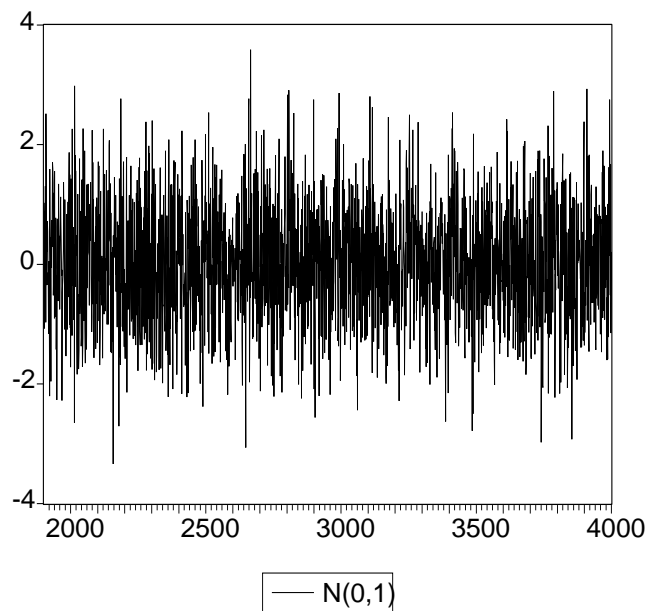
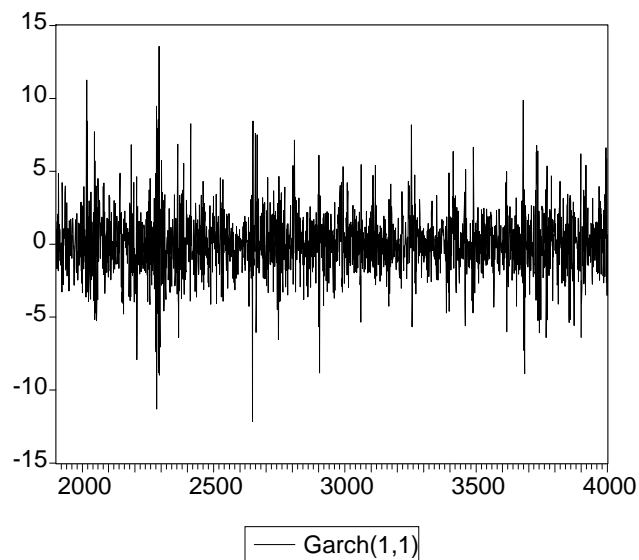


1. AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH)

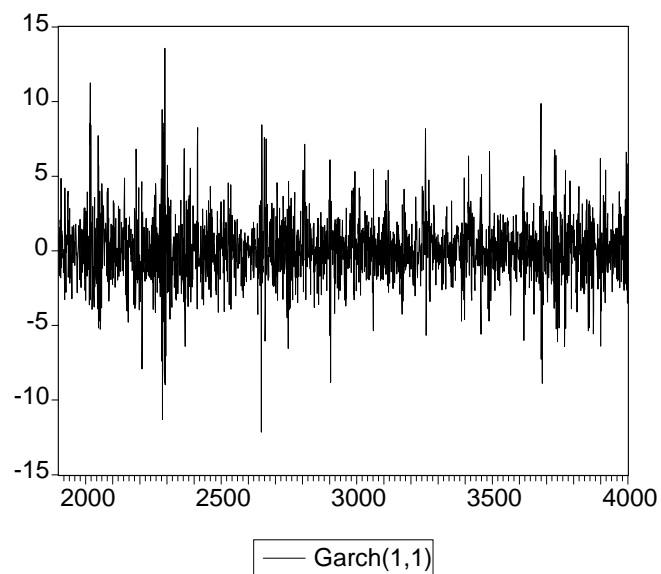
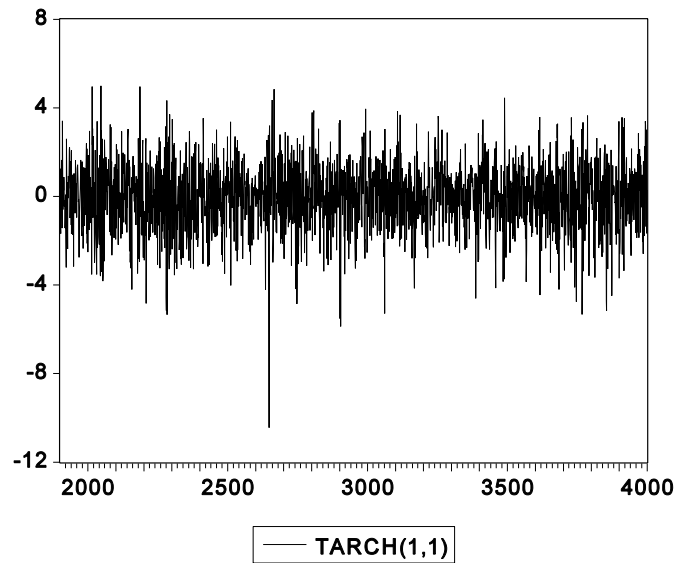
[1] EMPIRICAL REGULARITIES

[See Bollerslev, Engle and Nelsen, Handbook of Econometrics,4]

(1) Thick tails: Thicker than those of iid normal dist.



- (2) Volatility Clustering: Large changes tend to be followed by large changes.
- (3) Leverage effects: Changes in stock prices tend to be negatively related with changes in stock volatility.



(4) Non-trading periods effects:

Information that accumulates when financial markets are closed is reflected in prices after the market opens.

(5) Forecastable events effects:

Forecastable releases of important information (e.g., earnings announcement) are associated with high ex ante volatility.

(6) Volatility and serial correlation

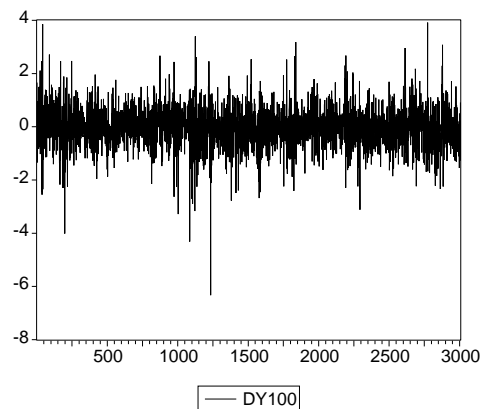
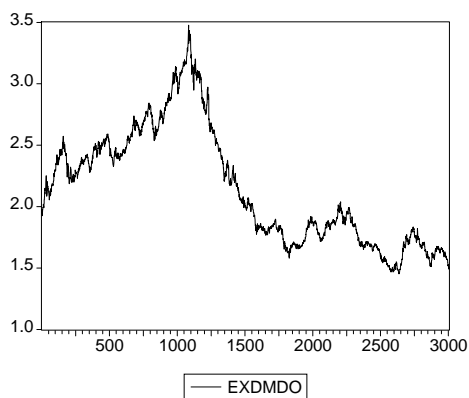
Inverse relation between volatility and serial correlation for US stock indices.

(7) Co-movements in volatilities

Commonality in volatility changes across stocks.

(8) Macroeconomic variables and volatility

Positive relation bet/w macro uncertainty and stock market volatility.



[2] MODEL, ESTIMATION AND SPECIFICATION TESTS

(1) BASIC MODEL

$$y_t = x_t \boldsymbol{\beta} + u_t, t = 1, 2, \dots, T; u_t = \sqrt{h_t} v_t,$$

- v_t iid $N(0,1)$: Need not be normal.

Can be t-distribution with $df = \mathbf{8}$

- $h_t = E(u_t^2 | \mathbf{S}_{t-1}) = \text{var}(u_t | \mathbf{S}_{t-1}) = h_t(\boldsymbol{\beta}, \mathbf{B})$,

where \mathbf{B} is a parameter vector determining volatility in y_t .

- Conditional variance of u_t given \mathbf{S}_{t-1} .
- h_t could be a function of some regressors, say, z_t .
- \mathbf{S}_{t-1} : Information set at time $t-1$

$$\mathbf{S}_{t-1} = \{x_{t-1}, x_{t-2}, \dots, y_{t-1}, y_{t-2}, \dots\}.$$

- $\mathbf{F}^2 = E(u_t^2) = \text{var}(u_t) = \text{Unconditional variance of } u_t$
= Unconditional mean of volatility.

NOTE:

- u_t should be serially uncorrelated.
- u_t^2 could be serially correlated.

(2) ESTIMATION

1) MLE

- Define the conditional pdf by $f(y_t|x_t, \mathbf{S}_{t-1}, \mathbf{2})$, where $\mathbf{2} = (\mathbf{1}, \mathbf{B})'$
- Assume v_t iid $N(0,1)$,

$$f(y_t|x_t, \mathbf{S}_{t-1}, \mathbf{2}) = \frac{1}{\sqrt{2\mathbf{B}h_t}} \exp\left[-\frac{(y_t - x_t\mathbf{1})^2}{2h_t}\right].$$

- $l_T(\mathbf{2}) = \mathbf{E}_t \ln[f(y_t|x_t, \mathbf{S}_{t-1}, \mathbf{2})]$
 $= -(T/2)\ln(2\mathbf{B}) - (1/2)\mathbf{E}_t \ln(h_t) - (1/2)\mathbf{E}_t (y_t - x_t\mathbf{1})^2/h_t.$
- $s_t = \ln[f(y_t|x_t, \mathbf{S}_{t-1}, \mathbf{2})]/\ln(2\mathbf{B})$.
- $H_T(\mathbf{2}) = \ln l_T(\mathbf{2})/\ln(2\mathbf{B})$; $B_T(\mathbf{2}) = \mathbf{E}_t s_t(\mathbf{2})/s_t(\mathbf{2}).$
- MLE: $\hat{\mathbf{2}} \sim N[\mathbf{2}, (-H_T(\hat{\mathbf{2}}))^{-1}] \sim N[\mathbf{2}, (B_T(\hat{\mathbf{2}}))^{-1}].$
- MLE with non-Gaussian v_t : See Hamilton (661-662).

2) MLE when normality assumption is violated

[QMLE: Bollerslev and Wooldridge, ER, 1992]

- MLE based on the normality assumption is still consistent.
- $\hat{\mathbf{2}} \sim N[0, (H_T(\hat{\mathbf{2}}))^{-1}B_T(\hat{\mathbf{2}})(H_T(\hat{\mathbf{2}}))^{-1}].$
- QMLE with other distributional assumptions
 [See Newey and Steigerwald (ECON, 1997)]

3) GMM

- Moment conditions:

$$E(x_t | u_t) = 0 ; E[(u_t^2 - h_t)u_{t-j}] = 0, j = 1, 2, \dots$$

- Not successful (Andersen and Sørensen, 1996, JBES).

(3) SPECIFICATION TESTING

1) LM test (Engle, 1982, ECONOMETRICA)

H_0 : No CH.

STEP 1: Do OLS on $y_t = x_t \beta + u_t$, and get \hat{u}_t .

STEP 2: Regress \hat{u}_t^2 on one, \hat{u}_{t-1}^2 , \hat{u}_{t-2}^2 , ..., \hat{u}_{t-q}^2 , and get R^2 .

STEP 3: $LM_T = TR^2 \xrightarrow{d} \chi^2(q)$, under H_0 : No CH.

2) Specification Tests based on standardized errors

- H_0 : Chosen model specification is correctly specified
- Let $e_t = \hat{u}_t / \sqrt{\hat{h}_t}$ (standardized residuals).
- If your choice of h_t is correctly specified, then e_t should be roughly iid.

- Testing autocorrelation of e_t or e_t^2 :

- AC (autocorrelation):
$$\frac{\mathbf{E}_{t-1}^{T \& J}(e_t | \bar{e})(e_{t+j} | \bar{e})/T}{\mathbf{E}_{t-1}^T(e_t | \bar{e})/T} = r_j/r_o,$$

where $r_j = \frac{1}{T} \sum_{t=1}^{T-j} (e_t - \bar{e})(e_{t+j} - \bar{e})$ and $r_o = \frac{1}{T} \sum_{t=1}^T (e_t - \bar{e})^2$.

- PAC (partial AC):
 - Regress e_t on one, $e_{t-1}, e_{t-2}, \dots, e_{t-j}$.
 - PAC of e_t and e_{t-j}

= coefficient of e_{t-j} in this regression of e_t .

- Under the hypothesis that e_t and e_{t-j} are uncorrelated,

$$\sqrt{T} \cdot \text{PAC} \xrightarrow{D} N(0,1)$$

- Ljung-Box Q-statistic:

$$Q_{LB} = T(T+2) \sum_{j=1}^p \frac{r_j^2/r_o^2}{T-j}$$

$\xrightarrow{D} \chi^2(p)$, under H_0 : no autocorrelation in e_t .

- Normality test (Jarque-Bera):

See Greene.

Note: (Ahn's worry)

- Is Q_{LB} a relevant test?
- In fact, there is no reason to believe that Q_{LB} is $\mathbf{P}^2(p)$.
- Bollerslev and Mikkelson's ad hoc solution:
(1996, Journal of Econometrics)
 - Use $\mathbf{P}^2(p - \# \text{ of ARCH and GARCH parameters})$.
 - Not theoretically relevant, but it works!!!

[3] ARCH(q) MODEL [Engle, 1982, ECON]

(1) Specification:

- $h_t = \mathbf{T} + \alpha_1 u_{t-1}^2 + \dots + \alpha_q u_{t-q}^2$.
- If (i) \mathbf{T}, α_j 's > 0 ; (ii) $\alpha_1 + \dots + \alpha_q < 1$, the model is stationary
[Sufficient, but not necessary]
- $\mathbf{F}^2 / \text{var}(u_t) = \mathbf{T} / (1 - \alpha_1 - \dots - \alpha_q)$ under (i) and (ii).
Even if $\alpha_1 + \dots + \alpha_q = 1$, the model could be still stationary and ergodic (so MLE is consistent and efficient).

- For MLE or QMLE,

$$h_t = \mathbf{T} + \alpha_1 (y_{t-1} - x_{t-1} \mathbf{\beta})^2 + \dots + \alpha_q (y_{t-q} - x_{t-q} \mathbf{\beta})^2.$$

$$\mathbf{2} = (\mathbf{\beta}, \mathbf{T}, \alpha_1, \dots, \alpha_q) \mathbf{N}$$

$$\text{Set } u_0^2 = \dots = u_{1-q}^2 = \mathbf{T}^{-1} \mathbf{E}_t (y_t - x_t \mathbf{\beta})^2.$$

- Can introduce some regressors in h_t : e.g.,

$$h_t = \mathbf{T} + \alpha_1 u_{t-1}^2 + \dots + \alpha_q u_{t-q}^2 + \lambda d_t, \text{ where } d_t \text{ is a dummy variable for Mondays.}$$

(2) Alternative representation

- Let $w_t = u_t^2 - h_t$:

$$u_t^2 = h_t + w_t = \mathbf{T} + \alpha_1 u_{t-1}^2 + \dots + \alpha_q u_{t-q}^2 + w_t.$$

$$\Upsilon u_t^2 - \text{AR}(q).$$

(3) Forecast of volatility

1) Let $u_{t\%s}^2 = E(u_{t\%s}^2 * u_t^2, u_{t\&1}^2, \dots)$: Optimal Predictor of future h_t .

$$2) u_{t\%1}^2 = \mathbf{T} + \beta_1 u_t^2 + \beta_2 u_{t-1}^2, \dots, \beta_q u_{t-q+1}^2;$$

$$u_{t\%2}^2 = \mathbf{T} + \beta_1 u_{t\%1}^2 + \beta_2 u_t^2 + \dots + \beta_q u_{t-q+2}^2;$$

$$u_{t\%3}^2 = \mathbf{T} + \beta_1 u_{t\%2}^2 + \beta_2 u_{t\%1}^2 + \beta_3 u_t^2 + \beta_q u_{t-q+1}^2;$$

: :

$$u_{t\%s}^2 = \mathbf{T} + \beta_1 u_{t\&1\%s}^2 + \beta_2 u_{t\&2\%s}^2 + \beta_3 u_{t\&3\%s}^2 + \dots + \beta_q u_{t\&q\%s}^2 .$$

$$u_{t\%s}^2 \hat{=} \mathbf{T} / (1! \beta_1! \dots! \beta_q), \text{ as } s \hat{=} 4.$$

[4] GENERALIZED ARCH [Bollerslev, 1986, JEC]

[Called GARCH(p,q)]

(1) Motivation

- q is usually too large when ARCH(q) is used.
- Need a parsimonious model.

(2) Specification

$$h_t = \mathbf{T} + \alpha_1 h_{t-1} + \dots + \alpha_p h_{t-p} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2 + \dots + \beta_q u_{t-q}^2.$$

- For MLE or QMLE,

$$\mathbf{2} = (\mathbf{T}, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q)$$

$$\text{Set } h_0 = \dots = h_{1-p} = u_0^2 = \dots = u_{1-q}^2 = \mathbf{T}^{-1} \mathbf{E}_t(y_t - x_t \mathbf{2})^2.$$

- Let $r = \max\{p, q\}$; and set $\beta_{q+1} = \dots = \beta_r = 0$ or $\alpha_{p+1} = \dots = \alpha_r = 0$.

$$h_t = \mathbf{T} + \alpha_1 h_{t-1} + \dots + \alpha_r h_{t-r} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2 + \dots + \beta_r u_{t-r}^2.$$

- If (i) $\mathbf{T}, \alpha_j, \beta_j > 0$ and (ii) $\mathbf{E}_{j-1}^r (\alpha_j + \beta_j) < 1$, the model is

stationary. [Sufficient, but not necessary.]

- $\mathbf{F}^2 = \text{var}(u_t^2) = \mathbf{T} / [1 - (\alpha_1 + \beta_1) - \dots - (\alpha_r + \beta_r)]$ (if (i) and (ii) hold).

(3) GARCH(1,1)

$$1) h_t = \mathbf{T} + \alpha h_{t-1} + \beta u_{t-1}^2 .$$

$$\begin{aligned} \bullet h_t &= \mathbf{T} + \alpha(\mathbf{T} + \alpha h_{t-2} + \beta u_{t-2}^2) + \beta u_{t-1}^2 \\ &= (1+\alpha)\mathbf{T} + \alpha^2 h_{t-2} + \beta(u_{t-1}^2 + \alpha u_{t-2}^2) \end{aligned}$$

$$= \mathbf{T}/(1-\alpha) + \beta \sum_{j=1}^{\infty} \alpha^{j-1} u_{t-j}^2 . \text{ [ARCH(4)]}$$

- Let $w_t = u_t^2 - h_t$: unforecastable volatility.

$$\begin{aligned} u_t^2 &= (u_t^2 - h_t) + h_t = w_t + [\mathbf{T} + \alpha h_{t-1} + \beta u_{t-1}^2] \\ &= w_t + \mathbf{T} + (\alpha + \beta)u_{t-1}^2 - \alpha(u_{t-1}^2 - h_{t-1}) \\ &= w_t + \mathbf{T} + (\alpha + \beta)u_{t-1}^2 - \alpha w_{t-1} \\ &= \mathbf{T} + (\alpha + \beta)u_{t-1}^2 + w_t - \alpha w_{t-1} \text{ [Like ARMA(1,1)]} \end{aligned}$$

$$\begin{aligned} \bullet h_t &= \mathbf{T} + \alpha h_{t-1} + \beta h_{t-1} v_{t-1}^2 = \mathbf{T} + h_{t-1}(\alpha + \beta v_{t-1}^2) / \mathbf{T} + h_{t-1} \geq_{t-1} \\ &= \mathbf{T}[1 + \sum_{j=1}^{\infty} \alpha^j \mathbf{A}_{t-1}^j (\alpha + \beta v_{t-1}^2)] \end{aligned}$$

Theorem: (Nelson, 1990, ECON)

GARCH(1,1) is stationary and ergodic iff $E[\ln(\alpha + \beta v_t)] < 0$. And,

$$\mathbf{T}/(1-\alpha) \neq \mathbf{F}^2 < 4.$$

Implication:

- Even if $\alpha + \beta = 1$, GARCH(1,1) can be stationary.
- MLE is still consistent and efficient.

2) Forecasting:

$$u_t^2 = \mathbf{T} + (\mathbf{*} + \mathbf{''})u_{t-1}^2 + w_t - \mathbf{*}w_{t-1}.$$

i) Let $u_{t\%s}^2 / E(u_{t\%s}^2 * u_t^2, u_{t\&1}^2, \dots, w_t^2, w_{t\&1}^2, \dots)$:

Optimal Predictor of h_{t+s} at time t .

$$w_{t\%s} / E(w_{t\%s} * u_t^2, \dots, w_t, \dots) = 0.$$

ii) $u_{t\%1}^2 = \mathbf{T} + (\mathbf{*} + \mathbf{''}_1)u_t^2 + w_{t+1} - \mathbf{*}w_t = \mathbf{T} + (\mathbf{*} + \mathbf{''})u_t^2 - \mathbf{*}w_t$;

$$u_{t\%2}^2 = \mathbf{T} + (\mathbf{*} + \mathbf{''}_1)u_{t\%1}^2$$

$$u_{t\%3}^2 = \mathbf{T} + (\mathbf{*} + \mathbf{''}_1)u_{t\%2}^2$$

:

$$u_{t\%s}^2 \overset{p}{\underset{6}{\approx}} \mathbf{T}/(1 - \mathbf{*} \mathbf{''}), \text{ as } s \underset{6}{\geq} 4.$$

3) Regarding $H_0: \mathbf{''} = 0$.

- Not possible to test for this hypothesis by Wald tests. Under H_0 , the model is not identified. [Under GARCH(1,0) with the stationarity conditions (i) and (ii), $h_t = h_{t-1} = \dots = \mathbf{F}^2 = \mathbf{T}/(1 - \mathbf{*})$.]
- See Andrews and Ploberger (1994, ECON) and B. Hansen (1996, ECON).

(4) GARCH(p,q)

$$h_t = \mathbf{T} + \alpha_1 h_{t-1} + \dots + \alpha_p h_{t-p} + \omega_1 u_{t-1}^2 + \omega_2 u_{t-2}^2 + \dots + \omega_q u_{t-q}^2.$$

• Let $r = \max\{p,q\}$; and set $\omega_{p+1} = \dots = \omega_r = 0$ or $\alpha_{q+1} = \dots = \alpha_r = 0$.

• $h_t = \mathbf{T} + \alpha_1 h_{t-1} + \dots + \alpha_r h_{t-r} + \omega_1 u_{t-1}^2 + \omega_2 u_{t-2}^2 + \dots + \omega_r u_{t-r}^2.$

• $u_t^2 = (u_t^2 - h_t) + h_t$ (let $w_t = u_t^2 - h_t$)

$$= w_t + [\mathbf{T} + \alpha_1 h_{t-1} + \dots + \alpha_r h_{t-r} + \omega_1 u_{t-1}^2 + \dots + \omega_r u_{t-r}^2]$$

$$= w_t + \mathbf{T} + (\alpha_1 + \omega_1) u_{t-1}^2 + \dots + (\alpha_r + \omega_r) u_{t-r}^2$$

$$- \alpha_1 (u_{t-1}^2 - h_{t-1}) - \dots - \alpha_r (u_{t-r}^2 - h_{t-r})$$

$$= \mathbf{T} + (\alpha_1 + \omega_1) u_{t-1}^2 + (\alpha_r + \omega_r) u_{t-r}^2$$

$$+ w_t - \alpha_1 w_{t-1} - \dots - \alpha_r w_{t-r}. \text{ (Like ARMA}(r,p)\text{)}$$

[5] INTEGRATED GARCH(p,q) [Bollerslev and Engle, ER, 1986]

[Called IGARCH]

- $$h_t = \omega + \alpha_1 h_{t-1} + \dots + \alpha_p h_{t-p} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2 + \dots + \beta_q u_{t-q}^2,$$

with $\mathbf{E} \alpha_j + \mathbf{E} \beta_j = 1$.

- Looks like nonstationary. But it could be stationary.
- Can test IGARCH(1,1) using a Wald statistic.

[We conjecture that Wald tests can be used for IGARCH(p,q).

But there is no formal proof.]

- $u_{t/s}^2 \overset{d}{\sim} \chi^2_1$ as $s \rightarrow \infty$.
- QMLE for IGARCH(1,1) is consistent and asymptotically normal under certain conditions (See Lumsdaine, 1996, ECON).

[6] Exponential GARCH [Nelson, 1991, ECONOMETRICA]
 [Called EGARCH]

(1) Motivation:

GARCH models do not capture leverage effects.

(2) Basic Model

$$\ln(h_t) = \mathbf{T} + \alpha_1 \ln(h_{t-1}) + \dots + \alpha_p \ln(h_{t-p}) + \beta_1 \mathbf{Q}_{t-1} + \beta_2 \mathbf{Q}_{t-2} + \dots + \beta_q \mathbf{Q}_{t-q},$$

where $\mathbf{Q}_t = |v_t| - E|v_t| + \gamma (v_t$ and v_t follows generalized error distribution (p. 668, Hamilton, or Nelson, 1990)

- $E|v_t| = \sqrt{2/\mathbf{B}}$.
- \mathbf{T} , α 's and β 's do not have to be positive.
- If $\gamma = 0$, $\mathbf{Q}_t = |v_t| - E|v_t|$.
 - Positive and negative v_t have the symmetric effects on h_t .
- If $-1 < \gamma < 0$:
 - If $v_t > 0$, $\mathbf{Q}_t = (1+\gamma)v_t - E|v_t| = (0 < 1+\gamma < 1) v_t - E|v_t|$
 - If $v_t < 0$, $\mathbf{Q}_t = (-1+\gamma)v_t - E|v_t| = (-2 < -1+\gamma < -1)v_t - E|v_t|$
 - Negative v_t has greater effects than positive v_t .
- If $\gamma < -1$:
 - If $v_t > 0$, $\mathbf{Q}_t = (1+\gamma)v_t - E|v_t| = (1+\gamma < 0) v_t - E|v_t|$
 - If $v_t < 0$, $\mathbf{Q}_t = (-1+\gamma)v_t - E|v_t| = (-1+\gamma < -2)v_t - E|v_t|$
 - Positive v_t reduces h_t .

(3) Conditions for stationarity and ergodicity:

$$E_j \sigma_j^2 < 4.$$

(4) Example:

• $r_t =$ (daily return on stock) - (daily interest rate on treasury bills)

• $r_t = \alpha_1 + \alpha_2 r_{t-1} + \beta h_t + u_t ;$

• $u_t = \sqrt{h_t} v_t ;$

• $\ln(h_t) - \mu_t = \alpha_1 (\ln(h_{t-1}) - \mu_{t-1}) + \alpha_2 (\ln(h_{t-2}) - \mu_{t-2}) + \sigma_1 \epsilon_{t-1} + \sigma_2 \epsilon_{t-2} .$

• $\mu_t = \mu + \ln(1 + \beta N_t)$, $N_t = \#$ of nontrading days bet/w (t-1) and t.

[7] COMPONENT GARCH(1,1)

[Engle and Lee (1999)?, Eviews Manual]

(1) Symmetric Component GARCH (1,1).

- Reconsider GARCH(1,1):
 - 1) $h_t = \mathbf{T} + \alpha h_{t-1} + \beta u_{t-1}^2$.
 - 2) $h_t = \mathbf{T} + \alpha(h_{t-1} - \mathbf{T}) + \beta(u_{t-1}^2 + \mathbf{T})$, where $\alpha = \mathbf{T} / (\mathbf{T} + \beta)$.
 - The equation 2) implies that in GARCH(1,1) h_t fluctuates around \mathbf{T} (mean reversion to \mathbf{T}).
 - The volatility measure h_t could have transitory (short-run) and permanent (long-run) components.
 - In GARCH(1,1), $(h_t - \mathbf{T})$ is the transitory component and \mathbf{T} is the permanent component.
- Symmetric Component GARCH (1,1)
 - Wish to allow the permanent component fluctuate over time.
 - $h_t = q_t + \alpha(h_{t-1} - q_{t-1}) + \beta(u_{t-1}^2 + q_{t-1})$;
 $q_t = \mathbf{T} + \delta(q_{t-1} - \mathbf{T}) + \mathbf{N}(u_{t-1}^2 + h_{t-1})$.
 - The transitory component $(h_t - q_t)$ fluctuates around zero and the long-run component fluctuates around \mathbf{T} .

- The above two equations imply:

$$h_t = \mathbf{T}' \left(\alpha_1 h_{t-1} + \alpha_2 h_{t-1} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2 \right),$$

where

$$\mathbf{T}' = (1 - \alpha_1 - \alpha_2) \mathbf{D} \mathbf{T}; \quad \alpha_1 = \alpha + \mathbf{D} \mathbf{N};$$

$$\alpha_2 = \alpha (\mathbf{D} + \beta_1 \mathbf{N}); \quad \beta_1 = \beta \mathbf{N}; \quad \beta_2 = \alpha (\mathbf{D} + \beta_1 \mathbf{N}).$$

- Thus, component GARCH(1,1) can be viewed as a restricted GARCH(2,2).

(2) Asymmetric Component GARCH(1,1)

- $h_t = \omega + \alpha_1 (h_{t-1} + q_{t-1}) + \alpha_2 (u_{t-1}^2 + q_{t-1}) + \beta_1 (u_{t-1}^2 + q_{t-1}) d_{t-1},$

where $d_t = 1$ iff $u_{t-1} < 0$.

[7] THRESHHOLD ARCH

[Glosten, Jagannathan and Runkle, JF, 1994; Called TARCH]

- $h_t = \mathbf{T} + \ast h_{t-1} + \mathbf{''} u_{t-1}^2 + \mathbf{(} (u_{t-1}^2) \mathbf{1}(u_{t-1} < 0),$
where \mathbf{T} , \ast , $\mathbf{''}$ and $\mathbf{(} > 0$.

[8] GARCH IN MEAN

[Engle, Lilien and Robins, 1987, ECON; Called GARCH-M]

$$y_t = x_t \mathbf{\$} + \mathbf{\$} h_t + u_t \text{ or } y_t = x_t \mathbf{\$} + \mathbf{\$} \sqrt{h_t} + u_t .$$

[9] MULTIVARIATE GARCH

(See Hamilton.)

[10] Applications -Eviews

(1) Estimation

STEP 1: Push **Objects/New Object**.

STEP 2: Choose **Equation**. Push OK button. Then, you are in **Equation Specification** box.
Go to **Equation Setting**, and Choose **ARCH**.

STEP 3: In **Equation Specification** box, type:

dy100 c

STEP 4: Go to **Equation Setting** and type:

2 1001

STEP 5: Click the **option button**. Increase the convergence rate (0.0001) and increase maxit to 1000. Choose algorithm and Heteroskedasticity-Robust Covariance matrix (for QMLE). Once you have chosen appropriate options, click the **ok** button.

STEP 6: Choose a specification and run the program.

(2) GARCH(1,2)

$$y_t = \beta + u_t; h_t = \omega + \alpha_1 h_{t-1} + \alpha_2 u_{t-1}^2 + \alpha_3 u_{t-2}^2$$

Dependent Variable: DY100

Method: ML - ARCH

Sample(adjusted): 2 1001

Included observations: 1000 after adjusting endpoints

Convergence achieved after 42 iterations

Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.049709	0.021991	2.260437	0.0238
Variance Equation				
C	0.010867	0.006700	1.622076	0.1048
ARCH(1)	0.032561	0.050742	0.641692	0.5211
ARCH(2)	0.015376	0.052303	0.293984	0.7688
GARCH(1)	0.932021	0.028271	32.96791	0.0000
R-squared	-0.000051	Mean dependent var		0.044314
Adjusted R-squared	-0.004071	S.D. dependent var		0.755497
S.E. of regression	0.757033	Akaike info criterion		2.205777
Sum squared resid	570.2337	Schwarz criterion		2.230316
Log likelihood	-1097.888	Durbin-Watson stat		2.137029

(3) TARCH(1,2)

$$y_t = \beta + u_t; h_t = \omega + \alpha_1 h_{t-1} + \beta_1 u_{t-1}^2 + \gamma_1(u_{t-1}^2) \mathbf{1}(u_{t-1} < 0) + \beta_2 u_{t-2}^2:$$

Dependent Variable: DY100

Method: ML - ARCH

Sample(adjusted): 2 1001

Included observations: 1000 after adjusting endpoints

Convergence achieved after 28 iterations

Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.047872	0.021862	2.189744	0.0285
Variance Equation				
C	0.007089	0.008815	0.804239	0.4213
ARCH(1)	0.028297	0.034449	0.821422	0.4114
(RESID<0)*ARCH(1)	0.004401	0.017584	0.250280	0.8024
GARCH(1)	1.312629	0.852429	1.539868	0.1236
GARCH(2)	-0.356171	0.803179	-0.443452	0.6574
R-squared	-0.000022	Mean dependent var		0.044314
Adjusted R-squared	-0.005053	S.D. dependent var		0.755497
S.E. of regression	0.757403	Akaike info criterion		2.207258
Sum squared resid	570.2172	Schwarz criterion		2.236705
Log likelihood	-1097.629	Durbin-Watson stat		2.137091

(4) EGRACH(1,2)

$$y_t = \beta + u_t;$$

$$\ln(h_t) = \alpha + \beta \ln(h_{t-1}) + \omega_1 \mathbf{Q}_{t-1} + \omega_2 \mathbf{Q}_{t-2}; \mathbf{Q}_{t-1} = |v_{t-1}| - E(|v_{t-1}|) + \epsilon_1 v_{t-1}; \mathbf{Q}_{t-2} = |v_{t-2}| - E(|v_{t-2}|) + \epsilon_2 v_{t-2}$$

$$Y \ln(h_t) = \alpha + \beta \ln(h_{t-1}) + \{\omega_1(|v_{t-1}| - E|v_{t-1}|) + \omega_1 \epsilon_1 v_{t-1}\} + \{\omega_2(|v_{t-2}| - E|v_{t-2}|) + \omega_2 \epsilon_2 v_{t-2}\}$$

Dependent Variable: DY100

Method: ML - ARCH

Sample(adjusted): 2 1001

Included observations: 1000 after adjusting endpoints

Convergence achieved after 47 iterations

Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.046725	0.022159	2.108595	0.0350
Variance Equation				
C	-0.040548	0.035358	-1.146777	0.2515
RES /SQR[GARCH](1)	0.045956	0.039668	1.158513	0.2467
RES/SQR[GARCH](1)	0.004615	0.009793	0.471314	0.6374
EGARCH(1)	1.563571	0.382723	4.085382	0.0000
EGARCH(2)	-0.571963	0.375954	-1.521364	0.1282
R-squared	-0.000010	Mean dependent var		0.044314
Adjusted R-squared	-0.005040	S.D. dependent var		0.755497
S.E. of regression	0.757398	Akaike info criterion		2.209685
Sum squared resid	570.2104	Schwarz criterion		2.239132
Log likelihood	-1098.843	Durbin-Watson stat		2.137116

(5) GARCH(1,2)-M with Variance

$$y_t = \alpha + \beta h_t + u_t;$$

$$h_t = \omega + \alpha_1 h_{t-1} + \alpha_2 u_{t-1}^2 + \alpha_3 u_{t-2}^2.$$

Dependent Variable: DY100

Method: ML - ARCH

Sample(adjusted): 2 1001

Included observations: 1000 after adjusting endpoints

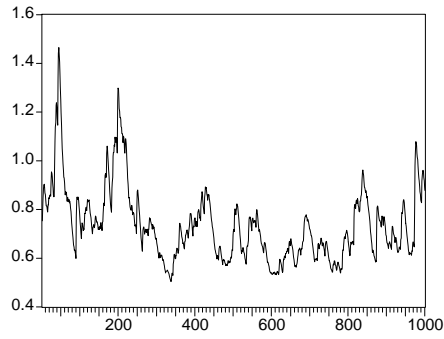
Convergence achieved after 46 iterations

Bollerslev-Wooldrige robust standard errors & covariance

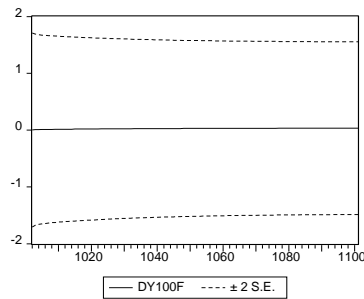
	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-0.210455	0.122764	-1.714315	0.0865
C	0.155385	0.064306	2.416321	0.0157
Variance Equation				
C	0.009611	0.007936	1.211117	0.2259
ARCH(1)	0.036125	0.027316	1.322476	0.1860
GARCH(1)	1.381466	0.506710	2.726347	0.0064
GARCH(2)	-0.434450	0.470056	-0.924252	0.3554
R-squared	0.005024	Mean dependent var		0.044314
Adjusted R-squared	0.000019	S.D. dependent var		0.755497
S.E. of regression	0.755490	Akaike info criterion		2.207335
Sum squared resid	567.3398	Schwarz criterion		2.236781
Log likelihood	-1097.667	F-statistic		1.003836
Durbin-Watson stat	2.146356	Prob(F-statistic)		0.414162

(6) Graph for $\sqrt{h_t}$.

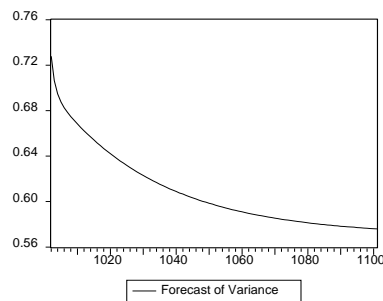
Go to **view/conditional SD graph**.



(7) Forecast: from 1002 to 1101



Forecast: DY100F	
Actual: DY100	
Forecast sample: 1002 1101	
Included observations: 100	
Root Mean Squared Error	0.853280
Mean Absolute Error	0.568194
Mean Abs. Percent Error	88.99422
Theil Inequality Coefficient	0.965964
Bias Proportion	0.002651
Variance Proportion	0.981123
Covariance Proportion	0.016226



(8) Wald Test:

If you wish to test multiple restrictions on parameters, go to **view/coefficient tests**.

(9) Specification tests based on standardized residuals

1) For Specification tests, go to **view/residual tests/correlogram-Q statistics**.

[About v_t . If your model is correctly specified, then the standardized residuals should be serially uncorrelated.]

Date: 04/03/00 Time: 16:49

Sample: 2 1001

Included observations: 1000

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
*	*	1	-0.065	-0.065	4.1978	0.040
.	.	2	0.010	0.006	4.2946	0.117
.	.	3	0.006	0.007	4.3312	0.228
.	.	4	0.049	0.050	6.7941	0.147
.	.	5	0.009	0.015	6.8678	0.231
.	.	6	-0.019	-0.018	7.2219	0.301
.	.	7	0.008	0.005	7.2852	0.400
.	.	8	-0.012	-0.014	7.4395	0.490
.	.	9	0.028	0.026	8.2582	0.508
.	.	10	0.011	0.016	8.3730	0.592
.	.	11	0.013	0.014	8.5381	0.664
.	.	12	-0.022	-0.020	9.0221	0.701
.	.	13	-0.036	-0.041	10.306	0.669
.	.	14	0.038	0.031	11.788	0.623
.	.	15	0.017	0.022	12.069	0.674

2) Go to **view/residual tests/correlogram squared residuals**.

[About v_t^2 . If your model is correctly specified, then squared standardized residuals should be serially uncorrelated.]

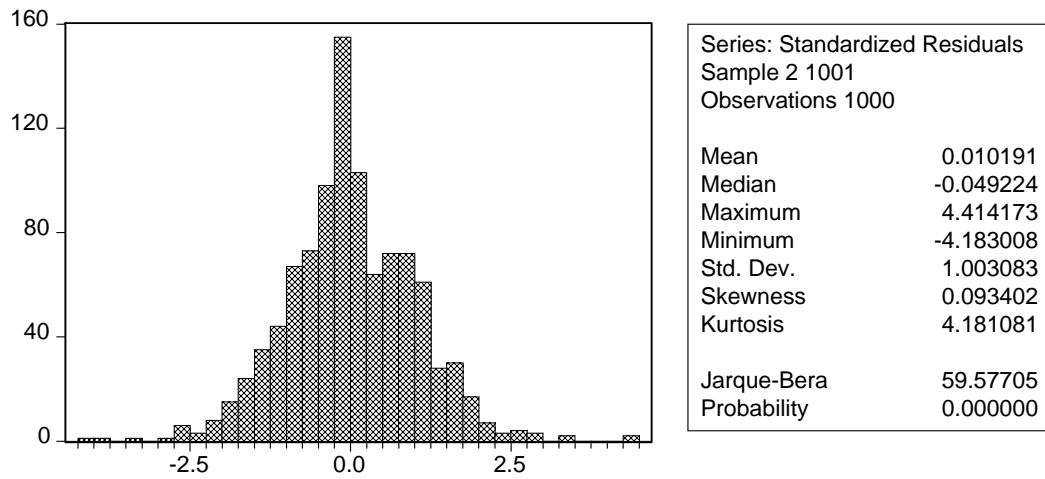
Date: 04/03/00 Time: 16:51

Sample: 2 1001

Included observations: 1000

Autocorrelation		Partial Correlation		AC	PAC	Q-Stat	Prob	
.		.		1	0.010	0.010	0.1052	0.746
.		.		2	-0.032	-0.032	1.1219	0.571
.		.		3	-0.016	-0.015	1.3787	0.711
.		.		4	0.036	0.035	2.6861	0.612
.		.		5	0.006	0.004	2.7185	0.743
.		.		6	-0.016	-0.015	2.9927	0.810
.		.		7	-0.011	-0.010	3.1200	0.874
.		.		8	0.012	0.010	3.2721	0.916
.		.		9	0.025	0.023	3.9068	0.917
.		.		10	0.008	0.009	3.9701	0.949
.		.		11	-0.041	-0.039	5.7012	0.893
.		.		12	-0.016	-0.015	5.9587	0.918
.		.		13	-0.042	-0.046	7.7774	0.858
.		.		14	-0.004	-0.006	7.7909	0.900
.		.		15	0.055	0.056	10.905	0.759
.		.		16	-0.026	-0.027	11.588	0.772

3) Go to **view/residual tests/histogram normality test**.



4) Go to **view/residual tests/ARCH LM tests**. Set Lag = 4.

[If you model is correctly specified, then standardized errors should not include any ARCH effects.]

ARCH Test:

F-statistic	0.668245	Probability	0.614110
Obs*R-squared	2.679241	Probability	0.612852