exist with lesser ones in the field of sense-perception, and not the reverse. In all this there is simply expressed a peculiar and profound connection of things. To demand at the present time a full elucidation of this matter, is to anticipate, in the manner of speculative philosophy, the results of all future special investigation, that is a perfect physical science. (Compare Appendix, XIX., p. 541.)

3. Views similar to those concerning time, are developed by Newton with respect to space and motion. We extract here a few passages which characterise his position.

"II. Absolute space, in its own nature and without regard to anything external, always remains similar and immovable.

"Relative space is some movable dimension or measure of absolute space, which our senses determine by its position with respect to other bodies, and which is commonly taken for immovable [absolute] space. . . .

"IV. Absolute motion is the translation of a body from one absolute place* to another absolute place; and relative motion, the translation from one relative place to another relative place. . . .

". . . And thus we use, in common affairs, instead of absolute places and motions, relative ones; and that without any inconvenience. But in physical disquisitions, we should abstract from the senses. For it may be that there is no body really at rest, to which the places and motions of others can be referred. . . .

"The effects by which absolute and relative motions

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* The place, or locus of a body, according to Newton, is not its position, but the part of space which it occupies. It is either absolute or relative.—Tran.
"are distinguished from one another, are centrifugal
forces, or those forces in circular motion which pro-
duce a tendency of recession from the axis. For in
a circular motion which is purely relative no such
forces exist; but in a true and absolute circular mo-
tion they do exist, and are greater or less according
to the quantity of the [absolute] motion.

"For instance. If a bucket, suspended by a long The rota-
cord, is so often turned about that finally the cord is
strongly twisted, then is filled with water, and held
at rest together with the water; and afterwards by
the action of a second force, it is suddenly set whirling
about the contrary way, and continues, while the
cord is untwisting itself, for some time in this mo-
tion; the surface of the water will at first be level,
just as it was before the vessel began to move; but,
subsequently, the vessel, by gradually communicat-
ing its motion to the water, will make it begin sens-
ibly to rotate, and the water will recede little by little
from the middle and rise up at the sides of the ves-
sel, its surface assuming a concave form. (This ex-
periment I have made myself.)

"... At first, when the relative motion of the wa-
ter in the vessel was greatest, that motion produced
no tendency whatever of recession from the axis; the
water made no endeavor to move towards the cir-
cumference, by rising at the sides of the vessel, but
remained level, and for that reason its true circular
motion had not yet begun. But afterwards, when
the relative motion of the water had decreased, the
rising of the water at the sides of the vessel indicated
an endeavor to recede from the axis; and this en-
deavor revealed the real circular motion of the water,
continually increasing, till it had reached its greatest
"point, when relatively the water was at rest in the
vessel . . . .

"It is indeed a matter of great difficulty to discover
and effectually to distinguish the true from the ap-
parent motions of particular bodies; for the parts of
that immovable space in which bodies actually move,
do not come under the observation of our senses.

"Yet the case is not altogether desperate; for there
exist to guide us certain marks, abstracted partly
from the apparent motions, which are the differences
of the true motions, and partly from the forces that
are the causes and effects of the true motions. If,
for instance, two globes, kept at a fixed distance
from one another by means of a cord that connects
them, be revolved about their common centre of
gravity, one might, from the simple tension of the
cord, discover the tendency of the globes to recede
from the axis of their motion, and on this basis the
quantity of their circular motion might be computed.
And if any equal forces should be simultaneously
impressed on alternate faces of the globes to augment
or diminish their circular motion, we might, from
the increase or decrease of the tension of the cord,
deduce the increment or decrement of their motion;
and it might also be found thence on what faces
forces would have to be impressed, in order that the
motion of the globes should be most augmented;
that is, their rear faces, or those which, in the cir-
cular motion, follow. But as soon as we knew which
faces followed, and consequently which preceded, we
should likewise know the direction of the motion.
In this way we might find both the quantity and the
direction of the circular motion, considered even in
an immense vacuum, where there was nothing ex-
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"ternal or sensible with which the globes could be compared."

4. It is scarcely necessary to remark that in the reflections here presented Newton has again acted contrary to his expressed intention only to investigate actual facts. No one is competent to predicate things about absolute space and absolute motion; they are pure things of thought, pure mental constructs, that cannot be produced in experience. All our principles of mechanics are, as we have shown in detail, experimental knowledge concerning the relative positions and motions of bodies. Even in the provinces in which they are now recognised as valid, they could not, and were not, admitted without previously being subjected to experimental tests. No one is warranted in extending these principles beyond the boundaries of experience. In fact, such an extension is meaningless, as no one possesses the requisite knowledge to make use of it.

Let us look at the matter in detail. When we say that a body $K$ alters its direction and velocity solely through the influence of another body $K'$, we have asserted a conception that it is impossible to come at unless other bodies $A$, $B$, $C$... are present with reference to which the motion of the body $K$ has been estimated. In reality, therefore, we are simply cognisant of a relation of the body $K$ to $A$, $B$, $C$... If now we suddenly neglect $A$, $B$, $C$... and attempt to speak of the deportment of the body $K$ in absolute space, we implicate ourselves in a twofold error. In the first place, we cannot know how $K$ would act in the absence of $A$, $B$, $C$...; and in the second place, every means would be wanting of forming a judgment of the behaviour of $K$ and of putting to the test what we had
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predicated,—which latter therefore would be bereft of all scientific significance.

Two bodies $K$ and $K'$, which gravitate toward each other, impart to each other in the direction of their line of junction accelerations inversely proportional to their masses $m$, $m'$. In this proposition is contained, not only a relation of the bodies $K$ and $K'$ to one another, but also a relation of them to other bodies. For the proposition asserts, not only that $K$ and $K'$ suffer with respect to one another the acceleration designated by $\kappa (m + m'/r^2)$, but also that $K$ experiences the acceleration $-\kappa m'/r^2$ and $K'$ the acceleration $+\kappa m/r^2$ in the direction of the line of junction; facts which can be ascertained only by the presence of other bodies.

The motion of a body $K$ can only be estimated by reference to other bodies $A$, $B$, $C$... But since we always have at our disposal a sufficient number of bodies, that are as respects each other relatively fixed, or only slowly change their positions, we are, in such reference, restricted to no one definite body and can alternately leave out of account now this one and now that one. In this way the conviction arose that these bodies are indifferent generally.

It might be, indeed, that the isolated bodies $A$, $B$, $C$... play merely a collateral rôle in the determination of the motion of the body $K$, and that this motion is determined by a medium in which $K$ exists. In such a case we should have to substitute this medium for Newton's absolute space. Newton certainly did not entertain this idea. Moreover, it is easily demonstrable that the atmosphere is not this motion-determinative medium. We should, therefore, have to picture to ourselves some other medium, filling, say, all space, with respect to the constitution of which and its kinetic
relations to the bodies placed in it we have at present no adequate knowledge. In itself such a state of things would not belong to the impossibilities. It is known, from recent hydrodynamical investigations, that a rigid body experiences resistance in a frictionless fluid only when its velocity changes. True, this result is derived theoretically from the notion of inertia; but it might, conversely, also be regarded as the primitive fact from which we have to start. Although, practically, and at present, nothing is to be accomplished with this conception, we might still hope to learn more in the future concerning this hypothetical medium; and from the point of view of science it would be in every respect a more valuable acquisition than the forlorn idea of absolute space. When we reflect that we cannot abolish the isolated bodies \( A, B, C \ldots \), that is, cannot determine by experiment whether the part they play is fundamental or collateral, that hitherto they have been the sole and only competent means of the orientation of motions and of the description of mechanical facts, it will be found expedient provisionally to regard all motions as determined by these bodies.

5. Let us now examine the point on which New- 

ton, apparently with sound reasons, rests his distinc-

tion of absolute and relative motion. If the earth is 

affected with an \textit{absolute} rotation about its axis, cen-

trifugal forces are set up in the earth: it assumes an 

oblate form, the acceleration of gravity is diminished 

at the equator, the plane of Foucault's pendulum ro-

tates, and so on. All these phenomena disappear if 

the earth is at rest and the other heavenly bodies are 

affected with absolute motion round it, such that the 

same \textit{relative} rotation is produced. This is, indeed, the 

case, if we start \textit{ab initio} from the idea of absolute space.
But if we take our stand on the basis of facts, we shall find we have knowledge only of relative spaces and motions. Relatively, not considering the unknown and neglected medium of space, the motions of the universe are the same whether we adopt the Ptolemaic or the Copernican mode of view. Both views are, indeed, equally correct; only the latter is more simple and more practical. The universe is not twice given, with an earth at rest and an earth in motion, but only once, with its relative motions, alone determinable. It is, accordingly, not permitted us to say how things would be if the earth did not rotate. We may interpret the one case that is given us, in different ways. If, however, we so interpret it that we come into conflict with experience, our interpretation is simply wrong. The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise.

Newton’s experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us, and our business is, to bring it into accord with the other facts known to us, and not with the arbitrary fictions of our imagination.

6. We can have no doubts concerning the significance of the law of inertia if we bear in mind the manner in which it was reached. To begin with, Galileo discovered the constancy of the velocity and direction.
of a body referred to terrestrial objects. Most terrestrial motions are of such brief duration and extent, that it is wholly unnecessary to take into account the earth’s rotation and the changes of its progressive velocity with respect to the celestial bodies. This consideration is found necessary only in the case of projectiles cast great distances, in the case of the vibrations of Foucault’s pendulum, and in similar instances. When now Newton sought to apply the mechanical principles discovered since Galileo’s time to the planetary system, he found that, so far as it is possible to form any estimate at all thereof, the planets, irrespectively of dynamic effects, appear to preserve their direction and velocity with respect to bodies of the universe that are very remote and as regards each other apparently fixed, the same as bodies moving on the earth do with respect to the fixed objects of the earth. The comportment of terrestrial bodies with respect to the earth is reducible to the comportment of the earth with respect to the remote heavenly bodies. If we were to assert that we knew more of moving objects than this their last-mentioned, experimentally-given comportment with respect to the celestial bodies, we should render ourselves culpable of a falsity. When, accordingly, we say, that a body preserves unchanged its direction and velocity in space, our assertion is nothing more or less than an abbreviated reference to the entire universe. The use of such an abbreviated expression is permitted the original author of the principle, because he knows, that as things are no difficulties stand in the way of carrying out its implied directions. But no remedy lies in his power, if difficulties of the kind mentioned present themselves; if, for example, the requisite, relatively fixed bodies are wanting.
of Newton's concept of "absolute" time intelligible. Mention is also made there (page 338) of the connexion obtaining between the concept of energy and the irreversibility of time, and the view is advanced that the entropy of the universe, if it could ever possibly be determined, would actually represent a species of absolute measure of time. I have finally to refer here also to the discussions of Petzoldt ("Das Gesetz der Eindeutigkeit," Vierteljährsschrift für wissenschaftliche Philosophie, 1894, page 146), to which I shall reply in another place.

**XX.**

(See page 238.)

Of the treatises which have appeared since 1883 on the law of inertia, all of which furnish welcome evidence of a heightened interest in this question, I can here only briefly mention that of Streintz (Physikalische Grundlagen der Mechanik, Leipsic, 1883) and that of L. Lange (Die geschichtliche Entwicklung des Bewegungsbegriffes, Leipsic, 1886).

The expression "absolute motion of translation" Streintz correctly pronounces as devoid of meaning and consequently declares certain analytical deductions, to which he refers, superfluous. On the other hand, with respect to rotation, Streintz accepts Newton's position, that absolute rotation can be distinguished from relative rotation. In this point of view, therefore, one can select every body not affected with absolute rotation as a body of reference for the expression of the law of inertia.

I cannot share this view. For me, only relative motions exist (Erhaltung der Arbeit, p. 48; Science of Mechanics, p. 229), and I can see, in this regard, no
distinction between rotation and translation. When a body moves relatively to the fixed stars, centrifugal forces are produced; when it moves relatively to some different body, and not relatively to the fixed stars, no centrifugal forces are produced. I have no objection to calling the first rotation “absolute” rotation, if it be remembered that nothing is meant by such a designation except relative rotation with respect to the fixed stars. Can we fix Newton’s bucket of water, rotate the fixed stars, and then prove the absence of centrifugal forces?

The experiment is impossible, the idea is meaningless, for the two cases are not, in sense-perception, distinguishable from each other. I accordingly regard these two cases as the same case and Newton’s distinction as an illusion (Science of Mechanics, page 232).

But the statement is correct that it is possible to find one’s bearings in a balloon shrouded in fog, by means of a body which does not rotate with respect to the fixed stars. But this is nothing more than an indirect orientation with respect to the fixed stars; it is a mechanical, substituted for an optical, orientation.

I wish to add the following remarks in answer to Streintz’s criticism of my view. My opinion is not to be confounded with that of Euler (Streintz, pp. 7, 50), who, as Lange has clearly shown, never arrived at any settled and intelligible opinion on the subject. Again, I never assumed that remote masses only, and not near ones, determine the velocity of a body (Streintz, p. 7); I simply spoke of an influence independent of distance. In the light of my expositions at pages 222–245, the unprejudiced and careful reader