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BAYESIAN TARGET DETECTION AND LOCALIZATION USING A DUAL-MODE SENSOR

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ABSTRACT

We investigate Bayesian methods and heuristics for configurable sensor management in a simple target detection and localization problem. In this problem, a target (if present) is located in one of M cells. A dual-mode sensor repeatedly interrogates either the entire search area (Mode A) or a single cell (Mode B); its performance in both modes is characterized by probabilities of correct detection and false alarm. We investigate several sensor control strategies, including the myopic optimal strategy which minimizes the probability of error for a single observation. All strategies are closed loop; the current sensor configuration depends on previous observations. Monte Carlo simulations show that the myopic optimal strategy gives the lowest probability of error for a fixed number of observations, while interrogating the cell with the highest probability of target present gives the lowest average number of observations needed to guarantee a fixed error probability.

1. INTRODUCTION

The management of configurable sensors to optimize overall system performance is an important issue in target detection/localization and target tracking systems. Bayesian decision theory provides a mathematical framework by which the sensor management problem can be addressed. In this paper, we apply Bayesian decision theory to the problem of choosing a sequence of sensor modalities for a dual-mode sensor in a simple target detection and localization problem. We attempt to find and characterize optimal and good sub-optimal rules for choosing sensor modalities.

We consider the following abstract scenario. A target may be present in an area to be searched by the sensor. The sensor is capable of operating in two modes: either interrogating the whole search area (Mode A) or interrogating a single cell in the search area (Mode B). In either mode, the sensor returns a binary-valued observation indicating

whether the target was detected in the interrogated area. Our objective is to correctly detect and locate the target from a sequence of observations collected by the sensor.

We represent this scenario as a sequential Bayesian decision problem. Initial knowledge about the target presence and location is represented by a prior probability distribution. Each sensor mode is characterized by a probability of target detection and a probability of false alarm; these probabilities will typically differ for the two modes. Observations are collected either a fixed number of times or until a stopping criterion is met. After each observation is obtained, the distribution of the target location is updated using Bayes theorem. Thus, the choice of sensor mode at a give time depends on previously collected observations.

We seek a decision rule that chooses sensor modes to optimize the system performance. A myopic optimal decision rule is one that optimally chooses the sensor mode given that a single observation will be collected. A sequential optimal decision rule is one that optimally chooses sensor modes for the collection of a sequence of observations. For this work, we measure optimality by one of two criteria: probability of error given a fixed number of observations, or number of observations needed to guarantee a given probability of error.

Sequential optimal decision rules can derived for this problem; unfortunately, their complexity increases exponentially with the length of the observation sequence. Thus, in this paper we develop myopic decision rules, but we evaluate the performance of these rules on sequences of observations.

This paper is structured as follows. Section 2 discusses the theoretical background of this problem and introduces related prior work. In Section 3, we give a mathematical formulation of the problem, and in Section 4, we derive the myopic optimal decision rule. We present several heuristic mode selection strategies in Section 5, and evaluate their performance through Monte Carlo simulation in Section 6. We present conclusions and areas for future work in Section 7.

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2. PREVIOUS WORK

Two areas of previous work are relevant to our work. The first is the formulation of this dual-mode sensor control problem using a Bayesian formalism. The second is the development and characterization of optimal sequential Bayesian decision rules for the sensor control problem.

The specific dual-mode sensor problem in this paper is very similar to and inspired by the two-mode problem investigated in [1, 2, 3], in which a two-mode sensor control problem was formulated and a loss function was defined. The loss function reflects the desired outcomes of the sensor control problem. The sensor probabilities of detection and false alarm in each mode were dependent on the prior probabilities of the target state. The myopic optimal decision rule that minimizes the expected loss for a single sensor use was obtained. This myopic rule was repeatedly applied to collect observations until the probability of error fell below a given threshold. This approach was illustrated through several simulation examples, but no statistical characterization of its performance was attempted. The myopic optimal decision rule was also extended to include a Markov target motion model[4].

Sequential optimal Bayesian decision rules that are directly applicable to a single-mode sensor control problem have been developed for sequential fault diagnosis[5] and dynamic search strategies[6]. In the context of fault detection and isolation, Raghavan *et. al.* have found a dynamic programming solution to a class of sequential Bayesian decision problems[5]; this work applies directly to a single-mode sensor control problem[7]. The complexity of the dynamic programming solution grows exponentially with the number of tests to be conducted and becomes computationally intractable for large problems. A suboptimal greedy (myopic) algorithm was shown to provide good performance.

Castanon has analyzed a simple class of dynamic search problems using dynamic programming[6]. His analysis can be applied to the single-mode sensor problem in which a fixed number of cells is sequentially interrogated with the goal of minimizing the probability of error. When the sensor probabilities of detection and false alarm are symmetric, the myopic optimal strategy is to interrogate either the most probable or second most probable cell; the sequential optimal strategy is to apply the myopic optimal strategy to collect each observation. When probabilities of detection and false alarm are not symmetric, finding the sequential optimal strategy is exponentially complex in the number of observations.

For the single-mode sensor, the problem of minimizing the number of observations needed to guarantee a given probability of correct decision can also be formulated using dynamic programming. To our knowledge, this problem has not been investigated for the single-mode sensor. It appears

that this problem exhibits the same exponential growth in complexity as the problem of reaching a decision in a fixed number of observations.

In [7], the performance of the myopic optimal decision rule and several heuristic decision rules is evaluated for the single-mode sensor problem using Monte Carlo simulation. The heuristics include choosing the most probable cell, the second most probable cell, the myopic optimal cell, and the cell that maximizes an expected discrimination measure. For a fixed number of observations, the myopic optimal decision rule gave the lowest probability of error of the four heuristics examined. Choosing to interrogate the most probable cell required the fewest number of observations to achieve a given probability of error.

3. MATHEMATICAL PROBLEM STATEMENT

We now formulate the dual-mode sensor control problem mathematically. This formulation allows us to extend the myopic optimal rule for the single-mode sensor problem to the dual-mode sensor problem.

In our formulation, a target is either present and located in one of M possible cells or absent; the target state is a discrete random variable denoted $X \in \{0, \dots, M\}$. A value of $X = 0$ indicates that the target is absent, while a value of $X = x \in \{1, \dots, M\}$ indicates that the target is in cell x . Our knowledge of the initial target state is represented by a prior probability distribution $p_X(x)$.

At each time epoch, the sensor can choose to operate in one of two modes. In Mode A, the sensor interrogates all cells simultaneously, and returns a value of $Y = 1$ if the target is detected in any cell and a value of $Y = 0$ if the target is not detected. In Mode B, the sensor interrogates one cell; the sensor returns a value of $Y = 1$ if the target is detected in the chosen cell and a value of $Y = 0$ if the target is not detected. The choice of sensor mode and, in Mode B, the cell to interrogate is denoted $d \in \{0, \dots, M\}$. A value of $d = 0$ indicates that the sensor is operating in Mode A, while a value of $d > 0$ indicates that the sensor is operating in Mode B and interrogating cell d .

The sensor performance in each mode is known and characterized by a probability of correct detection and false alarm. The probability of correct detection and false alarm in Mode A is p_{D_A} and p_{F_A} ; the probability of correct detection and false alarm in Mode B is p_{D_B} and p_{F_B} . For notational convenience, we define $\overline{p_{D_A}} = 1 - p_{D_A}$, $\overline{p_{F_A}} = 1 - p_{F_A}$, $\overline{p_{D_B}} = 1 - p_{D_B}$, and $\overline{p_{F_B}} = 1 - p_{F_B}$. The prob-

ability distribution of Y conditioned on X and d is

$$p_{Y|X,d}(y|x,d) = \begin{cases} \frac{p_{D_A}}{\overline{p_{D_A}}}, & y = 1 \text{ and } d = 0 \text{ and } x > 0 \\ \frac{\overline{p_{D_A}}}{\overline{p_{D_A}}}, & y = 0 \text{ and } d = 0 \text{ and } x > 0 \\ \frac{p_{F_A}}{\overline{p_{F_A}}}, & y = 1 \text{ and } d = 0 \text{ and } x = 0 \\ \frac{\overline{p_{F_A}}}{\overline{p_{F_A}}}, & y = 0 \text{ and } d = 0 \text{ and } x = 0 \\ \frac{p_{D_B}}{\overline{p_{D_B}}}, & y = 1 \text{ and } d > 0 \text{ and } x = d \\ \frac{\overline{p_{D_B}}}{\overline{p_{D_B}}}, & y = 0 \text{ and } d > 0 \text{ and } x = d \\ \frac{p_{F_B}}{\overline{p_{F_B}}}, & y = 1 \text{ and } d > 0 \text{ and } x \neq d \\ \frac{\overline{p_{F_B}}}{\overline{p_{F_B}}}, & y = 0 \text{ and } d > 0 \text{ and } x \neq d \end{cases}$$

The observed value y is used to update the probability distribution of X via Bayes theorem:

$$p_{X|Y,d}(x|y,d) = \frac{p_{Y|X,d}(y|x,d)p_X(x)}{\sum_{x'} p_{Y|X,d}(y|x',d)p_X(x')}$$

The process of choosing a mode and, for Mode B, a cell, obtaining an observation using the sensor, and updating the probability distribution using Bayes theorem is repeated either a fixed number of times or until the posterior probability of a cell conditioned on all observations exceeds a threshold close to one. At this point, the estimated target location is the cell whose posterior probability is largest; the estimated target location is denoted \hat{x} .

4. MYOPIC OPTIMAL RULE

We now present the myopic optimal (minimum probability of error) rule for choosing the mode and (in mode B) the cell to interrogate for a single observation. This decision rule is a straight-forward extension of the myopic optimal decision rule for a single-mode sensor[5, 6]. It depends only on the probability that no target is present and the probabilities of the most probable and second most probable cells; this is a consequence of the fact that p_{D_B} and p_{F_B} are independent of the cell chosen to interrogate.

Let \overline{m} and \hat{m} be the cell with largest probability and the cell with second largest probability:

$$\begin{aligned} \overline{m} &= \arg \max_{x>0} p_X(x) \\ \hat{m} &= \arg \max_{\substack{x>0 \\ x \neq \overline{m}}} p_X(x) \end{aligned}$$

Compute the following values:

$$L_0 = \max[\overline{p_{F_A}} p_X(0), \overline{p_{D_A}} p_X(\overline{m})] + \max[p_{F_A} p_X(0), p_{D_A} p_X(\overline{m})]$$

$$L_{\overline{m}} = \max[\overline{p_{F_B}} p_X(0), \overline{p_{D_B}} p_X(\overline{m}), \overline{p_{F_B}} p_X(\hat{m})] + \max[p_{F_B} p_X(0), p_{D_B} p_X(\overline{m}), p_{F_B} p_X(\hat{m})]$$

$$L_{\hat{m}} = \max[\overline{p_{F_B}} p_X(0), \overline{p_{F_B}} p_X(\overline{m}), \overline{p_{D_B}} p_X(\hat{m})] + \max[p_{F_B} p_X(0), p_{F_B} p_X(\overline{m}), p_{D_B} p_X(\hat{m})]$$

If $L_{\hat{m}} > L_{\overline{m}}$ and $L_{\hat{m}} > L_0$, set $d_0 = \hat{m}$. Otherwise, if $L_0 > L_{\overline{m}}$ and $L_0 > L_{\hat{m}}$, set $d_0 = 0$. Otherwise, set $d_0 = \overline{m}$.

5. HEURISTICS

In general, we wish to find an optimal sequence of sensor configurations to either minimize the probability of error or the number of observations collected. Unfortunately, the computational burden of finding the optimal sequence of observation grows exponentially with the number of observations. Thus, we consider several heuristics. These heuristics are motivated by the myopic optimal solution. Similar heuristics have been found to perform well for the simpler single mode problem in [7]. They are:

1. choose d_0 as the most probable value of X ,
2. choose $\overline{d_0}$ as the the second most probable value of X ,
3. choose the myopic optimal cell.

For the problem of minimizing the number of observations, we have investigated a fourth two-stage heuristic: repeatedly apply Mode A to determine whether a target is present, and then repeatedly apply Mode B to the most probable value of X ; Mode A is repeated until the probability of $X = 0$ either exceeds a threshold (in which case we decide that the target is absent) or drops below another threshold; in the second case, we then apply Mode B repeatedly to the most probable value of X until the probability of a cell exceeds the required threshold.

6. SIMULATION RESULTS

The performance of these heuristics has been evaluated using Monte Carlo simulations. In these simulations, we set p_{D_B} to 0.9 and p_{D_A} to 0.8 (Mode A has poorer detection performance than Mode B). We investigated values of p_{F_A} of 0.05, 0.15, and 0.25, and values of p_{F_B} ranging from 0.01 to 0.30. In the following, we present results for $p_{F_A} = 0.15$; these are typical of the other p_{F_A} values.

Figure 1 shows the average error rate for the three heuristics as a function of p_{F_B} . This error rate is averaged over 500,000 Monte Carlo simulation runs. Note that the myopic optimal decision rule consistently provides performance that is not worse, and often better, than choosing either the most probable or second most probable value for X ; however, the performance differences are not pronounced.

Figure 2 shows the average number of observations needed to achieve a probability of error less than 0.05 for the most probable, myopic optimal, and two-stage heuristics as a function of p_{F_B} . Results for the second most probable heuristic are not shown; on average, it requires more than twice

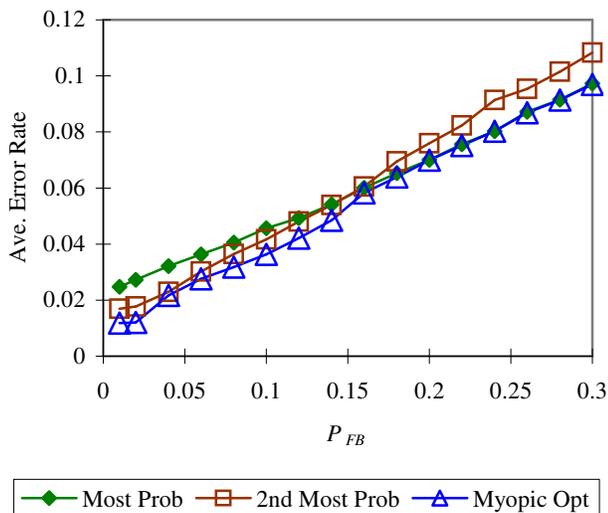


Fig. 1. Average error rate.

the number of observations used by the other heuristics to achieve the same probability of error. The number of observations is averaged over 500,000 Monte Carlo simulation runs. The heuristic of choosing the most probable value of X consistently has the best performance; it usually requires an average of one fewer observation than the two-stage algorithm. The myopic optimal heuristic performs equivalently to the most probable heuristic when p_{FB} is large because it usually chooses the most probable cell for these values of p_{FB} .

7. CONCLUSIONS

We have formulated a simple two-mode sensor control problem and investigated the performance of several myopic control strategies. We have found that the myopic optimal heuristic results in the lowest average error rate, and that choosing the most probable heuristic results in the lowest average number of observations. These findings are consistent with the results presented in [7] for the single-mode sensor problem.

One important unanswered question is the performance penalty (in terms of probability of error or number of needed observations) incurred by a myopic algorithm compared to an optimal sequential algorithm. We are currently investigating this performance tradeoff.

8. REFERENCES

[1] D. Cochran, D. Sinno, and A. Clausen, "Source detection and localization using a multi-mode detector: a

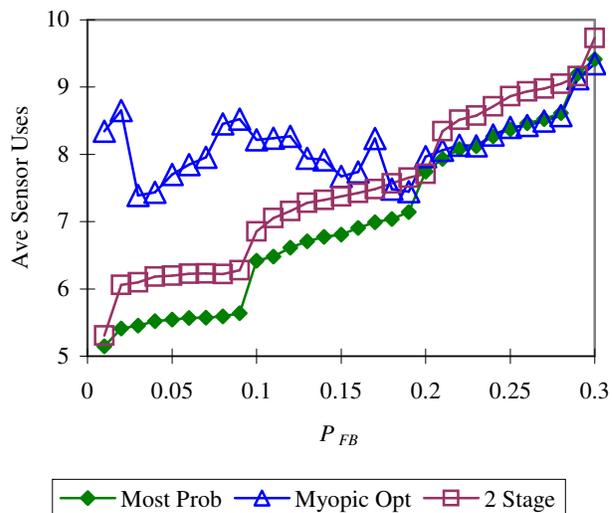


Fig. 2. Average number of observations required to obtain a probability of error less than 0.05.

bayesian approach," in *Proceedings of ICASSP99*, Mar. 1999, vol. 3, pp. 1173–1176.

- [2] D. Sinno, D. Cochran, and D. Morrell, "A Bayesian risk approach to multi-mode detection," in *U.S./Australia Joint Workshop on Defense Signal Processing*, Chicago, Aug. 1999.
- [3] D. Sinno, *Attentive Management of Configurable Sensor Systems*, Ph.D. thesis, Arizona State University, May 2000.
- [4] D. Sinno, D. Cochran, and D. Morrell, "Multi-mode detection with Markov target motion," in *Proceedings of the Third International Conference on Information Fusion, 2000*, July 2000, pp. 25–31.
- [5] V. Raghavan, M. Shakeri, and K. Pattipati, "Test sequencing algorithms with unreliable tests," *IEEE Trans. on Systems, Man, and Cybernetics-Part A: Systems and Human*, vol. 29, no. 4, pp. 347–357, July 1999.
- [6] D. Castanon, "Optimal search strategies in dynamic hypothesis testing," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 25, no. 7, pp. 1130–1138, July 1995.
- [7] D. Morrell and Ya Xue, "Analysis of and heuristics for sensor configuration in a simple target localization problem," in *Asilomar Conference on Systems, Signals, and Computers*, Nov. 2001.