HW 1 (Due May 28, Wednesday)

Q1. (10 points.) Let $\Sigma = aP_T + Q_T$, where a > 0. Show that $\Sigma^{-1} = a^{-1}P_T + Q_T$.

Q2. Consider the following panel data model:

$$y_i = X_i\beta + (e_T\alpha_i + \varepsilon_i),$$

where i = 1, ..., N, and all of the symbols are defined in the class notes. X contains k variables. Note that this model does not have time-invariant regressors. For the entire data, this model can be expressed as $y = X\beta + (V\alpha + \varepsilon)$, where $\alpha = (\alpha_1, ..., \alpha_N)'$. Answer the following questions (10 points on each).

(1) Show that

$$\begin{pmatrix} e_T\alpha_1 \\ e_T\alpha_2 \\ \vdots \\ e_T\alpha_N \end{pmatrix} = V\alpha .$$

(2) Assuming that σ_ε^2 is known, derive $Cov(\hat{\beta}_W)$.

(3) Assume that the α_i are random with $N(0, \sigma_\alpha^2)$ and uncorrelated with X_i. Assume that X_i contains only one regressor and both σ_α^2 and σ_ε^2 are known. Under these assumptions, show that $\text{var}(\hat{\beta}_{GLS}) < \text{var}(\hat{\beta}_W)$.