ECN 425 **EXERCISE 2** **DR. AHN**
ECONOMETRICS **FALL 2006**

Q1. (30 pts.; 5 pts. on each) Choose the one alternative that best completes the statement or answers the question.

1) Analyzing the behavior of unemployment rate across U.S. states in March of 2002 is an example of using

a) panel data. b) time series data. c) cross-section data. d) experimental data.

2) The expected value of a discrete random variable

a) is the outcome that is most likely to occur.
b) can be found by determining the 50% value in the p.d.f.
c) equals the population median.
d) is computed as a weighted average of the possible outcome of that random variable, where the weights are the probabilities of that outcome.

3) When you are testing a hypothesis against a two-sided alternative, then the alternative is written as

a) $\overline{Y} \neq \mu_{Y,o}$. b) $E(Y) \neq \mu_{Y,o}$. c) $E(Y) > \mu_{Y,o}$. d) $E(Y) = \mu_{Y,o}$.

4) An estimator $\hat{\mu}_Y$ of the population value μ_Y is unbiased if

a) $\hat{\mu}_Y = \mu_Y$.
b) $\overline{Y}$ has the smallest variance of all estimators.
c) $\overline{Y} \rightarrow_p \mu_Y$.
d) $E(\hat{\mu}_Y) = \mu_Y$

5) When the estimated slope coefficient in the simple regression model, $\hat{\beta}_1$, is zero, then

a) $R^2 = \overline{Y}$. b) $0 < R^2 < 1$. c) $R^2 = 0$. d) $R^2 >$ (RSS/TSS).

6) Heteroskedasticity means that

a) the values of X_i are different across different i.
b) the variance of the error term is not constant.
c) the observed units have different preferences.
d) agents are not all rational.

Q2. (20 pts.) The joint probability distribution of X and Y is given by the following table: (For example, f(x,y) = 0 when (x,y) = (5,10).)

X\Y	5	10
5	0.2	0
10	0.2	0.6

1) Find the marginal probability distribution of X.
2) Given X = 10, find the conditional mean of Y.

Q3. (5 pts.) Y is distributed χ_4^2. Find $\Pr(Y < 9.49)$.

Q4. (10 pts.) A population is distributed $N(500, 10000)$. Let $\bar{Y}$ be a sample mean computed using a random sample of size n = 100 from the population. Find $\Pr(\bar{Y} < 526)$.

Q5. (15 pts.) Consider the model: $Y_i = \beta_0 + \beta_1 X_i + u_i$. We have the following information:

$n = 10$; $\Sigma_i Y_i = 200$; $\Sigma_i X_i = 100$; $\Sigma_i Y_i^2 = 8000$; $\Sigma_i X_i^2 = 2000$; $\Sigma_i X_i Y_i = 3500$.

(1) Find the OLS estimates, $\widehat{\beta}_1$ and $\widehat{\beta}_2$. (10 pts.)
(2) Find R^2 (5 pts.)

Q6. (20 pts.; 10 pts. on each.) You have obtained measurements of height in inches of 29 female and 81 male students (*Studenth*) at your university. A regression of the height on a constant and a binary variable (*BFemme*), which takes a value of one for females and is zero otherwise, yields the following result:

$$\widehat{Studenth} = \underset{(0.3)}{71.0} - \underset{(1.00)}{4.84} \times BFemme, \quad R^2 = 0.40,\ SER = 2.0$$

(a) Interpret the estimated intercept and coefficient of $BFemme$.
(b) Test the hypothesis that females are shorter than males by 3 inches on average, at the 5% significance level, against the alternative hypothesis that females are shorter than males by more than 3 inches on average. Compute the appropriate statistic and p-value, and determine whether you would reject or do not reject H_o.

Answer Keys:

Q1. (30 pts.; 5 pts. on each) Choose the one alternative that best completes the statement or answers the question.

1) c; 2) d; 3) b; 4) d; 5) c; 6) b.

Q2. 1) For x = 5, $f_x(x) = 0.2 + 0 = 0.2$. For x = 10, $f_x(x) = 0.2 + 0.6 = 0.8$.
2) For y = 5; $f(y|x=10) = f(10,5)/f_x(10) = 0.2/0.8 = 0.25$;
For y = 10; $f(y|x=10) = f(10,10)/f_x(10) = 0.6/0.8 = 0.75$;
$E(y|x=10) = 5\times0.25 + 10\times0.75 = 1.25 + 7.5 = 8.75$.

Q3. $1 - \Pr(Y > 9.49) = 1 - 0.05 = 0.95$.

Q4. $\Pr(\bar{Y} < 526) = \Pr\left(\frac{\bar{Y} - 500}{\sqrt{10000/100}} < \frac{526 - 500}{\sqrt{10000/100}}\right) = \Pr(Z < 2.6) = 0.9953$.

Q5.

$\bar{X} = 100/10 = 10; \bar{Y} = 200/10 = 20$.

$\Sigma_i (X_i - \bar{X})^2 = 2000 - 10\times10^2 = 1000;$

$\Sigma_i (X_i - \bar{X})(Y_i - \bar{Y}) = 3500 - 10\times10\times20 = 1500;$

$\Sigma_i (Y_i - \bar{Y})^2 = 8000 - 10\times20^2 = 4000$

(1) $\hat{\beta}_1 = 1500/1000 = 1.5; \hat{\beta}_0 = 20 - 1.5\times10 = 5$

(2) $ESS = (1.5)^2 \times 1000 = 2250; TSS = 4000; R^2 = 2250/4000 = 0.5625$.

Q6.

(a) The average height of female students is smaller than the average height of male students by 4.84 inches.

(b) H_o: $\beta_1 = -3$ Vs. H_1: $\beta_1 < -3$
$t = (-4.84+3)/1.00=-1.84$; p-val = $\Pr(Z < -1.84) = 0.0329$; $\alpha = 0.05$
$p < \alpha \rightarrow$ Reject H_o in favor of H_1.

First Mid-Term Exam: (September 28, 2006, in class)

(1) Coverage:

- All materials in lecture notes.
- Chapters 1 – 5.
  Except:
  - Chapter 3.4 – 3.5
  - Section of "The t-statistic testing differences of means" (pp. 89-92).

(2) Materials you need for the exam:

- Calculator.
- The tables of N(0,1), χ^2, t(k) and F(k_1,k_2).
- One cheat sheet (Letter size. Use both sizes. Should be hand-written).

(3) The exercise questions in the textbook that you are not responsible for:

Chapter 2: (2.3), (2.16), (2.19)-(2.21), (2.23).
Chapter 3: (3.2) – (3.5), (3.7), (3.9), (3,10), (3.12), (3.13), (3.14), (3.16) c,d ; (3.17)-(3.21).
Chapter 4: (4.4), (4.8), (4.10)
Chapter 5: (5.6), (5.9), (5.11), (5.12), (5.15)