

## Chapter 36

# Equilibrium Sorting Models of Land Use and Residential Choice<sup>\*</sup>

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“Far better an approximate answer to the *right* question, which is often vague, than an *exact* answer to the wrong question, which can always be made precise.” —John Tukey (1962).

## **1. Introduction**

Americans are remarkably mobile. Since World War II, 18% of the United States population has moved to a new residence every year, on average. As Charles Tiebout (1956) famously observed, these movers face a public goods counterpart to the private market shopping trip. They choose among residential communities that differ in their housing prices and in their provision of amenities such as local public goods, urban attractions, and environmental services. The location choices that they make reveal features of their preferences. As heterogeneous households sort themselves across the urban landscape, their collective location choices will influence housing prices as well as the supply of amenities through a combination of voting, social interaction, and feedback effects. In order to better understand this two-way interaction between people and their surrounding environment, economists have developed equilibrium models of the sorting process.

Equilibrium sorting models begin with a formal description for the spatial landscape and the structure of household preferences. Utility-maximizing location choices are expressed as a function of the observable characteristics of households, houses, and communities, as well as structural parameters describing latent preferences. This functional relationship is then inverted, using the logic of revealed preferences to characterize the distribution of preferences in the population of households. Estimation results are used to calculate the willingness to pay for large scale changes in landscape amenities. Sorting models can also be used to simulate how people and markets will adjust to unexpected events and make predictions for “general equilibrium” benefit measures and future land use trends. This is a new and exciting framework for policy

evaluation that offers the potential to improve our understanding of land economics.

Compared to the standard quasi-experimental framework for describing how landscape changes affect housing prices, the development and estimation of a structural sorting model can seem intimidating. The analyst must be willing to collect additional data and think deeply about the economic forces that underlie market equilibria. Econometric identification may seem less transparent. It may be necessary to code the estimator from scratch, and the results may be viewed with skepticism by critics who dislike structural modeling. Despite these challenges, the potential insights from formulating, estimating, and interpreting an equilibrium sorting model far outweigh the learning costs. Put simply, the equilibrium sorting methodology allows us to provide approximate answers to the right questions about the relationships between land use, residential choice, and public policy.

This chapter summarizes the equilibrium sorting methodology. We have two main objectives. First, we intend to make the empirical models accessible to economists who are new to the literature. Thus, we provide more detail about data sets and estimators than one finds in the typical journal article. Our second objective is to clarify the relationship between the newer structural models of the sorting process and the older reduced-form models of hedonic equilibria that have long served as a workhorse for economic analysis of land use and household location choice. We argue that the two frameworks are inseparable. Hedonic price functions describe sorting equilibria, and what we learn about the sorting process influences how we interpret hedonic price functions.

We intend this chapter to be more pragmatic than previous efforts to characterize the literature. Considerable space is devoted to: (i) empirical descriptions for the spatial landscape and household preferences; (ii) econometric procedures for estimating structural preference param-

ters; and (iii) procedures for simulating how markets adjust to unexpected events. This leaves us with less space to cover historical background, systematically catalog empirical results, or recommend directions for future research. Readers interested in these topics are directed to Palmquist (2005), Klaiber and Smith (2009), Epple, Gordon, and Sieg (2010), and Kuminoff, Smith, and Timmins (2010).

The chapter proceeds as follows. Section 2 begins with a general description for the spatial landscape that nests empirical hedonic and sorting models. Then we define the household's location choice problem, characterize a sorting equilibrium, and briefly summarize results on existence and uniqueness. In section 3 we move from theory to practice. Focusing on the two predominant microeconomic frameworks—the “pure characteristics model” (Epple and Sieg 1999) and the “random utility model” (Bayer, McMillan, and Ruben 2004)—we explain how to build an empirical sorting model and estimate structural parameters. Data sets, modeling assumptions, and econometric procedures are covered. Section 4 explains how the estimation results can be used to simulate how people and markets would adjust to an unexpected event. Many of the insights gleaned from the estimation and simulation of sorting models also have important implications for hedonic estimation. Section 5 summarizes insights on the causes and consequences of omitted variable bias, benefit measurement, and the interpretation of land value capitalization effects. Finally, section 6 concludes.

## **2. Conceptual Framework**

### *2.1. The Spatial Landscape*

Consider a metropolitan region comprised of  $j = 1, \dots, J$  housing communities, each of which

contains  $N_j$  houses.<sup>1</sup> The region is assumed to be sufficiently small that most working households could relocate anywhere in the region without having to move to a different job. At the same time, the region is assumed to be self-contained in the sense that few households would consider living outside the region. Some regions that meet these criteria may be small and isolated, such as the Grand Junction metro area in western Colorado. Others may be large and integrated, such as the San Francisco-Oakland-San Jose consolidated metropolitan statistical area, containing more than 4 million people spread out over several interconnected cities and suburbs.

Within the region, each housing community provides a unique bundle of amenities,  $g_j$ . “Amenities” are defined broadly to include any non-marketed goods and services that matter to households. Examples include local public goods produced from property tax revenue (public education, police and fire protection), environmental services (air quality, microclimate), proximity to urban attractions (central business district, shopping, dining), and the demographic composition of the community (race, age, wealth). Within a community, individual houses differ in their structural characteristics. The vector  $h_{n_j}$  will be used to describe the physical attributes of a particular house,  $n$ , located in community  $j$ . Examples include the square footage of the house, the number of bedrooms, and the quality of building materials.

Households are heterogeneous. They differ in terms of their incomes ( $y$ ), preferences ( $\alpha$ ), and demographic characteristics ( $d$ ). Each household will maximize its utility by choosing a specific house in its preferred community. We use  $n_j$  to denote the household’s simultaneous choice of a community and a house within that community:

$$(1) \quad \max_{n_j} U_i(g_j, h_{n_j}, b, \alpha_i) \text{ subject to } y_i = b + P_{n_j}.$$

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<sup>1</sup> The terms “community” and “neighborhood” are used interchangeably in the literature.

In the budget constraint, the price of the numeraire commodity ( $b$ ) is normalized to 1 and  $P_{n_j}$  represents the annualized after-tax price of housing.

The collective location choices made by the population of households may influence the spatial distribution of amenities. For example, as open space gets converted to urban development new opportunities for dining and nightlife may emerge, along with increased traffic and air pollution. Homeowners may be asked to vote on assessments to fund the preservation of remaining open space or to support public schools. The academic performance of students in those schools may depend on the incomes and education levels of their parents. While we do not model these mechanisms formally, it is important to keep them in mind because they create a need for instruments in econometric estimation.

Finally, three assumptions are typically maintained in order to reduce the amount of friction in the market. First, everyone is assumed to have perfect information about the spatial landscape. Second, everyone is assumed to face the same schedule of prices. Finally, households are assumed to be freely mobile.

## 2.2. *Characterizing a Sorting Equilibrium*

In a sorting equilibrium, prices, physical housing characteristics, amenities, and location choices are all defined such that no household could improve its utility by moving and each household occupies exactly one house. Equation (2) provides a formal statement of this condition.

$$(2) \quad U_i(g_j, h_{n_j}, y_i - P_{n_j}, \alpha_i) \geq U_i(g_k, h_{m_k}, y_i - P_{m_k}, \alpha_i) \quad \forall \quad i, m, k : P_{m_k} < y_i,$$

$$\text{and } \sum_{i, n_j} A_{i, n_j} = 1 \quad \forall \quad i, n_j,$$

where  $A_{i, n_j}$  is an indicator variable that equals 1 *if and only if* household  $i$  occupies house  $n$  in

community  $j$ . While we suppress temporal subscripts, equation (2) is best viewed as a single-period snapshot of market outcomes. It may or may not be a long-run steady state. Current incomes and preferences may reflect temporary factors. Credit may be unusually easy (or difficult) to obtain. The average household may be unusually optimistic (or pessimistic) about the future asset value of housing. Budget constraints may reflect other transitory macroeconomic or microeconomic shocks. As these factors change over time, so will the features of the sorting equilibrium.

With a few mild restrictions on preferences, the market outcomes from a sorting equilibrium can be described by a hedonic price function. If  $U_i(g_j, h_{n_j}, b, \alpha_i)$  is continuously differentiable, monotonically increasing in the numeraire, and Lipschitz continuous, then theorem 1 from Bajari and Benkard (2005) can be invoked to prove that equilibrium prices must be functionally related to housing characteristics and amenities,  $P_{n_j} = P(g_j, h_{n_j})$ .<sup>2</sup> This result places less discipline on the price function than Rosen's (1974) hedonic model. Households are not assumed to be free to choose continuous quantities of each amenity. Nor is the market assumed to be perfectly competitive. In fact, Bajari and Benkard demonstrate that no assumptions about the supply side of the market are needed to prove that equilibrium can be described by a price function.

Relaxing the assumptions of Rosen's model has costs and benefits. The main benefit is a more realistic description of the spatial landscape. While households may be able to purchase approximately continuous quantities of physical housing characteristics, the same is not true for landscape amenities. Air quality changes discretely from air basin to air basin; test scores change discretely from school district to school district; and some communities are adjacent to open space, while others are not. The cost of relaxing Rosen's continuity assumption is that we

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<sup>2</sup> While Bajari and Benkard (2005) treat non-price attributes as exogenous, it is straightforward to extend their result to the case of endogenous amenities by assuming that households ignore their own contributions to each amenity.

lose the ability to translate the price function gradient into measures of the marginal willingness to pay for amenities. Nevertheless, we shall see that the price function still plays an important role in estimation.

A second, stronger, restriction that has proven useful in characterizing sorting equilibria is the single crossing condition. Single crossing helps to characterize the ways in which households sort themselves across locations according to their heterogeneous incomes and preferences. To see the intuition, consider the simplest form of preference heterogeneity—vertical differentiation. In a “vertical” model, households differ only in their preferences for housing “quality” relative to the numeraire. They are assumed to agree on a ranking of locations by overall quality,  $q = f(g, h)$ . Given this assumption, equation (3) defines the slope of an indirect indifference curve in  $(q, p)$  space.

$$(3) \quad M(q, p, \alpha, y) = \left\langle \frac{dp}{dq} \Big|_{V = \bar{V}} \right\rangle .$$

If  $M$  is monotonically increasing in  $(y | \alpha)$  and  $(\alpha | y)$ , then indifference curves in the  $(q, p)$  plane will satisfy single crossing in  $y$  and  $\alpha$ . Under this condition, any sorting equilibrium must satisfy three properties: *boundary indifference*, *increasing bundles*, and *stratification*.<sup>3</sup>

To interpret the three properties, it is useful to first order locations by quality. Without loss of generality, let the ordering be defined such that  $q_1 < \dots < q_R$ . The *increasing bundles* property requires that households must pay for the amenities provided by higher ranked locations through higher housing prices:  $P_1 < \dots < P_R$ . *Stratification* requires that households are stratified across the  $R$  locations by  $(\alpha | y)$  and  $(y | \alpha)$ . In other words, all else constant, households in

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<sup>3</sup> For additional background on the role of single-crossing conditions in equilibrium sorting models see Epple and Romer (1991), Epple and Sieg (1999), and Kuminoff, Smith, and Timmins (2010).



higher ranked locations will have higher income and stronger preferences for amenities. Finally, *boundary indifference* defines the set of values for  $(\alpha, y)$  that would make a household exactly indifferent between locations  $r$  and  $r+1$ .

Figure 1 provides a simple illustration of a sorting equilibrium that satisfies the three properties. Consistent with increasing bundles, the price ranking of communities matches the ranking by overall amenity provision. The figure partitions  $(\alpha, y)$  space into three cells corresponding to  $(\alpha, y)$  combinations that rationalize the choice of each community. For example, community 1 would maximize utility for any household with values for income and preferences in the lower left cell of the partition. The boundaries between adjacent cells define the  $(\alpha, y)$  combinations that would make a household exactly indifferent between the corresponding communities. To see how households are stratified across communities notice that, conditional on preferences, wealthier households choose communities with more public goods. Likewise, conditional on income, households with stronger preferences choose communities with more public goods. This two-dimensional stratification is consistent with Tiebout's (1956) reasoning and helps to explain why we sometimes observe low-income households living in high-amenity communities and high-income households living in low-amenity communities.

Stratification, increasing bundles, and boundary indifference are particularly helpful in estimating the class of pure characteristics models covered in section 3.2. The single crossing condition is sufficient, but not necessary, to guarantee that a sorting equilibrium will satisfy these properties. In addition to providing a simple characterization of equilibrium, the single crossing condition can help to guarantee that equilibria exist.

### 2.3. *Existence and Uniqueness*

General proofs of existence and uniqueness require fairly strong restrictions on preferences and amenities. One strategy is to assume that households have identical preferences ( $\alpha_i = \alpha, \forall i$ ) so they differ only in their incomes. In this case, the single crossing condition makes it possible to prove existence in the presence of an endogenously determined amenity (Ellickson 1971, Westoff 1977). Another strategy is to allow preference heterogeneity but rule out social interactions (Nechyba 1997). Bayer and Timmins (2005) develop a third approach. They smooth the preference function by adding an idiosyncratic iid shock to utility. This allows them to prove existence in a setting where households with heterogeneous preferences for exogenous amenities share a common marginal utility for a single endogenous amenity. Whether the equilibrium is unique depends on whether marginal utility is positive or negative.

In the presence of more complex preference structures, analysts have used numerical simulations to demonstrate that equilibria *may* exist (Epple and Platt 1998, Sieg, Smith, Banzhaf, and Walsh 2004, Walsh 2007, Klaiber and Phaneuf 2010, Kuminoff and Jarrah 2010, Kuminoff 2011). Despite the lack of general proofs for existence and uniqueness, the empirical literature has moved forward with preference structures that allow considerable heterogeneity and acknowledge the potential endogeneity of amenities. Analysts simply assume that the available data reflect an equilibrium. Then they write down a utility function and estimate values for the structural parameters that justify those data as an equilibrium.

### 3. Estimation

Moving from theory to estimation requires three sources of information: (i) a definition for the choice set; (ii) a parametric representation of the preference function; and (iii) assumptions for the statistical distributions used to characterize sources of unobserved heterogeneity. While specific modeling choices differ from study to study, most applications can be grouped into two

broad frameworks: random utility models (RUM) based on Bayer, McMillan, and Reuben (2004) and pure characteristics models (PCM) based on Epple and Sieg (1999).<sup>4</sup>

The RUM and PCM frameworks provide alternative characterizations of the same sorting equilibrium. They require data on the same core variables: prices, housing characteristics, household demographics, and spatially delineated amenities. Data sources vary. Housing prices and structural characteristics are typically drawn from the same sources as the hedonic literature—assessor databases or the U.S. Census of Housing. Data on consumer demographics are typically drawn from the Census of Population. Data on amenities have been drawn from a variety of federal and state government agencies. While it is possible to calibrate empirical models using aggregate data, the rule of thumb is to use the highest resolution micro data that are available.<sup>5</sup>

### 3.1. *The Random Utility Framework*

The random utility framework builds on McFadden’s (1974) seminal discrete choice model. Bayer, McMillan, and Reuben (2004) developed the first application to residential sorting, using data from the San Francisco area. A key feature of their application is the recognition that both housing prices and amenities may be endogenous in the estimation process. Consider housing prices. Unobserved attributes of communities that make them more desirable also increase the demand to locate there. *Ceteris paribus*, equilibrium prices must be higher in more desirable communities. The implication of this logic is the need to use instrumental variables to disentangle the correlation between equilibrium housing prices and unobserved amenities. A similar argument applies to amenities that are endogenously determined through the sorting process.

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<sup>4</sup> Ferreyra (2007) proposed a third “general equilibrium” approach building on earlier work by Nechevba (1997, 1999, 2000).

<sup>5</sup> Sieg et al. (2004), Bayer, Ferreira, and McMillan (2007), and Klaiber and Phaneuf (2010) provide particularly detailed descriptions of how their data sets were assembled.

Bayer, McMillan, and Reuben (2004) show that the structure of the sorting model itself can help to overcome these econometric challenges.

Subsequent applications refined the RUM framework and used it to estimate preferences for school quality (Bayer, Ferreira, and McMillan 2007), land use (Klaiber and Phaneuf 2010), and air quality (Tra 2010). A distinguishing characteristic of these applications is the way they define locations as particular “types” of housing. For example, Klaiber and Phaneuf define a housing type as a unique (house size, time period, community), rather than an individual house. This aggregation follows from Berry, Linton, and Pakes (2004) who demonstrate that consistent estimation for this class of RUM model requires the number of consumers to exceed the number of choice alternatives. Bayer, Ferreira, and McMillan (2007) use micro-census data on individual houses while Tra uses information on sampled houses contained within Census public use microdata areas (PUMAs) grouped by common housing characteristics. Each of these approaches either implicitly or explicitly aggregates individual houses into “housing types within communities” that form the choice set.

### *3.1.1. Parameterization of the Model*

Parameterization of the model begins by dividing utility into observed and unobserved components. A location-specific unobservable,  $\xi$ , is used to represent housing characteristics and amenities that are observed by households, but not the analyst. Additionally, an “error” term,  $\epsilon$ , is added, recognizing that households may have idiosyncratic preferences for each location.

The utility a household receives from choosing a particular housing type,  $t$ , in community  $j$ , is usually expressed as a linear function of its attributes,

$$(4) \quad V_{tj}^i = \alpha_h^i h_{tj} + \alpha_g^i g_j + \alpha_p^i p_{tj} + \xi_{tj} + \epsilon_{tj}^i .$$

The way that communities are subdivided into housing types varies from study to study. At one extreme, a type could be defined as precisely as an individual house. At the opposite extreme, a type could be defined as coarsely as the mean or median house in a particular community. Most studies use definitions between these extremes for reasons that we discuss in the context of the mechanics of the estimator. Meanwhile, communities are often defined using Census aggregates, such as PUMAs, tracts, or block groups.

Three features of (4) are worth noting. First, the  $i$  superscripts on  $\alpha$  allow households to differ in their relative preferences for different attributes. This generalizes the “vertical” preference structure introduced earlier and is often referred to as “horizontal” differentiation.<sup>6</sup> Second, the marginal utility of income is implicitly assumed to be constant. It is suppressed in (4) as is the custom in random utility models.<sup>7</sup> Lastly, while the choice of a location is deterministic from the perspective of each household, assuming a statistical distribution for the idiosyncratic term,  $\epsilon_{t_j}^i$ , makes it possible to derive a closed-form expression for the share of households who choose each housing type.

Assuming  $\epsilon_{t_j}^i$  is distributed according to an iid Type I extreme value distribution produces a familiar logit expression for the probability that household  $i$  chooses each housing type,

$$(5) \quad Pr_{t_j}^i = \frac{\exp(\alpha_h^i h_{t_j} + \alpha_g^i g_j + \alpha_p^i p_{t_j} + \xi_{t_j}^i)}{\sum_{s,k} \exp(\alpha_h^i h_{s_k} + \alpha_g^i g_k + \alpha_p^i p_{s_k} + \xi_{s_k}^i)}.$$

Aggregating (5) over  $i = 1, \dots, I$  households generates the expected share of households choosing a particular housing type,

$$(6) \quad \sigma_{t_j} = \frac{1}{I} \sum_i Pr_{t_j}^i.$$

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<sup>6</sup> The “vertical” and “horizontal” terminology is adapted from Lancaster (1979).

<sup>7</sup> Tra (2010) includes a non-linear income term of  $\ln(y^i - p_h)$  which preserves the budget constraint but presents difficulties for welfare measurement (McFadden, 1999; Herriges and Kling, 1999).

This share forms the foundation for market clearing in the model. There is no direct assignment of individual households to specific housing types. Instead, equilibrium is characterized using the predicted share of households selecting each housing type.

Ensuring market clearing requires the predicted share of households choosing each housing type must be identical to the observed share for that type. In other words, housing supply must equal housing demand. This condition is satisfied by the inclusion of the alternative specific unobservables,  $\xi_{t_j}$ . Given a distributional assumption for  $\epsilon_{t_j}^i$ , Berry (1994) demonstrates that including a complete set of alternative specific unobservables results in predicted and observed market shares coinciding as a necessary condition for maximum likelihood estimation.<sup>8</sup>

### 3.1.2. Estimation Procedures

Recall that the specification for utility in (4) allows for horizontal preference heterogeneity. Past applications have taken advantage of this flexibility by decomposing each preference parameter,  $\alpha^i$ , into the sum of a constant component and a component that varies along observable demographic characteristics of households:

$$(7) \quad \alpha^i = \alpha^0 + \alpha^1 d^i .$$

Using this decomposition, the utility function can be expanded as (8).

$$(8) \quad V_{t_j}^i = \alpha_h^0 h_{t_j} + \alpha_h^1 d^i h_{t_j} + \alpha_g^0 g_j + \alpha_g^1 d^i g_j + \alpha_p^0 p_{t_j} + \alpha_p^1 d^i p_{t_j} + \xi_{t_j} + \epsilon_{t_j}^i .$$

All of the structural parameters in (8) can be recovered using a two-stage approach to estimation.

The first stage recovers parameters that vary with household demographic characteristics ( $\alpha_h^1, \alpha_g^1, \alpha_p^1$ ) as well as the mean indirect utility for each alternative ( $\theta_{t_j}$ ). The second stage uses

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<sup>8</sup> This property holds for the linear exponential family of models that includes conditional logit.

the first-stage estimate for mean indirect utility to recover preference parameters common to all households  $(\alpha_h^0, \alpha_g^0, \alpha_p^0)$ . This partitioning is shown in equations (9a) and (9b),

$$(9a) \quad V_{t_j}^i = \alpha_h^1 d^i h_{t_j} + \alpha_g^1 d^i g_j + \alpha_p^1 d^i p_{t_j} + \theta_{t_j} + \epsilon_{t_j}^i$$

$$(9b) \quad \hat{\theta}_{t_j} = \alpha + \alpha_h^0 h_{t_j} + \alpha_g^0 g_j + \alpha_p^0 p_{t_j} + \xi_{t_j},$$

using the script-free  $\alpha$  term in (9b) to represent an intercept.<sup>9</sup>

In principle, the parameters in (9a) could be estimated using a standard conditional logit model. For many applications, however, the number of housing types is large, making gradient-based maximum likelihood estimation burdensome due to the propagation of mean indirect utility parameters. To reduce this computational burden, past studies have relied on the results from Berry (1994). Specifically, a contraction mapping algorithm enables recovery of estimates for each mean indirect utility parameter  $(\hat{\theta}_{t_j})$  without using gradient based searches. This computational “trick” speeds model convergence significantly.<sup>10</sup>

Second stage estimation of (9b) raises several econometric issues. First, because the dependent variable consists of *estimated* mean indirect utilities from (9a), some additional criteria must be satisfied to establish consistency and asymptotic normality (Berry, Linton, and Pakes 2004). Let  $T = \sum_{t,j} t_j$  represent the total number of distinct housing types. Consistency and asymptotic normality are defined as  $T \rightarrow \infty$ . To guarantee consistency, the number of households must grow large relative to the number of types:  $\frac{T \log T}{I} \rightarrow 0$ . Asymptotic normality requires the

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<sup>9</sup> An intercept is included to account for the normalization that occurs in first stage estimation. Evaluating differences in utility prevents recovery of the full  $j=1 \dots J$  mean indirect utility parameters. In practice, researchers often normalize by setting the first mean indirect utility parameter equal to zero and estimate the remaining  $J-1$  parameters.

<sup>10</sup> The standard contraction mapping routine is:  $\theta_{t_j}^{s+1} = \theta_{t_j}^s - \ln \left( \sum_i \frac{Pr_{t_j}^i}{\sigma_{t_j}} \right)$ , where  $s$  indexes iteration.

additional restriction that  $\frac{T^2}{I}$  is bounded. These two requirements help motivate the characterization of housing types.

Consistency cannot be established if  $t$  is defined as an individual house as this results in  $T = I$ . At the same time, it seems important to recognize that the prices and structural characteristics of houses vary within the Census aggregates used to define housing communities. Empirical studies have sought a middle ground that addresses both issues. For example, Klaiber and Phaneuf (2010) use square footage to divide the houses in each Census block group into “small”, “medium”, and “large” terciles. Then they define housing types using the median values of structural housing characteristics, amenities, and prices for the houses in each block group and size category.

Another econometric issue is that prices, and potentially amenities, are likely correlated with the error term in (9b), confounding OLS estimation. A popular instrumentation strategy is to exploit the logic of the sorting process to form an “optimal” instrument (Bayer and Timmins 2007). The insight behind the IV strategy explained by Bayer and Timmins is to utilize the variation in prices that reflects exogenous characteristics of distant locations. Such instruments are relevant because the equilibrium levels of endogenous attributes at each location are influenced by the attributes of all other locations through the sorting equilibrium. Their validity relies on the assumption that the analyst can identify “exogenous” attributes at distant locations that are uncorrelated with  $\xi_{t_j}$ .

If we treat amenities as exogenous and employ the Bayer-Timmins instrument for price, Klaiber and Phaneuf (2010) demonstrate that the two-step estimator can proceed as follows:

**Step 0.** Estimate (9a) to obtain parameter estimates.

**Step 1.** Make a guess for the coefficient on price,  $\alpha_p^{0*}$ .



- Step 2.** Move the term  $\alpha_p^{0*} p_{t_j}$  in (9b) to the left hand side and add additional control variables (denoted by tildes) formed within rings around each choice alternative to the right hand side of the modified (9b).
- Step 3.** Estimate the modified (9b) from step 2 via OLS and set the residual to zero.
- Step 4.** Calculate the mean indirect utility implied by step 3, denoting it by  $\tilde{\theta}_{t_j}$ .
- Step 5.** Use the first stage estimates from step 0 along with the initial guess for the coefficient on price and the estimates obtained in step 4 to solve for the set of prices,  $\rho_h^{iv}$  such that aggregate predicted shares exactly equal observed shares of each alternative  $t_j$ .
- Step 7.** Perform IV estimation of (9b) using  $p_{t_j}^{IV}$  as an instrument.
- Step 8.** Use the estimate of  $\alpha_p^0$  from step 7 to iterate starting at step 1 until the estimate of  $\alpha_p^0$  converges.

### 3.2. *The Pure Characteristics Framework*

Most of the recent empirical models developed within the pure characteristics framework build on earlier work by Dennis Epple and his co-authors (e.g. Epple, Filimon, and Romer 1984, 1993, Epple and Romer 1991). These studies introduced a CES specification for preferences as an example. Epple and Platt (1998) calibrated the CES function to data on housing market outcomes, and Epple and Sieg (1999) developed a structural estimator. Their approach to estimation was refined in subsequent work by Sieg et al. (2002, 2004). The PCM framework has since been used to investigate the benefits of numerous amenities including landscape attributes in Portland, Oregon (Wu and Cho 2003), air quality in Southern California (Smith et al. 2004) and Northern California (Kuminoff 2009), open space in the Raleigh-Durham area of North Carolina (Walsh 2007), and school quality in Phoenix, Arizona (Klaiber and Smith 2010).

### 3.2.1. Parameterization of the Model

One of the distinguishing features of the PCM framework is a mixed discrete-continuous depiction of the choice set. Households are assumed to be free to choose continuous quantities of physical housing characteristics in each of a discrete number of residential communities. Under this assumption, the location choice process can be characterized by the choice of a community. Conditional on that choice, a household will select a house with the optimal combination of physical characteristics.

Sieg et al. (2002) illustrate how the discrete-continuous representation for the choice set influences how we define the “price of housing” in an indirect utility function. They demonstrate that as long as  $h_{n_j}$  enters utility through a separable sub-function that is homogeneous of degree 1, housing expenditures can be expressed as the product of a price index and a quantity index,  $P_{n_j} = q(h_{n_j}) \cdot p(g_j)$ . In this case,  $p_j = p(g_j)$  replaces  $P_{n_j}$  in the indirect utility function. Equation (10) steps through this logic.

$$\begin{aligned}
 & U[g_j, h_{n_j}(g_j, P_{n_j}, \alpha_i, y_i), y_i - P_{n_j}(g_j, h_{n_j}), \alpha_i] \\
 (10) \quad & = U[g_j, h_{n_j}(g_j, q(h_{n_j}) \cdot p_j(g_j), \alpha_i, y_i), y_i - q(h_{n_j}) \cdot p_j(g_j), \alpha_i] \\
 & = V(g_j, p_j, \alpha_i, y_i).
 \end{aligned}$$

The first equality follows from Sieg et al. (2002). The second equality simply rewrites utility in indirect terms. The “price of housing” in each community,  $p_j$ , represents the implicit price (per unit of  $q$ ) to consume the bundle of nonmarket amenities provided by that community.

The same assumptions that allow Sieg et al. (2002) to factor housing expenditures into price and quantity indices also support a strategy to estimate  $p_1, \dots, p_j$  from a hedonic regression. Taking logs of the expenditure function yields a general expression for an estimable hedonic

model,

$$(11) \quad \ln(P_{n_j}) = \ln[q(h_{n_j})] + \ln[p_j(g_j)] .$$

Given an assumption for the functional form of the quantity index, micro data on housing sales can be used to recover  $\hat{p}_1, \dots, \hat{p}_j$  as community-specific fixed effects in a regression of log sale prices on housing characteristics.<sup>11</sup> Normalizing the smallest fixed effect to equal one produces the prices that enter the discrete-choice model of community selection.

Equation (12) illustrates the CES specification for preferences. It describes the utility that household  $i$  obtains from living in community  $j$ .

$$(12) \quad V_{i,j} = \left\{ \alpha_i (G_j)^\rho + \left[ \exp\left(\frac{(y_i)^{1-\nu} - 1}{1-\nu}\right) \exp\left(-\frac{\beta P_j^{\eta+1} - 1}{1+\eta}\right) \right]^\rho \right\}^{\frac{1}{\rho}} ,$$

where  $G_j = \gamma_1 g_{1,j} + \dots + \gamma_{R-1} g_{R-1,j} + \xi_j$ , and  $F(\alpha, y) \sim \text{lognormal}$ .

This indirect utility function does not correspond to any closed-form expression for direct utility, but it has several useful properties. It recognizes that physical housing characteristics may not be perfect substitutes for amenities. It also generates a convenient Cobb-Douglas specification for the demand for housing.<sup>12</sup> Finally, the CES specification maps directly into the underlying theory from section 2.2. It yields parametric expressions for boundary indifference, stratification, and increasing bundles that serve as the basis for the estimation algorithm.<sup>13</sup>

The first term inside the CES nest represents utility from amenities. Households obtain utility from a linear index of amenities provided by each community. They are assumed to agree

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<sup>11</sup> For example, if the quantity index is assumed to be multiplicative then the regression is a simple linear-in-logs specification with fixed effects for communities. Data on the transaction prices of actual housing sales are converted to annualized values by adapting the formula from Poterba (1992).

<sup>12</sup> This follows from the exponential form of the term in square brackets.

<sup>13</sup> For additional discussion of the properties of CES specifications for utility in sorting model see Epple, Filimon, and Romer (1984, 1993), Epple and Romer (1991), Epple and Platt (1998), Epple and Sieg (1999), and Sieg et al. (2002).

on a common set of weights for the amenities in the index  $(\gamma_1, \dots, \gamma_{R-1})$  but they differ in their overall preferences for amenities relative to the private good component of housing and the numeraire  $(\alpha_i)$ . Of the  $R$  amenities in the index,  $R-1$  are observable.  $g_{R,j} = \xi_j$  represents the composite effect of community-specific attributes that are observed by households but not the analyst. As in the RUM model,  $\xi$  varies across choices but is restricted to be the same for every household.<sup>14</sup> This is an example of what Berry and Pakes (2007) label the “pure characteristics” approach to modeling choice among differentiated objects. Utility is defined purely over the characteristics of communities; there is no idiosyncratic location-household-specific  $\varepsilon_{ij}$  shock.

The second term inside the CES nest represents utility from the private good component of housing. Households are assumed to share the same elasticity of substitution between amenities and private goods ( $\rho$ ), and the same demand parameters for the private good component of housing: price elasticity ( $\eta$ ), income elasticity ( $\nu$ ), and demand intercept ( $\beta$ ). Applying Roy’s Identity to (12) yields a simple expression for the demand for housing,

$$(13) \quad q_i = \beta p_j^\eta y_i^\nu.$$

While households share a common set of demand parameters, notice that individual demand varies with income.

A key feature of the CES specification in (12) is that preferences are vertical. Since households have identical relative preferences for  $g_1, \dots, g_R$ , they agree on the ranking of communities by the  $G$  index. Given the expected signs for the housing demand parameters ( $\rho$ ,

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<sup>14</sup> While the RUM and PCM frameworks both use  $\xi$  to represent choice-specific unobserved attributes, they will generally recover different estimates for  $\xi$  due to the different spatial scales at which they define the variable and due to their different specifications for preferences.

$\beta > 0, \eta < 0, \nu > 0$ ), preferences satisfy single crossing if  $\rho < 0$ .<sup>15</sup> This makes it possible to describe how households sort themselves across communities in equilibrium. To see this, first order communities by price:  $p_1 < p_2 < \dots < p_J$ . *Increasing bundles* implies  $G_1 < G_2 < \dots < G_J$ . Equation (14) uses *boundary indifference* to implicitly define the  $(\alpha, y)$  combinations that make a household exactly indifferent between  $j$  and  $j+1$ .

$$(14) \quad \ln(\alpha_i) - \rho \left( \frac{y_i^{1-\nu} - 1}{1-\nu} \right) = \ln \left( \frac{Q_{j+1} - Q_j}{G_j^\rho - G_{j+1}^\rho} \right) = B_{j,j+1}, \quad \text{where} \quad Q_j = \exp \left[ -\frac{\rho}{1+\eta} (\beta p_j^{\eta+1} - 1) \right].$$

Notice that all of the heterogeneity in income and preferences appears to the left of the equality. The *stratification* property implies that any household with income and preference such that:  $\ln(\alpha_i) - \rho \left[ (y_i^{1-\nu} - 1) / (1-\nu) \right] < B_{j,j+1}$  will prefer community  $j$  to every higher ranked community:  $j+1, j+2, \dots, J$ . Therefore, the left side of (14) can be used to characterize the sorting of households into communities. This result plays an important role in the mechanics of the estimator.

### 3.2.2. Estimation Procedures

Estimation procedures vary slightly from study to study. Here we describe the simulated GMM approach developed by Sieg et al. (2004).

Treating the first-stage estimates for housing prices as known constants, the GMM estimator can be used to recover all of the structural parameters. Let  $\theta$  represent a vector of these parameters,  $\theta = [\beta, \eta, \nu, \rho, \mu_\alpha, \mu_y, \sigma_\alpha, \sigma_y, \lambda, G_1, \gamma_1, \dots, \gamma_{R-1}]$ . Equation (15) defines the GMM objective function, where  $z$  is a set of instruments,  $m$  represents the moment conditions, and  $A$  is the covariance matrix of moments.

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<sup>15</sup> Empirical studies have been unanimous in confirming the expected signs of these four parameters.

$$(15) \quad \theta = \arg \min_{\theta \in \Theta} \left\{ \frac{1}{J} \sum_{j=1}^J z_j m_j(\theta) \right\}' A^{-1} \left\{ \frac{1}{J} \sum_{j=1}^J z_j m_j(\theta) \right\}.$$

Sieg et al. demonstrate that the seven moment conditions in (16) can be used to identify all the parameters in  $\theta$ .<sup>16</sup>

$$(16) \quad m_j(\theta) = \left\{ \begin{array}{l} \tilde{G}_j - \gamma_1 g_{1,1} - \dots - \gamma_{R-1} \cdot g_{R-1,j} \\ y_j^{25} - \tilde{y}_j^{25} \\ y_j^{50} - \tilde{y}_j^{50} \\ y_j^{75} - \tilde{y}_j^{75} \\ \ln P_{n \in j}^{25} - \ln \beta - (\eta + 1) \ln p_j - \nu \ln \tilde{y}_j^{25} \\ \ln P_{n \in j}^{50} - \ln \beta - (\eta + 1) \ln p_j - \nu \ln \tilde{y}_j^{50} \\ \ln P_{n \in j}^{75} - \ln \beta - (\eta + 1) \ln p_j - \nu \ln \tilde{y}_j^{75} \end{array} \right\}.$$

The first moment condition is based on the level of amenity provision. Given a value for overall provision of amenities in the cheapest community,  $G_1$ , the sorting behavior implied by vertical differentiation allows  $G_2, \dots, G_J$  to be defined recursively. The predictions for  $G_1, \dots, G_J$  are then used to identify the (constant) weights in the amenity index. The residual to the moment condition defines the composite unobserved amenity in each community ( $\xi_1, \dots, \xi_J$ ).

The next three moment conditions are based on the model's prediction for the distribution of income. Under the maintained assumptions on preferences, the information in  $\theta$  can be used to simulate community-specific income distributions. Three of the moment conditions match the 25<sup>th</sup>, 50<sup>th</sup>, and 75<sup>th</sup> quantiles from the simulated distributions of income in each community ( $\tilde{y}_j^{25}, \tilde{y}_j^{50}, \tilde{y}_j^{75}$ ) to their empirical counterparts ( $y_j^{25}, y_j^{50}, y_j^{75}$ ).

The last three moment conditions use the simulated income distributions to match predicted and observed quantiles from the distribution of housing expenditures in each community.

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<sup>16</sup> The particular moment conditions selected by Sieg et al. are somewhat arbitrary. In principle, one could use fewer moment conditions and additional instruments. Alternatively, one could develop moment conditions based on different quantiles of the distributions of income and housing expenditures.

The expenditure moments are obtained by multiplying (13) by price and taking logs.

Instruments are required to address endogeneity in the moment condition based on provision of amenities. The problem is that observed and unobserved amenities may be correlated. If households sort themselves across communities according to their income and preferences for a seemingly exogenous amenity—air quality for example—their location choices may influence the levels of other endogenous amenities such as public school quality, inducing correlation between them. PCM applications have followed Epple and Sieg (1999) in developing instruments from monotonic functions of each community’s rank in the price index. These instruments will be valid as long as unobserved amenities are of second order importance; i.e. if they affect households’ location choices without affecting the price rank of a community. The relevance of the instruments stems from the expectation that communities with higher levels of observed amenities will tend to be higher in the price ranking.

The mechanics of the simulated GMM estimator are straightforward. It can be implemented using a Nelder-Mead algorithm that iterates over the following steps.

**Step 1.** Select a starting value for  $\theta = [\beta, \eta, \nu, \rho, \mu_\alpha, \mu_y, \sigma_\alpha, \sigma_y, \lambda, G_1, \gamma_1, \dots, \gamma_{R-1}]$

**Step 2.** Draw  $I$  “households” from  $F(\alpha, y) \sim \text{lognormal}$ . In some applications,  $I$  is set to the actual population of the study region. In other cases, it is scaled down by an order of magnitude to reduce computational demands.

**Step 3.** Calculate  $K_i = \ln(\alpha_i) - \rho \left( \frac{y_i^{1-\nu} - 1}{1-\nu} \right)$  for all  $i = 1, \dots, I$  and use it to sort households in ascending order. Epple and Sieg (1999) demonstrate that the vertical model implies that, in any equilibrium, households will sort themselves across communities

according to  $K_i$ , such that households with higher values for  $K_i$  will always locate in higher ranked communities.

**Step 4.** Sort households across communities. Let  $S_1, \dots, S_J$  represent the observed population counts of each community such that  $\sum_j S_j = I$ . Starting with the lowest  $K_i$ , assign the first  $S_1$  households to community 1. Then assign the next  $S_2$  households to community 2, and so on.

**Step 5.** Given  $G_1$ , solve for  $G_2$  to make the boundary person between communities 1 and 2 indifferent between them. Then given  $G_2$ , solve for  $G_3$ , and so on....

**Step 6.** Calculate  $\hat{y}_j^{25}, \hat{y}_j^{50}, \hat{y}_j^{75}$  for each community.

**Step 7.** Use  $\hat{y}_j^{25}, \hat{y}_j^{50}, \hat{y}_j^{75}$ , and  $G_2(\hat{\theta}), \dots, G_J(\hat{\theta})$  and  $\hat{\theta}$  to evaluate the GMM objective function (15). If the minimization criteria of the numerical algorithm are satisfied, stop. If not, update  $\theta$  and return to step 2.

### 3.3. Comparing the RUM and PCM Frameworks

The RUM and PCM frameworks are each capable of explaining a given data set as a sorting equilibrium. This makes it difficult to compare the two models based on in-sample performance. In our opinion, neither model is strictly preferred to the other. Each has some features that seem flexible and others that seem restrictive.

PCM models provide a relatively flexible preference function, recognizing that public and private goods are not perfect substitutes. They also embed a budget constraint. The identifying assumption is that each household is able to afford a subset of houses in the community where it actually locates, and in the communities that are adjacent in the price ranking. In contrast, the PCM maintains a relatively strong assumption about the importance of unobserved



amenities. Unobserved amenities that influence the price ranking of communities threaten the validity of the rank-based instruments.

Advantages of the RUM model include its relatively flexible characterization of the choice set. It recognizes that zoning regulations may prevent homebuyers from choosing continuous quantities of housing characteristics. Moreover, the instruments proposed by Bayer and Timmins (2007) are robust to the presence of unobserved amenities that influence the price ranking of communities. Yet, the RUM model also makes strong assumptions. The linear specification for utility assumes amenities and structural housing characteristics are perfect substitutes. Likewise, every household is assumed to be capable of purchasing every house.

Both frameworks maintain strong assumptions about preference heterogeneity. The PCM's vertical characterization fails to recognize that households are likely to differ in their relative preferences for landscape amenities. Households with young children may be primarily concerned about public school quality, for example, whereas retirees may place more weight on proximity to golf courses. RUM models are capable of recognizing these tradeoffs. However, that flexibility comes at a cost. The RUM model's flexible treatment of preference heterogeneity is enabled by its strong assumption that every household's preferences for the unobserved attributes of every house happen to be drawn from the same iid Type I extreme value distribution. Kuminoff (2009) illustrates how the two frameworks present a bias-variance tradeoff. By restricting the extent of preference heterogeneity, the PCM introduces some bias. The RUM framework relaxes the restriction that causes the bias, but it does so in a way that increases the scope for distributional assumptions to influence the results.

It is important to keep in mind that the "flexible" and "restrictive" assumptions of RUM and PCM models are not inexorably linked to either framework. They reflect modeling deci-

sions embedded in the original estimators developed by Epple and Sieg (1999) and Bayer, McMillen, and Reuben (2004). A clever econometrician could mix, match, and alter the features of the two models to develop new estimators. That said, no amount of econometric cleverness can ever identify the true behavioral model with absolute certainty. Perhaps the best way to evaluate the validity of a sorting model is to test its out-of-sample predictions for how people and markets will adjust to unexpected changes in the spatial landscape.

#### **4. Evaluating the Benefits of Large Scale Changes in the Spatial Landscape**

Estimates for the structural parameters of a RUM or PCM model can be used to develop theoretically consistent predictions for the distribution of benefits from large-scale changes in the spatial distribution of prices or amenities. One can easily calculate partial equilibrium measures of willingness to pay (WTP) for a prospective policy change. The model can also be used to simulate the transition to the new equilibrium that would follow the introduction of the policy. Comparing the ex-ante and ex-post equilibria makes it possible to predict migration patterns, capitalization effects, changes in the levels of endogenous amenities, and the corresponding “general equilibrium” measures of WTP. In this section, we define “partial” and “general” equilibrium benefit measures and then discuss how to close the model and solve for a new equilibrium.

##### *4.1. Benefit measurement*

Consider a policy that changes the supply of a single amenity in community  $j$  from  $g_{j1}$  to  $g_{j1}^*$ . A partial equilibrium measure of the willingness to pay for this change,  $WTP_{PE}$ , holds constant all other features of the equilibrium.<sup>17</sup> In contrast, a general equilibrium measure,  $WTP_{GE}$ , accounts for potential changes in housing prices, location choices, and the levels of other endogenous

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<sup>17</sup> Calculation of partial equilibrium benefit measures differs between the PCM and RUM frameworks. In the PCM, (17a) is inverted to calculate WTP directly. In the RUM, the idiosyncratic error term means that WTP must be defined as an expected value using a version of the usual log-sum rule.

amenities. Equations (17a)–(17b) formalize this distinction,

$$(17a) \quad V(\alpha_i, y_i - WTP_{PE}, g_{j1}^*, g_{j\sim 1}, h_{n_j}, P_{n_j}) = V(\alpha_i, y_i, g_{j1}, g_{j\sim 1}, h_{n_j}, P_{n_j})$$

$$(17b) \quad V(\alpha_i, y_i - WTP_{GE}, g_{k1}^*, g_{k\sim 1}^*, h_{m_k}, P_{m_k}^*) = V(\alpha_i, y_i, g_{j1}, g_{j\sim 1}, h_{n_j}, P_{n_j}),$$

where  $g_j = [g_{j1}, g_{j\sim 1}]$ . In (17b) the change in subscripts from  $n_j$  to  $m_k$  recognizes that households may respond to the change by moving to a new location. The asterisk superscripts on  $g_{k\sim 1}^*$  and  $P_{m_k}^*$  recognize that, as people re-sort, their behavior may affect the levels of other endogenous amenities, and prices may need to adjust to clear the market. As  $\Delta g_{j1} = g_{j1}^* - g_{j1}$  grows or impacts a larger number of households, it becomes increasingly important to model general equilibrium feedback effects. Overall, the richness in this characterization for how people interact with their surrounding environment makes the general equilibrium sorting model a powerful framework for policy evaluation.

#### 4.2. Closing the Model

The RUM and PCM estimators essentially characterize housing demand, treating the supply of housing as fixed. However, solving for a new equilibrium requires characterizing both supply and demand, as well as any sources of friction in the market. Thus, to close the model the analyst must define the supply of housing, formalize their assumptions about moving costs, write down production functions for endogenous amenities, and clarify whether households are treated as owners or renters. The way that each of these issues is treated varies from application to application. However, three general trends are worth discussing.

First, land use policies often play dual roles. They simultaneously enhance open space amenities and they restrict urban development. As such, a new land use policy targeting the current supply of an amenity may also influence the future supply of housing. While equilibrium

sorting models are capable of modeling this connection, few applications have done so. Instead, the supply of housing is usually treated as fixed or defined by a constant-elasticity assumption (e.g. Sieg et al. 2004; Smith et al. 2004; Klaiber and Phaneuf 2010; Kuminoff 2011). This approach simplifies computation of the new equilibrium, but risks overlooking important policy implications. Future research that models the impacts of land use policies on both amenities and housing supply would be a welcome addition to the literature. Walsh (2007) provides an initial example of how this can be done.

Second, the initial general equilibrium applications have mostly treated households as being freely mobile. In our experience, this assumption tends to produce a good deal of consternation among seminar audiences. Anyone who has gone through the process of moving to a new house is all too familiar with the costs involved: physical costs, search costs, time costs, borrowing costs, and the psychological cost of adjusting to a new environment. The good news is that the structure of a sorting model makes it straightforward to utilize prior information about moving costs (Kuminoff 2009). For example, Kuminoff (2011) models the changes in commuting costs and wage rates that occur when working households alter their job and/or house locations. Likewise, Bayer, Keohane, and Timmins (2009) demonstrate that some moving costs can be estimated using related information, such as the location of an individual's hometown.

Finally, all of the applications we discuss in this chapter treat households as renters. Capital gains from housing sales are assumed to be captured by absentee landowners. This approach simplifies computation of the new equilibrium, but abstracts from issues that matter to policymakers. Many policies are effectively enacted on the owners of capital, especially policies influencing individual tax treatment. With this in mind, future research that builds changes in assets into the budget constraint would be another useful addition to the literature.

#### 4.3. Solving for a New Equilibrium in a Random Utility Model

Solving for a new equilibrium in the RUM framework requires calculating housing prices, location choices, and the levels of endogenous amenities such that housing supply and housing demand equate in all locations. Klaiber and Phaneuf (2010) describe the solution process for the special case where amenities are exogenous. The basic idea is to iterate over price changes until the predicted market shares for each housing type equal the supply of housing for that type. The steps are as follows:

- Step 1.** Given the new spatial distribution of amenities, use the estimated preference parameters to calculate the aggregate demand for each housing type,  $\sigma_{t_j}^{d,0}$ , where  $d$  stands for “demand” and 0 indicates this is the initial iteration of the algorithm.
- Step 2.** Determine whether excess demand ( $\sigma_{t_j}^{d,0} > \sigma_{t_j}^s$ ) or excess supply ( $\sigma_{t_j}^{d,0} < \sigma_{t_j}^s$ ) exists for each housing type.
- Step 3.** For types with excess demand, increase prices by a small percentage. Decrease prices by a small percentage for types with excess supply.<sup>18</sup>
- Step 4.** Using the new prices, recalculate the aggregate housing demand for each type,  $\sigma_{t_j}^{d,1}$ .
- Step 5.** Continue iterating over steps 2-4 until  $\sigma_{t_j}^d = \sigma_{t_j}^s$  for every type.

#### 4.4. Solving for a New Equilibrium in the Pure Characteristics Model

As in the RUM framework, it is straightforward to solve for a new PCM equilibrium in the special case where amenities are exogenous. The “vertical” restriction on preference heterogeneity allows the problem to be formulated as a one-dimensional rootfinding problem. To see this, first

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<sup>18</sup> A weighted average of previous and new prices can help to prevent oscillation in convergence. The magnitude of price changes can be weighted to be proportional to the difference in observed shares to speed convergence.

recall that communities will always be ordered by their equilibrium housing prices and provisions of public goods:  $p_1 < p_2 < \dots < p_J$  and  $G_1 < G_2 < \dots < G_J$ . Following a shock to public goods, the new equilibrium price ranking must be identical to the new ranking by  $G$ . Using this fact, the solution algorithm proceeds as follows:

- Step 1.** Make a guess for the new price of housing in the cheapest community,  $p_1^*$ .
- Step 2.** Use the left side of (14) to sort households into community 1 until total housing demand equals supply, aggregating over (13) to calculate demand.
- Step 3.** Use the last household sorted into community 1 to solve for the value of  $p_2^*$  that satisfies (14).
- Step 4.** Repeat steps 2-3 for communities 2 through  $J$ , or until all households are assigned to communities.
- Step 5.** If there is excess housing supply in community  $J$ , increase  $p_1^*$  and return to step 2. If there is excess demand, decrease  $p_1^*$  and return to step 2.

This recursive structure effectively reduces the simulation to a one-dimensional problem where the new equilibrium price of housing in community 1 is adjusted until the market clears in community  $J$ .

#### *4.5. Endogenous Amenities*

RUM and PCM solution algorithms can be modified to recognize that, as households re-sort, their behavior can affect the supply of endogenous amenities. The way this is modeled is context-specific. We briefly describe three examples, each of which finds that endogenous adjustment of amenities is important for characterizing the impacts of a prospective policy.

Klaiber and Smith (2010) use a PCM to evaluate the general equilibrium implications of

reductions in teaching staff in Maricopa County (Arizona) school districts. School quality is measured using the student-teacher ratio. Mandated reductions in teaching staff reduce school quality, inducing some households to move. As households with school age children move, the number of students in each school district changes, which feeds back into the student teacher ratio, inducing additional households to move....and so on until prices, location choices, and the student-teacher ratio all converge in equilibrium.

Walsh (2007) uses a PCM to investigate the impact of public open space preservation on households and urbanization in Wake County, North Carolina. He endogenizes the supply of housing by recognizing that privately owned farmland will tend to be developed as the demand for housing increases. As a result, land preservation policies can have unintended consequences. Suppose that public funds are used to purchase a small amount of scenic open space near a residential neighborhood. If the amenities associated with the preserved parcels increase the demand for housing in the neighborhood, it may actually accelerate the rate at which the remaining privately owned open space is developed.

Finally, Bayer and McMillan (2005) use a RUM to assess the role of households' preferences for several amenities, including the demographics of their neighbors. Measures of demographic composition, such as average income, average education, and neighborhood population shares by race, are directly determined by the sorting process. As a result, a public policy that influences an exogenous amenity is shown to be capable of altering neighborhood demographic composition.

## **5. Implications for Hedonic Estimation**

Since hedonic and sorting models describe the same underlying equilibrium, advances in the sorting literature also improve our understanding of the challenges associated with using re-

duced-form hedonic regressions to evaluate the benefits of prospective changes in the spatial landscape. We briefly summarize three ways in which the theory, estimation, and simulation of sorting models has clarified the challenges with hedonic estimation.

### *5.1. The Economics of Omitted Variable Bias*

Omitted variables systematically confound the identification of conventional hedonic regressions. This stylized fact has motivated an entire sub-literature on quasi-experimental approaches to estimation (Parmeter and Pope, 2011). The experimentalist perspective is that the analyst never observes all of the landscape amenities that are correlated with the amenity of interest. Breaking the correlation requires instruments that effectively randomize the amenity “treatment”. The equilibrium sorting literature complements the experimentalist perspective by providing an explanation for omitted variable bias and suggesting further implications for benefit measurement.

If people choose where to live based, in part, on their heterogeneous incomes and preferences for amenities, then their location choices will influence the long run levels of endogenous amenities (Ferreyra 2007, Walsh 2007, Epple and Ferreyra 2008, Bayer and McMillan 2010). Under single-crossing restrictions on preferences, it is natural to expect multiple amenities to be spatially correlated. As wealthier households move to areas with nice microclimates and low crime rates, for example, they may vote to pass special assessments that enhance local public education. If data on microclimates and crime rates are unavailable, then conventional hedonic estimates of the MWTP for school quality will tend to be biased upward. This logic helps to explain why quasi-experimental estimates of the MWTP for school quality are typically less than half the size of estimates from conventional hedonic regressions (Black 1999; Bayer, Ferreira, McMillan 2007; Kuminoff and Pope 2012).



Endogenous amenities present an additional challenge for benefit measurement. A public policy that alters the spatial distribution of one amenity may influence the long run levels of other endogenous amenities. In this case, hedonic price functions do not provide enough information to evaluate the welfare implications of the policy.

## 5.2. *Benefit Measurement and Policy Evaluation*

The empirical hedonic literature is mostly limited to estimating the willingness to pay for marginal changes in amenities.<sup>19</sup> However, estimates for average MWTP are often used to approximate the benefits from prospective policies that would produce non-marginal changes. Sorting models underscore the limitations of this strategy and provide a means to address them.

Hedonic and sorting models tend to generate similar estimates for average MWTP. For example, Sieg et al. (2004) find that the average MWTP for reduced ozone concentrations is approximately \$67 (1990 dollars) per household in the Los Angeles metro area. This figure is well within the range of estimates from comparable hedonic studies (\$8 to \$181).<sup>20</sup> Bayer, Ferreira, and McMillan (2007) provide a more refined comparison. Using the same data and the same quasi-experimental identification strategy, they find that hedonic and RUM estimates of the average MWTP for school quality differ by less than 14%. However, average MWTP is rarely a sufficient statistic for policy evaluation. Policymakers care about distributional implications. Moreover, developing credible benefit measures requires recognizing the demand is less than perfectly elastic and that people may react to the policy by adjusting their behavior.

Heterogeneity in preferences and the supply of amenities can lead to wide benefit distri-

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<sup>19</sup> Rosen's (1974) original vision for hedonic demand estimation remains unfulfilled due to the difficulty with identifying demand curves (Bartik 1987, Epple 1987).

<sup>20</sup> Klaiber and Phaneuf (2010) provide a more detailed comparison. Using the same data set (but different controls for omitted variables) they find that hedonic and sorting models produce very similar estimates of MWTP for some types of open space (\$30 versus \$28 for a 1% increase in local parks) and very different estimates for other types of open space (-\$277 versus \$618 for a 1% increase in agricultural preserves).

butions. For example, Sieg et al. (2004) find that the average *marginal* WTP for air quality in Los Angeles county is twice as large as in neighboring Ventura county. When they evaluate the *non-marginal* ozone reductions that actually occurred between 1990 and 1995, the difference in WTP between Los Angeles and Ventura increases to 800%! This difference arises from a combination of lower baseline levels of ozone in Ventura, a smaller reduction in Ventura between 1990 and 1995, and heterogeneity in preferences and income. Predicted adjustments to housing prices and location choices also have significant welfare implications. Partial and general equilibrium benefit measures differ by over 100% for the average Ventura household.

### *5.3. The Wedge between Capitalization Effects and Benefit Measures*

Public policies or unexpected events that shock the spatial distribution of an amenity can also be used to identify the rate at which that amenity is capitalized into property values. The quasi-experimental branch of the hedonic literature has focused on developing clever research designs for identifying these “capitalization effects” (see Parmeter and Pope [2011] for examples). These studies typically reformulate the price function within a panel data framework, using first differences, fixed effects, or difference-in-difference estimators. The resulting estimates for capitalization effects are interesting, but they cannot be interpreted as benefit measures unless we are prepared to make a series of heroic assumptions about people and markets.

One of the key maintained assumptions that make it possible to interpret marginal capitalization effects as measures of MWTP is that the gradient of the hedonic price function is constant over the duration of the study. This assumption effectively requires demand curves for the amenity to be perfectly elastic. If demand is downward sloping, the adjustment to a new sorting equilibrium will generally produce a wedge between the marginal capitalization effect and the MWTP. The size of the wedge will depend on the distribution of income and preferences, the

supply response, and concomitant changes to the landscape over the duration of the study.

The wedge between capitalization and willingness to pay can be very large. Kuminoff and Pope (2012) find that capitalization effects for reported changes in public school quality tend to differ from quasi-experimental measures of ex-ante and ex-post MWTP by more than 100%. Likewise, Klaiber and Smith (2011) find it difficult to predict the size or the direction of the bias in using capitalization effects to approximate the benefits of non-marginal changes. These findings reinforce the earlier theoretical results of Lind (1973) and Starrett (1981), as well as simulation results from Sieg et al. (2004) and Smith et al. (2004), where predicted changes in housing prices bear little resemblance to predicted changes in benefits. Thus, the collective evidence from the sorting literature suggests that capitalization effects for amenities are best interpreted literally, as a statistical description of changes in housing asset values.

## **6. Conclusions**

Equilibrium sorting models provide a powerful framework for modeling the two-way interaction between people and their surrounding environment. They have tremendous potential for policy evaluation. The Clean Air Act, the Clean Water Act, and the Superfund program are examples of major public policies designed to produce large scale changes in the spatial distribution of nonmarket amenities. We would like to understand their distributional implications and be able to predict how new policies will affect consumer welfare and market outcomes. Equilibrium sorting models are the first revealed preference framework capable of meeting this task, while recognizing that people adapt to changes in their surrounding environment.

Like every revealed preference framework, sorting models rely on maintained assumptions about the structure of consumer preferences. This means their predictions for benefit measures, housing market outcomes, and the evolution of the surrounding landscape are best

viewed as approximations. How accurate are these approximations? The ability to answer this question is one of the novelties of the literature. Sorting models make testable predictions for market and nonmarket outcomes! Thus, the same types of natural experiments and policy discontinuities that have been used to develop instruments for reduced-form hedonic models could also be used to test a sorting model's predictions for property value capitalization effects and migration patterns. Future evidence on external validity would help to refine the current generation of estimators and continue to advance the literature.

Finally, our objective has been to provide an introductory guide to sorting models for empirical analysts. We have tried to be clear about the subtleties of the microeconomic models and the mechanics of estimation and simulation procedures. Nevertheless, our own experience has been that the most effective way to learn a sorting model is to "get your hands dirty". Readers who are up to the challenge can find examples of data and code on our webpages.

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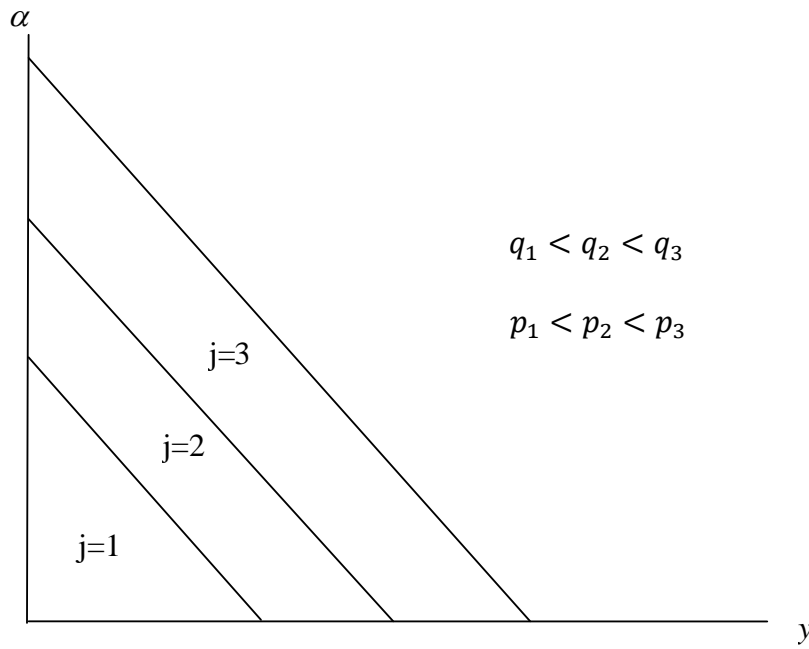
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*Figure 1.* Partition of Households into Communities by Preferences and Income