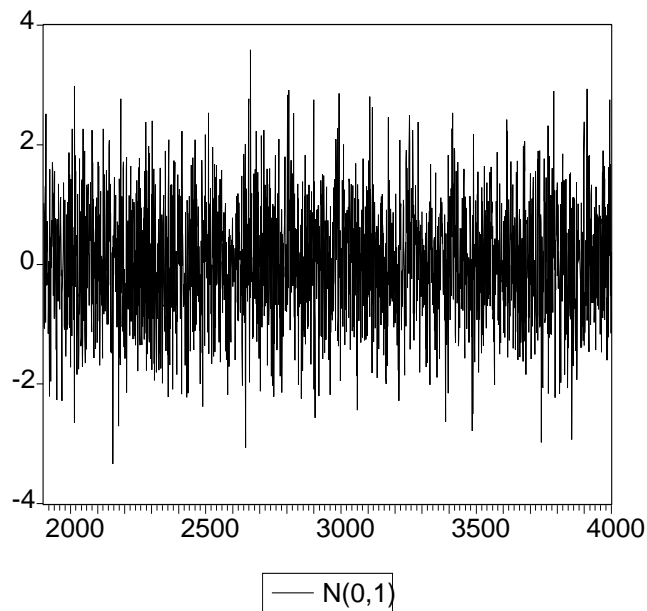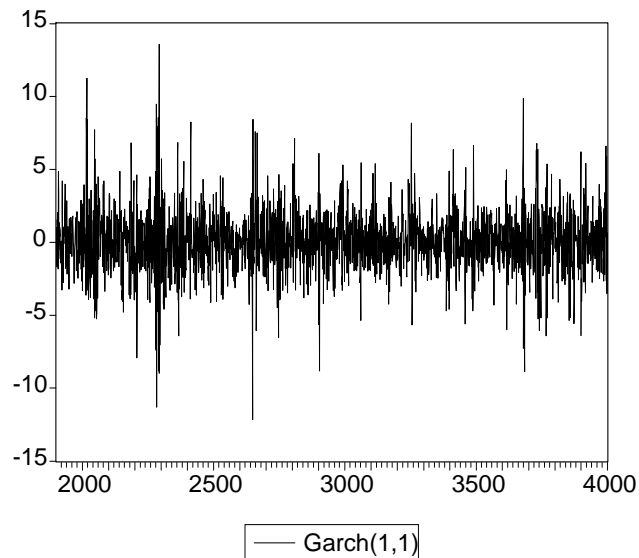# 1. AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH)

[1] EMPIRICAL REGULARITIES
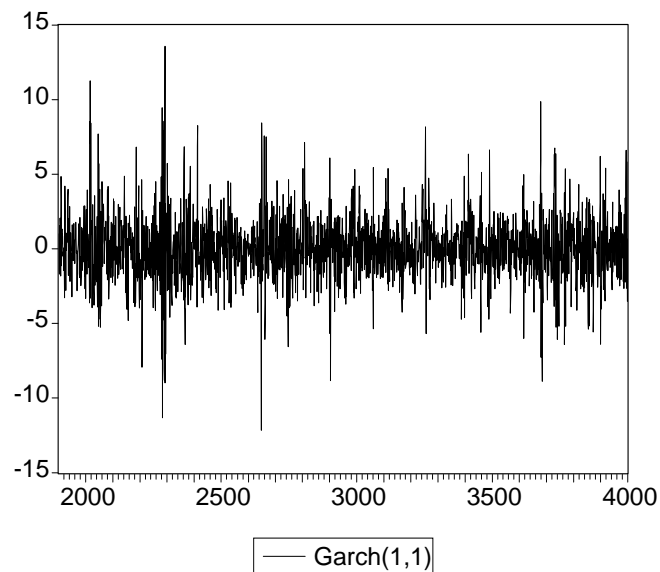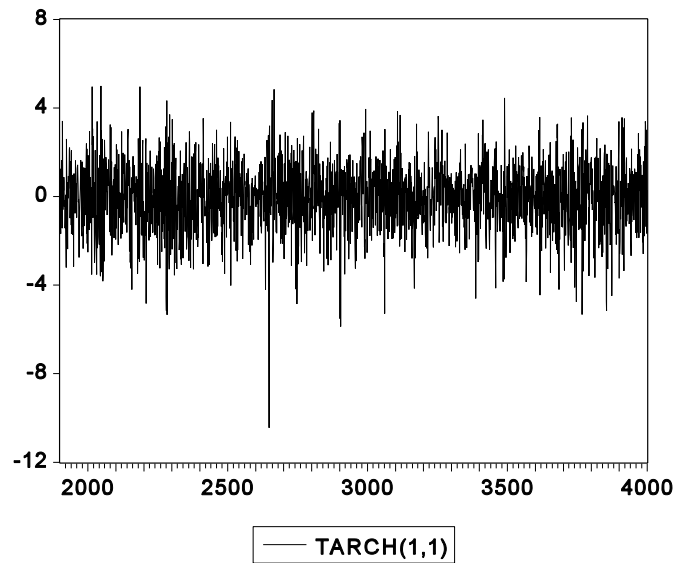
[See Bollerslev, Engle and Nelsen, Handbook of Econometrics,4]

(1)   Thick tails: Thicker than those of iid normal dist.

Garch(1,1)

N(0,1)

(2)   Volatility Clustering: Large changes tend to be followed by large changes.

(3)   Leverage effects: Changes in stock prices tend to be negatively related with changes in stock volatility.

TARCH(1,1)

Garch(1,1)

(4)   Non-trading periods effects:

Information that accumulates when financial markets are closed is reflected in prices after the market opens.

(5)   Forecastable events effects:

Forecastable releases of important information (e.g., earnings announcement) are associated with high ex ante volatility.
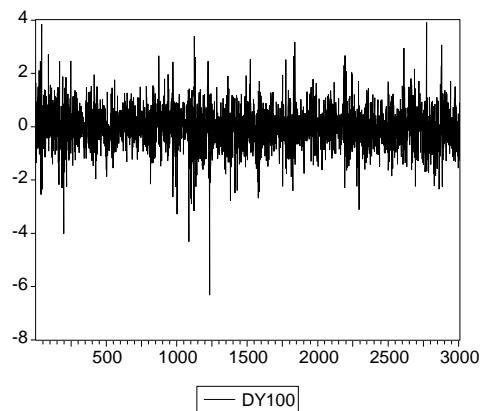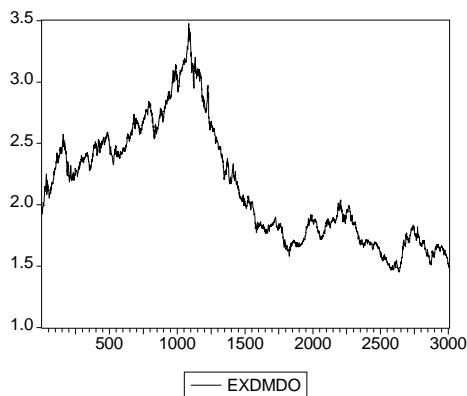
(6)   Volatility and serial correlation

Inverse relation between volatility and serial correlation for US stock indices.

(7)   Co-movements in volatilities

Commonality in volatility changes across stocks.

(8)   Macroeconomic variables and volatility

Positive relation bet/w macro uncertainty and stock market volatility.

[2] MODEL, ESTIMATION AND SPECIFICATION TESTS

(1) BASIC MODEL

$$y_t = x_t \$ + u_t, \ t = 1, 2, \ldots, T; \ u_t = \sqrt{h_t}\, v_t,$$

- v_t iid N(0,1): Need not be normal.

   Can be t-distribution with df = 8.

- $h_t = E(u_t^2 | S_{t-1}) = var(u_t | S_{t-1}) = h_t(\$,B),$

   where B is a parameter vector determining volatility in y_t.

   - Conditional variance of u_t given S_{t-1}.

   - h_t could be a function of some regressors, say, z_t.

   - S_{t-1}: Information set at time t-1

      $$S_{t-1} = \{x_{t-1}, x_{t-2}, \ldots, y_{t-1}, y_{t-2}, \ldots \}.$$

   - $F^2 = E(u_t^2) = var(u_t) =$ Unconditional variance of u_t

      = Unconditional mean of volatility.

NOTE:

- u_t should be serially uncorrelated.

- u_t^2 could be serially correlated.

(2) ESTIMATION

1) MLE

- Define the conditional pdf by $f(y_t|x_t, S_{t-1}, \theta)$, where $\theta = (\phi', \beta')'$

- Assume v_t iid $N(0,1)$,

$$f(y_t{}^*x_t, S_{t\&1}, \theta) = \frac{1}{\sqrt{2\pi h_t}}\exp\left[\&\frac{(y_t ! \; x_t\$)^2}{2h_t}\right].$$

- $l_T(\theta) = E_t \ln[f(y_t|x_t, S_{t-1}, \theta)]$

$$= -(T/2)\ln(2\pi) - (\tfrac{1}{2})E_t \ln(h_t) - (\tfrac{1}{2})E_t(y_t - x_t\$)^2/h_t .$$

- $s_t = \partial \ln[f(y_t|x_t, S_{t-1}, \theta)]/\partial \theta$.

- $H_T(\theta) = \partial^2 l_T(\theta)/\partial \theta \partial \theta'$; $B_T(\theta) = E_t s_t(\theta) s'_t(\theta)$.

- MLE: $\hat\theta . N[\theta, (-H_T(\hat\theta))^{-1}] . N[\theta, (B_T(\hat\theta))^{-1}]$.

- MLE with non-Gaussian v_t: See Hamilton (661-662).

2) MLE when normality assumption is violated

[QMLE: Bollerslev and Wooldridge, ER, 1992]

- MLE based on the normality assumption is still consistent.

- $\hat\theta . N[0, (H_T(\hat\theta))^{-1}B_T(\hat\theta)(H_T(\hat\theta))^{-1}]$.

- QMLE with other distributional assumptions

   [See Newey and Steigerwald (ECON, 1997)]

3) GMM

- Moment conditions:

$$E(x_t \cdot u_t) = 0 \; ; \; E[(u_t^2 - h_t)u_{t-j}] = 0, j = 1, 2, ...$$

- Not successful (Andersen and Sørensen, 1996, JBES).

(3)   SPECIFICATION TESTING

1) LM test (Engle, 1982, ECONOMETRICA)

H_o: No CH.

STEP 1: Do OLS on $y_t = x_t \$ + u_t$, and get $\hat{u}_t$.

STEP 2: Regress $\hat{u}_t^2$ on one, $\hat{u}_{t-1}^2$, $\hat{u}_{t-2}^2$, ... , $\hat{u}_{t-q}^2$, and get R^2.

STEP 3: $LM_T = TR^2 \to^d P^2(q)$, under H_o: No CH.

2) Specification Tests based on standardized errors

- H_o: Chosen model specification is correctly specified

- Let $e_t = \hat{u}_t / \sqrt{\hat{h}_t}$ (standardized residuals).

- If your choice of h_t is correctly specified, then e_t should be roughly iid.

- Testing autocorrelation of e_t or e_t^2:

  - AC (autocorrelation):

  $$\frac{E_{t'1}^{T\&J}(e_t - \bar{e})(e_{t\&j} - \bar{e})/T}{E_{t'1}^{T}(e_t - \bar{e})/T} = r_j/r_o ,$$

    where $r_j = {}'\sum_{t'1}^{T\&J}(e_t - \bar{e})(e_{t\&j} - \bar{e})/T$ and $r_o = {}'\sum_{t'1}^{T\&J}(e_t - \bar{e})^2/T$.

- PAC (partial AC):

  - Regress e_t on one, e_{t-1}, e_{t-2}, ... , e_{t-J}.

  - PAC of e_t and e_{t-J}

    = coefficient of e_{t-J} in this regression of e_t.

  - Under the hypothesis that e_t and e_{t-J} are uncorrelated,

    $$\sqrt{T} \cdot PAC \sim N(0,1)$$

- Ljung-Box Q-statistic:

  $$Q_{LB} = T(T+2) \; E_{j'1}^{p} \frac{r_j^2/r_o^2}{T - j} .$$

    6d $P^2(p)$, under H_o: no autocorrelation in e_t.

- Normality test (Jarque-Bera):

  See Greene.

Note: (Ahn's worry)

- Is Q_{LB} a relevant test?

- In fact, there is no reason to believe that Q_{LB} is $P^2(p)$.

- Bollerslev and Mikkelson's ad hoc solution:

    (1996, Journal of Econometrics)

    - Use $P^2(p$ - # of ARCH and GARCH parameters).

    - Not theoretically relevant, but it works!!!

[3] ARCH(q) MODEL [Engle, 1982, ECON]

(1) Specification:

- $h_t = \omega + \alpha_1 u_{t-1}^2 + ... + \alpha_q u_{t-q}^2$.

  - If (i) ω, α_j's > 0; (ii) $\alpha_1 + ... + \alpha_q < 1$, the model is stationary

    [Sufficient, but not necessary]

  - $\sigma^2 / var(u_t) = \omega/(1 - \alpha_1 - ... - \alpha_q)$ under (i) and (ii).

    Even if $\alpha_1 + ... + \alpha_q = 1$, the model could be still stationary

    and ergodic (so MLE is consistent and efficient).

  - For MLE or QMLE,

    $h_t = \omega + \alpha_1 (y_{t-1} - x_{t-1}\beta)^2 + ... + \alpha_q (y_{t-q} - x_{t-q}\beta)^2$.

    $\theta = (\beta', \omega, \alpha_1, ..., \alpha_q)'$

    Set $u_0^2 = ... = u_{1-q}^2 = T^{-1} E_t (y_t - x_t\beta)^2$.

  - Can introduce some regressors in h_t: e.g.,

    $h_t = \omega + \alpha_1 u_{t-1}^2 + ... + \alpha_q u_{t-q}^2 + \gamma d_t$, where d_t is a dummy

    variable for Mondays.

(2) Alternative representation

- Let $w_t = u_t^2 - h_t$:

  $u_t^2 = h_t + w_t = \omega + \alpha_1 u_{t-1}^2 + ... + \alpha_q u_{t-q}^2 + w_t$.

    $\Rightarrow u_t^2 \sim AR(q)$.

(3)　Forecast of volatility

1) Let $u^2_{t+s|t} = E(u^2_{t+s} | u^2_t, u^2_{t-1}, ...)$: Optimal Predictor of future h_t.

2) $u^2_{t+1|t} = \omega + \alpha_1 u_t^2 + \alpha_2 u_{t-1}^2 + ... + \alpha_q u_{t-q+1}^2$;

$u^2_{t+2|t} = \omega + \alpha_1 u^2_{t+1|t} + \alpha_2 u_t^2 + ... + \alpha_q u_{t-q+2}^2$;

$u^2_{t+3|t} = \omega + \alpha_1 u^2_{t+2|t} + \alpha_2 u^2_{t+1|t} + \alpha_3 u_t^2 + \alpha_q u_{t-q+1}^2$;

: :

$u^2_{t+s|t} = \omega + \alpha_1 u^2_{t+1+s|t} + \alpha_2 u^2_{t+2+s|t} + \alpha_3 u^2_{t+3+s|t} + ... + \alpha_q u^2_{t+q+s|t}$.

$u^2_{t+s|t} \overset{p}{\to} \omega / (1 - \alpha_1 - ... - \alpha_q)$, as $s \to \infty$.

[4]   GENERALIZED ARCH [Bollerslev, 1986, JEC]

[Called GARCH(p,q)]

(1)   Motivation

• q is usually too large when ARCH(q) is used.

• Need a parsimonious model.

(2)   Specification

$$h_t = \omega + \alpha_1 h_{t-1} + ... + \alpha_p h_{t-p} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2 + ... + \beta_q u_{t-q}^2 .$$

- For MLE or QMLE,

$$\theta = (\omega, \alpha_1,...,\alpha_p, \beta_1,...,\beta_q)'$$

Set $h_0 = ... = h_{1-p} = u_0^2 = ... = u_{1-q}^2 = T^{-1} E_t (y_t - x_t \beta)^2 .$

- Let $r = \max\{p,q\}$; and set $\beta_{q+1} = ... = \beta_r = 0$ or $\alpha_{p+1} = ... = \alpha_r = 0$.

$$h_t = \omega + \alpha_1 h_{t-1} + ... + \alpha_r h_{t-r} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2 + ... + \beta_r u_{t-r}^2 .$$

- If (i) $\omega, \alpha_j, \beta_j > 0$ and (ii) $\sum_{j=1}^{r} (\alpha_j + \beta_j) < 1$, the model is

stationary. [Sufficient, but not necessary.]

- $\sigma^2 = var(u_t^2) = \omega /[1-(\alpha_1+\beta_1)-...-(\alpha_r+\beta_r)]$ (if (i) and (ii) hold).

(3) GARCH(1,1)

1) $h_t = T + {}^*h_{t-1} + {}^"u_{t-1}^2$.

  • $h_t = T + {}^*(T + {}^*h_{t-2} + {}^"u_{t-2}^2) + {}^"u_{t-1}^2$

    $= (1 + {}^*)T + {}^{*2}h_{t-2} + {}^"(u_{t-1}^2 + {}^*u_{t-2}^2)$

    $= T/(1-{}^*) + {}^" E_{j'\ 1}^4\ {}^{*j\&1}u_{t\&j}^2$. [ARCH(4)]

  • Let $w_t = u_t^2 - h_t$: unforecastable volatility.

    $u_t^2 = (u_t^2 - h_t) + h_t = w_t + [T + {}^*h_{t-1} + {}^"u_{t-1}^2]$

      $= w_t + T + ({}^*+{}^")u_{t-1}^2 - {}^*(u_{t-1}^2 - h_{t-1})$

      $= w_t + T + ({}^*+{}^")u_{t-1}^2 - {}^*w_{t-1}$

      $= T + ({}^*+{}^")u_{t-1}^2 + w_t - {}^*w_{t-1}$ [Like ARMA(1,1)]

  • $h_t = T + {}^*h_{t-1} + {}^"h_{t-1}v_{t-1}^2 = T + h_{t-1}({}^*+{}^"v_{t-1}) / T + h_{t-1}\rangle_{t-1}$

    $= T[1 + E_{j'\ 1}^4 A_{i'\ 1}^j ({}^*+{}^"v_{t-1})]$

Theorem: (Nelson, 1990, ECON)

  GARCH(1,1) is stationary and ergodic iff $E[\ln({}^*+{}^"v_t)] < 0$. And,

    $T/(1-{}^*) \# F^2 < 4$.

Implication:

  • Even if ${}^* + {}^" = 1$, GARCH(1,1) can be stationary.

  • MLE is still consistent and efficient.

2) Forecasting:

$$u_t^2 = \mathsf{T} + (^* +\text{''})u_{t-1}^2 + w_t - {}^*w_{t-1}.$$

i) Let $u_{t\%s^*t}^2 \; / \; E(u_{t\%s}^2 {}^*u_t^2, u_{t\&1}^2, \dots, w_t^2, w_{t\&1}^2, \dots):$

Optimal Predictor of h_{t+s} at time t.

$$w_{t\%s^*t} \; / \; E(w_{t\%s}{}^*u_t^2, \dots, w_t, \dots) = 0.$$

ii) $u_{t\%1^*t}^2 = \mathsf{T} + (^* +\text{''}_1)u_t^2 + w_{t+1^*t} - {}^*w_t = \mathsf{T} + (^* +\text{''})u_t^2 - {}^*w_t;$

$$u_{t\%2^*t}^2 = \mathsf{T} + (^* +\text{''}_1)u_{t\%1^*t}^2;$$

$$u_{t\%3^*t}^2 = \mathsf{T} + (^* +\text{''}_1)u_{t\%2^*t}^2;$$

$$\vdots$$

$$u_{t\%s^*t}^2 \overset{p}{6} \mathsf{T}/(1 \; ^* \; \text{''}), \text{ as } s\; 6\; 4.$$

3) Regarding H_o: $\text{''} = 0.$

- Not possible to test for this hypothesis by Wald tests. Under H_o, the model is not identified. [Under GARCH(1,0) with the stationarity conditions (i) and (ii), $h_t = h_{t-1} = \dots = \mathsf{F}^2 = \mathsf{T}/(1-^*).$]

- See Andrews and Ploberger (1994, ECON) and B. Hansen (1996, ECON).

(4) GARCH(p,q)

$$h_t = T + {}^*_1 h_{t-1} + \ldots + {}^*_p h_{t-p} + {}''_1 u_{t-1}{}^2 + {}''_2 u_{t-2}{}^2 + \ldots + {}''_q u_{t-q}{}^2 .$$

- Let $r = \max\{p,q\}$; and set $''_{p+1} = \ldots = ''_r = 0$ or $*_{q+1} = \ldots = *_r = 0$.

- $h_t = T + {}^*_1 h_{t-1} + \ldots + {}^*_r h_{t-r} + {}''_1 u_{t-1}{}^2 + {}''_2 u_{t-2}{}^2 + \ldots + {}''_r u_{t-r}{}^2 .$

- $u_t{}^2 \quad = (u_t{}^2 - h_t) + h_t \quad$ (let $w_t = u_t{}^2 - h_t$)

$$= w_t + [T + {}^*_1 h_{t-1} + \ldots + {}^*_r h_{t-r} + {}''_1 u_{t-1}{}^2 + \ldots + {}''_r u_{t-r}{}^2]$$

$$= w_t + T + ({}^*_1 + {}''_1) u_{t-1}{}^2 + \ldots + ({}^*_r + {}''_r) u_{t-r}{}^2$$

$$- {}^*_1 (u_{t-1}{}^2 - h_{t-1}) - \ldots - {}^*_r (u_{t-r}{}^2 - h_{t-r})$$

$$= T + ({}^*_1 + {}''_1) u_{t-1}{}^2 + ({}^*_r + {}''_r) u_{t-r}{}^2$$

$$+ w_t - {}^*_1 w_{t-1} - \ldots - {}^*_r w_{t-r}. \quad \text{(Like ARMA(r,p))}$$

[5] INTEGRATED GARCH(p,q) [Bollerslev and Engle, ER, 1986]

[Called IGARCH]

- $h_t = \top + *_1 h_{t-1} + ... + *_p h_{t-p} + "_1 u_{t-1}^2 + "_2 u_{t-2}^2 + ... + "_q u_{t-q}^2$,

  with $E_j *_j + E_j "_j = 1$.

- Looks like nonstationary. But it could be stationary.

- Can test IGARCH(1,1) using a Wald statistic.

  [We conjecture that Wald tests can be used for IGARCH(p,q).

  But there is no formal proof.]

- $u_{t\%*t}^2$ 6 4 as s 6 4.

- QMLE for IGARCH(1,1) is consistent and asymptotically normal under certain conditions (See Lumsdaine, 1996, ECON).

[6] Exponential GARCH [Nelson, 1991, ECONOMETRICA]
[Called EGARCH]

(1) Motivation:

GARCH models do not capture leverage effects.

(2) Basic Model

$$\ln(h_t) = \top + {}^*_1\ln(h_{t-1}) + ... + {}^*_p\ln(h_{t-p}) + {}^"_1 O_{t-1} + {}^"_2 O_{t-2} + ... + {}^"_q O_{t-q},$$

where $O_t = |v_t| - E|v_t| + ($ v_t and v_t follows generalized error

distribution (p. 668, Hamilton, or Nelson, 1990)

- $E|v_t| = \sqrt{2/\beta}$.

- $\top$, *'s and $^"$'s do not have to be positive.

- If $(= 0$, $O_t = |v_t| - E|v_t|$.

  - Positive and negative v_t have the symmetric effects on h_t.

- If $-1 < (< 0$:

  - If $v_t > 0$, $O_t = (1+( )v_t - E|v_t| = (0 < 1+(< 1)$ $v_t - E|v_t|$

  - If $v_t < 0$, $O_t = (-1+( )v_t - E|v_t| = (-2 < -1+(< -1)v_t - E|v_t|$

  - Negative v_t has greater effects than positive v_t.

- If $(< -1$:

  - If $v_t > 0$, $O_t = (1+( )v_t - E|v_t| = (1+(< 0)$ $v_t - E|v_t|$

  - If $v_t < 0$, $O_t = (-1+( )v_t - E|v_t| = (-1+(< -2)v_t - E|v_t|$

  - Postive v_t reduces h_t.

(3) Conditions for stationarity and ergodicity:

$$E_j \Pi_j < 4.$$

(4) Example:

- r_t = (daily return on stock) - (daily interest rate on treasury bills)

  - $r_t = \$_1 + \$_2 r_{t-1} + \delta h_t + u_t$;

  - $u_t = \sqrt{h_t}\, v_t$;

  - $\ln(h_t) - \mu_t = *_1(\ln(h_{t-1}) - \mu_{t-1}) + *_2(\ln(h_{t-2}) - \mu_{t-2}) + \Pi_1 O_{t-1} + \Pi_2 O_{t-2}$.

  - $\mu_t = \mu + \ln(1 + \delta N_t)$, N_t = # of nontrading days bet/w (t-1) and t.

[7] COMPONENT GARCH(1,1)

(1) Symmetric Component GARCH (1,1).

- Reconsider GARCH(1,1):
  - 1) $h_t = \top + {}^* h_{t-1} + {}'' u_{t-1}^2$.
  - 2) $h_t{}' \top \% {}^*(h_{t\&1} \& \top) \% {}''(u_{t\&1}^2 \& \top)$, where $\top{}' T/(1 \& {}'' \& {}^*)$.
  - The equation 2) implies that in GARCH(1,1) h_t fluctuates around $\top$ (mean reversion to $\top$).
  - The valitility measure h_t could have transitory (short-run) and permanent (long-run) components.
  - In GARCH(1,1), $(h_t \& \top)$ is the transitory component and $\top$ is the pernament component.

- Symmetric Component GARCH (1,1)
  - Wish to allow the permanent component fluctuate over time.
  - $h_t{}' q_t \% {}^*(h_{t\&1} \& q_{t\&1}) \% {}''(u_{t\&1}^2 \& q_{t\&1})$;
    $q_t{}' \top \% D(q_{t\&1} \& \top) \% N(u_{t\&1}^2 \& h_{t\&1})$.
  - The transitory component $(h_t \& q_t)$ fluctuates around zero and the long-run component fluctuates around $\top$.

- The above two equations imply:

$$h_t \approx \varpi + \alpha_1 h_{t-1} + \alpha_2 h_{t-1} + \beta_1 u_{t-1}^2 + \beta_2 u_{t-2}^2,$$

where

$$\varpi = (1 - \alpha - \beta)(1 - \rho)\varpi \ ; \ \alpha_1 = \alpha + \rho - \phi \ ;$$

$$\alpha_2 = -(\alpha\rho - (\alpha+\beta)\phi) \ ; \ \beta_1 = \beta + \phi \ ; \ \beta_2 = -(\beta\rho + (\alpha+\beta)\phi).$$

- Thus, component GARCH(1,1) can be viewed as a restricted GARCH(2,2).

(2)   Asymmetric Component GARCH(1,1)

- $h_t \approx q_t + \alpha(h_{t-1} - q_{t-1}) + \beta(u_{t-1}^2 - q_{t-1}) + \gamma(u_{t-1}^2 - q_{t-1})d_{t-1},$

  where $d_t = 1$ iff $u_{t-1} < 0$.

[7] THRESHHOLD ARCH

[Glosten, Jagannathan and Runkle, JF, 1994; Called TARCH]

- $h_t = \top + {}^{\star}h_{t-1} + " u_{t-1}{}^2 + ((u_{t-1}{}^2)\mathbf{1}(u_{t-1} < 0),$

  where $\top$, *, $"$ and $(> 0.$

[8] GARCH IN MEAN

[Engle, Lilien and Robins, 1987, ECON; Called GARCH-M]

$y_t = x_t\$ + 8h_t + u_t$ or $y_t = x_t\$ + 8\sqrt{h_t} + u_t$.

[9] MULTIVARIATE GARCH

(See Hamilton.)

[10] Applications -Eviews

(1) Estimation

STEP 1: Push **Objects/New Object**.

STEP 2: Choose **Equation**. Push OK button. Then, you are in **Equation Specification** box.
Go to **Equation Setting**, and Choose **ARCH**.

STEP 3: In **Equation Specification** box, type:

dy100 c

STEP 4: Go to **Equation Setting** and type:

2 1001

STEP 5: Click the **option buttom**. Increase the convergence rate (0.0001) and increase maxit
to 1000. Choose algorithm and Heteroskedasticity-Robust Covariance matrix (for
QMLE). Once you have chosen appropriate options, click the **ok** buttom.

STEP 6: Choose a specification and run the program.

(2) GARCH(1,2)

$$y_t = \$ + u_t; \quad h_t = \top + {}^*_1 h_{t-1} + {}^{"}_1 u_{t-1}^2 + {}^{"}_2 u_{t-2}^2:$$

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 42 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.049709	0.021991	2.260437	0.0238

Variance Equation				
C	0.010867	0.006700	1.622076	0.1048
ARCH(1)	0.032561	0.050742	0.641692	0.5211
ARCH(2)	0.015376	0.052303	0.293984	0.7688
GARCH(1)	0.932021	0.028271	32.96791	0.0000

R-squared	-0.000051	Mean dependent var	0.044314
Adjusted R-squared	-0.004071	S.D. dependent var	0.755497
S.E. of regression	0.757033	Akaike info criterion	2.205777
Sum squared resid	570.2337	Schwarz criterion	2.230316
Log likelihood	-1097.888	Durbin-Watson stat	2.137029

(3) TARCH(1,2)

$$y_t = \$ + u_t; \quad h_t = T + {}^*{}_1 h_{t-1} + "{}_1 u_{t-1}{}^2 + ({}_1(u_{t-1}{}^2)\mathbf{1}(u_{t-1} < 0) + "{}_2 u_{t-2}{}^2:$$

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 28 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.047872	0.021862	2.189744	0.0285
Variance Equation				
C	0.007089	0.008815	0.804239	0.4213
ARCH(1)	0.028297	0.034449	0.821422	0.4114
(RESID<0)*ARCH(1)	0.004401	0.017584	0.250280	0.8024
GARCH(1)	1.312629	0.852429	1.539868	0.1236
GARCH(2)	-0.356171	0.803179	-0.443452	0.6574

R-squared	-0.000022	Mean dependent var	0.044314
Adjusted R-squared	-0.005053	S.D. dependent var	0.755497
S.E. of regression	0.757403	Akaike info criterion	2.207258
Sum squared resid	570.2172	Schwarz criterion	2.236705
Log likelihood	-1097.629	Durbin-Watson stat	2.137091

ARCH-24

(4) EGRACH(1,2)

$y_t = \$ + u_t;$

$\ln(h_t) = T + {}^*\ln(h_{t-1}) + "\ _1 O_{t-1} + "\ _2 O_{t-2}; O_{t-1} = |v_{t-1}| - E(|v_{t-1}|) + (\ _1 v_{t-1}; O_{t-2} = |v_{t-2}| - E|v_{t-2}| + (\ _2 v_{t-2}$

$Y\ \ln(h_t) = T + {}^*\ln(h_{t-1}) + \{"\ _1(|v_{t-1}| - E|v_{t-1}|) + "\ _1(\ _1 v_{t-2}\} + \{"\ _2(|v_{t-2}| - E|v_{t-2}|) + "\ _2(\ _2 v_{t-2}\}$

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 47 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.046725	0.022159	2.108595	0.0350

Variance Equation				
C	-0.040548	0.035358	-1.146777	0.2515
|RES|/SQR[GARCH](1)	0.045956	0.039668	1.158513	0.2467
RES/SQR[GARCH](1)	0.004615	0.009793	0.471314	0.6374
EGARCH(1)	1.563571	0.382723	4.085382	0.0000
EGARCH(2)	-0.571963	0.375954	-1.521364	0.1282

R-squared	-0.000010	Mean dependent var	0.044314
Adjusted R-squared	-0.005040	S.D. dependent var	0.755497
S.E. of regression	0.757398	Akaike info criterion	2.209685
Sum squared resid	570.2104	Schwarz criterion	2.239132
Log likelihood	-1098.843	Durbin-Watson stat	2.137116

(5) GARCH(1,2)-M with Variance

$y_t = \$ + 8h_t + u_t;$

$h_t = \top + {}^*h_{t-1} + {}^{\prime\prime}{}_1 u_{t-1}{}^2 + {}^{\prime\prime}{}_2 u_{t-2}{}^2 .$

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 46 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-0.210455	0.122764	-1.714315	0.0865
C	0.155385	0.064306	2.416321	0.0157

Variance Equation				
C	0.009611	0.007936	1.211117	0.2259
ARCH(1)	0.036125	0.027316	1.322476	0.1860
GARCH(1)	1.381466	0.506710	2.726347	0.0064
GARCH(2)	-0.434450	0.470056	-0.924252	0.3554

R-squared	0.005024	Mean dependent var	0.044314
Adjusted R-squared	0.000019	S.D. dependent var	0.755497
S.E. of regression	0.755490	Akaike info criterion	2.207335
Sum squared resid	567.3398	Schwarz criterion	2.236781
Log likelihood	-1097.667	F-statistic	1.003836
Durbin-Watson stat	2.146356	Prob(F-statistic)	0.414162

(6) Graph for $\sqrt{h_t}$.

Go to **view/conditional SD graph**.
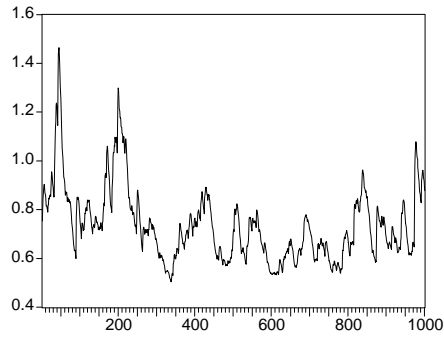
(7) Forecast: from 1002 to 1101
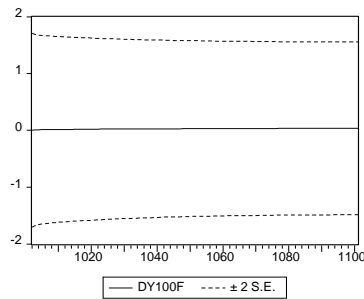
Forecast: DY100F
Actual: DY100
Forecast sample: 1002 1101
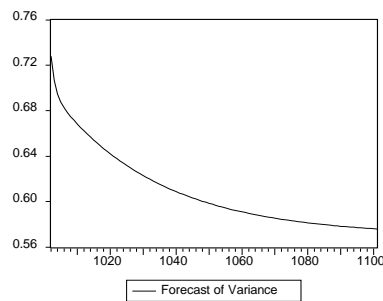Included observations: 100

Root Mean Squared Error	0.853280
Mean Absolute Error	0.568194
Mean Abs. Percent Error	88.99422
Theil Inequality Coefficient	0.965964
Bias Proportion	0.002651
Variance Proportion	0.981123
Covariance Proportion	0.016226

(8) Wald Test:

If you wish to test multiple restrictions on parameters, go to **view/coefficient tests**.

(9) Specification tests based on standardized residuals

1) For Specification tests, go to **view/residual tests/correlogram-Q statististics**.

[About v_t. If your model is correctly specified, then the standardized residuals should be serially uncorrelated.]

Date: 04/03/00   Time: 16:49
Sample: 2 1001
Included observations: 1000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
*| |	*| |	1	-0.065	-0.065	4.1978	0.040
.| |	.| |	2	0.010	0.006	4.2946	0.117
.| |	.| |	3	0.006	0.007	4.3312	0.228
.| |	.| |	4	0.049	0.050	6.7941	0.147
.| |	.| |	5	0.009	0.015	6.8678	0.231
.| |	.| |	6	-0.019	-0.018	7.2219	0.301
.| |	.| |	7	0.008	0.005	7.2852	0.400
.| |	.| |	8	-0.012	-0.014	7.4395	0.490
.| |	.| |	9	0.028	0.026	8.2582	0.508
.| |	.| |	10	0.011	0.016	8.3730	0.592
.| |	.| |	11	0.013	0.014	8.5381	0.664
.| |	.| |	12	-0.022	-0.020	9.0221	0.701
.| |	.| |	13	-0.036	-0.041	10.306	0.669
.| |	.| |	14	0.038	0.031	11.788	0.623
.| |	.| |	15	0.017	0.022	12.069	0.674

2) Go to **view/residual tests/correlogram squared residuals**.

[About v_t^2. If your model is correctly specified, then squared standardized residuals should be serially uncorrelated.]
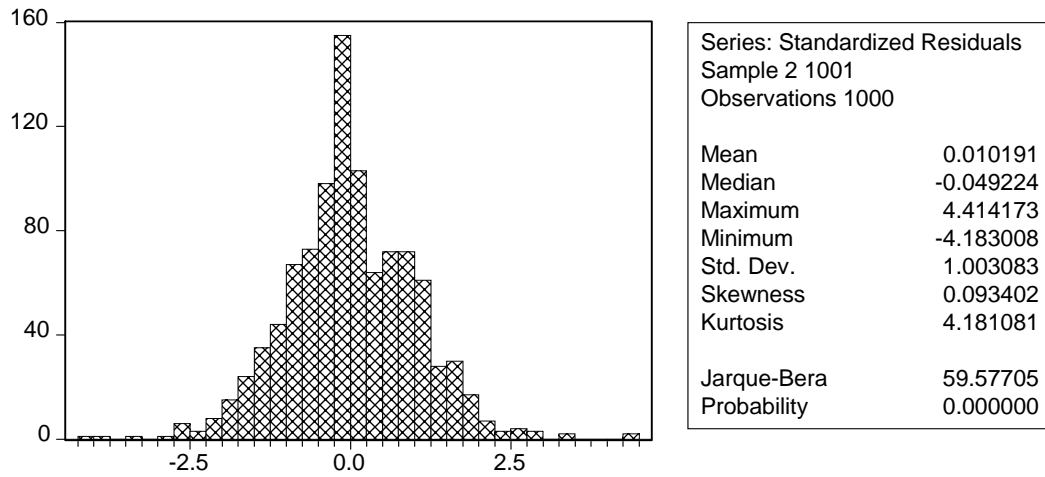
Date: 04/03/00   Time: 16:51
Sample: 2 1001
Included observations: 1000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.| |	.| |	1	0.010	0.010	0.1052	0.746
.| |	.| |	2	-0.032	-0.032	1.1219	0.571
.| |	.| |	3	-0.016	-0.015	1.3787	0.711
.| |	.| |	4	0.036	0.035	2.6861	0.612
.| |	.| |	5	0.006	0.004	2.7185	0.743
.| |	.| |	6	-0.016	-0.015	2.9927	0.810
.| |	.| |	7	-0.011	-0.010	3.1200	0.874
.| |	.| |	8	0.012	0.010	3.2721	0.916
.| |	.| |	9	0.025	0.023	3.9068	0.917
.| |	.| |	10	0.008	0.009	3.9701	0.949
.| |	.| |	11	-0.041	-0.039	5.7012	0.893
.| |	.| |	12	-0.016	-0.015	5.9587	0.918
.| |	.| |	13	-0.042	-0.046	7.7774	0.858
.| |	.| |	14	-0.004	-0.006	7.7909	0.900
.| |	.| |	15	0.055	0.056	10.905	0.759
.| |	.| |	16	-0.026	-0.027	11.588	0.772

3) Go to **view/residual tests/histogram normality test**.

Series: Standardized Residuals	
Sample 2 1001	
Observations 1000	
Mean	0.010191
Median	-0.049224
Maximum	4.414173
Minimum	-4.183008
Std. Dev.	1.003083
Skewness	0.093402
Kurtosis	4.181081
Jarque-Bera	59.57705
Probability	0.000000

4) Go to **view/residual tests/ARCH LM tests**. Set Lag = 4.

[If you model is correctly specified, then standardized errors should not include any ARCH effects.]

ARCH Test:

F-statistic	0.668245	Probability	0.614110
Obs*R-squared	2.679241	Probability	0.612852