sion 8) that they will never be treated throughout as if they will be related differently to the earth only the two incommensurable motions of either sun in all these treatises, an identity is derived—a unique total can occur only once through

uous Questio de proportione driti ad costam ejusdem (see n. 59), the same conclusion derived from the general claim that motions of sun and moon were able and they once entered into opposition, never through all this occur again in exactly the us if a total lunar eclipse or it could never happen again. positio quod motus solis res- ri et motus lunae respecto sui lurabiles, sicut est verisimile, jus oppositum nondum est de- dic quod centrum solis et cen- dide temporibus describerunt

sequences and conclusions about other planets would make it obvious that planetary tables involving conjunctions, oppositions, and other aspects could never be represented by numbers if the celestial motions are incommensurable (II.419–26).

Part III

With the conclusion of all formal propositions concerned with incommensurable and incommensurable motions, Oresme now seeks to fulfill his earlier promise (I.66–67) to determine whether or not the celestial motions are actually incommensurable. Instead of a formal discussion of this important issue in keeping with the mathematical and propositional character of the two preceding parts, Oresme resorts, in Part III, to a debate form set within a personal dream in which Apollo and his Muses and Sciences appear before Oresme. The dramatis personae are four in number: Oresme, the mortal onlooker; Apollo, the judge; and two protagonists, personifications of two of Apollo’s sciences, Arithmetic, representing the side of commensurability, and Geometry, defending incommensurability. Oresme, convinced by Apollo at the outset that mortal man is incapable of acquiring exact knowledge about sensible things (III.10–31) and therefore impotent to determine whether or not the celestial motions are commensurable, 105 begs Apollo to enlighten him. Turning to Arithmetic and Geometry, Apollo orders them to “teach him what he asks” (III.50).
Arithmetic, who speaks first, offers a number of "basic arguments" to each of which Geometry will later respond. Their respective positions are summarized by bringing together Arithmetic's argument, followed immediately by Geometry's counterargument. At least four specific proposals and counterproposals may be distinguished: 106

1. ARITHMETIC: Rational ratios are better than irrational ratios because they produce pleasure, whereas irrational ratios cause offensive effects (III.84–86, 100–102).
   GEOMETRY: True, rational ratios do produce a certain beauty in the world; but if they were united harmoniously with irrational ratios this would be far better, since from such a union there would result a rich variety of wonderful effects (III.340–42).

2. ARITHMETIC: Arithmetic is the firstborn of the mathematical sciences and her numbers are most worthy to represent the relationships between the celestial motions (III.135–47).
   GEOMETRY: Geometry represents magnitude in general, which includes numbers as a special category. Thus, Geometry embraces not only numerical or rational ratios but much more, so that it not only furnishes as much beauty to the heavens as does Arithmetic, but adds much more splendor (III.360–67).

3. ARITHMETIC: Only if the celestial motions are commensurable and represented by rational ratios can the music produced by the moving spheres be harmonious concords; if these ratios were irrational, the music would be discordant (III.148–53).
   GEOMETRY: If there really is celestial music, it would not vary as the velocities of the celestial motions but, rather, as volumes of the celestial spheres, or in some other way (III.384–88). But even if celestial music resulted from the celestial motions themselves, there is no evidence for assuming that the principal harmonic concordances would be produced. Furthermore, no one has yet determined whether celestial music is sensible or merely intelligible (III.392–99). But if it is sensible and created by fixed and rational ratios, it would be monotonous; only infinite variation is capable of producing interesting sounds (III.402–6). 107

106. Oresme's quotations and sources for these arguments will be found in the footnotes to the translation. In most instances, there is further discussion in the Commentary.

107. In III.375–97, Oresme, whose genuine opinions are represented by Geometry, uses the widely accepted traditional account of the Pythagorean discovery of the principal musical concordances to offer a more consistent and plausible alternative to the equally widely held Pythagorean conviction that ratios of planetary velocities produce the principal musical concordances (see above, p. 19, n. 28). Thus Oresme repudiates the view expressed earlier by Arithmetic (III.160–67), who was quoting from Cicero's Dream of Scipio in the Macrobian version and commentary. Oresme's counterargument is simple—if we assume that the principal musical concordances arise from the ratios of planetary velocities, consequences directly at variance with observable celestial phenomena would result (see I.463–80 and III.388–94). But even if musical concordances are produced in the sky, they would not arise from planetary velocities—since Oresme believes that these are probably incommensurable.

It is in seeking for an alternative explanation that Oresme utilizes and applies the traditional and false Pythagorean "experiment...
4. ARITHMETIC: Unless the celestial motions are commensurable, astronomical prediction and tables, as well as knowledge of future events, would be impossible (III.263–66, 275). Terrestrial effects would fail to repeat (III.304–11) and a Great Year could not occur (III.298–303).

GEOMETRY: But if all celestial motions are commensurable, conjunctions and other astronomical aspects could occur only in a certain finite number of fixed and different places in the sky (III.421–28). Thus a peculiar consequence of commensurable celestial motions is that certain places in the sky would be preferred over other places. Would it not be better that such events be capable of occurring anywhere in the sky (III.420–34)? Man cannot attain to exact knowledge of astronomical phenomena and must rest content with approximations (III.435–40). Indeed, acquisition of exact knowledge would serve to discourage man from making continual observations (III.442–44); and if man had precise knowledge of future celestial positions he would become like the immortal gods themselves, a repugnant thought (III.451–54). Fortunately, this is not likely to pose problems since, as shown elsewhere by mathematical demonstration, it is probable that the celestial motions are incommensurable (III.457–66).

Upon completion of the two orations, Oresme is bewildered and unable to determine the truth, which seems to agree with each side (III.469–74). Apollo, sensing Oresme’s uneasiness and frustration, generously offers a final and true judgment on the issue, when the dream terminates and Oresme awakens (III.474–81).

It seems natural to inquire why Oresme chose a purely literary framework in

---

4. ARITHMETIC: Unless the celestial motions are commensurable, astronomical predictions and tables, as well as knowledge of future events, would be impossible (III.263–66, 275). Terrestrial effects would fail to repeat (III.304–11) and a Great Year could not occur (III.298–303).

GEOMETRY: But if all celestial motions are commensurable, conjunctions and other astronomical aspects could occur only in a certain finite number of fixed and different places in the sky (III.421–28). Thus a peculiar consequence of commensurable celestial motions is that certain places in the sky would be preferred over other places. Would it not be better that such events be capable of occurring anywhere in the sky (III.420–34)? Man cannot attain to exact knowledge of astronomical phenomena and must rest content with approximations (III.435–40). Indeed, acquisition of exact knowledge would serve to discourage man from making continual observations (III.442–44); and if man had precise knowledge of future celestial positions he would become like the immortal gods themselves, a repugnant thought (III.451–54). Fortunately, this is not likely to pose problems since, as shown elsewhere by mathematical demonstration, it is probable that the celestial motions are incommensurable (III.457–66).

Upon completion of the two orations, Oresme is bewildered and unable to determine the truth, which seems to agree with each side (III.469–74). Apollo, sensing Oresme’s uneasiness and frustration, generously offers a final and true judgment on the issue, when the dream terminates and Oresme awakens (III.474–81).

It seems natural to inquire why Oresme chose a purely literary framework in...
which to argue a scientific question in a treatise which up to this point had been of a highly technical character. Any reasonable attempt to answer this question must turn on the fact that Oresme was perfectly aware that no precise solution or conclusive demonstration could decide whether or not the celestial motions are commensurable. In emphasizing man’s inability to acquire exact knowledge about sensible things (III.10–31)—and this includes the movements of the celestial motions—Apollo convinces Oresme that no precise mathematical solution or demonstration could ever decide an issue that depended upon physical and sensible, rather than mathematical, criteria (III.32–37). The slightest alteration in the physical relations between any two or more physical things, however small and undetectable, would alter their true mathematical ratio or relationship but go unnoticed by those compelled to rely solely on sense perception.®

Or conversely, if men attain exactness, they could not prove that they had done so. Thus, for example, even if men could and did produce an exact and absolutely true calendar, they would not know for certain that they had done so (III.29–31). Although we might resort to probable arguments in favor of one or the other hypothesis, such arguments would not answer satisfactorily the fundamental question raised here in the De commensurabilitate—namely, is it physically true that celestial motions are commensurable; or are some, or all, incommensurable? A categorical, not a probable, reply was required, since Oresme had himself stated that “it is necessary that all celestial motions be either mutually commensurable, or that some be mutually incommensurable” (1.62–64). Because no solution was possible, a formal scholastic discussion of the issue would have been fruitless. For this reason, it appears that Oresme wished to avoid a serious formal consideration of this insoluble problem.

Nevertheless, since physical, astronomical, and astrological arguments could be seriously influenced by the choice between the two alternatives—even if that choice were made in terms of probability and persuasiveness—Oresme realized how important it was to put before his readers the strongest arguments available for each of these two hypotheses. To achieve this, he hit upon the idea of utilizing a literary form that would yet allow him to expound his argument in the cdents of the celestial oration, form of the appearance of the rapt attention as the impressively on each

But if the realist with a desire to avoid favored and deemed did he choose to emb urge a continuous rebuke to Oresme an through the senses, oration in behalf of it followed by Geometers. Would it not have been dialogue or conversant, subtle thrust and ed separate discourse:

However more attr: it is one that Oresme’s conversational dialogue arguments would 1 and immediate denial. Arithmetic claims that ratios “no one could effects beforehand. In unknown and even would have emphasize to the immortal gods know beforehand that low a certain degree, exact point or instant tins insists that without be in a state of perp asserting that “it woul

110. For a brief consideration of De commensurabilitate, III.9–17 and 34–36, see Maier, Metaphysische Hintergründe, p. 402, n. 37. In deliberating the commensurability and incommensurability of celestial motions, Oresme expressed a similar idea in his later Le Livre du ciel et du monde, bk. 1, chap. 29, 44c (Menut, Oresme du ciel, p. 197): “Now, since it is impossible to know absolutely whether the number of all the stars is even or odd, in the same way the mortal men in the world, dead or alive or to be, could not discover nor know by their natural lights for certain whether all the movements of the heavens are commensurable or incommensurable, for by the part of a movement which would be imperceptible to the senses, even if it were a hundred thousand times larger, two such movements or similar motions could be incommensurable and yet could appear to be commensurable.”

111. Oresme was surely continuous dialogue’ as a cero’s De natura deorum,
Summary and Analysis of *De commensurabilitate*

...ing a literary form that did not commit him, to a definite position and which would yet allow him to propagandize effectively in behalf of his genuine conviction that the celestial motions were probably incommensurable. The debate, or oration, form of Part III served this purpose admirably, for it permitted him the appearance of objectivity in the guise of a seeker after truth listening with rapt attention as the arguments of Apollo’s immortal muses were marshaled impressively on each side of this great issue.

But if the realization that a categorical solution was impossible, coupled with a desire to avoid obvious and direct identification with an opinion he favored and deemed probable, led Oresme to present Part III as a debate, why did he choose to employ two separate orations set within a dream rather than utilize a continuous but imaginary dialogue? Why is it that, after Apollo’s rebuke to Oresme and all mortals on the futility of seeking exact knowledge through the senses, Arithmetic steps forward and delivers an uninterrupted oration in behalf of the commensurability of the celestial motions, and is then followed by Geometry’s continuous plea in behalf of incommensurability? Would it not have been more stimulating and exciting to have had a continuous dialogue or conversation employing argument and immediate counterargument, subtle thrust and counterthrust, rather than long-winded and uninterrupted separate discourses?111

However more attractive and exciting the former procedure would have been it is one that Oresme might consciously have avoided since in a continuous conversational dialogue on this particular issue the arguments and counterarguments would have resulted in little more than a sequence of claims and immediate denials with little dramatic interplay of ideas. For example, Arithmetic claims that if the celestial motions were not represented by rational ratios “no one could ever foresee aspects, or predict conjunctions, or learn of effects beforehand. Indeed, astronomy would lie hidden [from us] in every age, unknown and even unknowable,...” (III.263–66). In response, Geometry would have emphasized that knowledge of future events would make man akin to the immortal gods, an unworthy thought (III.451–54); “it is enough to know beforehand that a future conjunction, or eclipse, of this mobile falls below a certain degree, minute, second, or third; nor is it necessary to predict the exact point or instant of time [in which these occur]...” (III.435–38). Arithmetic insists that without precise knowledge of the celestial motions, men would be in a state of perpetual ignorance (III.270); Geometry would counter by asserting that “it would be better that something should always be known about

---

111. Oresme was surely familiar with the continuous dialogue as exemplified by Cicero’s *De natura deorum*, Chalcidius’s translation of Plato’s *Timaeus*—both of which he quotes directly—and Adelard of Bath’s *Questiones naturales.*
them, while, at the same time, something should always remain unknown, so that it may be investigated further” (III.444–46). Throughout Part III, Arithmetic lauds the beauty and harmony of the world founded on celestial motions that are regular and related rationally to produce a never-ending sequence of identical but magnificent celestial and terrestrial effects, while Geometry emphasizes the wonders of an unending sequence of unique and new effects stemming from the incommensurability of these same motions (see III.406–15).

Since a definite solution to this problem was beyond reach, a conversation or dialogue centering on such claims could not have been properly joined to yield any dramatic impact, but would have produced instead a series of tedious assertions and denials without substantive argument. To avoid so sterile a format, Oresme wisely chose to have each side represented by one uninterrupted oration. In this way, maximum effectiveness could be achieved as each side gathered all its arguments for consecutive presentation, so that not only was the greatest cumulative impact possible, but a considerable dramatic element was permitted to develop. Arithmetic could present, in rapid succession, one authoritative and rhetorical appeal after another. An impressive array of quotations from poets and philosophers could be paraded before the reader. In her turn, Geometry was free to reply in kind with arguments and appeals drawn from cosmic diversity.

Finally, we must consider why Oresme chose to have these orations occur within the framework of his own dream. The explanation seems obvious. It provided a convenient means of sidestepping the need for supplying a definite solution to an issue for which man could supply no categorical solution. None could be derived from the rhetorical appeals of Arithmetic and Geometry, for they served only to confuse Oresme, who remarks that “truth seemed consonant with each side” (III.469–70). Apollo explains that the great debate by these two “most illustrious mothers of evident truth” (III.475) had been but an amusing exercise and that now he, Apollo, would pronounce a final and true judgment. But “the dream vanishes, the conclusion is left in doubt” (III.480) and the reader remains in ignorance on this weighty question.

As a vehicle, the dream was a convenient literary device for avoiding a positive stand. Anyone could easily understand the abrupt and frustrating interruption of a dream in which some significant utterance or act was on the verge of occurrence. But the interrupted dream serves an ulterior motive. Not only has this rude awakening robbed us of a unique opportunity to acquire absolute truth, but it has made us dependent on the appeals, such as they are, of Arithmetic and Geometry for whatever insight we may acquire into this problem.

The reader now realizes that at best he must formulate his own judgment solely on the basis of these two orations, which incorporate the only kinds of arguments and appeals that men can know and understand. Compelled to re-turn to the orations to the probability—and are incommensurable. evident in a general w and Geometry to have the fact that Geometrization that emerges from foundation on which us that it has been “de tionibus proportionum’ designated, it is more surable, just as it is not a multitude of number. Thus any two unknow be incommensurable

112. In Part III of them bilitate, Oresme could natel cited his De propor nium, or, for that matter own works. It would have Oresme introduced qu titles of, his own works, of Apollo, Arithmetic, o ver, Arithmetic and Gmitted to quote freely an from the likes of Cicero thius, and many others). I this reason that Oresme only that the propositio been “demonstrated else the treatise, where such a involved, Oresme did cit nibus by name (see II.201

113. There is little dou tended to impress upon commensurabilitate his co tial motions are probably This interpretation gain from the fact that in the 1 du ciel et du monde, Or commensurabilitate as the offered several reasons show the probable into the celestial motions. (c mouemens du ciel sc
ways remain unknown, so throughout Part III, Arithmetic
ended on celestial motions never-ending sequence of
s, while Geometry empha-
and new effects stemming
see III.406–15).
reach, a conversation or
been properly joined to
instead a series of tedious
To avoid so sterile a for-
by one uninterrupted
be achieved as each side
on, so that not only was
erable dramatic element
in rapid succession, one
pressive array of quo-
before the reader. In her
ents and appeals drawn
ve these orations occur-
tion seems obvious. It
for supplying a definite
egical solution. None
etic and Geometry, for
ruth seemed consonant
act debate by these two
een but an amusing
al and true judgment.
ft” (III.480) and the
for avoiding a positive
rustrating interruption
on the verge of occur-
ve. Not only has this
quire absolute truth,
are, of Arithmetic and
roblem.
t his own judgment
ate the only kinds of
nd. Compelled to re-
turn to the orations themselves, there is little doubt that the reader will settle for
the probability—and this is all he can now hope for—that the celestial motions
are incommensurable. And this is precisely what Oresme has planned. This is
evident in a general way from the fact that Arithmetic was made to speak first
and Geometry to have the last word. More important, and even conclusive, is
the fact that Geometry alone invokes and appeals to a mathematical demonstra-
tion that emerges from all the rhetoric and appeals to authority as the one sound
foundation on which to base a decision. As her final argument, Geometry tells
us that it has been “demonstrated elsewhere” (III.459)—in Oresme’s De propor-
tionibus proportionum
— that “when any two unknown magnitudes have been
designated, it is more probable that they are incommensurable than commen-
surable, just as it is more probable that any unknown [number] proposed from
a multitude of numbers would be non-perfect rather than perfect” (III.459–63).
Thus any two unknown celestial (as well as terrestrial) motions would probably
be incommensurable and their ratio of velocities irrational.113 This is beyond

112. In Part III of the De commensura-
bilitate, Oresme could not have appropri-
ately cited his De proportionibus proportio-
num, or, for that matter, any other of his
own works. It would have been strange had
Oresme introduced quotations from, or
titles of, his own works, into the discourses
of Apollo, Arithmetic, or Geometry (how-
ever, Arithmetic and Geometry were per-
mitted to quote freely and anachronistically
from the likes of Cicero, Macrobius, Boe-
thius, and many others). It is presumably for
this reason that Oresme has Geometry say
only that the proposition in question has
been “demonstrated elsewhere.” Earlier in
the treatise, where such awkwardness was not
involved, Oresme did cite his De proportio-
nibus by name (see II.201).
113. There is little doubt that Oresme in-
tended to impress upon readers of the De
commensurabilitate his conviction that celes-
tial motions are probably incommensurable.
This interpretation gains strong support
from the fact that in the much later Le Livre
du ciel et du monde, Oresme cited the De
commensurabilitate as the place where he had
offered several reasonable arguments to
show the probable incommensurability of
the celestial motions. (“Et que aucun des
mouvements du ciel soient incommensu-
rables, ce est plus vraisemblable que / (44d)
n’est l’opposé, si comme je monstray jadis
par plusieurs persuasions en un traité in-
titulé De commensurabilitate vel incommen-
surabilitate motuum cieli.”—Menut, Oresme
du ciel, p. 196; see also V. P. Zoubow, “Nicole
Oresme et la musique,” Mediaeval and Ren-
aissance Studies, vol. 5 [1961], pp. 102–3.)
It is, however, surprising that Oresme would
cite the De commensurabilitate, which con-
tained only “plusieurs persuasions,” rather
than the De proportionibus proportionum,
which is the only treatise known thus far
where he offered mathematical arguments to
demonstrate not only the probable incom-
mensurability of the celestial motions, but
also the more general claim that any two
ratios involving physical or mathematical
magnitudes are probably incommensurable.
Indeed, in the De commensurabilitate itself,
Oresme cites another treatise (III.458–66;
see above, n. 112)—almost certainly his De
proportionibus—for demonstrative support
of the general claim, and by implication the
specific claim, about celestial incommensur-
ability; it is again the De proportionibus pro-
portionum to which Oresme appeals explic-
tively in support of the same two claims in his
Questiones de sphaera (for the text of this
passage, see Grant, Oresme PP.AP, p. 63,
Introduction, Chapter 2

...Therefore, if any ratios were sought, it is would be irrational and its surable"—(ibid., pp. 249, 293).

Thus far, all this is pu and it is not until Chapter...the final proposition of Oresme applies Chapter 3 celestial motion and, inductive magnitudes in generation that ratios of velocit as ratios of ratios (ibid., p. 51), Oresme prove that represent changeable magnitudes truly reflect behavior of ratios describ Proposition X.

In order to represent a velocities arising from resistance, Oresme adopted accepted function formu Bradwardine in 1328 (ibid we may represent as $F_2:V_2$ where $F$ signifies motive power, and $V$ velocity. Terrestrial motion, Oresme ratios of force and resistan to celestial motion however, such celestial ve...“from the ratio of the quations or circles described, of the times in which they 293). Thus if we know th traversed by any two pla (the distances are measur out by radius vectors), their ratio of velocities $V_2:V_1$, where $S$ is disturbances are equal but the times, the ratio of velocit as the ratio of the tin $V_2:V_1$ when $S_2 = S_1$. Since ratios of distances, ties are interrelated in described, the ratio of velocit $F_2:R_2 = (F_1:R_1)^{p/q}$ by $S_2:S_1$ when $T_2 = T_1;T_1;T_1$ when $S_2 = S_1$. But possible ratios is used a
Summary and Analysis of *De commensurabilitate*

Introduction, Chapter 2

s an antecedent and con-

cedent proclaims (ibid.,
7) that for any given se-

cral qualification see ibid.,

itional thing than rational

be formed. This is shown
ple (ibid., pp. 246-48,
ving 100 rational ratios

When any two of them-
tially, the possible con-
tios of greater inequality
ich only 25 are rational
 of the remainder are irra-
 y, the ratio of irrational
 of ratios is 197:1, a dis-
crease as more ratios
 248, lines 335-58; the
de are given in a practica
rop. XI on pp. 254-58,

which is actually the
osition X, asserts that
 ratios are probably in-
 is follows immediately
 or it is based on the
 any two unknown ra-
 would be incommensu-
 ble only by an irrational
 re are more irrational
 of ratios. As Oresme

dermark is made in the
 re about perfect and
 (III.461-63): "...we
 y numbers are taken in
 perfect or cube numbers
 r numbers and as more
 the series the greater
 be to cube numbers or
 numbers. Thus if there
 id such information as
 eat it is, and whether
 wholly unknown...it
 such an unknown
 u cube number....

d here about num-
 ratios of rational ra-
 efore, since there are
 s than others, under-
sense [of ratios of ra-

tios].... Therefore, if any unknown ratio of
 ratios were sought, it is probable that it
 would be irrational and its ratios incommensur-
able"—(ibid., pp. 249, 251).

Thus far, all this is purely mathematical
 and it is not until Chapter 4, Proposition VII,
 the final proposition of the treatise, that
 Oresme applies Chapter 3, Proposition X to
 celestial motion and, indeed, to all continu-
 ous magnitudes in general. On the assump-
 tion that ratios of velocities are relatable just
 as ratios of ratios (ibid., p. 286, lines 334-37
 and p. 51), Oresme proceeds as if all ratios
 that represent changeable and continuous
 magnitudes truly reflect the mathematical
 behavior of ratios described in Chapter 3,
 Proposition X.

In order to represent a ratio of terrestrial
 velocities arising from ratios of force to re-
 sistance, Oresme adopted and used a widely
 accepted function formulated by Thomas
 Bradwardine in 1238 (ibid., p. 17) and which
 we may represent as \( F_2:R_2 = (F_1:R_1)^{v_2/v_1} \),
 where \( F \) signifies motive force, \( R \) resistive
 power, and \( V \) velocity. By analogy with
 terrestrial motion, Oresme says that such ra-
 tos of force and resistance can also be ap-
 plied to celestial motions. Kinematically,
 however, such celestial velocities are derived
 "from the ratio of the quantities of the mo-
 tions or circles described, and from the ratio
 of the times in which they revolve" (ibid.,
 p. 293). Thus if we know the ratio of distances
 traversed by any two planets in equal times
 (the distances are measured by angles swept
 out by radius vectors), we can determine
 their ratio of velocities, since \( S_2:S_1 = V_2:V_1 \),
 where \( S \) is distance; or, if the dis-
 tances are equal but traversed in unequal
 times, the ratio of velocities will be inversely
 as the ratio of the times—i.e., \( T_1:T_2 =
 V_2:V_1 \) when \( S_2 = S_1 \) (ibid., pp. 52-53).

Since ratios of distances, times, and veloc-
 ities are interrelated in the manner just de-
 scribed, the ratio of velocities in the for-
 mulation \( F_2:R_2 = (F_1:R_1)^{v_2/v_1} \), could be replaced
 by \( S_2:S_1 \) when \( T_2 = T_1 \), or replaced by
 \( T_1:T_2 \) when \( S_2 = S_1 \). But whichever of these
 possible ratios is used as the exponent, we
 have in each instance a ratio of ratios. And
 since Chapter 3, Proposition X had demon-
 strated in a purely mathematical context the
 probability that any two unknown ratios
 selected at random from a proposed chosen
 sequence (ibid., p. 41, n. 54) would probably
 be incommensurable—i.e., only relatible by
 an irrational exponent and constituting an
 irrational ratio of ratios—Oresme now be-
 lies it justifiable to extend the application
 of this demonstration to ratios of ratios rep-
 resenting both terrestrial and celestial phis-
 ical magnitudes. Hence we have a straight-
 forward extension to physics and cosmology
 of results obtained in pure mathematics.

As a first move in this direction, Oresme
 generalizes the range of application of the
 mathematical probability argument to em-
 brace all continuous magnitudes: "When
 there have been proposed any two things
 whatever acquirable [or traversable] by a
 continuous motion and whose ratio is un-
 known, it is probable that they are incom-
 mensurable. And if more are proposed, it is
 more probable that any [one of them] is in-
 commensurable to any [other]. The same
 thing can be said of two times and of any
 continuous quantities whatever" (ibid., p.
 303). It follows from this that any proposed,
 but unknown, ratio of terrestrial or celestial
 velocities would probably be incommensur-
 able, so that Oresme could conclude: "When
 two motions of celestial bodies have been
 proposed, it is probable that they would be
 incommensurable, and most probable that
 any celestial motion would be incommensur-
 able to the motion of any other [celestial]
 sphere:..." (ibid., p. 305). And in what may
 be a reference to the *Ad paucam respetientes*,
 which in its corrected and expanded form
 probably became the *De commensurabilitate*.
 Oresme, a few lines below, declares that
 "...many very beautiful propositions that I
 arranged at another time follow, and I intend
 to demonstrate them more perfectly later,
 in the last chapter,..." (ibid.; despite Oresme's
 reference to the last chapter of the *De pro-
 portionibus*, I have argued [ibid., pp. 76-80]
 that this refers to the *Ad paucam* and, there-
question the strongest argument found in Part III and, since Oresme chose to conclude the orations with it, he leaves little doubt that he intended to guide the reader to his viewpoint\textsuperscript{114} as skillfully and unobtrusively as possible.

If the interpretation given above is a reasonable reflection of the considerations that induced Oresme to choose the particular format of Part III, it is obvious that it represents a carefully constructed climax to a sophisticated and unusual treatise. But quite apart from the structure of Part III, we are impressed by the parade of quotations and the evident learning that is displayed here. Although on occasion Oresme composed a poem or became lyrical and enthusiastic on some particular idea or concept, nowhere else in his known works does there appear anything comparable to the literary effort of Part III. A striking, but upon reflection not really unexpected, feature of the concluding part of the \textit{De commensurabilitate} is the fact that nowhere does Oresme inject any Christian

\textit{(Note 113 continued)}

fore, indirectly to the \textit{De commensurabilitate}).

The passage in \textit{De commensurabilitate}, III. 457–65, and the arguments of Geometry in general, may have been in Oresme's mind when, in his \textit{Livre de divinations}, he says that astronomy (actually the word \textit{astrologie} is used, but the terms are synonymous in the context of the discussion) "can be adequately known but it cannot be known precisely and with punctual exactness, as I have shown in my treatise on the Measurement of the Movements of the Heavens \textit{[La Mesure des mouvements du ciel]} and have proved by reason founded on mathematical demonstration."—G. W. Coopland, ed. and trans., \textit{Nicole Oresme and the Astrologers} (Cambridge, Mass., 1952), p. 55. Since the title that Oresme cites here has only a general relationship to the title of the \textit{De commensurabilitate}, it is possible that the \textit{Livre de divinations} was written prior to the \textit{De commensurabilitate} and we have here a reference to Oresme's \textit{Ad paucæ respicientes}, which in one manuscript is called \textit{Tractatus brevis et utilis de proportionatate motuum celestium} (see Grant, \textit{Oresme PPAP}, p. 380; this title does not precisely translate as \textit{La Mesure des mouvements du ciel} and I have argued that there is no concrete evidence for believing that it was the proper title of the \textit{Ad paucæ respicientes}, which is a title that I formulated from the opening words of the treatise). In my view the reference is to one of these two treatises, but the title cited by Oresme seems definitely to preclude the \textit{De proportionibus proportionum} as the intended work, even though it would have been the most appropriate reference—provided, of course, that it antedates the \textit{Livre de divinations}—for it is only in the \textit{De proportionibus} that Oresme "proved" that we cannot obtain precise knowledge of celestial motions and positions.

114. From note 113 and the frequent use made of this key concept in many of his other treatises (e.g., \textit{De proportionibus proportionum}, \textit{Ad paucæ respicientes}, \textit{Questiones de sphera}, \textit{Questiones super de celo}, \textit{Quaestiones super geometriam Euclidis}, the \textit{Quodlibeta}, and \textit{Le Livre du ciel et du monde}), there is no doubt whatever of Oresme's conviction that the celestial motions were probably incommensurable. But in the \textit{De commensurabilitate}, where he had to give the appearance of impartiality, this conviction could not be made explicit. I have tried to show, however, that the arguments and appeals are so formulated as to convince the reader that the celestial motions are probably incommensurable. Indeed, in his \textit{Le Livre du ciel et du monde}, he tells us that he had shown this in the \textit{De commensurabilitate} (the statement appears near the beginning of the immediately preceding note).

115. See Stahl, trans., \textit{Ma
since Oresme chose to be intended to guide the reader as possible. On of the considerations of Part III, it is obvious that the context of the splayed here. Although col and enthusiastic on 10n works does there art III. A striking, but excluding part of the De ne inject any Christian words of the treatise). In ence is to one of these two titles cited by Oresme seems to be the De proportionibus the intended work, even have been the most appro-provided, of course, that it 4. de divinacions—for it is proportionibus that Oresme e cannot obtain precise tions and positions. 113 and the frequent use of this concept in many of his De proportionibus propor-questions, Questions de super de celo, Quaestiones Euclidis, the Quadrilaterals, Quaestiones et du monde, there is no -Oresme's conviction that ions were probably inbut in the De commen he had to give the ap-spatially, this conviction could cit. I have tried to show, arguments and appeals are to convince the reader that ions are probably incom-115, in his Le Livre du ciel et as that he had shown this nabilitate (the statement beginning of the immedi-115. See Stahl, trans., Macrobius, p. 8.