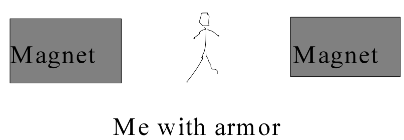
TOPIC II
EQUILIBRIUM MODELS

[1] Concept of Equilibrium

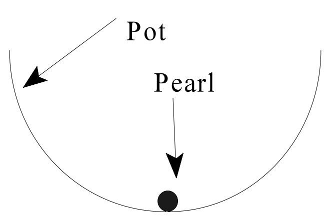
- Equilibrium:

  A state without tendency to change.

EX 1:

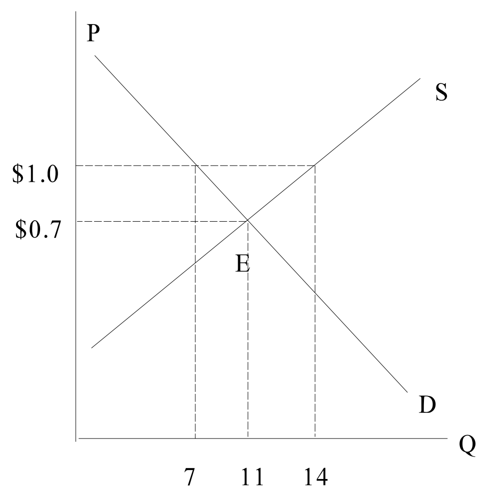


EX 2:

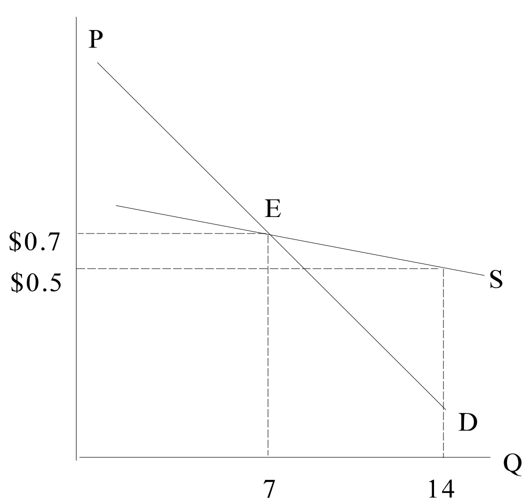


- Types of Equilibrium
  - Stable Equilibrium: Once out of equilibrium, there is a tendency to go back.
  - Unstable Equilibrium: No return.

EX 1: Stable Equil.



EX 2: Unstable Equil.



[2] Partial Market Equilibrium

- Look at only a market.

(1) Linear Demand and Supply Curves

- $Q_d = a + bP$, $a > 0$ and $b < 0$
- $Q_s = c + dP$, $c \leq 0$ and $d > 0$
- $Q_d = Q_s$ (equilibrium condition)

- Wish to solve for equilibrium price ($\bar{P}$) and quantity ($\bar{Q}$).

1) $\bar{Q} = a + b\bar{P}$

2) $\bar{Q} = c + d\bar{P}$

1) → 2): $a + b\bar{P} = c + d\bar{P}$

→ $b\bar{P} - d\bar{P} = c - a$

→ $(b-d)\bar{P} = c - a$

3) $\bar{P} = \dfrac{c-a}{b-d} > 0.$

3) → 1): $\bar{Q} = a + b\dfrac{c-d}{b-d} = a\dfrac{b-d}{b-d} + b\dfrac{c-d}{b-d} = \dfrac{a(b-d)+b(c-a)}{b-d}$

4) $\bar{Q} = \dfrac{bc-ad}{b-d} > 0$ (?)

Digression to Quadratic equations:

- $ax^2 + bx + c = 0$, $a \neq 0$.
- solutions: $\bar{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac < 0$, no solution exists.

EX 1: $x^2 + 4x + 5 = 0$.

→ $b^2 - 4ac = 4^2 - 4 \cdot 1 \cdot 5 = 15 - 20 = -4 < 0$.

→ No solution.

EX 2: $x^2 + 4x - 5 = 0$

→ $\bar{x} = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{-4 \pm \sqrt{36}}{2} = \frac{-4 \pm 6}{2}$

$= 1$ or -5.

- A simple way to find solutions for $x^2 + bx + c = 0$
  - Find a solution, say p.
  - Then, the other solution, say q, must be equal to c/p.

EX 3: $x^2 + 4x - 5 = 0$

→ Find a solution by substituting some numbers.

→ get $p = 1$ → $q = -5/1 = -5$.

EX 4: $-x^2 + 4 = 0$.

→ $x^2 - 4 = 0$.

→ A solution is $p = 2$. → $q = -4/2 = -2$.

- Some useful formulas:
  - $x^2 + bx + c = (x-p)(x-q)$, where p and q are solutions for $x^2+bx+c = 0$.
  - $x^2 - p^2 = (x+p)(x-p)$.

End of Digression

(2) Nonlinear Demand and Supply Curves

- $Q_d = 4 - P^2$; $Q_s = 4P - 1$; $Q_d = Q_s$
- Assume $P > 0$ and $Q > 0$.

→ 1) $\bar{Q} = 4 - \bar{P}^2$;

2) $\bar{Q} = 4P - 1$

2) → 1): $4\bar{P} - 1 = 4 - \bar{P}^2$ → $\bar{P}^2 + 4\bar{P} - 5 = 0$

3) $\bar{P} = 1$ and $\bar{P} = -5$ (drop)

3) → 1):

4) $\bar{Q} = 3$.

EX: Textbook, P. 45, Q1 - Q5.

Digression to Higher-Degree Polynomial Equations

- $p_n(x) = a_o + a_1x + a_2x^2 + ... + a_nx^n = 0.$
  - First, find a solution $\bar{x}_1 \ni p_n(\bar{x}_1) = 0$.
  - Then, $\exists\ h_{n-1}(x) \ni p_n(x) = (x-\bar{x}_1)h_{n-1}(x)$.
  - Find a solution $\bar{x}_2$ such that $h_{n-1}(\bar{x}_2) = 0$.
  - Then, $\exists\ g_{n-2}(x) \ni h_{n-1}(x) = (x-\bar{x}_2)g_{n-2}(x)$.
  - Repeat this procedure until you find $\bar{x}_1, ... , \bar{x}_n$.
- How can we find $h_{n-1}(x)$, $g_{n-2}(x)$?

EX: $x^3 - x^2 - 4x + 4 = 0$

- First, $\bar{x}_1 = 1$.

  → $x^3 - x^2 -4x + 4 = (x-1)h_2(x)$

$$
\begin{array}{lcl}
 & & x^2 \quad\quad -4 \\
x - 1 & : & x^3 - x^2 - 4x + 4 \\
 & & x^3 - x^2 \\
 & & \quad\quad\quad -4x + 4 \\
 & & \quad\quad\quad -4x + 4 \\
 & & \quad\quad\quad\quad\quad\quad 0
\end{array}
$$

- $x^3 - x^2 - 4x + 4 = (x-1)(x^2-4) = (x-1)(x-2)(x+2) = 0.$
- $\bar{x} = 1$ or 2 or -2.

End of Digression

[3] General Equilibrium Model

- Look at multiple markets concurrently.

- Market 1:
  - $Q_{d1} = 10 - 2P_1 + P_2$
  - $Q_{s1} = -2 + 3P_1$
  - $Q_{d1} = Q_{s1}$

- Market 2:
  - $Q_{d2} = 15 + P_1 - P_2$
  - $Q_{s2} = -1 + 2P_2$
  - $Q_{d2} = Q_{s2}$

- Wish to find $\overline{Q}_1$, $\overline{Q}_2$, $\overline{P}_1$ and $\overline{P}_2$.

- At equilibrium:

1) $\overline{Q}_1 = 10 - 2\overline{P}_1 + \overline{P}_2$

2) $\overline{Q}_1 = -2 + 3\overline{P}_1$

3) $\overline{Q}_2 = 15 + \overline{P}_1 - \overline{P}_2$

4) $\overline{Q}_2 = -1 + 2\overline{P}_2$.

- Solution:

2) → 1): $-2 + 3\overline{P}_1 = 10 - 2\overline{P}_1 + \overline{P}_2$

5) $5\overline{P}_1 - \overline{P}_2 = 12$

3) → 4): $15 + \overline{P}_1 - \overline{P}_2 = -1 + 2\overline{P}_2$

→ $\overline{P}_1 - 3\overline{P}_2 = -16$

6) $\overline{P}_1 = 3\overline{P}_2 - 16$

6) → 5): $5(3\overline{P}_2 - 16) - \overline{P}_2 = 12$

→ $14\overline{P}_2 = 92$

7) $\overline{P}_2 = 92/14 = 46/7$

7) → 6):

8) $\overline{P}_1 = 3(46/7) - 16 = 26/7$

8) → 2):

9) $\overline{Q}_1 = -2 + 3(26/7) = 64/7$

7) → 4):

10) $\overline{Q}_2 = -1 + 2(46/7) = 85/7$.

EX: Textbook, P.51, Q3.

[4] National-Income Analysis

1) $Y = C + I_o + G_o$ (subscript "o" means "exogenous".)

2) $C = a + b(Y-T)$

(b = marginal propensity to consume; a = autonomous spendings; $0 < b < 1$; $a > 0$)

3) $T = t_oY$ ($0 < t_o < 1$)

- Wish to find $\bar{Y}$, $\bar{C}$ and $\bar{T}$.
- 3 endogenous variables and 3 equations → solvable.
- Solution:

[Start with simplest equations.]

3) → 2):

4) $\bar{C} = a + b(\bar{Y} - t_o\bar{Y}) = a + b(1-t_o)\bar{Y}$.

[Count endogenous variables in the new equation and find an equation (among the equations in the original system) which has the same endogenous variables.]

4) → 1): $\bar{Y} = a + b(1-t_o)\bar{Y} + I_o + G_o$

→ $[1-b(1-t_o)]\bar{Y} = a + I_o + G_o$

5) $$\bar{Y} = \frac{a + I_o + G_o}{1 - b(1-t_o)}$$

5) → 3):

6) $\bar{T} = t_o \bar{Y} = \dfrac{t_o(a + I_o + G_o)}{1 - b(1 - t_o)}$

5) → 4):

7) $\bar{C} = a + b(1\text{-}t_o) \dfrac{a + I_o + G_o}{1 - b(1 - t_o)}$.

- Look at 5):
  - $m \equiv \dfrac{1}{1 - b(1 - t_o)}$ is called a multiplier.
  - When G_o increases by \$1 billion, income increases by \$m billions.
  - As t_o increases, m falls.

[5] Balanced Budget Model

1) $Y = C + I_o + G_o$

2) $C = a + b(Y - T)$

3) $T = G_o$.

From 3):

4) $\bar{T} = G_o$.

4) → 2):

5) $\bar{C} = a + b(\bar{Y} - G_o)$

5) → 1): $\bar{Y} = a + b(\bar{Y} - G_o) + I_o + G_o$

→ $(1-b)\bar{Y} = a + I_o + (1-b)G_o$

6) $\bar{Y} = \dfrac{a + I_o}{1 - b} + G_o$.

6) → 5):

7) $\bar{C} = a + b\dfrac{a + I_o}{1 - b}$.

EX: Textbook, Q1 - Q3.