ECN 726 | **ASSIGNMENT 3** | **S. C. AHN**
Fall 2006 | **DUE November 7 (Tuesday)**

1. (20 pts.; 10 on each.) Consider the following single equation in a simultaneous equations system:

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + \varepsilon_1 = Z_1\delta_1 + \varepsilon_1,$$

where y_1, Y_1, X_1 and ε_1 are $T\times1$, $T\times p_1$, $T\times p_2$, and $T\times1$, respectively. Y_1 and X_1 are the matrices of the included endogenous and exogenous regressors, respectively. Assume that the error terms in ε_1 are i.i.d. over different t. Let $X = [X_1,X_1^*]$ be the $T\times q$ ($q > p_1+p_2$) matrix of the exogenous regressors that appear in the system. Assume that the first column of X_1 is the $T\times1$ vector of ones.

(1) Let $e = (e_1, \ldots, e_T)'$ be the $T\times1$ vector of the 2SLS residuals. Show that $\Sigma_t e_t = 0$.
(2) Let $\hat{e}$ and $\ddot{e}$ be the $T\times1$ vectors of the fitted values and residuals, respectively, from an OLS regression of e on X. Using the fact that $\hat{e}'\hat{e} + \ddot{e}'\ddot{e} = e'e$, show that the J-test based on 2SLS is a special case of Hansen's GMM specification test (GMM, J-test).

2. (15 pts.) Consider the following equations:

$$y_1 = \gamma_{31}y_3 + \beta_{11}x_1 + \varepsilon_1$$
$$y_2 = \gamma_{12}y_1 + \gamma_{32}y_3 + \beta_{22}x_2 + \varepsilon_2$$
$$y_3 = \gamma_{13}y_1 + \varepsilon_3$$

Examine the identification of each equation, assuming $\gamma_{31} + 4\beta_{11} = 2$.

2. (50 pts.; 10 pts. on each.) A simultaneous equations model is given:

(A) $y_1 = \gamma_{21}y_2 + \beta_{11} + \beta_{21}x_2 + \beta_{31}x_3 + \varepsilon_1$;
(B) $y_2 = \gamma_{12}y_1 + \beta_{12} + \beta_{42}x_4 + \varepsilon_2$.

To estimate this model, use the data set named sem.db, which is available from Dr. Ahn's web page. The data set contains 100 observations on 5 variables (y1, y2, x2, x3 and x4). Using this data set, construct a GAUSS program that can do the followings:

(1) Estimate models (A) and (B) by 2SLS. Report the variable names, estimated coefficients, standard errors, t statistics and R^2.
(2) Evaluate the quality of the instrumental variables for each equation.
(3) Can you test the specification of the first equation? If so, report your test result. How about the second equation?
(4) Estimate models (A) and (B) by (two-step) 3SLS. Report the variable names, estimated coefficients, standard errors, and t statistics.
(5) You will observe that the 3SLS estimates for equation (B) are identical to the 2SLS results obtained from (1). Provide a proper explanation for these phenomena.

3. (15 pts.) Consider the following model:

$$HWORK = \beta_1 + \beta_2 EDU + \beta_3 \exp(LRATE) + \beta_4 \exp(LOFINC) + \varepsilon.$$

Assume that the error term is heteroskedastic (but not autocorrelated). Using mwemp.db (data description is given in p. SEM-35), construct a GAUSS program that can do the followings.

(1) (10 pts.) Estimate the model by continuous-updating GMM using (1,AGE,EXPP,EDU,TENURE,REGS) as instruments. Report the parameter names, estimated parameter values, standard errors, and t-statistics. (You may fail to get converged results depending on your choice of starting values of the parameters. If you fail, try different starting values.)

(2) (5 pts.) Test the model specification at 5% of significance level. Report your test statistic, degrees of freedom and p-value.