TOPIC IV
CALCULUS

[1] Limit

- Suppose that we have a function $y = f(x)$.

  What would happen to y as $x \to x_o$?

Definition:

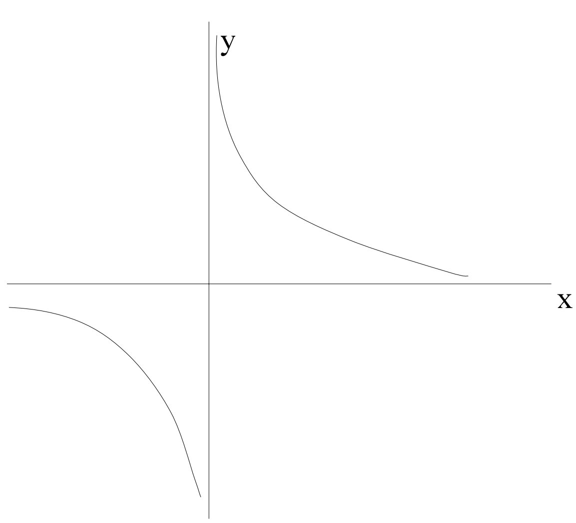
Let $y = f(x)$. Then, the limit value of y as $x \to a$ is denoted by $\lim_{x\to a} f(x)$.

EX 1: $y = 1 + 2x$.

As $x \to 0$, $y \to 1$. $\Rightarrow \lim_{x\to 0} y = 1$.

EX 2: $y = 1/x$, $x \neq 0$.

As $x \to 0$, $y \to \infty$ or $-\infty$ $\Rightarrow$ Limit does not exists.



As $x \to 0$ from the right, $y \to \infty$.

$$\Rightarrow \lim_{x\to o^+} \frac{1}{x} = \infty.$$

As $x \to 0$ from the left, $y \to -\infty$.

$$\Rightarrow \lim_{x\to 0^-} \frac{1}{x} = -\infty.$$

EX 3: $y = \frac{1}{x-1}$.

- $\lim_{x\to 2} y = 1$;
- $\lim_{x\to 1^+} y = \infty$; • $\lim_{x\to 1^-} y = -\infty$.
- $\lim_{x\to\infty} y = 0$; • $\lim_{x\to -\infty} y = 0$.

EX 4: $y = \frac{1-x^2}{1-x}$, $x \neq 1$.

- $\lim_{x\to 1} y = \frac{0}{0} = 0$ (?)

$\Rightarrow$ Nope!!!.

$\Rightarrow \lim_{x\to 1} \frac{1-x^2}{1-x} = \lim_{x\to 1} \frac{(1-x)(1+x)}{1-x} = \lim_{x\to 1}(1+x) = 2$!!!

EX 5: $y = \frac{x^2}{x}$, $x \neq 0$.

- $\lim_{x\to 0} y = \lim_{x\to 0} x = 0$.
- $\lim_{x\to\infty} y = \lim_{x\to\infty} x = \infty$.

Note:

- Before computing a limit, better to simplify the function f.
- Although f(x) may not be defined at $x = x_o$, f(x) could have a limit.

EX 6: $y = \frac{x+a}{x}$.

- $\lim_{x\to\infty} y = \lim_{x\to\infty}(1 + \frac{a}{x}) = 1$.

Theorem:

Suppose that lim f(x) and lim g(x) exists. (Here, lim = $\lim_{x\to x_o}$.) Then,

(R1) lim [f(x) ± g(x)] = lim f(x) ± lim g(x).

(R2) lim[f(x)g(x)] = [lim f(x)][lim g(x)]

(R3) lim[f(x)/g(x)] = [lim f(x)]/[lim g(x)], if lim g(x) ≠ 0.

EX: $y = e^x/x^2$.

- $\lim_{x\to\infty} y = [\lim_{x\to\infty} e^x]/[\lim_{x\to\infty} x^2]$ (?)

  ⇒ Nope!!!, since $\lim_{x\to\infty} e^x$ and $\lim_{x\to\infty} x^2$ do not exist.

  ⇒ In fact, $\lim_{x\to\infty} e^x/x^2 = \infty$.

- $\lim_{x\to 0} y = [\lim_{x\to 0} e^x]/[\lim_{x\to 0} x^2]$ (?)

  ⇒ Nope!!!, since $\lim_{x\to 0} x^2 = 0$.

  ⇒ In fact, $\lim_{x\to 0} y = \infty$.

EX: $y = (x^2-1)/(x-1)$, $x \neq 1$.

- $\lim_{x\to 1} y = [\lim (x^2-1)]/[\lim (x-1)]$ (?)

  ⇒ Nope!!!, since $\lim_{x\to 1}(x-1) = 0$.

  ⇒ In fact, $\lim_{x\to 1} y = \lim_{x\to 1}(x+1) = 2$.

EX: $y = (x^2+2)/(x^2+2x+2)$.

- $\lim_{x\to\infty} y = [\lim_{x\to\infty}(x^2+2)]/[\lim_{x\to\infty}(x^2+2x+2)]$ (?)

  $\Rightarrow$ Nope!!!, since $\lim_{x\to\infty}(x^2+2)$ does not exists.

- $\lim_{x\to\infty} \frac{x^2+2}{x^2+2x+2} = \lim_{x\to\infty} \frac{1+2/x^2}{1+2/x+2/x^2}$

$$= [\lim (1+2/x^2)]/[\lim (1+2/x+2/x^2)]$$

$$= 1.$$

L'Hôpital's Theorem

Suppose that we have two functions f(x) and g(x). Suppose:

$\lim f(x) = 0$ and $\lim g(x) = 0$,

or, $\lim f(x) = \pm \infty$ and $\lim g(x) = \pm \infty$.

Then, $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$.

[2] Derivative

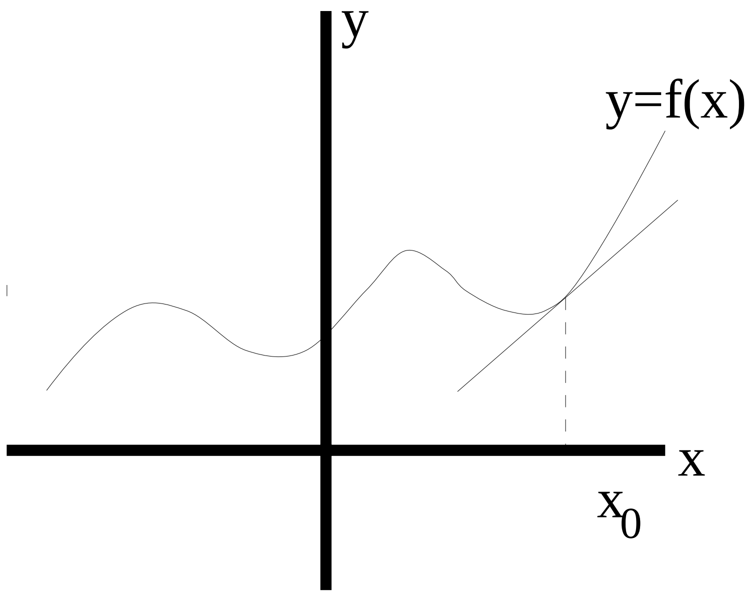
- $y = f(x)$
- We want to know changes in y (Δy) when x changes by Δx.
- rate of change = $\frac{\Delta y}{\Delta x}$
- $$\left.\frac{\Delta y}{\Delta x}\right|_{x=x_0} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$
- $$\frac{dy}{dx} = y' = f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=x_0} = \text{derivative evaluated at } x_0 = f'(x_0)$$

Note:

If $\lim\limits_{\Delta x \to 0^+} \neq \lim\limits_{\Delta x \to 0^-}$, then, we say that derivative does not exist.

Question: What is derivative?



- / : tangent line at $x = x_0$
- $f'(x_0)$: measures slope of tangent line

[3] Continuity vs. differentiability

Definition:

Consider a function y = f(x). Suppose:

1) f is defined at x_0;

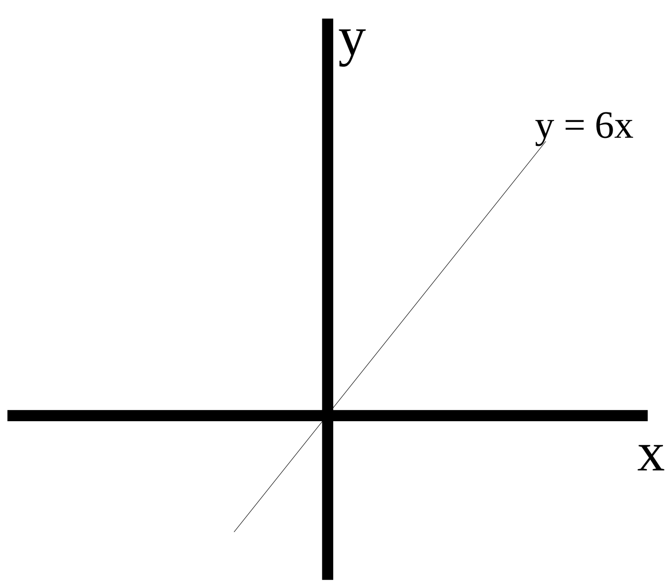
2) $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = \lim_{x \to x_0} f(x)$;

3) $\lim_{x \to x_0} f(x) = f(x_0)$.

Then, f is called continuous at $x = x_0$.

EX 1) y = f(x) = 6x, for all x.

- Is this function continuous at x = 1?



1) f(1) = 6;

2) $\lim_{x \to 1} y = 6$;

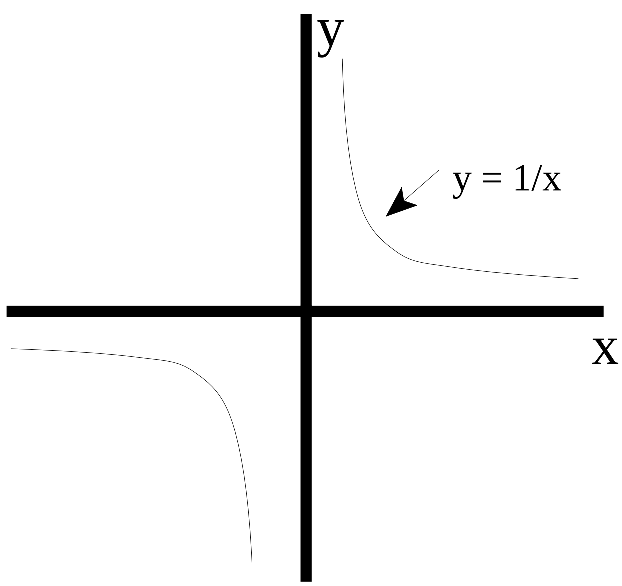
3) $\lim_{x \to 1} y = f(1)$

⇒ Continuous.

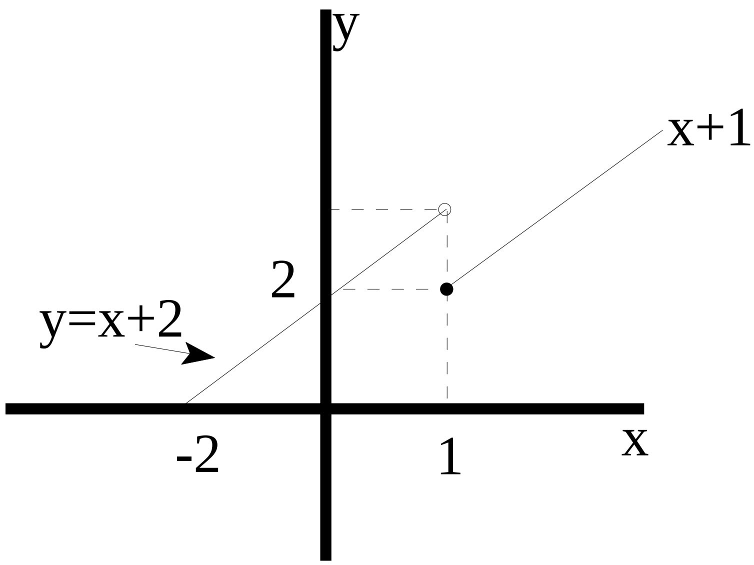
EX 2) $y = \dfrac{1}{x}$

• Is this function continuous at x = 0

1) f(0) not defined.



⇒ y is not continuous at x = 0.



EX 3) $y = \begin{cases} x+1 & \text{if } x \geq 1 \\ x+2 & \text{if } x < 1 \end{cases}$

1) f(1) = 2.

2) $\lim_{x \to 1^+} f(x) = 2; \ \lim_{x \to 1^-} f(x) = 3$

$\Rightarrow$ Not continuous.

Definition:

Suppose that at x_0,

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \text{ exists.}$$

Then, we say thaty f is differentiable at x_0.

Note:

- If $\lim_{\Delta x \to 0^+} \neq \lim_{\Delta x \to 0^-}$, we say that derivative does not exist.

Theorem:

If f is differentiable at x_0, then, f is continuous at x_0.

<proof>

Note that:

$$f(x) - f(x_0) = \frac{f(x) - f(x_0)}{x - x_0} \cdot (x - x_0).$$

Then,

$$\lim_{x \to x_0} [f(x) - f(x_0)] = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \cdot (x - x_0) = f'(x_0) \cdot 0 = 0.$$

Thus, $\lim_{x \to x_0} f(x) - f(x_0) = 0 \Rightarrow \lim_{x \to x_0} f(x) = f(x_o)$.

Corollary:

If f is not continuous, then f is not differentiable.

EX 1) $y = \begin{cases} x+1 & x \geq 1 \\ x+2 & x < 1 \end{cases}$

- Not continuous at x = 1

$\Rightarrow$ not differentiable.

Note:

- If a function is continuous, it may or may not be differentiable.

Definition:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

EX 1) |-4| = 4; |4| = 4

EX2) y = f(x) = |x-2| + 1

$$\Rightarrow y = \begin{cases} x-1 & \text{if } x \geq 2 \\ -x+3 & \text{if } x < 2 \end{cases}$$

$\Rightarrow$ Continuous?

1) f(2) defined and f(2) = 1

2) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = 1$

3) $\lim_{x \to 2} f(x) = f(2)$.

$\Rightarrow$ So, f is continuous.

$\Rightarrow$ Differentiable?

$$\lim_{\Delta x \to 0^+} \frac{f(2 + \Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{\{(2 + \Delta x) - 1\} - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0^+} \frac{\Delta x}{\Delta x} = 1$$

$$\lim_{\Delta x \to 0^-} \frac{f(2 + \Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{\{-(2 + \Delta x) + 3\} - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0^-} -\frac{\Delta x}{\Delta x} = -1$$

⇒ not differentiable

[4] Rules for Derivatives

(1) $y = f(x) = k.$

$$\frac{dy}{dx} = 0.$$

(2) $y = ax^n$

$y' = anx^{n-1}$

EX 1) $y = 3x^6$

$\Rightarrow y' = 3 \times 6 \times x^{6-1} = 18x^5$

EX 2) $y = 3\sqrt{x} = 3x^{\frac{1}{2}}$

$$\Rightarrow y' = 3 \times \frac{1}{2} \times x^{\frac{1}{2}-1} = \frac{3}{2}x^{-\frac{1}{2}} = \frac{3}{2}\frac{1}{x^{\frac{1}{2}}} = \frac{3}{2\sqrt{x}}$$

(3) $$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x) = f'(x) + g'(x)$$

EX) $y = 2x^2 + x$

$$\Rightarrow \frac{dy}{dx} = 4x + 1.$$

(4) $$\frac{d}{dx}(f(x)\,g(x)) = \left(\frac{d}{dx}f(x)\right)g(x) + f(x)\left(\frac{d}{dx}g(x)\right)$$

(5) $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\left(\frac{d}{dx}f(x)\right)g(x) - f(x)\left(\frac{d}{dx}g(x)\right)}{[g(x)]^2}.$$

EX) $f(x) = 6x^2 + 6x$

$g(x) = 3x^3 + 6x^2$

$\Rightarrow f'(x) = 12x + 6;\ g'(x) = 9x^2 + 12x$

$$\Rightarrow \frac{d}{dx}\left(f(x)\cdot g(x)\right) = (12x + 6)(3x^3 + 6x^2) + (6x^2 + 6x)(9x^2 + 12x)$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{(12x + 6)(3x^3 + 6x^2) - (6x^2 + 6x)(9x^2 + 12x)}{(3x^3 + 6x^2)^2}$$

[5] Chain Rule

- $x \to y \to z$
  - $z = f(y)$
  - $y = g(x)$
  - $\dfrac{dz}{dx} = \dfrac{dz}{dy}\dfrac{dy}{dx} = f'(y) \cdot g'(x)$

- $x \to y \to z \to w$
  - $\dfrac{dw}{dx} = \dfrac{dw}{dz}\dfrac{dz}{dy}\dfrac{dy}{dx}.$

EX 1) $z = 3y^2$; $y = 2x + 5$

$\Rightarrow x \to y \to z$

$\Rightarrow \dfrac{dz}{dx} = \dfrac{dz}{dy}\dfrac{dy}{dx} = (6y) \bullet 2 = 12y = 12(2x + 5) = 24x + 60$

$\Rightarrow \left.\dfrac{dz}{dx}\right|_{x=1} = 24 \times 1 + 60 = 84$

EX 2) $y = 3(2x^2+1)^3$.

$\Rightarrow$ Set $z = 2x^2 + 1$.

$\Rightarrow$ Then, $y = 3z^3$; $z = 2x^2 + 1$.

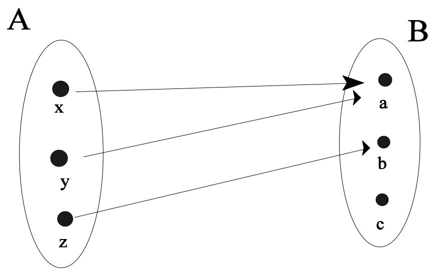
$\Rightarrow dy/dx = (dy/dz)(dz/dx) = (9z^2)(4x) = 36xz^2 = 36x(2x^2+1)^2$.

EX 3) $TR = f(Q)$; $Q = g(L)$

$\Rightarrow \dfrac{dTR}{dL} = \dfrac{dTR}{dQ} \cdot \dfrac{dQ}{dL} = MR \bullet MP_L = MRP_L$

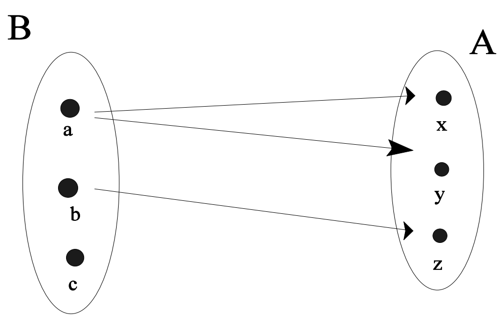
[6] Inverse Function

(EX 1) Consider the following example.

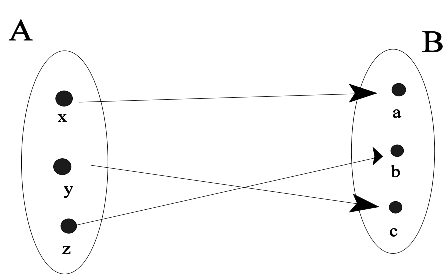


This is a function.

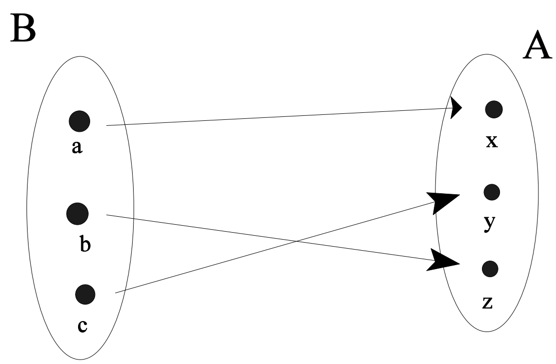
- Looking at the reversed relation:



(EX 2) Consider the following example.



Now, look at the reversed relation:



Definition:

If the reversed relation is also a function, we say that there exists the inverse function, and we denote it by f^{-1}.

Definition:

A function $y = f(x)$ is strictly increasing (decreasing) iff $f(x_1) > f(x_2)$, whenever $x_1 > (<)\ x_2$.

Theorem:

If a function $y = f(x)$ is strictly increasing or strictly decresing, then $x = f^{-1}(y)$ (inverse function) exists.

Note:

(1) “Function”: If you know what happened yesterday (cause), you can predict what will happen today (result).

(2) “Inverse function”: If you know what happens today (result), you can figure out what happened yesterday (cause).

EX) $y = f(x) = 5x + 25, -\infty < x < \infty$.

$\Rightarrow x = y/5 - 5 = f^{-1}(y)$.

Theorem:

Suppose that a function y = f(x) has an inverse function. Then,

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} .$$

EX) For the above example,

- dy/dx = 5 ; dx/dy = 1/5.
- Clearly, dx/dy = 1/(dy/dx).

[7] Partial Differentiation

- Consider a function:

  $y = f(x_1,x_2,...,x_n)$,

  where x_i's are independent variables and y is a dependent variables.

  Suppose we would like to know the rate of change of y when x_1 changes.

Definition:

$$\frac{\partial y}{\partial x_1} = \frac{dy}{dx_1}\Big| ceteris\ \ paribus = \lim_{\Delta x_1 \to \infty} \frac{\Delta y}{\Delta x_1} .$$

(Here, we treat all other variables as constant!!!!)

EX 1) $y = 3x_1^2 + x_1x_2 + 4x_2^2$

- $\partial y/\partial x_1 = 6x_1 + x_2$.
- $\partial y/\partial x_2 = x_1 + 8x_2$.

EX 2) $y = (x_1 + 4)(3x_1 + 4x_2)$.

- $\partial y/\partial x_1 = [\partial(x_1+4)/\partial x_1](3x_1+4x_2)$

  $+ (x_1+4)[\partial(3x_1+4x_2)/\partial x_1]$

  $= 3x_1 + 4x_2 + (x_1+4)\ 3 = 6x_1 + 4x_2 + 12.$
- $\partial y/\partial x_2 = 4x_1 + 16$ (Show this at home.)

EX 3) $y = \dfrac{x_1x_2^2}{x_1 - x_2}$.

- $$\frac{\partial y}{\partial x_1} = \frac{\frac{\partial(x_1 x_2^2)}{\partial x_1}(x_1 - x_2) \; - \; (x_1 x_2^2)\frac{\partial(x_1 - x_2)}{\partial x_1}}{(x_1 - x_2)^2}$$
$$= \frac{x_2^2(x_1 - x_2) \; - \; x_1 x_2^2(1)}{(x_1 - x_2)^2} = \frac{-x_2^3}{(x_1 - x_2)^2} \; .$$

- Simliarly,

  $\partial y/\partial x_2 = (2x_1^2 x_2 - x_1 x_2^2)/(x_1 - x_2)^2$.

[8] Jacobian Determinant

- There are n equations:

$$y_1 = f^1(x_1, x_2, \ldots x_n, x_{n+1}, x_{n+2}, \ldots x_m)$$

$$y_2 = f^2(x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_m)$$

$$\vdots$$

$$y_n = f^n(x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_m).$$

- Let's treat $x_{n+1}, \ldots x_m$ as given. (Treat them as constants)
- Sometimes, we wish to know whether all the functions are distinctive, that is, whether there are redundant functions or not.
- For this case, we use Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_n} \end{pmatrix}$$

- If $|J| = 0$, we say that the functions are functionally dependent.
- If $|J| \neq 0$, we say that the functions are functionally independent.

EX 1) $y_1 = 2x_1^2 + x_2^3;\ y_2 = x_1^2 x_2$

$$\frac{\partial y_1}{\partial x_1} = 2 \times 2x_1 + 0; \quad \frac{\partial y_1}{\partial x_2} = 3x_2^2$$

$$\frac{\partial y_2}{\partial x_1} = 2x_1x_2 \quad ; \quad \frac{\partial y_2}{\partial x_2} = x_1^2$$

$$\Rightarrow |J| = \begin{vmatrix} 4x_1 & 3x_2^2 \\ 2x_1x_2 & x_1^2 \end{vmatrix} = 4x_1^3 - 6x_1x_2^3 \neq 0$$

EX 2) $y_1 = x_1 + 2x_2$

$$\Rightarrow \quad y_2 = x_1^2 + 4x_1x_2 + 4x_2^2 = (x_1 + 2x_2)^2$$

$$\Rightarrow \quad |J| = \begin{vmatrix} 1 & 2 \\ 2x_1 + 4x_2 & 4x_1 + 8x_2 \end{vmatrix} = 4x_1 + 8x_2 - 2(2x_1 + 4x_2) = 0$$

[9] Differentials

- $\dfrac{dy}{dx}$: derivatives

- dy, dx : differentials (small changes)

- $\dfrac{dy}{dx} = f'(x) \Rightarrow dy = f'(x)dx$

EX) $y = 3x^2 + 7x$

$\Rightarrow \dfrac{dy}{dx} = f'(x) = 6x + 7$

$\Rightarrow dy = (6x + 7)dx$

One important application of differentials:

- ε_d (price-elasticity of demand)

$$= \frac{\%\ \text{changes in demand}}{\%\ \text{changes in price}}$$

= % changes in demand when price changes by 1%

$$= \frac{\frac{\Delta Q_d}{Q_d}}{\frac{\Delta P}{P}} \approx \frac{\frac{dQ_d}{Q_d}}{\frac{dP}{P}} = \frac{dQ}{dP} \cdot \frac{P}{Q_d}$$

EX) $Q_d = 100 - 2P$

$$\Rightarrow \frac{dQ_d}{dP} = -2$$

$$\Rightarrow \frac{P}{Q_d} = \frac{P}{100 - 2P}$$

$$\Rightarrow \varepsilon_d = (-2) \times \frac{P}{100 - 2P} = \frac{P}{P - 50}$$

$$\Rightarrow \text{When } P = 10,\ \varepsilon_d = \frac{10}{10 - 50} = -\frac{1}{4} = -0.25$$

Total differentiation:

- $y = f(x_1, \ldots x_n)$
- Denote $f_j = \frac{\partial y}{\partial x_j}$
- Then,
    - $dy = f_1 dx_1 + f_2 dx_2 + \ldots + f_n dx_n$
    - $dy\Big|_{dx_2 = \cdots = dx_n = 0} = f_1 dx_1.$

EX) $U = U(x_1, x_2)$

$$\Rightarrow dU = \frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = U_1 dx_1 + U_2 dx_2$$

⇒ Suppose that dU = 0. Then, $U_1dx_1 + U_2dx_2 = 0$

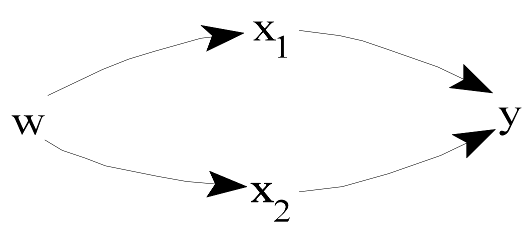
⇒ $\left.\frac{dx_2}{dx_1}\right|_{dU = 0} = -\frac{U_1}{U_2}$ (slope of indifference curve)

EX) $y = x_1^2 + 2x_1x_2 + 4x_2^2$

⇒ $dy = (2x_1 + 2x_2)dx_1 + (2x_1 + 8x_2)dx2$

[10] Total derivatives

- CASE I: $y = f(x_1,x_2)$; $x_1 = g(w)$; $x_2 = h(w)$



- $$\frac{dy}{dw} = \frac{\partial y}{\partial x_1}\frac{dx_1}{dw} + \frac{\partial y}{\partial x_2}\frac{dx_2}{dw}$$

EX 1) $y = x_1^2 x_2^3$; $x_1 = 6w + 6$; $x_2 = 3w^2 + 4w$

$$\Rightarrow \frac{dy}{dw} = \frac{\partial y}{\partial x_1}\frac{dx_1}{dw} + \frac{\partial y}{\partial x_2}\frac{dx_2}{dw}$$

$$= (2x_1x_2^3)6 + (3x_1^2x_2^2)(6w + 4)$$

$$= 12(6w+6)(3w^2+4w) + 3(6w+6)^2(3w^2+4w)^2(6w+4)$$

$$= 72(w+1)(3w^2+4w) + 108(w+1)^2(3w^2+4w)^2(6w+4).$$

EX 2) $y = (6w+6)^2(3w^2+4w)^3$

$\Rightarrow$ Set $x_1 = 6w + 6$; $x_2 = 3w^2 + 4w$.

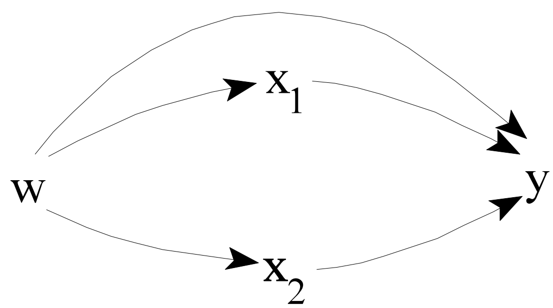
$$\Rightarrow \frac{dy}{dw} = \frac{\partial y}{\partial x_1}\frac{dx_1}{dw} + \frac{\partial y}{\partial x_2}\frac{dx_2}{dw}$$

- CASE II:

  $y = f(x_1, x_2, w)$ — ①

  $x_1 = g(w)$ — ②

  $x_2 = h(w)$ — ③



- $$\frac{dy}{dw} = \frac{\partial f}{\partial x_1}\frac{dx_1}{dw} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dw} + \frac{\partial f}{\partial w}$$

Note:

- dy/dw = total derivative.
- $\partial f/\partial w$ = partial derivative from ①

EX 1) $y = x_1^2 + x_2 + w$; $x_1 = 6w$; $x_2 = 3w^2$

$$\Rightarrow \frac{dy}{dw} = \frac{\partial y}{\partial x_1}\frac{dx_1}{dw} + \frac{\partial y}{\partial x_2}\frac{dx_2}{dw} + \frac{\partial y}{\partial w}$$

$$= (2x_1) \times 6 + 1 \times 6w + 1 = 12(6w) + 6w + 1 = 78w + 1.$$

EX 2) $y = 3x - w^2$; $x = 2w^2 + w + 4$

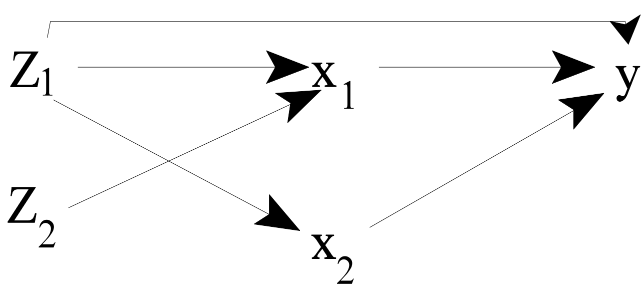
Show that $\frac{dy}{dw} = 10w + 3.$

Total Partial Derivatives:

$y = f(x_1, x_2, z_1, z_2)$ — ①

$x_1 = g(z_1, z_2)$ — ②

$x_2 = h(z_1, z_2)$ — ③

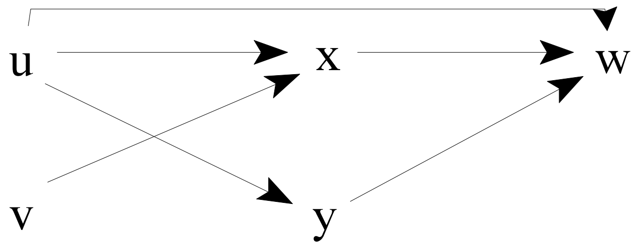


- Want to know $\left.\frac{dy}{dz_1}\right|_{dz_2 = 0} = \frac{\partial y}{\partial z_1}$ (in textbook, $\frac{\xi y}{\xi z_1}$)

- $\frac{\partial y}{\partial z_1} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial z_1} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial z_1} + \frac{\partial f}{\partial z_1}$

fr.① fr. ② fr.① fr.③ fr.①

EX 3) $w = ax^2 + bxy + cu$; $x = \alpha u + \beta v$; $y = \gamma u$



$$\frac{\partial w}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial u}.$$

$$= (2ax + by)\alpha + (bx)\gamma + c$$

$$= 2a\alpha x + b\alpha y + b\gamma x + c$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} = (2ax + by) \bullet \beta$$

$$dw = \frac{\partial w}{\partial u}du + \frac{\partial w}{\partial v}dv$$