- **I recommend you to solve the following exercise questions in the textbook.**
- **The answers are also given below.**
- **When you have questions about any of the problems listed below, contact Ms. Maggie Dimitrova. Her office hours are M. 12:40-1:30 and W. 3:00-3:50.**

1. Introduction and Set Theory
   1) p. 17: Q2 - Q8.
   2) p. 23: Q1 - Q6.

2. Equilibrium Model
   3) p. 40: Q1 - Q5.
   4) p. 45: Q1 - Q5.
   5) p. 51: Q3.
   6) p. 53: Q1 - Q3.

3. Linear Algebra
   7) pp. 66-67: Q1(d); Q4(b); Q5(d).
   8) pp. 78-79: Q3.
   9) pp. 81-82: Q1(a),(c); Q2(b),(d); Q3(a).
   10) p. 87: Q2; Q4; Q6.
   11) p. 98: Q1(c),(f); Q2; Q3; Q4.
   12) p. 103: Q2; Q3; Q4(a); Q6(a).
   13) p. 107: Q2(a),(b); Q4(a).
   14) p. 112: Q1(a); Q2; Q3(d).
   15) p. 115: Q1; Q2.
   16) p. 124: Q4.

4. Calculus
   17) p. 131: Q1; Q3.
   18) p. 140: Q1 - Q3.
   19) p. 144: Q3(a).
   20) pp. 153-154: Q1; Q4; Q5.
   21) p. 159: Q1(a),(c); Q2(a),(c); Q3(c),(d),(e).
   22) p. 169: Q1; Q2; Q3(f); Q7(d); Q8(d).
   23) pp. 173-174: Q1; Q3(a); Q4; Q6(a).
   24) pp. 177-178: Q1(a),(d); Q2(a),(d); Q4; Q5.
   25) p. 186: Q1(a),(b).

5. Comparative Statics.
   26) pp. 193-194: Q1(a); Q2; Q3; Q4.
   27) pp. 195-196: Q1(a); Q2(b); Q3; Q5.
   28) p. 198: Q2(a).
   29) pp. 203-204: Q2(a); Q2(b); Q3; Q4(b).
   30) p. 214: Q2(a); Q3; Q5; Q7.
   31) pp. 225-226: Q1; Q2; Q4; Q5; Q6.

6. Unconstrained Optimization I.

32) p. 239: Q1(d); Q2(b); Q4.
33) pp. 244-245: Q1(c); Q2.
34) pp. 253-254: Q1(c),(d); Q3(b),(c,(d),(e); Q6.
35) p. 267: Q2(c).
36) pp. 273-274: Q3(c).
37) pp. 282-283: Q3(a), Q4(b).
38) p. 287: Q1(b); Q2(e); Q3(f); Q4.
39) pp. 291-292: Q4; Q5.
40) pp. 297-298: Q1(e),(f),(g),(h); Q3(e),(f),(g),(h); Q4(d),(e),(f).
41) pp. 301-302: Q1.
42) pp. 305-306: Q1(a),(e); Q2; Q3; Q4; Q7(a); Q9.

7. Unconstrained Optimization II.
43) pp. 318-319: Q1; Q4.
44) p. 331: Q1(a); Q4(a),(e); Q6(a).
45) p. 337: Q2; Q4.
46) p. 352: Q1; Q5; Q6.

8. Optimization with Equality Constraints.
47) p. 378: Q1; Q3.
48) p. 399-400: Q4(b),(c); Q8(b).
49) pp. 409-410: Q2.
50) pp. 417-418: Q1(b),(d); Q3(a); Q4; Q6(a); Q8.
51) pp. 430-431: Q2; Q5(a); Q8; Q9(a).

ANSWERS TO HOMEWORK PROBLEMS

1. Introduction and Set Theory.

(1) p.17
Q2. True statements are d), f), g) and h).
Q3. a) {2,4,6,7}; b) {2,4,6}; c) {2,6}; d) {2}; e) {2}; f) {2,4,6}.
Q4. All are valid.
Q7. $\varnothing$, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}
Q8. 16 subsets

2) p.23
Q1. a) {(3,a), (3,b), (6,a), (6,b), (9,a), (9,b)}
b) {(a,m), (a,n), (b,m), (b,n)}
c) {(m,3), (m,6), (m,9), (n,3), (n,6), (n,9)}
Q2. {(3,a,m),(3,a,n),(3,b,m),(3,b,n),(6,a,m),(6,a,n),(6,b,m),(6,b,n),(9,a,m),(9,a,n),(9,b,m), (9,b,n)}
Q3. No. When S1=S2
Q4. None.
Q5. Range = {y/8$\leq$y$\leq$17}
Q6. The range is the set of all nonpositive real numbers.

2. Equilibrium Model.

(3) p.40
Q1. P=29/9, Q=158/9
Q2. a) P=61/9, Q=276/9; b) P=36/7, Q=138/7
Q4. No

(4) p.45
Q1. a) $x_1 = 5$, $x_2 = 3$; b) $x_1 = 4$, $x_2 = -2$
Q2. a) $x_1 = 5$, $x_2 = 2$; b) $x_1 = 4$, $x_2 = -2$
Q3. a) $P_1 = 1$; $P_2 = -5$; b) $x_1 = 2$, $x_2 = x_3 = -2$; c) $x_1 = 4$, $x_2 = 1$, $x_3 = 2$; d) $x_1 = 0$, $x_2 = 4$, $x_3 = 1$.
Q4. $x^3 - 10x^2 + 11x + 70 = 0$
Q5. a) P=1, Q=2; b) $P=\sqrt{5}$, $Q=3$.

(5) p. 51
Q3. $P_1 = \frac{57}{17}; P_2 = \frac{59}{17}; Q_1 = \frac{194}{17}; Q_2 = \frac{143}{17}$.

(6) p.53
Q1. a) Three endogenous variables;
b) $Y = \frac{a - bd + I_0 + G_0}{1 - b(1-t)}$, $T = \frac{d(1-b) + t(a + I_0 + G_0)}{1 - b(1-t)}$, $C = \frac{a - bd + b(1-t)(I_0 + G_0)}{1 - b(1-t)}$.
Q2. a) Y, C and G; c) $Y = \frac{a - bT_0 + I_0}{1 - b - g}$; d) b+g$\neq$1.
Q3. Y=121, C=91.

3. Linear Algebra.

(7) p.66-67

Q1. d) $\begin{bmatrix} 20 & 18 \\ 24 & -6 \end{bmatrix}$.

Q4. b) $[7a+c \quad 2b+4c]$

Q5. d) $a_1 + a_2x + a_3x^2 + ... + a_nx^{n-1}$.

(9) p. 81-82

Q1. a) $\begin{bmatrix} -1 & 8 & 7 \\ 0 & -2 & 4 \end{bmatrix}$; c) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Q2. b) $\begin{bmatrix} 39 \\ -12 \end{bmatrix}$; d) $\begin{bmatrix} -x_1 & 8x_1 - 2x_2 & 7x_1 + 4x_2 \end{bmatrix}$

Q3. a) 4*3

(10) p. 87

Q4. D and F are inverses and E and G are inverses

Q6. A and (X'X) must be square

(11). p. 98

Q1. c) 0; f) 8xy+2x-30

Q2. +, -, +, -, -.

Q3. $|M_a| = \begin{vmatrix} e & f \\ h & i \end{vmatrix}$; $|M_b| = \begin{vmatrix} d & f \\ g & i \end{vmatrix}$; $|M_f| = \begin{vmatrix} a & b \\ g & h \end{vmatrix}$; $|C_a| = |M_a|$; $|C_b| = -|M_b|$; $|C_f| = -|M_f|$.

Q4. a) 72; b) -81

(12) p. 103

Q3. a) property IV; b) property III

Q4. a) nonsingular

Q6. a) no

(13) p. 107

Q2. a) $A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$; b) $B^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ -9 & 1 \end{bmatrix}$.

Q4. a) $E^{-1} = \frac{1}{8}\begin{bmatrix} 3 & 2 & -9 \\ -1 & 2 & -5 \\ -6 & -4 & 26 \end{bmatrix}$

(14) p.112

Q1. a) $x_1 = 5, x_2 = 2$

Q2. $A^{-1}=\frac{1}{7}\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix}$

Q3. d) $x=\frac{1}{2}(b+c), y=\frac{1}{2}(a+c), z=\frac{1}{2}(a+b)$

(15) p. 115

Q1. $\begin{bmatrix}Y\\ C\\ T\end{bmatrix}=\frac{1}{1-b+bt}\begin{bmatrix}I_0+G_0+a-bd\\ b(1-t)(I_0+G_0)+a-bd\\ t(I_0+G_0)+at+d(1-b)\end{bmatrix}$

Q2. $\begin{bmatrix}Y\\ C\\ G\end{bmatrix}=\frac{1}{1-b-g}\begin{bmatrix}I_0+a-bT_0\\ bI_0+(1-g)(a-bT_0)\\ g(I_0+a-bT_0)\end{bmatrix}$

(16) p. 124

Q4. d) $\begin{bmatrix}0.95 & -0.25 & -0.34\\ -0.33 & 0.9 & -0.12\\ -0.19 & -0.38 & 1.00\end{bmatrix}*\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}1800\\ 200\\ 900\end{bmatrix}$

4. Calculus

(17) p. 131

Q1. a) $\frac{\Delta y}{\Delta x}=8x+4\Delta x$; b) 8x; c)$f'(3)=24$, $f''(4)=32$

Q3. a) $\frac{\Delta y}{\Delta x}=5$, a constant function; b) No, $\frac{dy}{dx}=5$

(18) p. 140

Q1. Left-side limit = right-side limit=15. Yes.

Q2. a) 8; b) 20; c) $a^2 + 4a + 8$

Q3. a) 5; b) 5

(19) p. 144

Q3. a) $-7 \prec x \prec 5$

(20) pp. 153-154

Q4. a) no; b) no; c) 9

Q5. No.

(21) p. 159

Q1. a) $13x^{12}$; c) $42x^5$

Q2. a) $4x^{-5}$; c) $36w^3$

Q3. c) f'(1) = 10, f'(2) = 10/8; d) f'(1) = 1, f'(2) = $2^{1/3}$; e) f'(1) = 2, f'(2) = $2^{1/3}$.

(22) p. 169
Q1. $VC = Q^3 - 5Q^2 + 14Q$; $MC = 3Q^2 - 10Q + 14$.
Q2. $MC = 3Q^2 - 8Q + 214$.
Q3. f) $\frac{x^2-3}{x^2}$
Q7. d) $\frac{acx^2+2adx-bc}{(cx+d)^2}$
Q8. d) $\frac{-b}{x^2}$

(23) p. 173-174
Q1. $-6x(5-x^2)^2$
Q3. a) $18x(3x^2-13)^2$
Q4. $-32(16x+3)^{-3}$
Q6. a) yes

(24) p. 177-178
Q1. a) $\frac{\partial y}{\partial x_1}=6x_1^2-22x_1x_2$; $\frac{\partial y}{\partial x_2}=-11x_1^2+6x_2$; d) $\frac{\partial y}{\partial x_1}=\frac{4}{x_2-2}$.
Q2. a) $f_x = 2x + 5y$; $f_y = 5x - 3y^2$; d) $f_x=\frac{x^2+1}{x^2y}$, $f_y=-\frac{x^2-1}{xy^2}$.
Q4. $MPP_K=(o.3)96K^{-0.7}L^{0.7}$, $MPP_L=(o.7)96K^{0.3}L^{-0.3}$
Q5. a) $U_1=2(x_1+2)(x_2+3)^3$, $U_2=3(x_1+2)^2(x_2+3)^3$; b) $U_1(3,3)=2160$

(25) p. 186
Q1. a) the functions are dependent; b) the functions are independent

5. Comparative statics.

(26) pp. 193-194
Q1. a) $dy = -3(x^2+1)dx$
Q2. $\varepsilon_{MY}=\frac{dM/dY}{M/Y}$
Q3. a) $\frac{dC}{dY}=b$, $\frac{C}{Y}=(a+bY)/Y$.
Q4. a) no; b) Q=k/P

(27) pp. 195-196
Q1. a) $dz=(6x+y)dx+(x-6y^2)dy$

Q2. b) $dy = 2\left(\frac{x_2}{x_1+x_2}\right)^2 dx_1 + 2\left(\frac{x_1}{x_1+x_2}\right)^2 dx_2$

Q3. $\varepsilon_{QP} = \frac{2bP^2}{(a+bP^2+R^{1/2})}$, $\varepsilon_{QR} = \frac{R^{1/2}}{2(a+bP^2+R^{1/2})}$.

Q5. $\varepsilon_{XP} = \frac{-2}{Y_f^{1/2}P^2+1}$.

(28) p. 198

Q2. a) $dy = \frac{x_2 dx_1 - x_1 dx_2}{(x_1+x_2)^2}$

(29) pp. 203-204

Q2. a) $3t^2+60t-21$; b) $14t+1$

Q3. $[a\alpha A/K + b\beta A/L + A'(t)]K^{\alpha}L^{\beta}$

Q4. b) $\frac{\partial W}{\partial u} = 10uf_1 + f_2$, $\frac{\partial W}{\partial v} = 3f_1 - 12v^2 f_2$.

(30) p. 214

Q2. a) yes (explain)

Q3. Yes.

Q5. -1

Q7. $\frac{\sigma Y}{\sigma \gamma} = \frac{-\beta}{1-\beta+\beta\delta}$, $\frac{\sigma Y}{\sigma \delta} = \frac{-\beta Y}{1-\beta+\beta\delta}$

(31) pp 225-226

Q1. b) the implicit function theorem is applicable; c) $\frac{1}{S'+T'-I'}$

Q2. a) $F(P;Y_0;T_0) = D(P,Y_0) - S(P,T_0)$

b) the implicit function theorem is applicable and the identity is $D(P,Y_0) - S(P,T_0) = 0$.

c) $\frac{\partial P}{\partial Y_0} = \frac{D_{Y_0}}{D_P - S_P}$; $\frac{\partial P}{\partial T_0} = \frac{-S_{T_0}}{D_P - S_P}$.

d) $\frac{\sigma Q}{\sigma Y_0} = \frac{\sigma S}{\sigma P}\left(\frac{\sigma P}{\sigma Y_0}\right)$, $\frac{\sigma Q}{\sigma T_0} = \frac{\sigma D}{\sigma P}\left(\frac{\sigma P}{\sigma T_0}\right)$

Q4. a) $F(P;t_0,Q_{s0}) = D(P,t_0) - Q_{s0} = 0$.

b) the implicit function theorem applies.

c) $\frac{\partial P}{\partial t_0} = -\frac{\frac{\partial D}{\partial t_0}}{\frac{\partial D}{\partial P}}$

Q5. $\frac{\partial Y}{\partial M_{s0}} = -\frac{I' - S_i}{|J|}$, $\frac{\partial i}{\partial M_{s0}} = -\frac{S_Y + M'}{|J|}$

Q6. a) yes; b) kY+L(i); c) $\frac{\partial Y}{\partial M_{s0}} = \frac{I'}{|J|} > 0; \frac{\partial i}{\partial M_{s0}} = \frac{1 - C'}{|J|} < 0; \frac{\partial Y}{\partial G_{s0}} = \frac{L'}{|J|} > 0; \frac{\partial i}{\partial G_0} = -\frac{k}{|J|} > 0.$

6. Unconstrained Optimization I.

(32) p. 239
Q1. d) f(1) = -1 is a relative minimum
Q2. b) f(1)=31/3 is an inflection point
Q4. a) $M = \phi'(x)$, $A = \phi(x)/x$;
c) they intersect when the average curve reaches a peak or a trough
d) $\varepsilon = 1$

(33) pp. 244-245
Q1. c) $f'(x) = 2(1-x)^{-2}$, $f''(x) = 4(1-x)^{-3}$, $f'''(x) = 12(1-x)^{-4}$.
Q2. (a) and (d).

(34) pp. 253-254
Q1. c) f(1)= 16/3 is a maximum. f(5) = -16/3 is a minimum
d) there exists no relative extremum
Q3. b) $R = 100Q - Q^2$; c) $\pi = -1/3Q^3 + 6Q^2 - 11Q - 50$; d) Q=11; e) maximum profit is $111\frac{1}{3}$.
Q6. a) Q=f(L), $R = P_0 f(L)$, $C = W_0 L + F$, $\pi = P_0 f(L) - W_0 L - F$; b) $P_0 f'(L) = W_0$; c) if f''(L) < 0

(35) p. 267
Q2. c) f(3)=7

(36) pp. 273-274
Q3. c) $\frac{dy}{dt} = -12e^{-2t}$

(37) pp. 282-283
Q3. a) $10e^{0.15}$
Q4. b) 0.03

(38) p. 287
Q1. b) -4
Q2. e)6
Q3. f)3
Q4. (a) and (c) are valid and (b) and (d) are not.

(39) pp. 291-292

Q4. a) $y=e^{(3\ln 8)t}$; b) $y=2e^{(2\ln 7)t}$; c) $y=5e^{(\ln 5)t}$; d) $y=2e^{(4\ln 15)t}$

Q5. a) $t=\frac{\ln(y)}{\ln 7}$; b) $t=\frac{\ln(3y)}{\ln 8}$; c) $t=\frac{3\ln(9y)}{\ln 15}$; d) $t=\frac{2\ln(y)}{\ln 10}$

(40) pp. 297-298

Q1. e) $(2ax+b)e^{2x^2+bx+c}$; f) $(x+1)e^{x}$; g) $2x(x+1)e^{2x}$; h) $a(bx+1)e^{bx+c}$

Q3. e) $\frac{dy}{dx}=\frac{1}{x(1+x)}$; f) $\frac{dy}{dx}=\frac{1-9x}{x(1-x)}$; g) $\frac{dy}{dx}=\frac{1}{x(1+x)}$; h) $\frac{dy}{dx}=10x^{3}(1+4\ln x)$

Q4. d) $\frac{dy}{dx}=\frac{2}{x\ln 7}$; e) $\frac{dy}{dx}=\frac{16x}{(8x^{2}+3)\ln 2}$; f) $\frac{dy}{dx}=\frac{x}{\ln 3}+2x\log_3 x$

(41) pp. 301-302

Q1. $t=1/r^{2}$

(42) pp. 305-306

Q1. a) $r_y=2/t$; e) $r_y=1/t-\ln 3$

Q2. $r_H=b\ln(2)$, $r_C=a$, $r_{C/H}=a-b\ln(2)$

Q3. $r_y=kr_x$

7. Unconstrained Optimization II.

(43) pp. 318-319

Q1. x=y=0, z=3 is a minimum.

Q4. x=y=0, z=4 is a minimum.

(44) p. 331

Q1. a) $q=4u^{2}+4uv+3v^{2}$

Q4. a) $q=\begin{bmatrix}u & v\end{bmatrix}*\begin{bmatrix}3 & -2\\ -2 & 7\end{bmatrix}*\begin{bmatrix}u\\ v\end{bmatrix}$; e) $q=\begin{bmatrix}u_1 & u_2 & u_3\end{bmatrix}*\begin{bmatrix}3 & -1 & 2\\ -1 & 5 & -1\\ 2 & -1 & 4\end{bmatrix}*\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$.

Q6. a) $r_1,r_2=\frac{1}{2}(7\pm\sqrt{17})$

(45) p. 337

Q2. $x_1=x_2=x_3=0$ $z=29$ is a maximum.

Q4. x=0, y=0, w=1, z=2-e.

(46) p. 352

Q1. a) strictly convex; b) strictly convex; c) strictly convex.

Q5. a) a disc; b) yes

Q6. a) not a convex set; b) a convex set; c) a convex set; d) a convex set

8. Optimization with Equality Constraints.

(47) p. 378

Q1. a) x=1, y=1/2, z=1/2; b) x=6, y=2, z=36; c) x=1, y=5, z=-19; d) x=-1/2, y=1/2, z=27/4

(48) p. 399-400

Q4. b) linear, therefore, both quasiconcave and quasiconvex; c) the function is quasiconvex

Q8. b) the function is quasiconcave

(49) pp. 409-410

Q2. a) $Z=(x+2)(y+1)+\lambda(B-xP_x-yP_y)$.

b) $\lambda=\dfrac{B+2P_x+P_y}{2P_xP_y}$, $x=\dfrac{B-2P_x+P_y}{2P_x}$, $y=\dfrac{B+2P_x-P_y}{2P_y}$.

c) it is satisfied

d) the results are consistent with the preceding problem

(50) pp. 417-418

Q1. b) homogeneous of degree one; d) homogeneous of degree one.

Q4. Yes, they are true

Q8. a) $j^2Q=g(jK,jL)$; b) $Q=L^2\phi(k)$; c) $MPP_K=L\phi'(k)$; d) it is homogeneous of degree 1 in K & L.

(51) pp. 430-431

Q2. It will plot as a straight line through the origin. The elasticity of substitution is unity.

Q5. a) $\dfrac{Lf_L}{Kf_K}=\dfrac{1-\delta}{\delta}\left(\dfrac{K}{L}\right)^{\rho}$.

Q8. a) 7; b) 1; c) ln5-1; d) 0.