of HML is allowed to vary. Only the results for HML are reported to save space. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Relative Bias

	$\rho_{HML} = 0$	$\rho_{HML} = 0.3$	$\rho_{HML} = 0.5$	$\rho_{HML} = 0.9$	$\rho_{HML} = 0.95$
	HML	**HML**	**HML**	**HML**	**HML**
OLS	-0.2	-0.2	-0.3	-0.6	-0.6
GLS-SH	-0.1	-0.2	-0.2	-0.5	-0.6

Rejection rate (size) of the *t*-test for H_o: price = true value

	$\rho_{HML} = 0$	$\rho_{HML} = 0.3$	$\rho_{HML} = 0.5$	$\rho_{HML} = 0.9$	$\rho_{HML} = 0.95$
	HML	**HML**	**HML**	**HML**	**HML**
OLS-FM	5.6	17.2	29.1	68.4	79.2
OLS-SH	6.7	8.0	9.2	23.9	37.1
GLS-SH	6.8	7.7	9.2	23.6	36.8

Rejection rate (power) of the *t*-test for Ho: price = 0

	$\rho_{HML} = 0$	$\rho_{HML} = 0.3$	$\rho_{HML} = 0.5$	$\rho_{HML} = 0.9$	$\rho_{HML} = 0.95$
	HML	**HML**	**HML**	**HML**	**HML**
OLS-FM	94.7	71.9	50.8	11.9	8.5
OLS-SH	93.3	68.8	47.7	11.9	8.0
GLS-SH	93.1	68.5	47.9	11.8	7.8

Table 9
Simulations with Near-Multicollinear Betas: 100 Portfolios

The data generating process used is the same as the process used for Table 4, except that the betas of VW and HML are generated with different levels of correlation reflected in four different values of the multicollinearity coefficient (MC). Only the results for VW and HML are reported since they are the only ones that change. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices and the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Relative Bias

	MC = 1		MC = 5		MC = 10		MC = 15	
	VW	**HML**	**VW**	**HML**	**VW**	**HML**	**VW**	**HML**
OLS	0.4	0.2	-2.4	-1.8	-8.9	-6.5	-17.7	-13.1
GLS-SH	0.4	0.1	-2.3	-1.7	-8.5	-6.3	-17.0	-12.6

Rejection rate (size) of the *t*-test for H_o: price = true value

	MC = 1		MC = 5		MC = 10		MC = 15	
	VW	**HML**	**VW**	**HML**	**VW**	**HML**	**VW**	**HML**
OLS-FM	6.2	6.7	6.4	5.8	5.9	5.7	5.7	5.4
OLS-SH	6.0	6.4	6.3	5.6	5.8	5.3	5.5	5.2
GLS-SH	5.7	6.4	6.9	6.2	6.7	5.7	7.0	6.5

Rejection rate (power) of the *t*-test for H_o: price = 0

	MC = 1		MC = 5		MC = 10		MC = 15	
	VW	**HML**	**VW**	**HML**	**VW**	**HML**	**VW**	**HML**
OLS-FM	39.7	59.7	34.5	59.7	24.7	59.6	15.9	57.1
OLS-SH	39.7	55.5	30.3	60.2	22.3	60.7	15.1	57.1
GLS-SH	39.1	56.4	31.0	59.4	19.5	59.4	14.4	57.1

Table 10
Simulations with Near-Constant Betas: 100 Portfolios

The data generating process used is the same as the process used for Table 4, except that different standard deviation values are used for the VW betas in order to have a corresponding Near-Constant Beta measures *VB* = 0.13, 0.54 and 0.97. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

PANEL A: Fama and French (1993) risk prices

Relative Bias

	VB_{VW} = 0.13				VB_{VW} = 0.57				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS	0.0	0.4	0.2	0.2	0.0	-0.1	0.2	0.2	-3.0	-18.8	0.2	0.2
GLS-SH	0.0	0.4	0.4	0.1	0.0	-0.1	0.4	0.1	-2.8	-17.8	0.4	0.1

Rejection rate (size) of the *t*-test for H_o: price = true value

	VB_{VW} = 0.13				VB_{VW} = 0.57				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	4.8	6.2	5.3	6.7	5.6	6.3	4.9	6.4	8.2	4.1	5.0	6.5
OLS-SH	4.3	6.0	4.8	6.4	4.6	5.8	4.3	6.4	7.7	3.4	4.4	6.3
GLS-SH	15.7	6.2	5.2	6.4	17.3	6.2	5.2	6.4	20.5	7.9	5.2	6.5

Rejection rate (power) of the *t*-test for H_o: price = 0

	VB_{VW} = 0.13				VB_{VW} = 0.57				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	100.0	39.7	28.1	59.7	100.0	35.8	24.5	57.3	100.0	18.1	25.2	64.1
OLS-SH	100.0	39.7	26.0	55.5	100.0	38.0	26.0	55.7	100.0	17.0	25.5	64.0
GLS-SH	100.0	39.1	25.5	56.4	100.0	34.9	24.1	56.2	100.0	23.4	25.4	63.7

Table 11
Simulations with Near-Constant and Near Multicollinear Betas: 100 Portfolios

The data generating process used is the same as the process used for Table 10, except that the betas of VW and HML are generated with correlation reflected in the value of the multicollinearity coefficient MC=5. Different standard deviation values are used for the VW betas in order to have a corresponding Near-Constant Beta measures *VB* = 0.13, 0.54 and 0.97. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Multicollinearity coefficient, MC=5

Relative Bias

	VB_{VW} = 0.13				VB_{VW} = 0.54				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS	0.0	-2.4	0.2	-1.8	-0.0	-6.1	0.2	-1.8	-10.1	-68.3	0.2	-3.2
GLS-SH	0.0	-2.3	0.3	-1.7	-0.0	-5.9	0.3	-1.7	-10.0	-68.0	0.3	-3.2

Rejection rate (size) of the *t*-test for H_o: price = true value

	VB_{VW} = 0.13				VB_{VW} = 0.57				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	4.8	6.4	4.1	5.8	5.1	6.7	4.9	6.2	14.4	7.3	4.9	6.1
OLS-SH	4.3	6.3	4.3	5.6	4.4	6.5	4.4	5.9	13.6	6.7	4.3	6.0
GLS-SH	14.3	6.9	4.5	6.2	15.4	7.4	5.1	6.5	26.5	15.2	5.1	6.3

Rejection rate (power) of the *t*-test for H_o: price = 0

	VB_{VW} = 0.13				VB_{VW} = 0.57				VB_{VW} = 0.97			
	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML	Cst.	VW	SMB	HML
OLS-FM	100	34.5	28.3	59.7	100	8.3	17.5	90.3	91.9	2.5	17.5	89.9
OLS-SH	100	30.3	26.0	60.2	100	7.5	17.2	90.2	91.5	2.2	17.1	89.0
GLS-SH	100	31.0	25.5	59.4	100	11.4	16.5	89.7	94.0	2.8	16.3	91.4

Table 12
Simulations with Persistent Factors: 100 Portfolios

The data generating process used is the same as the process used for Table 4, except that the AR(1) coefficient (ρ_{HML}) of HML is allowed to vary. Only the results for HML are reported to save space. Reported are the relative biases (%) in the OLS and GLS-SH estimators of risk prices, the empirical sizes and power (%) of the two-tailed *t*-tests based on three TP estimators (OLS-FM, OLS-SH and, GLS-SH). The empirical sizes and power of the *t* tests are computed by the same ways that are used for Table 4.

Relative Bias

	$\rho_{HML} = 0$	$\rho_{HML} = 0.3$	$\rho_{HML} = 0.5$	$\rho_{HML} = 0.9$	$\rho_{HML} = 0.95$
	HML	**HML**	**HML**	**HML**	**HML**
OLS	0.2	0.3	0.4	1.6	3.0
GLS-SH	0.1	0.3	0.4	1.6	3.0

Rejection rate (size) of the *t*-test for H_o: price = true value

	$\rho_{HML} = 0$	$\rho_{HML} = 0.3$	$\rho_{HML} = 0.5$	$\rho_{HML} = 0.9$	$\rho_{HML} = 0.95$
	HML	**HML**	**HML**	**HML**	**HML**
OLS-FM	6.7	15.0	25.4	63.8	74.2
OLS-SH	6.4	7.2	8.1	21.2	33.2
GLS-SH	6.4	7.2	8.1	21.3	33.1

Rejection rate (power) of the *t*-test for H_o: price = 0

	$\rho_{HML} = 0$	$\rho_{HML} = 0.3$	$\rho_{HML} = 0.5$	$\rho_{HML} = 0.9$	$\rho_{HML} = 0.95$
	HML	**HML**	**HML**	**HML**	**HML**
OLS-FM	39.7	33.0	20.9	7.8	7.6
OLS-SH	39.7	31.5	21.9	8.6	6.4
GLS-SH	39.1	31.8	21.9	8.6	6.5