TOPIC I
INTRODUCTION AND SET THEORY

[1] Introduction

- Economics Vs. Mathematical Economics.
  - "Income positively affects consumption. Consumption level can never be negative. The marginal propensity consume is less than one."

    → $C = a + bY$, where $a > 0$ and $0 < b < 1$.
  - Pro: "Mathematics is language." (P. A. Samuelson)
  - Con: "Mathematical economics is too mechanical."

- What is the goal of economics?
  - Predict and explain economic decisions and phenomena.
    - → Economics is a science.
    - → Then, why not mathematics?

- Structure of (mathematical) economics as a science
  1. Hypothesis (theory explaining a phenomenon)
     - Assumptions → conclusions

       (causes) (results)
     - Deductive
  2. Test (Econometrics)

- Check validity of theories using real data.
- Deductive + Inductive.

- Mathematical economic model
  - Equations (for assumptions).
  - Prove equations.
  - Get solutions (conclusion).

EX: A Simple Keynesian Model

No government (no tax and no government spending)

Investment level (I) is exogenously given.

- Assumptions

1) $C = a + bY$, $a > 0$ and $0 < b < 1$.

2) $Y = C + I$

→ Parameters (constants): a, b (given from heaven).

→ Exogenous variables: I (Given externally to the model).

→ Endogenous variables: C and Y

(which can be solved given exo. var. and par.).

→ Solutions: $\overline{C}$ and $\overline{Y}$.

- Solve the equations (by Elimination of Variables):

2) → 1):

$$C = a + b(C+I)$$

$$\rightarrow (1-b)C = a + bI$$

3) $C = \frac{a+bI}{1-b}$.

3) → 1):

$$Y = \frac{a+bI}{1-b} + I = \frac{a}{1-b} + \frac{1}{1-b}I.$$

- Solutions:

$$\bar{C} = \frac{a+bI}{1-b};\ \bar{Y} = \frac{a}{1-b} + \frac{1}{1-b}I.$$

→ When investment level increases by $1, income level increases by 1/(1-b).

→ If b = 0.5, income increases by $2.

→ Since 1/(1-b) (multiplier) is greater than one, income increases by more than $1.

[2] Set Theory

(1) Set

Definition: (Set)

A set is a collection of distinct objects.

EX 1: S = a set of students' names in the ECN 485 class

= {actual names ... } (enumeration)

= $\{x \mid x$ = a student's name in ECN 485$\}$ (descriptive)

EX 2:

$\mathbb{N} = \{1,2, \ldots , 3\} = \{x \mid x = \text{natural number}\}$

$\mathbb{I} = \{\ldots,-1,0,1,\ldots\} = \{x \mid x = \text{integer}\}$

$Q = \{1/2,1/3,\ldots\} = \{x \mid x = \text{rational number}\}$

$\mathbb{R} = \{1/2, \sqrt{2},\ldots\} = \{x \mid x = \text{real number}\}$

Definition:

"$\in$" means "belongs to (a set)" or "is an element of (a set)".

EX 1: $A = \{a, b, c\}$

→ $a \in A$.

EX 2: Wish to describe a set of all real numbers greater than 2.

→ $J = \{x \mid x > 2, x \in \mathbb{R}\}$.

(2) Finite and Infinite Set

- $A = \{1,2,3\} \rightarrow$ finite set.
- $\mathbb{R} = \{x \mid x = \text{real number}\} \rightarrow$ infinite set

(3) Countable and Uncountable Sets

- $A = \{1,2,3\} \rightarrow$ countable.
- $\mathbb{N}$ = the set of natural numbers $\rightarrow$ infinite but countable.
- $\mathbb{I}$ = the set of integers $\rightarrow$ infinite and (surprisingly) countable.
- $\mathbb{R}$ = the set of real number $\rightarrow$ (surprisingly) uncountable.

(4) Subsets

Definition:

Let S_1 and S_2 be sets. S_1 is said to be a subset of S_2, $S_1 \subset S_2$, iff (if and only iff) $\forall a \in S_1 \Rightarrow a \in S_2$.

EX 1: $A = \{1,2,3\}$; $B = \{1,2\}$

$B \subset A$; $A \subset A$; $\varnothing$ (null set) $\subset A$.

# of possible subsets.

If a set A has n elements, the number of possible subsets is 2^n.

EX: $A = \{1,2\}$

$\rightarrow \varnothing$, {1}, {2} and {1,2} $\rightarrow 4 = 2^2$.

B = {1,2,3}

$\rightarrow \varnothing$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3} $\rightarrow 8 = 2^3$.

(5) Set Algebra

Definition:

$S_1 = S_2$ iff $S_1 \subset S_2$ and $S_2 \subset S_1$.

EX: A = {1,2,3}; B = {3,2,1}

$\rightarrow$ Order does not matter.

Definition:

$A \cup B = \{a \mid a \in A \text{ or } a \in B\}$ (union)

$A \cap B = \{a \mid a \in A \text{ and } a \in B\}$ (intersection)

EX: A = {1,2,3}, B = {3,4}

$\rightarrow$ $A \cup B = \{1,2,3,4\}$; $A \cap B = \{3\}$

Definition:

A and B are called disjoint iff $A \cap B = \varnothing$.

EX: A = {1,2}, B = {3,4}

Definition:

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

EX 1: $A = \{1,2,3\}$, $B = \{3,4,5\}$, $C = \{6,7\}$

$A - B = ?$

$A - C = ?$

EX 2: Let A and B be two sets.

Show that if $A \cap B = \varnothing$, $A - B = A$.

Proof:

We first show that $A - B \subset A$.

$\forall x \in A - B \Rightarrow x \in A$ and $\notin B \Rightarrow x \in A$.

We now show that $A \subset A - B$.

$\forall x \in A \Rightarrow x \in A$ and $x \notin B$ (since $A \cap B = \varnothing$) $\Rightarrow x \in A - B$.

Definition:

A universal set is a set of interest.

EX 1: $U = \{1,2,...\}$; $U = \{x \mid x = \text{real number}\}$

Definition: (Complementary set)

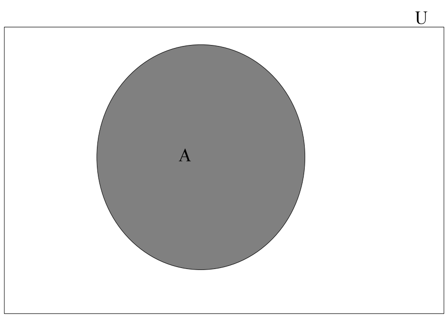
Let $A \subset U$. Then, $A^C = U - A = \{x \mid x \in U \text{ and } x \notin A\}$

EX: U = {1,2,3,4}, A = {1,2}

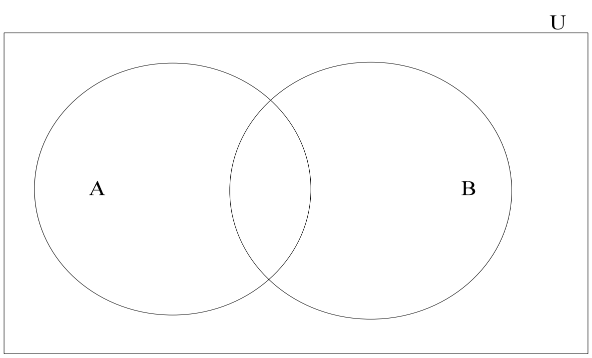
→ $A^C = U - A = \{3,4\}$.

(6) Venn Diagram

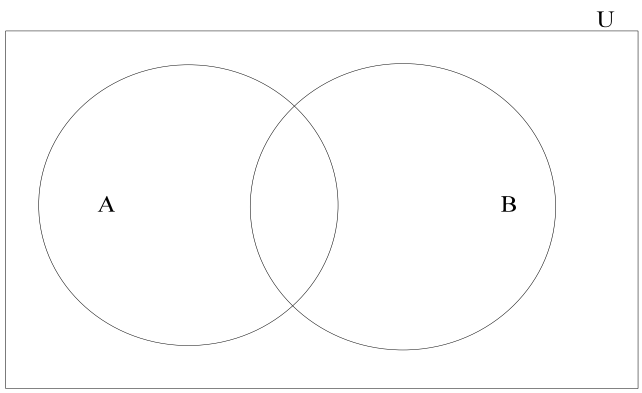
EX 1:



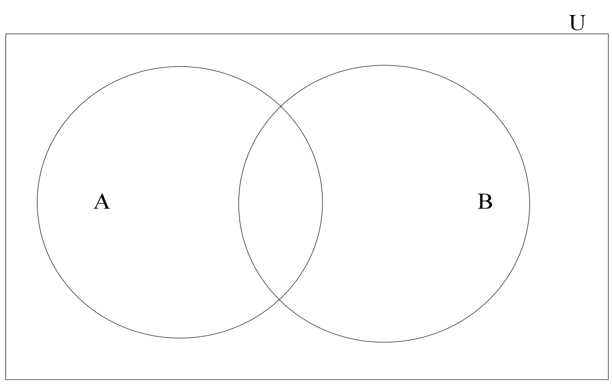
EX 2: $A \cup B$

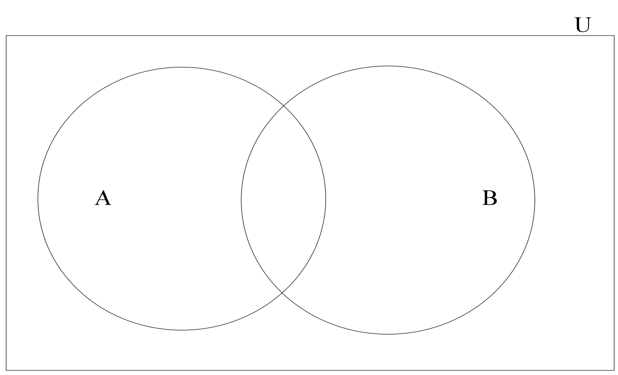


EX 3: $A \cap B$

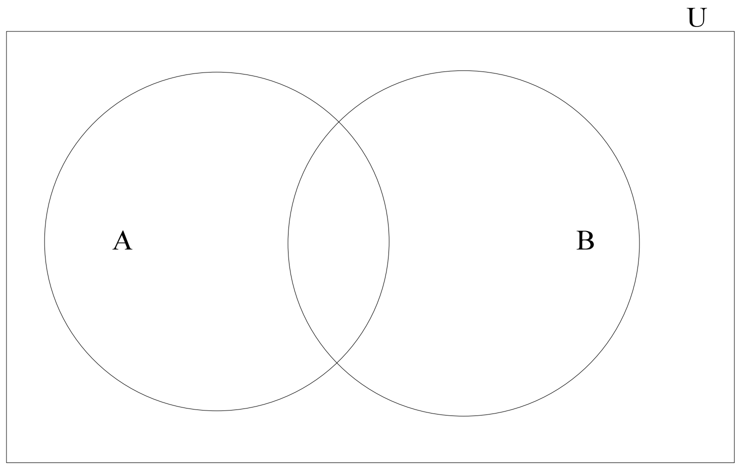


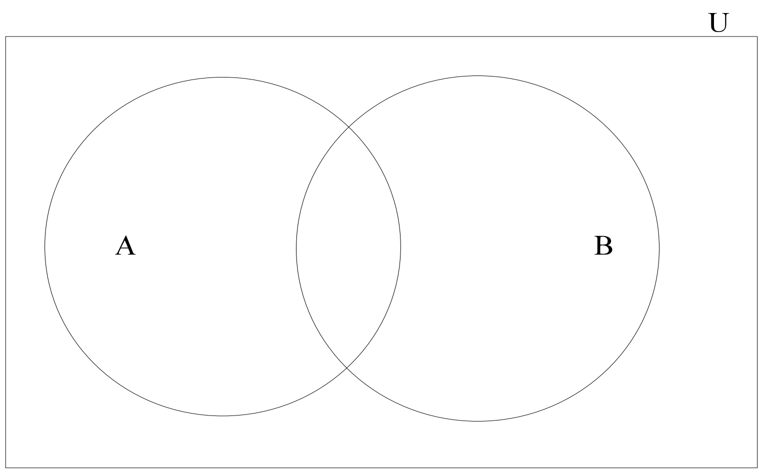
EX 4: $A - B = A - (A \cap B)$





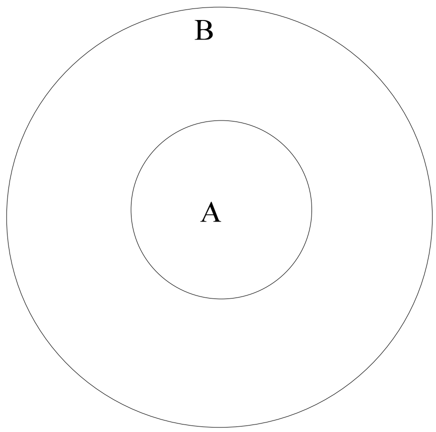
EX 5: $A - B = A \cap B^C$





EX 6: $A \subset B$

$\rightarrow \quad A \cup B = B, A \cap B = A.$



(7) Three Laws of Operation

A. Commutative Law

$$A \cup B = B \cup A; A \cap B = B \cap A$$

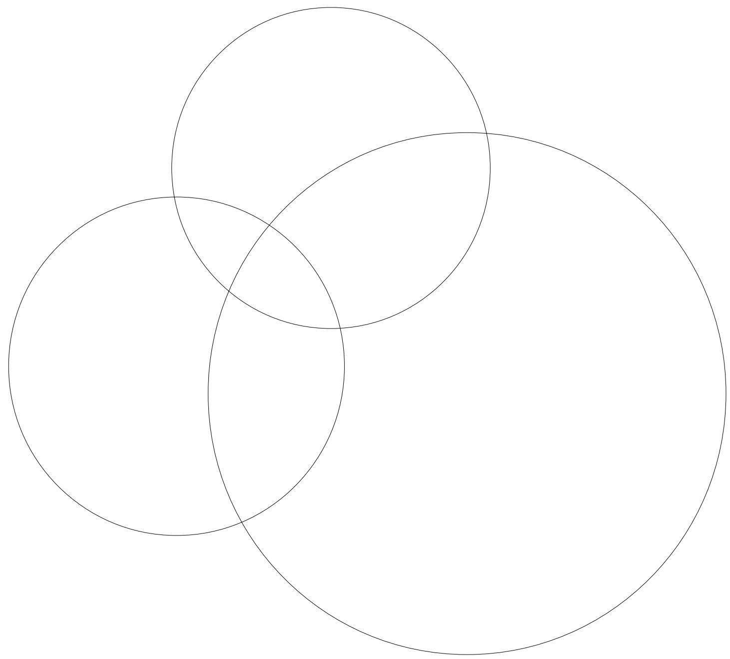
B. Associative Law

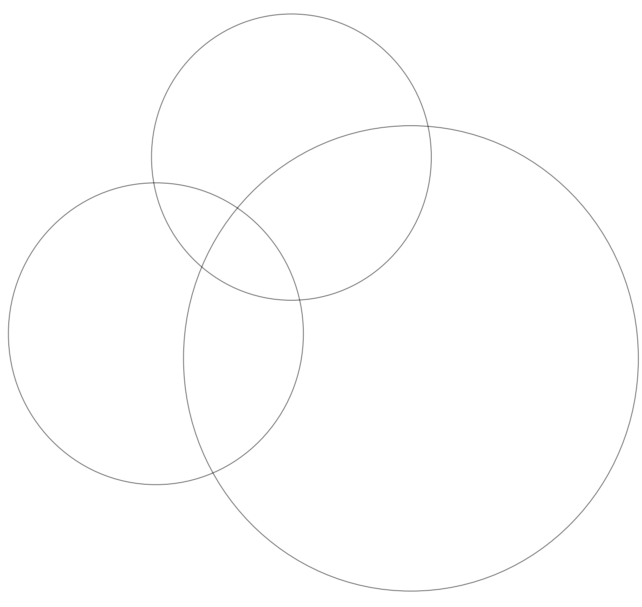
$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

C. Distributive Law

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) .$$



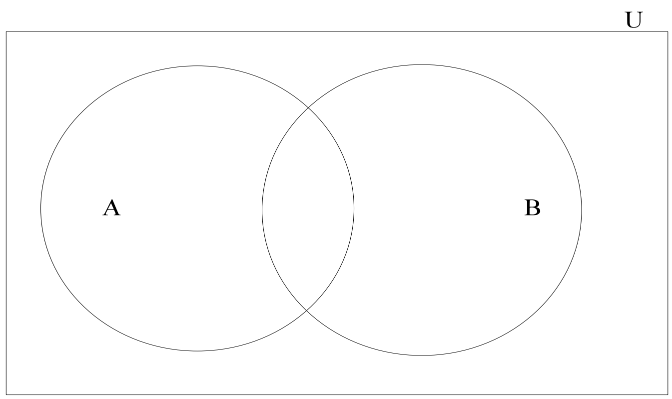


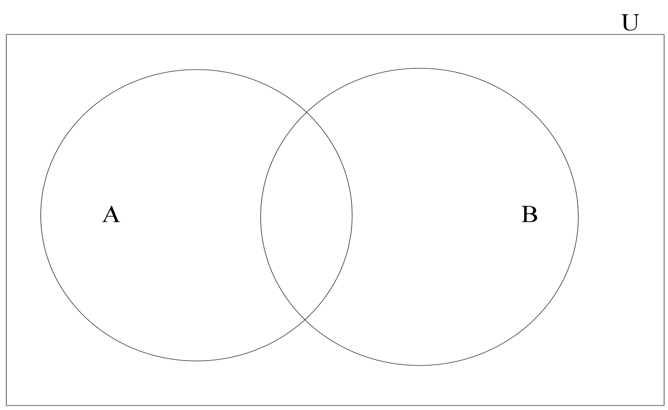
EX: Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(8) De Morgan's Law

$$(A \cap B)^C = A^C \cup B^C .$$

$$(A \cup B)^C = A^C \cap B^C .$$





EX: Show $(A \cup B)^C = A^C \cap B^C$.

EX: Textbook, p. 17, Q2 - Q8.

[3] Relations and Functions

(1) Ordered Set

- For usual sets, the order of elements do not matter: {a,b} = {b,a}
- Ordered set = A set in which the order of elements does matter.

  → Use notation (a,b).

EX: (age, weight) = (19,127)

(2) Cartesian Product

Definition: Cartesian Product (A set of ordered sets)

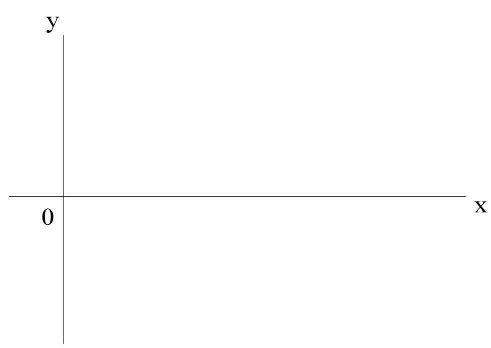
$A \times B = \{(x,y) | x \in A \text{ and } y \in B\}$.

EX 1: A = {ko,jn} and B = {su,ki,st}

→ A × B = {(ko,su), (ko,ki), (ko,st), (jn,su), (jn,ki), (jn,st)}.

EX 2: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) | x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$

→ 2-dimensional Euclidean space.



EX 3: $A \times B \times C = \{(x,y,z) \mid x \in A \text{ and } y \in B \text{ and } z \in C\}$.

EX 4: $\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times .... \times \mathbb{R}$ (n times)

(3) Relation

Definition: (Relation)

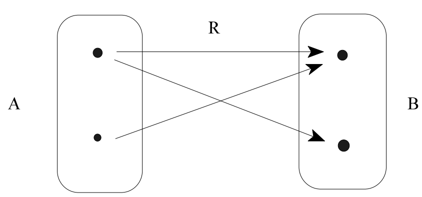
A relation R from A to B is a subset of $A \times B$. Usually it is denoted by:

$R: A \to B$.

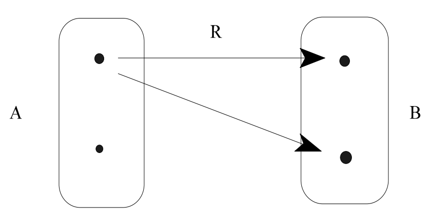
EX: $A = \{a,b\}$, $B = \{x,y\}$.

$\to$ $A \times B = \{(a,x),(a,y),(b,x),(b,y)\}$

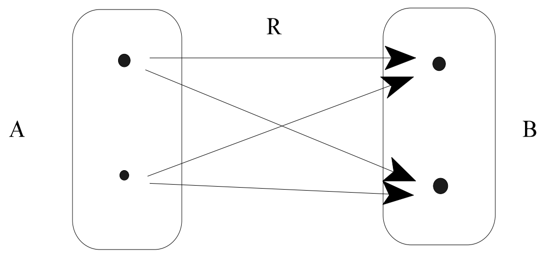
1) $\{(a,x),(a,y),(b,x)\}$



2) $R = \{(a,x),(a,y)\}$



3) $A \times B$

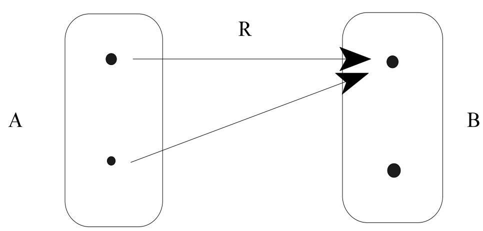


Definition: (Domain and Image)

$\text{Dom}(R) = \{a \mid (a,b) \in R\}$

$\text{Im}(R) = \{b \mid (a,b) \in R\}$ (or called range)

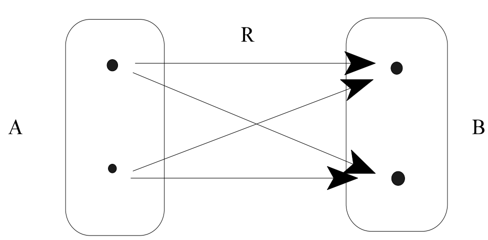
EX: $A = \{a,b\}$, $B = \{x,y\}$



$R = \{(a,x),(b,x)\}$

$\text{Dom}(R) = \{a,b\}$

$\text{Im}(R) = \{x\}$



$R = A \times B$.

$\text{Dom}(R) = \{a,b\}$

$\text{Im}(R) = \{x,y\}$

(4) Function

Definition:

Let X and Y be two sets. Let f be a relation from X to Y ($f: X \to Y$). For each $x \in X$, $\exists |\ y \in Y \ni (x,y) \in f$. Then, f is called a function.

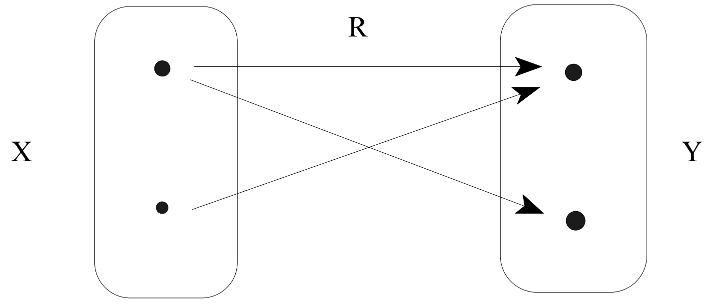
Note:

- A relation f is a function, if
  1) Dom(f) = X ["Any cause has a result."];
  2) If $(x,y_1) \in f$ and $(x,y_2) \in f$, then, $y_1 = y_2$
     ["One cause has only one result."].

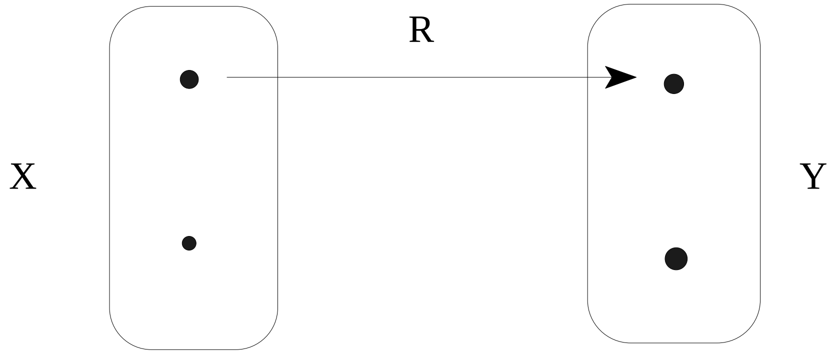
  Then, we write y = f(x).
- x is called independent variable and y is called dependent variable.

EX 1:

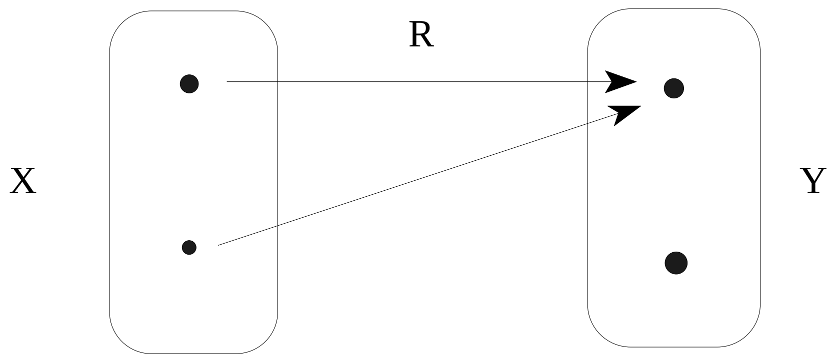
$X = \{x_1,x_2\}$; $Y = \{y_1,y_2\}$



Not a function.



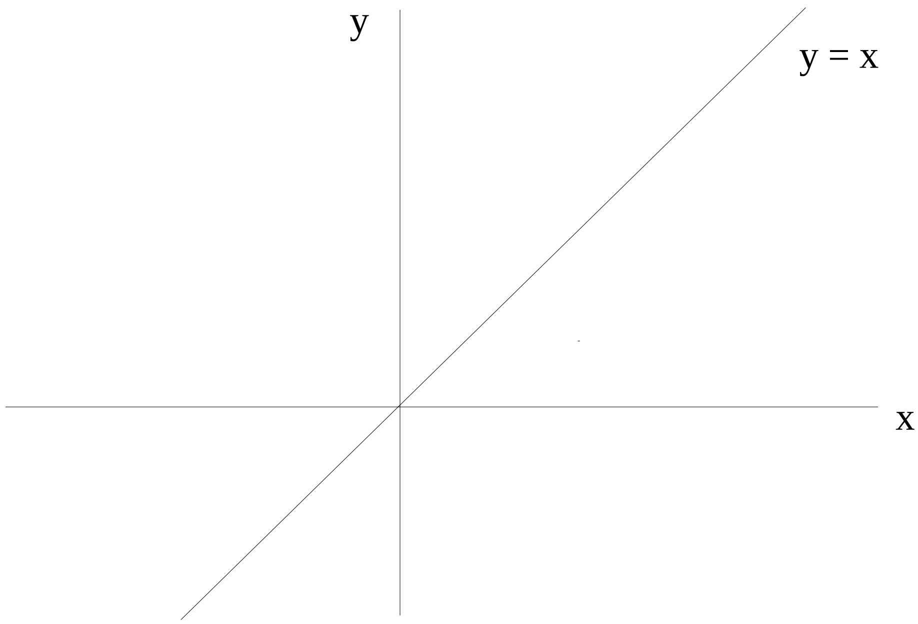
Not a
function



A function.

EX 2:

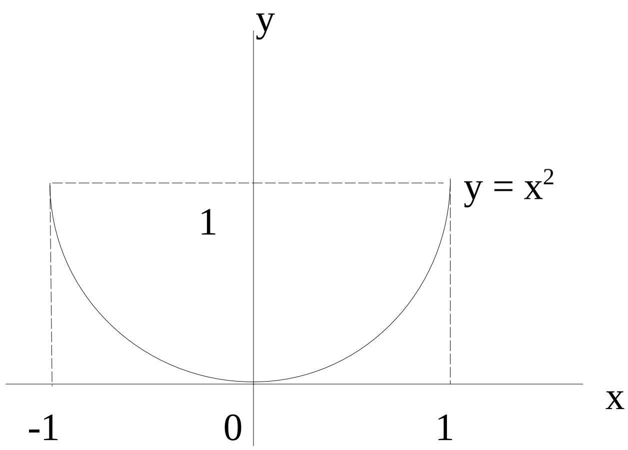
$S = \{(x,y\} \mid y \leq x, (x,y) \in \mathbb{R}^2\} \rightarrow$ Not a function.



$Q = \{(x,y) \mid y = x, (x,y) \in \mathbb{R}^2\} \rightarrow$ A function.

EX 3:

$y = x^2, -1 \le x \le 1.$



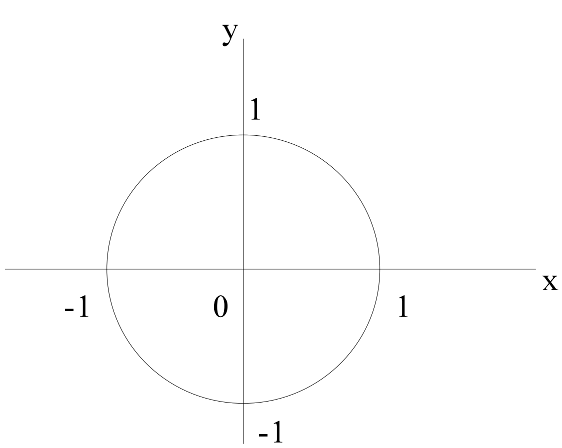
A function.

Dom(f) = $\{x | -1 \le x \le 1\}$

Im(f) = $\{y | 0 \le y \le 1\}$

EX 4:

$x^2 + y^2 = 1, -1 \le x \le 1.$



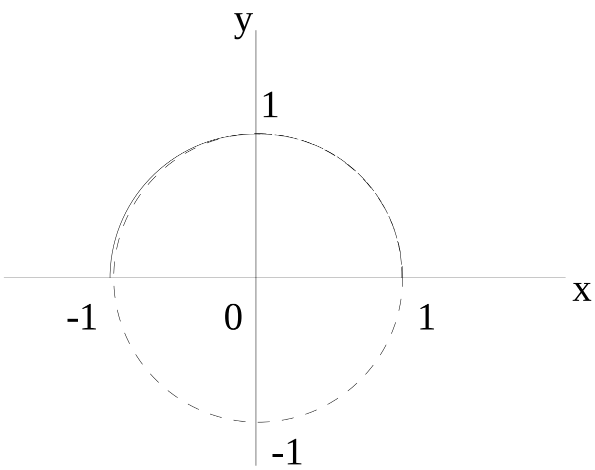
Not a function.

Since x = 0 is related with

different values of y (-1 and 1).

EX 5:

$x^2 + y^2 = 1$, $-1 \le x \le 1$ and $0 \le y \le 1$.



A function.

(5) Types of Function

1) Constant function:

$y \equiv f(x) = 7.$

2) Polynomial function:

- $y \equiv f(x) = a_o + a_1x + a_2x^2 + ... + a_nx^n.$

  [n'th order polynomial function]

- constant f^n: $y = a_o$
- linear f^n: $y = a_o + a_1x$
- quadratic f^n: $y = a_o + a_1x + a_2x^2$
- cubic f^n: $y = a_o + a_1x + a_2x^2 + a_3x^3$

3) Rational function

- $y = \dfrac{x-1}{x^2+2x+4}$.

- $y = \dfrac{a}{x}$ (or xy = a: rectangular hypobola).

(6) Extension of Function

1) f: $A \times B \to C$

Let A = {bm,gm}, B = {a+,a-}, C = {d,nd}

f = {((bm,a+),nd), ((bm,a-),d), ((gm,a+),d), ((gm,a-),nd)}

(bm,a+) → nd

(bm,a-) → d

(gm,a+) → d

(gm,a-) → nd

2) f: $\mathbb{R}^2 \to \mathbb{R}$.

f: $(x,y) \to z$ [$z = x^2 + y^2$, $z = x^2 + xy + y^2$]

EX: Textbook, p. 23, Q1 - Q6; p. 29, Q1, Q3, Q5, Q6.