ECON 425 **EXERCISE 1** **Dr. AHN**

Q1. A random variable X has the following pdf:

x	0	1	2	3	4
f(x)	b	2b	3b	4b	5b

1) What is the value of b? Why?
2) Find the $P(X \leq 3)$.
3) Find E(x).

Q2. The joint probability distribution of X and Y is given by the following table: (For example, f(4,9) = 0.)

x\y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

1) Find the marginal pdfs of X and Y.
2) Find var(2x+3y).

Q3. Let X stand for the rate of return on a security (say, IBM) and Y the rate of return on another security (say, General Motors). Let $\mu_X = \mu_Y = 0.5$, $\sigma_X^2 = 4$, $\sigma_Y^2 = 9$ and corr(x,y) = -0.8.

1) Find E[0.5x+0.5y] and var[0.5x+0.5y]. [Hint: $E[0.5x+0.5y] = 0.5 \bullet E(x)+0.5 \bullet E(y)$ and $var[0.5x+0.5y] = (0.5)^2 \bullet var(x) + (0.5)^2 \bullet var(y) + 2 \bullet (0.5) \bullet (0.5) \bullet cov(x,y)$; $cov(x,y) = corr(x,y) \bullet \sigma_X \sigma_Y$.]
2) Is it better to invest equally in the two securities (i.e., diversify) than in either security exclusively? (Hint: Investors consider both expected rate of return and risk.) Explain in detail why or why not.

Q4. Let $Y \sim \chi^2(5)$.

1) Find a such that $P(Y > a) = 0.05$.
2) Find c such that $P(Y < c) = 0.9$.

Q5. Let the two random variables, X_1 and X_2, are i.i.d. with N(0,1). Find $P(X_1^2+X_2^2 > 9.21)$.

Q6. Consider the three random variables, X, Y, and Z. Assume that all of them are stochastically independent. Let X be N(0,1); Y be $\chi^2(5)$; Z be $\chi^2(4)$.

1) Find $\Pr\left(\frac{X}{\sqrt{Y/5}} > 2.57\right)$.

2) Find $\Pr\left(\frac{\sqrt{Y/5}}{\sqrt{Z/4}} > 2.5020\right)$.

Answers:

1. 1) b = 1/15, since $\Sigma_x f(x) = 1$.
   2) $\Pr(X=0) + \Pr(X=1) + \Pr(X=2) + \Pr(X=3) = 2/3$.
   3) 8/3.
2. 1) $f_x(2) = 1/4$; $f_X(4) = 1/2$; $f_X(6) = 1/4$; $f_Y(1) = 1/2$; $f_Y(3) = 1/3$; $f_Y(9) = 1/6$.
   2) 80.
3. 1) $E(0.5x+0.5y) = 0.5 \cdot E(x)+0.5 \cdot E(y) = 0.5$
   $\text{var}(0.5x+0.5y) = (0.5)^2 \cdot 4+(0.5)^2 \cdot 9+2 \cdot (0.5) \cdot (0.5) \cdot 2 \cdot 3 \cdot (-0.8) = 0.85$.
   2) Observe that $E[(1/2) \cdot x + (1/2) \cdot y] = E(x) = E(y) = 0.5$. Thus, the two investment strategies give you the same expected return. However, $\text{var}(1/2 \cdot x+1/2 \cdot y) = 0.85$, $\text{var}(X) = 4$ and $\text{var}(Y) = 9$. So, investing equally in the two securities is less risky than investing in one security exclusively.
4. 1) a = 11.07
   2) $\Pr(Y > c) = 1 - 0.9 = 0.1. \rightarrow c = 9.24$.
5. $\Pr[\chi^2(2) > 9.21] = 0.01$. [Hint: Note that $X_1^2+X_2^2$ is $\chi^2(2)$.]
6. 1) $\Pr[t(5) > 2.57] = 0.025$;
   2) $$\Pr\left(\sqrt{F(5,4)} > 2.5020\right) = \Pr\left(F(5,4) > 2.5020^2\right) = \Pr(F(5,4) > 6.26) = 0.05.$$